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Transverse relaxation of spin-polarized ^{3}He gas due to a magnetic field gradient

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The presence of a magnetic field gradient in a sample cell containing spin-polarized ^{3}He gas will cause an increased transverse relaxation rate. The origin of this relaxation mechanism is the loss of phase coherence of the atoms due to the fluctuating magnetic field seen by the atoms as they diffuse throughout the cell. The Redfield theory of relaxation due to randomly fluctuating magnetic fields is applied to yield expressions for the transverse relaxation rate for spherical and cylindrical sample cells. Experiments performed with an experimental ^{3}He free-precession magnetometer give results in agreement with the theory.

I. INTRODUCTION

The longitudinal relaxation of polarized nuclear spins in a gas due to a magnetic field gradient has been studied by Schearer and Walters[1] and by Gamblin and Carver[2] a number of years ago. In these papers a theory of longitudinal relaxation is developed that yields a relaxation rate in agreement with experiments.

The work on the transverse relaxation problem has been more recent. Richards *et al.*,[3] in a paper on a ^{3}He nuclear Zeeman maser, note that the transverse magnetization decays exponentially with a time constant given by

$$\frac{1}{T_2}=\langle(\Delta\omega)^2\rangle\tau_D\ , \tag{1}$$

where $\langle(\Delta\omega)^2\rangle$ is the average of $\gamma^2(H_z-\langle H_z\rangle)^2$ over the cell volume and τ_D is a characteristic diffusion time of the order R^2/D for a spherical sample cell of radius R and self-diffusion constant D. In the expression for $\langle(\Delta\omega)^2\rangle$, γ represents the gyromagnetic ratio of the spin species, H_z is the magnetic field within the cell, and $\langle H_z\rangle$ is the volume average. For a simple gradient field the relaxation rate is then

$$\frac{1}{T_2}=\frac{k\gamma^2R^4|\nabla H_z|^2}{5D}\ , \tag{2}$$

where k is an experimentally determined constant of order unity. Cates, Schaefer, and Happer[4] have developed a comprehensive theory of relaxation due to field inhomogeneities for both T_1 and T_2. The theory is valid for all pressures and field values. In the high-pressure limit (for which the diffusion time is long compared to the precession time), Cates, Schaefer, and Happer give the result

$$\frac{1}{T_2}=\frac{8\gamma^2R^4|\nabla H_z|^2}{175D} \tag{3}$$

for the transverse relaxation rate for a spherical cell. The theory is extended in a paper by Cates *et al.*[5] to include the effects of oscillating magnetic fields. In experiments performed by Cates *et al.*[5] on ^{129}Xe gas the values of the transverse relaxation in a rotating frame are found to be in good agreement with theoretical predictions.

In this paper the Redfield theory, as described by Slichter,[6] is applied to find the transverse relaxation rate due to a magnetic field gradient for spin-polarized atoms in a gas enclosed within a cylindrical or spherical cell. Experiments have been performed with polarized ^{3}He gas and the results are compared with the theory.

II. THEORY

The transverse relaxation due to a field gradient is caused by polarized atoms in the sample cell at various positions precessing at slightly different rates. Suppose the magnetic field in the cell may be described by a constant field $\mathbf{H}_0$ and a uniform gradient $\nabla\mathbf{H}$

$$\mathbf{H}(\mathbf{r})=\mathbf{H}_0+\nabla\mathbf{H}\cdot\mathbf{r}\ . \tag{4}$$

The origin of the coordinate system is taken as the center of the cell with the z axis along $\mathbf{H}_0$. According to Slichter,[6] the motion of the spin $\langle\mathbf{I}\rangle$, neglecting source terms, is given by

$$\frac{d}{dt}\langle I_x\rangle=\gamma(\langle\mathbf{I}\rangle\times\mathbf{H}_0)_x-\frac{\gamma^2}{2}[S_{Hy}(\omega_0)+S_{Hz}(0)]\langle I_x\rangle\ , \tag{5}$$

$$\frac{d}{dt}\langle I_y\rangle=\gamma(\langle\mathbf{I}\rangle\times\mathbf{H}_0)_y-\frac{\gamma^2}{2}[S_{Hx}(\omega_0)+S_{Hz}(0)]\langle I_y\rangle\ , \tag{6}$$

$$\frac{d}{dt}\langle I_z\rangle=\gamma(\langle\mathbf{I}\rangle\times\mathbf{H}_0)_z-\frac{\gamma^2}{2}[S_{Hx}(\omega_0)+S_{Hy}(\omega_0)]\langle I_z\rangle\ , \tag{7}$$

where

$$S_{Hx}(\omega)=\int_{-\infty}^{\infty}\langle H_x(t)H_x(t+\tau)\rangle e^{-i\omega\tau}d\tau \tag{8}$$

and similar expressions exist for $S_{Hy}(\omega)$ and $S_{Hz}(\omega)$. The average in Eq. (8) is taken over the atoms of the sample cell. In (5)–(8), ω_0 is γH_0. From (7) we can identify the longitudinal relaxation rate as the decay constant for $\langle I_z\rangle$

$$\frac{1}{T_1}=\frac{\gamma^2}{2}[S_{Hx}(\omega_0)+S_{Hy}(\omega_0)] , \tag{9}$$

and the transverse relaxation rate as the decay constant for $\langle I_x+iI_y\rangle$

$$\frac{1}{T_2}=\frac{\gamma^2}{4}[S_{Hx}(\omega_0)+S_{Hy}(\omega_0)]+\frac{\gamma^2}{2}S_{Hz}(0) . \tag{10}$$

The autocorrelation function of (8) may be written as

$$\begin{aligned}\langle H_x(t)H_x(t+\tau)\rangle=&\left[\frac{\partial H_x}{\partial x}\right]^2\langle x(t)x(t+\tau)\rangle\\&+\left[\frac{\partial H_x}{\partial y}\right]^2\langle y(t)y(t+\tau)\rangle\\&+\left[\frac{\partial H_x}{\partial z}\right]^2\langle z(t)z(t+\tau)\rangle ,\end{aligned} \tag{11}$$

with similar expressions for H_y and H_z. Independence of the motion in the three coordinate directions is assumed. Equation (11) states that the gradient relaxation rates are determined by the positional autocorrelation functions such as $\langle x(t)x(t+\tau)\rangle$ or the corresponding spectrum $S_x(\omega)$. Determination of the spectrum $S_x(\omega)$ is straightforward for the frequency much larger than $1/\tau_D$. If the average time between collisions is τ_C, then we can write the autocorrelation function for the x velocity as[7]

$$\langle u_x(t)u_x(t+\tau)\rangle=\langle u_x^2\rangle e^{-\frac{\tau}{\tau_C}} , \tag{12}$$

where $\langle u_x^2\rangle=kT/M$. Here k is Boltzmann's constant, T is the absolute temperature, and M is the ^{3}He mass. The spectrum corresponding to (12) is

$$S_{ux}(\omega)=\frac{2\langle u_x^2\rangle\tau_C}{1+\omega^2\tau_C^2} , \tag{13}$$

and since $u_x(t)=dx(t)/dt$

$$S_x(\omega)=\frac{2\langle u_x^2\rangle\tau_C}{\omega^2(1+\omega^2\tau_C^2)} . \tag{14}$$

From (11)

$$S_{Hx}(\omega)=|\nabla H_x|^2S_x(\omega) , \tag{15}$$

and similar expressions may be written for $S_{Hy}(\omega)$ and $S_{Hz}(\omega)$. The expression for the longitudinal relaxation rate is then

$$\frac{1}{T_1}=\frac{\gamma^2}{2}(|\nabla H_x|^2+|\nabla H_y|^2)S_x(\omega_0) . \tag{16}$$

We expect (16) to be valid if the distance that the atoms diffuse in time $1/\omega_0$ [i.e., $(2D/\omega_0)^{1/2}$] is small compared to the cell dimensions. For representative experimental conditions of a 0.5-G field and a 10-Torr pressure, this diffusion distance is 0.16 cm, which may be compared to the cell diameter of 5.08 cm. Schearer and Walters[1] show that (16) is quite accurate for τ_C chosen as $(2.2\pm0.2)\times10^{-7}$ sec for 1 Torr at 300 K.

From (10) and (11) the transverse relaxation rate $1/T_2$ may be written as

$$\begin{aligned}\frac{1}{T_2}=\frac{1}{2T_1}+\frac{\gamma^2}{2}\Bigg[&\left[\frac{\partial H_z}{\partial x}\right]^2S_x(0)+\left[\frac{\partial H_z}{\partial y}\right]^2S_y(0)\\&+\left[\frac{\partial H_z}{\partial z}\right]^2S_z(0)\Bigg] .\end{aligned} \tag{17}$$

It is clear that, because of the factors $S_x(0)$, etc., the effect of diffusion cannot be neglected as in (16). The kernel of the problem is hence to find the autocorrelation function $\langle x(t)x(t+\tau)\rangle$ for times comparable to the diffusion time. The autocorrelation function for x based on times t and t_0 may be written as

$$\langle xx_0\rangle=\int\int p(\mathbf{r},t;\mathbf{r}_0,t_0)xx_0d^3r\,d^3r_0 , \tag{18}$$

where $p(\mathbf{r},t;\mathbf{r}_0,t_0)$ is the joint probability density of an atom being at position $\mathbf{r}$ at time t and being at position $\mathbf{r}_0$ at time t_0. For concreteness we assume that $t>t_0$. The joint probability density can be factored into a single probability density $p(\mathbf{r}_0,t_0)$ (which is uniform) and the conditional probability density $p(\mathbf{r},t|\mathbf{r}_0,t_0)$

$$p(\mathbf{r},t;\mathbf{r}_0,t_0)=p(\mathbf{r}_0,t_0)p(\mathbf{r},t|\mathbf{r}_0,t_0) . \tag{19}$$

For times $|t-t_0|$ much greater than the collision time τ_C, the conditional density obeys the diffusion equation[7]

$$D\nabla^2p(\mathbf{r},t|\mathbf{r}_0,t_0)=\frac{\partial}{\partial t}p(\mathbf{r},t|\mathbf{r}_0,t_0) \tag{20}$$

subject to the reflection boundary condition

$$\nabla p(\mathbf{r}_S,t|\mathbf{r}_0,t_0)\cdot\mathbf{n}=0 , \tag{21}$$

where $\mathbf{r}_S$ is on the cell wall and $\mathbf{n}$ is an outwardly directed normal unit vector to the cell wall. The initial condition is

$$p(\mathbf{r},t_0|\mathbf{r}_0,t_0)=\delta(\mathbf{r}-\mathbf{r}_0) . \tag{22}$$

We note that $\mathbf{r}$ and t are viewed as the independent variables in (20)–(22), with $\mathbf{r}_0$ and t_0 considered as parameters.

Equations (20)–(22) have been solved for a few simple geometries. Suppose that the cell is cylindrical (i.e., of uniform cross section) with ends that are plane and normal to the cell axis. Let x be the cell axis so that the end planes occur at $x=\pm L/2$. Initially we assume that ∇H_z is in the x direction. The conditional density is found to be

$$p(x,t|x_0,t_0)=\frac{1}{L}+\frac{2}{L}\sum_{n=1,3,\ldots}^{\infty} e^{-\frac{n^2\pi^2 D|t-t_0|}{L^2}}\left[\sin\frac{n\pi x}{L}\right]\sin\frac{n\pi x_0}{L}$$
$$+\frac{2}{L}\sum_{n=2,4,\ldots}^{\infty} e^{-\frac{n^2\pi^2 D|t-t_0|}{L^2}}\left[\cos\frac{n\pi x}{L}\right]\cos\frac{n\pi x_0}{L}\ . \tag{23}$$

Then

$$\langle xx_0\rangle=\int_{-L/2}^{L/2} x_0 p(x_0,t_0)dx_0\int_{-L/2}^{L/2} x p(x,t|x_0,t_0)dx$$
$$=\frac{8L^2}{\pi^4}\sum_{n=1,3,\ldots}^{\infty}\frac{1}{n^4}e^{-\frac{n^2\pi^2 D|t-t_0|}{L^2}}\ . \tag{24}$$

In (24) $p(x_0,t_0)$ has the uniform density $1/L$. Then

$$S_x(0)=\int_{-\infty}^{\infty}\langle x(t)x(t+\tau)\rangle d\tau$$
$$=\frac{16L^4}{\pi^6 D}\sum_{n=1,3,\ldots}^{\infty}\frac{1}{n^6}$$
$$=\frac{L^4}{60D}\ , \tag{25}$$

and the expression for the transverse gradient relaxation rate is

$$\frac{1}{T_2}=\frac{1}{2T_1}+\frac{\gamma^2 L^4}{120D}\left[\frac{\partial H_z}{\partial x}\right]^2\ . \tag{26}$$

In (25) the summation was replaced with $\pi^6/960$ from Gradshteyn and Ryzhik.[8]

Next we consider a cylindrical cell for which the cross section is circular in the y-z plane with radius a. Suppose that ∇H_z falls along the y axis. For this case the conditional density may be written

$$p(\mathbf{r},t|\mathbf{r}_0,t_0)=\frac{1}{2\pi a^2}\sum_{n,k}A_{nk}J_n\left[\frac{x_{nk}\rho}{a}\right]J_n\left[\frac{x_{nk}\rho}{a}\right]$$
$$\times e^{in(\phi-\phi_0)}e^{-(x_{nk}^2 D|t-t_0|/a^2)}\ . \tag{27}$$

The (ρ,ϕ) coordinate system is defined by $y=\rho\cos\phi$ and $z=\rho\sin\phi$. The quantities x_{nk} are zeroes of the first derivative of the Bessel function J_n

$$\frac{d}{dx}J_n(x_{nk})=0\ , \tag{28}$$

and A_{nk} are given by

$$A_{nk}=\frac{a^2}{\int_0^a J_n^2(x_{nk}\rho/a)\rho d\rho}\ . \tag{29}$$

The y autocorrelation function is found from (27) to be

$$\langle yy_0\rangle=a^2\sum_k A_{1k}B_k e^{-\frac{x_{1k}^2 D|t-t_0|}{a^2}}\ , \tag{30}$$

where

$$B_k=\frac{1}{a^6}\left[\int_0^a\rho^2 J_1\left[\frac{x_{1k}\rho}{a}\right]d\rho\right]^2\ . \tag{31}$$

With the autocorrelation function $\langle yy_0\rangle$ we can determine $S_y(0)$ and $1/T_2$,

$$\frac{1}{T_2}=\frac{1}{2T_1}+\frac{\gamma^2 a^4}{D}\left[\frac{\partial H_z}{\partial y}\right]^2\sum_k\frac{A_{1k}B_k}{x_{1k}^2}\ . \tag{32}$$

Manipulation of Bessel function identities yields (except for A_{00})

$$A_{nk}=\frac{2x_{nk}^2}{J_n^2(x_{nk})(x_{nk}^2-n^2)}\ , \tag{33}$$

$$B_k=\frac{J_1^2(x_{1k})}{x_{1k}^4}\ . \tag{34}$$

The summation of (32) may be written

$$\sum_k\frac{A_{1k}B_k}{x_{1k}^2}=2\sum_k\frac{1}{x_{1k}^4(x_{1k}^2-1)}=\frac{7}{96}\ . \tag{35}$$

The transverse relaxation rate is then

$$\frac{1}{T_2}=\frac{1}{2T_1}+\frac{7\gamma^2 a^4}{96D}\left[\frac{\partial H_z}{\partial y}\right]^2\ . \tag{36}$$

From (26) and (36) one can write the relaxation rate for a general cylindrical cell of length L and radius a

$$\frac{1}{T_2}=\frac{1}{2T_1}+\frac{\gamma^2 L^4}{120D}\left[\frac{\partial H_z}{\partial x}\right]^2+\frac{7\gamma^2 a^4}{96D}\left[\frac{\partial H_z}{\partial y}\right]^2\ . \tag{37}$$

The derivation of (37) depends on the fact that $\langle x(t)y(t')\rangle=0$. One should note that (37) still applies if $\mathbf{H}_0$ is not directed along the z axis (which is perpendicular to the cell axis). For this case H_z in (37) should be replaced with the component of $\mathbf{H}$ along $\mathbf{H}_0$.

We next consider the spherical cell of radius R subjected to a gradient along the z axis. A spherical coordinate system is chosen, for which $x=r(\sin\theta)\cos\phi$, $y=r(\sin\theta)\sin\phi$, and $z=r(\cos\theta)$. From (20)–(22) the conditional density for this case is

$$p(\mathbf{r},t|\mathbf{r}_0,t_0)=\frac{1}{R^3}\sum_{n,l,m}A_{ln}j_l\left[\frac{x_{ln}r}{R}\right]j_l\left[\frac{x_{ln}r_0}{R}\right]$$
$$\times Y_{lm}^*(\theta,\phi)Y_{lm}(\theta_0,\phi_0)$$
$$\times e^{-\frac{x_{ln}^2 D|t-t_0|}{R^2}}\ , \tag{38}$$

where

$$A_{ln}=\frac{R^3}{\int_0^R r^2 j_l^2(x_{ln}r/R)\,dr} . \tag{39}$$

In (38), Y_{lm} are spherical harmonics, j_l represent spherical Bessel functions, and the values x_{ln} satisfy

$$\frac{d}{dx} j_l(x_{ln})=0 . \tag{40}$$

The z autocorrelation function is found to be

$$\langle zz_0\rangle = R^2 \sum_n A_{\mathrm{ln}} B_n e^{-\frac{x_{\mathrm{ln}}^2 D|t-t_0|}{R^2}} , \tag{41}$$

where

$$B_n=\frac{1}{R^8}\left[\int_0^R r^3 j_1\left[\frac{x_{\mathrm{ln}}r}{R}\right]dr\right]^2 . \tag{42}$$

The expression for $1/T_2$ is hence

$$\frac{1}{T_2}=\frac{1}{2T_1}+\frac{\gamma^2 R^4}{D}\left[\frac{\partial H_z}{\partial z}\right]^2 \sum_n \frac{A_{\mathrm{ln}}B_n}{x_{\mathrm{ln}}^2} . \tag{43}$$

The values of A_{ln} and B_n are found in the Appendix to Ref. 4:

$$A_{ln}=\frac{2x_{ln}^2}{j_l(x_{ln})[x_{ln}^2-l(l+1)]} , \tag{44}$$

$$B_n=\frac{j_1^2(x_{\mathrm{ln}})}{x_{\mathrm{ln}}^4} . \tag{45}$$

The value of the summation in (43) is also given in Ref. 4:

$$\sum_n \frac{A_{\mathrm{ln}}B_n}{x_{\mathrm{ln}}^2}=2\sum_n \frac{1}{x_{\mathrm{ln}}^4(x_{\mathrm{ln}}^2-2)}=\frac{8}{175} . \tag{46}$$

The traverse relaxation time is hence

$$\frac{1}{T_2}=\frac{1}{2T_1}+\frac{8\gamma^2 R^4}{175D}\left[\frac{\partial H_z}{\partial z}\right]^2 . \tag{47}$$

For the general case where the gradient in H_z is not along the z axis (i.e., the direction of $\mathbf{H}_0$), then $(\partial H_z/\partial z)^2$ in (47) should be replaced with $|\nabla H_z|^2$. This result is in agreement with Cates, Schaefer, and Happer[4] for the high-pressure limit.

III. EXPERIMENT

Experiments have been performed to measure the transverse relaxation of spin-polarized ^{3}He gas due to a magnetic field gradient using an experimental ^{3}He free-precession magnetometer in the Earth's field (of 0.53 G). The experimental magnetometer consisted of a ^{3}He cell (cylindrical or spherical), a ^{4}He lamp, a lens to collimate the lamp light and direct it into the cell, a circular polarizer, a 12.7-cm-diam set of Helmholtz coils, and two many-turn pickup coils adjacent to the cell. Oscillators provided the voltage to drive the electrodeless discharges in the cell and lamp. The magnetometer was oriented so that the light from the lamp was directed along the Earth's field $\mathbf{H}_0$. The Helmholtz coil axis and the pickup coil axis were perpendicular to $\mathbf{H}_0$ and to each other. The operation of the magnetometer proceeded in three stages. In the first stage the cell and lamp electrical discharges were established. The discharge in the cell resulted in a population of $2\,^3S_1$ metastable atoms. These atoms absorbed some of the circularly polarized $2\,^3S-2\,^3P$ radiation from the lamp and became partially polarized. Through spin-exchange collisions of polarized metastable atoms and ground-level atoms, the ground-level nuclear spin became polarized.[9] After several minutes a nuclear spin polarization of 1%–3% accumulated along the $\mathbf{H}_0$ direction. At this point the lamp and cell discharges were extinguished. In the second stage a small (relative to H_0) oscillating magnetic field from the Helmholtz coil was used to rotate the cell polarization so that precession would begin. This required a few seconds. During the third stage the precessing cell polarization induced an oscillatory voltage in the pickup coils. The free precession continued for a time interval of convenient length, usually of the order of T_2. After the free precession was started, the magnetic field gradient was established and the precession signal amplitude was periodically measured. In the experiments, a high impedance amplifier to the pickup coils was used in order to eliminate the possibility of radiation damping.[10] The amplified signal amplitude was measured with a Hewlett-Packard 5420A Spectrum Analyzer.

The magnetic field gradient was provided by two ten-turn, 8.26-cm-diam coils. These were placed 1.83 m apart on a (magnetic) east-west line with the magnetometer centered between them. A sketch of the magnetometer and the gradient-producing coils is given in Fig. 1. The coils were connected to a power supply so that the magnetic moment associated with one coil was parallel to $\mathbf{H}_0$ and the other antiparallel. This arrangement was chosen to produce the gradients $\partial H_z/\partial x$ and $\partial H_x/\partial z$ (at the center of the cell) with the other gradients being small relative to $\partial H_z/\partial x$. To a first approximation, the gradient-producing coils may be considered as point magnetic dipoles. In this case the gradients other than $\partial H_z/\partial x$ and $\partial H_x/\partial z$ are identically zero. To a second

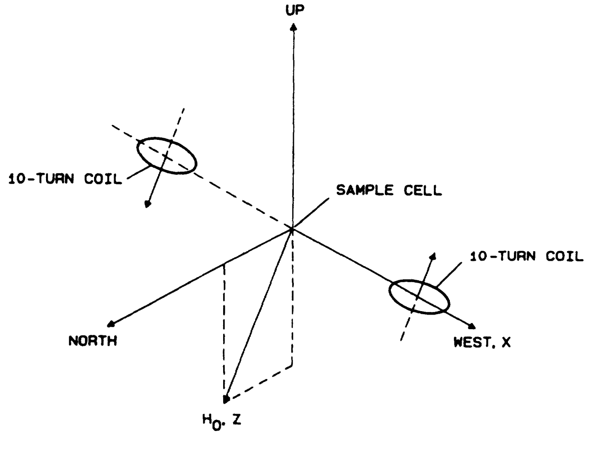

FIG. 1. Sketch showing placement of ^{3}He magnetometer sample cell and magnetic gradient producing coils.

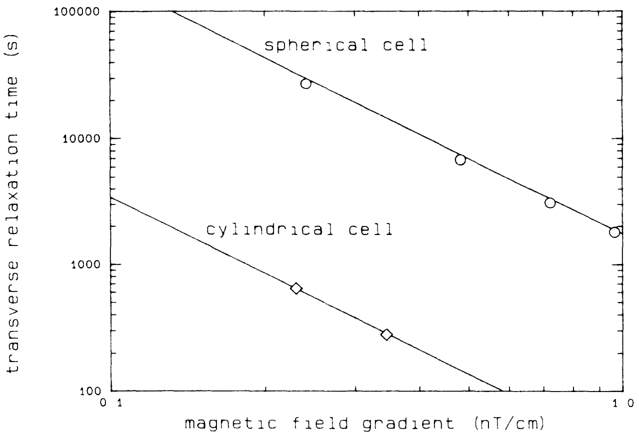

FIG. 2. Transverse relaxation time of spin-polarized ^{3}He as a function of the magnetic field gradient. Curves are shown for a 5-Torr, 12.35-cm-long cylindrical cell and a 10-Torr, 4.96-cm-diam spherical cell.

approximation, terms of the order of $(b/x)^2 \partial H_z/\partial x$ must be added to the first-approximation gradient values. Here b is the coil radius and x is the separation between the cell center and coil center. Because $(b/x)^2$ is small (0.002) and the square of the gradients appears in the $1/T_2$ expressions, the dipole approximation is considered adequate and all gradients other than $\partial H_z/\partial x$ and $\partial H_x/\partial z$ are neglected. The experiments were performed in a building constructed of nonmagnetic materials for use in magnetometer testing. The naturally occurring magnetic gradient inside the building was of the order of 0.03 nT/cm.

During the course of an experiment, precession signal amplitudes were recorded regularly at a convenient time interval (typically 1 min). After the data were collected, a maximum-likelihood estimate of the measured transverse relaxation time T_{2M} was found.

The experimental results are displayed in Fig. 2, in which the relaxation time T_2 is plotted as a function of the gradient $\partial H_z/\partial x$. The experimental points are indicated by circles or diamonds. The theoretical curves, shown as solid lines, were calculated using a ^{3}He diffusion constant of 1370.2 cm^2/s (for 1 Torr and 300 K).[11]

The lower curve of Fig. 2 applies to a 5-Torr, 12.35-cm-long cylindrical cell with its axis oriented along the x direction. The indicated relaxation time plotted in Fig. 2 is the relaxation time due to gradient relaxation alone; the effect of normal relaxation with zero gradient has been removed. Hence, the value of T_2 plotted in Fig. 2 is found from

$$\frac{1}{T_2} = \frac{1}{T_{2M}} - \frac{1}{T_{20}} , \qquad (48)$$

where T_{2M} is the total measured relaxation time and T_{20} is the zero-gradient (wall) relaxation time. The measured zero-gradient relaxation time for the cylindrical cell was 5347 s. The two experimental values differ from the calculated values using Eq. (37) by an rms average of 1.38%. In the calculation of the theoretical value, the $1/(2T_1)$ was ignored. The calculated T_1 relaxation time for the cylindrical cell subjected to a 1-nT/cm gradient is 2×10^7 s. The results for a 10-Torr, 4.96-cm-diam spherical cell are shown in the upper curve of Fig. 2. The zero-gradient relaxation time was 53 509 s. For this cell the experimental values differed from the calculated values of Eq. (47) by 7.35%. The agreement of the experiments and the theory is considered to be reasonably good.

It might be noted that if the summations in Eqs. (25), (32), and (43) are approximated by only the first terms, the resulting errors are less than 0.15%, which is small compared to experimental errors. For a more complicated cell geometry which may require a numerical estimation of the diffusion time, a one-term approximation for the autocorrelation function appears quite adequate.

IV. CONCLUSIONS

Theoretical expressions for the transverse relaxation rate of a spin-polarized gas due to a magnetic field gradient have been found for cylindrical and spherical sample cells. The expression for the spherical cells agrees with that of Cates, Schaefer, and Happer[4] (for the high-pressure limit). Experiments have been performed with spin-polarized ^{3}He gas confined within a cylindrical and a spherical cell. The measured values of gradient relaxation time agree well with the theoretical values.

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