Generation of bound states in a continuum

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We show that a result derived previously by Lami and Rahman [Phys. Rev. A **34**, 3908 (1986)] whereby bound states can be created in a continuum, contains an algebraic error. We present the correction and discuss some implications.

In complicated atomic and molecular systems, many different reaction and excitation channels between individual states and groups of states interfere constructively and destructively. This interference, induced when coupling mixes the interacting channels, can lead to the existence of bound states in the continuum[1–10] and population trapping.[11–20] Investigations of these phenomena may provide insight into processes necessary to achieve selective laser chemistry.

Recently, Lami and Rahman[3,4] (hereafter referred to as LR) published a pair of papers that examined the conditions under which discrete states can exist within the energy range occupied by a continuum. They demonstrate the somewhat surprising result that under rather general conditions on two or more external fields, there exists an equal number of bound states within the energy region of the continuum. The fact that a long-lived state can be created above the dissociation threshold signals that there may be hope for selective laser chemistry. With this result, they proceed to investigate how such a state might be detected using quantum-beat spectroscopy. It is at this point that the analysis they present breaks down quantitatively. In this note, we derive the correct quantitative result and present a brief discussion of the implications it has for the qualitative features on which LR elaborate.

Lami and Rahman begin with a system as shown in Fig. 1 and project the Hamiltonian thus obtained onto the discrete states, thereby producing an effective Hamiltonian from which a set of dressed states may be obtained analytically. Their effective Hamiltonian is

$$H^{\mathrm{eff}}=\begin{bmatrix} -i\gamma_1+\delta & -(\gamma_1\gamma_a)^{1/2}(q_1+i) & -(\gamma_1\gamma_2)^{1/2}(q_{12}+i) \\ -(\gamma_1\gamma_a)^{1/2}(q_1+i) & -i\gamma_a & -(\gamma_a\gamma_2)^{1/2}(q_2+i) \\ -(\gamma_1\gamma_2)^{1/2}(q_{12}+i) & -(\gamma_a\gamma_2)^{1/2}(q_2+i) & -i\gamma_2+\delta' \end{bmatrix}, \tag{1}$$

where δ is the detuning of laser 1 from the $|1\rangle\rightarrow|2\rangle$ resonance; δ' is the detuning of laser 2 from the $|3\rangle\rightarrow|2\rangle$ resonance; q_1, q_{12}, and q_2 are the Fano parameters derived in Refs. 1 and 2, and γ_1, γ_a, and γ_2 are the widths of the corresponding effective resonances.

This effective Hamiltonian (we will subsequently drop the superscript and all future references to the Hamiltonian will be understood to refer to this effective Hamiltonian) has the special property that its real and imaginary parts commute

$$\begin{aligned} H&=R+i\Delta\ , \\ 0&=\Delta R-R\Delta\ . \end{aligned} \tag{2}$$

This allows the eigenvalues for the Hamiltonian to be decomposed as the sum of the eigenvalues for the real and imaginary parts computed separately. Using this, we find immediately that the imaginary part has two zero eigenvalues. The eigenvalues of Δ are therefore

$$\lambda_I=\{0,0,-\Gamma\}\ , \tag{3}$$

where $\Gamma=\gamma_1+\gamma_2+\gamma_a$. Using the nonzero eigenvalue, we can find the corresponding eigenvector for Δ,

$$U_3=\frac{1}{\Gamma}\begin{bmatrix} \gamma_2^{1/2} \\ \gamma_a^{1/2} \\ \gamma_2^{1/2} \end{bmatrix}\ . \tag{4}$$

U_3 can then be used to compute the corresponding real eigenvalue for the Hamiltonian as well as conditions on δ and δ'. We compute the real part of the eigenvalue from row 2 of R in Eq. (2) as

$$\lambda_{3,R}=-q_1\gamma_1-q_2\gamma_2\ . \tag{5}$$

This can then be substituted into rows 1 and 3 of R to get expressions for δ and δ',

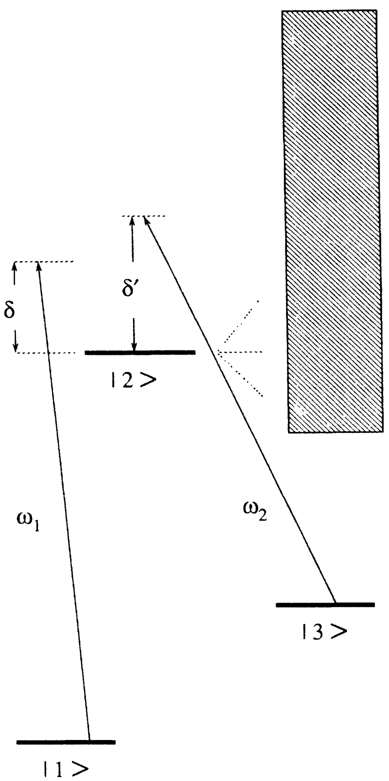

FIG. 1. This figure shows the system described by Lami and Rahman. There are two bound states and a continuum coupled internally to a resonance. Transitions to the resonance are accomplished by means of two lasers at frequencies ω_1 and ω_2.

$$\delta = q_1(\gamma_a - \gamma_1) - \gamma_2(q_2 - q_{12}) , \tag{6a}$$

$$\delta' = q_2(\gamma_a - \gamma_2) - \gamma_1(q_1 - q_{12}) . \tag{6b}$$

These are identical to the critical values of δ and δ' obtained by LR. The known eigenvalue can then be used to reduce the secular equation of H^{eff} from third order to second order, which is amenable to the quadratic formula. This reduction also checks whether the value just produced in Eq. (5) is an eigenvalue of the effective Hamiltonian. A little algebra shows that this is indeed the case, and the reduced characteristic equation gives the remaining two *real* eigenvalues as

$$\begin{aligned}\lambda_{1,2} = \tfrac{1}{2}\{ & (\delta + \delta' + q_1\gamma_1 + q_2\gamma_2) \\ & \pm [(\delta + \delta' + q_1\gamma_1 + q_2\gamma_2)^2 \\ & + 4(q_1^2\gamma_1\gamma_a + q_2^2\gamma_a\gamma_2 - q_1 q_2 \gamma_a^2 - q_1 q_{12}\gamma_1\gamma_a \\ & - q_2 q_{12}\gamma_a\gamma_2)]^{1/2}\} . \end{aligned} \tag{7}$$

The values above differ from those given by Lami and Rahman in two ways. First, their values include a term which is dimensionally different from the other terms in their expression. Second, even though our values have a somewhat simpler form than those previously presented, the overall qualitative results deduced by LR appear to remain intact. There are, however, certain quantitative differences that need further elaboration.

As LR point out, the energy values and quantum-beat frequencies that result from the preceding calculation are not obvious without resort to graphical methods. In Fig. 2, we show the beat frequency as a function of the atomic parameter q_{12}. The parameters used here are the same as for Fig. 5 of Ref. 4. Figure 2 shows what seems to be a strong hyperbolic dependence of the beat frequency, which quickly reaches the asymptotic region of the hyperbola. This is in sharp contrast to the conclusion of LR that the beat frequency grows quadratically in q_{12}. Thus the beat frequency is considerably less sensitive

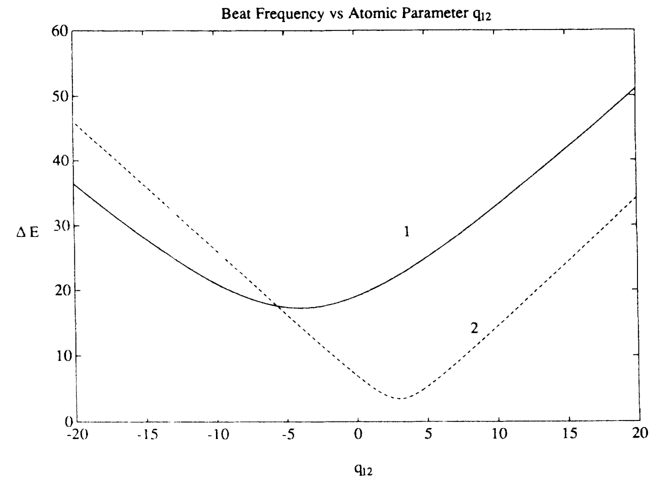

FIG. 2. The beat frequency $\lambda_1 - \lambda_2$ as a function of the parameter q_{12} with $\gamma_1 = \gamma_a = \gamma_2 = 1$. Curve 1 has $q_1 = 1$ and $q_2 = -9$ and curve 2 has $q_1 = 2$ and $q_2 = 4$. This graph uses the same parameter values as given in Fig. 5 of Ref. 4.

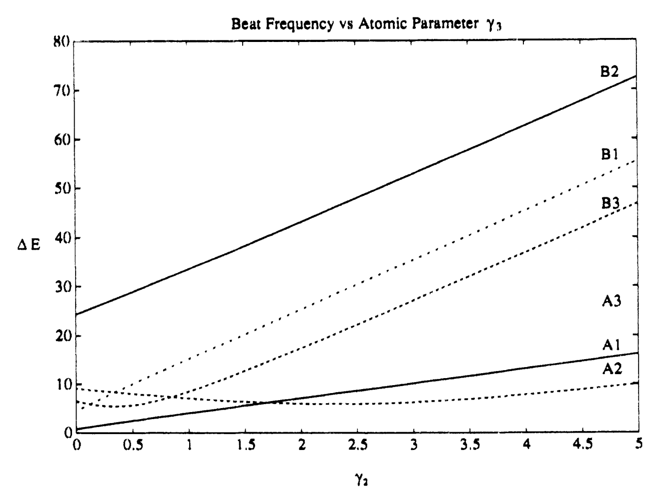

FIG. 3. The beat frequency as a function of γ_2. The parameter values for these six graphs are the same as those given in Fig. 6 of Ref. 4. These values are $\gamma_1 = 0.1$ and $\gamma_a = 1.0$ for all the curves. Curves $A1$, $A2$, and $A3$ have $q_{12} = 3$, 12, and -6, respectively with $q_1 = 1$ and $q_2 = 4$ while curves $B1$, $B2$, and $B3$ have $q_{12} = -4$, 14, and -14 with $q_1 = 1$ and $q_2 = -9$.

than was previously thought. Such is also the case for the beat frequency as a function of the width γ_2. Figure 3 shows the beat frequency as a function of γ_2 for the same parameters given in Fig. 6 of Ref. 4. Here again, the curves are less sensitive to the value of γ_2 than in LR, so that the beat frequency increases in the approximately linear manner of LR, but at a lesser slope. Hence it appears that a wider range of atomic parameters are available than one would conclude from LR for which the beat frequency is below a certain threshold. This does not alter the fact that the conditions for creation of a bound state within the energy region of a continuum, the critical values δ and δ', remain unchanged.

We have shown that the quantitative results given by LR contain an algebraic error, the correction of which significantly alters the functional form of the resulting beat frequency. Even though this is the case, the overall qualitative results remain intact.

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