

Pulsation of $1\omega_0$ and $2\omega_0$ emission for laser-produced plasmas. II. Theory

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It is demonstrated that stimulated Brillouin scattering (SBS) saturated by a nonlinear shift of the wave number of the ion-acoustic wave can be responsible for temporal oscillations of the radiation reflected from an expanding laser-produced plasma. Repetitive amplification and quenching of the SBS result in rapid motion of the reflection point, thus causing large sweeps of the phase of the emitted light. Both temporal pulsations and modulation of time-integrated spectra of the back-reflected radiation and its harmonics are in accord with experimental observations.

I. INTRODUCTION

Two of our recent papers^{1,2} described extensive experimental measurements of the time-integrated¹ and time-resolved² emission spectra at frequencies near the laser frequency ($1\omega_0$) and its second harmonic ($2\omega_0$) from plasmas created by focusing moderate intensity ($>10^4$ W/cm²) Nd-laser pulses onto solid targets. The experiments demonstrated that the time-integrated spectra showed random spectral modulations whose character was essentially unaffected by changes in the laser (pulse duration, laser intensity, etc.) and target parameters, while time-resolved measurements showed that the emission occurred in bursts of duration 10–20 psec separated by 20–50 psec. The characteristics of the temporal bursts were also insensitive to the same parameters, suggesting that a clear link existed between these two phenomena.

We showed¹ that the spectral modulation and the overall characteristics of the time-integrated spectra could be adequately explained if it was assumed that the emission within a relatively narrow band underwent rapid phase modulation (equivalent to a nonlinear Doppler shift) during the laser pulse. It was also shown that profile modification induced by the ponderomotive force was capable of inducing sufficient phase modulation of the reflected laser light to generate the experimentally observed spectra. In the light of these observations we undertook a series of experiments where we time resolved the emission spectra to search for temporal sweeping of the emission frequency. This was in fact observed,² although the temporal structure was considerably more complex than the simplest case consistent with the theory presented in Ref. 1.

A search of the literature demonstrated that rapid temporal bursts characterized emission from laser produced plasmas in a very wide range of conditions. Not only did $1\omega_0$ and $2\omega_0$ emission pulsate, but also $\frac{3}{2}\omega_0$ emission (originating from the two-plasmon decay instability); emission near $\omega_0/2$ (Raman); and visible harmonics (more than the tenth, etc.) of the laser frequency in experiments using CO₂ lasers. Bursts were observed in experiments using uv (351-nm) to infrared (10.6- μ m) lasers em-

phasizing that the phenomenon leading to the bursts was little affected by laser frequency. Taken in conjunction with evidence from our experiments that the bursts were little affected by laser and target parameters, it seemed logical to search for a universal phenomenon that could occur in laser-produced plasmas and lead to both modulation and pulsations of the emission.

As a starting point for this paper, where we attempt to explain the origin of the bursts and the phase modulation, we refer the reader to the time-integrated and time-resolved spectra in Figs. 1(b) and 7(a), respectively, of paper I (Ref. 2) of this series as a summary of the data included in Refs. 1 and 2. Both spectra were of second-harmonic emission from Nd-laser-produced plasmas, the time-integrated spectrum being recorded using a 70-psec duration laser pulse at an intensity of $\approx 10^{16}$ W/cm² on a plane glass target. The emission at 45° to the target normal was passed through a 1-m grating spectrograph (resolution 0.02 Å) and recorded using Reticon diode array detector. All the structure in the time-integrated spectrum represents “real” spectral modulation since the detector noise was insignificant. Figure 7(a) of Ref. 2 shows a time-resolved $2\omega_0$ spectrum obtained using a 220-psec duration laser pulse at an intensity of 4.3×10^5 W/cm², again using a glass target. In this case the grating in the spectrograph was carefully apertured to provide good temporal resolution (≈ 10 psec) and the spectrum recorded using an Imacon 500 streak camera. Careful inspection of this figure reveals that the bursts of emission are not parallel, indicating that the frequency is sweeping more or less continuously during each burst. (Interpretation of the time-resolved spectrum is complicated by pincushion distortion in the image intensifier used with the streak camera, however, the fact that the bursts are not parallel is unequivocal evidence of frequency sweeping since this is not a feature of the distortion.)

Our observations^{1,2} clearly suggest that the burst phenomenon and the spectral modulations are intimately linked—a conclusion that fits comfortably within our model involving phase modulation. This link is illustrated by the following argument where the process by which the phase modulated time-resolved spectrum constructs the modulated time-integrated spectrum via manipulation in Fourier space. This construction involves *multi-*

plication in Fourier space of two components, one representing the spectrum of phase event and the other representing the amplitude modulation. The multiplication operation preserves the dominant character of the Fourier transform of the amplitude function that generates the spectral modulations.

Before introducing our model to explain the observations, we consider that it is essential to summarize the other observations of burst phenomena and the previous explanations that have been offered in the literature.

II. SUMMARY OF PREVIOUS EXPERIMENTS REPORTING TEMPORAL PULSATIONS OF HARMONIC EMISSION

Tarvin and Schroeder³ time resolved the spectrum of the reflected laser light from microballoon targets irradiated with 500-psec duration 1.06- μm pulses at intensities between 4×10^{14} W/cm² and 10^{16} W/cm². They observed that the overall spectrum displayed a time-varying blue shift but that it was also modulated in both time and frequency. The blue shift was interpreted as reflecting the trajectory of the critical surface via the Doppler effect while the spectral modulations were attributed as reflecting a range of velocities with which different parts of the critical surface moved. The rapid (≈ 20 -psec duration) bursts of reflected power were apparently interpreted as due to "coherence effects" in the collection optics.

Jackel, Perry, and Lubin⁴ simultaneously streaked the $2\omega_0$ and $\frac{3}{2}\omega_0$ emission (in the direction perpendicular to the axes of the heating beams) from a Nd-laser-produced plasma in an attempt to measure the velocity of the critical and quarter critical density surfaces. During this experiment they found that both these harmonics were irregularly modulated in time with a spacing between the intensity minima of 40–60 psec. No explanation for these bursts was offered.

Carter, Sim, and Hughes⁵ investigated the $2\omega_0$ emission from a D-T filled microballoon irradiated using $F = 1$ optics by two 100-psec duration counterpropagating beams from a Nd laser at an intensity of $\approx 10^6$ W/cm². Streak records showed intense spectrally ($\leq 1\text{-\AA}$) and temporally (< 10 -psec) resolution limited spots of emission. Various explanations were offered including oscillatory motion of the critical surface driven by the pondermotive force; modifications of the dielectric function of the plasma by bursts of hot electrons generated at the critical surface; and rapid variations in the direction of the emission due to rippling of the critical surface. These explanations were, however, admitted by the authors to be rather tentative.

Recent observations of $2\omega_0$ emission by Xu, Zhan, and Zhao⁶ using 200-psec duration Nd-laser pulses to irradiate planar targets at intensities up to 10^{15} W/cm² revealed that the emission was often temporally modulated with the discrete bursts separated by 30–50 psec. They offered an explanation similar to that proposed by Andreev *et al.*^{7,8} involving the growth and collapse of caviton structures arising from resonant electrostatic fields near the critical surface.

Gray *et al.*⁹ time resolved the spectrum of $1\omega_0$ and $2\omega_0$

emission from plane or spherical targets irradiated using a single beam Nd-glass laser producing 1.6-nsec duration pulses focused using $F = 1$ optics to intensities up to 5×10^{15} W/cm². The $1\omega_0$ spectra were found to pulsate on a time scale of about 80 psec (the temporal resolution of the recording system was, however, only 40 psec) and this phenomenon was stated to be "not well understood" by the authors. Pulsations were also observed in the $2\omega_0$ spectra for laser intensities above 10^{15} W/cm², with the bursts in this case lasting < 15 psec and separated by 30–40 psec. The authors suggested that one possible explanation for these pulsations was that the ablation flow across the critical surface was unstable giving rise to strong local density fluctuations. They supposed that the time scale of these fluctuations τ was given by $\tau \approx L_m/c_s$, where $L_m \approx 3 \mu\text{m}$ is the scale length of the modulations and $c_s \approx 2 \times 10^7$ cm/sec is the sound speed, hence obtaining reasonable agreement with the observations.

The same explanation was preferred by Carter *et al.*¹⁰ to explain pulsations of $\frac{3}{2}\omega_0$ emission less than 10 psec in duration observed in experiments where two counterpropagating 100-psec duration Nd-laser pulses irradiated microballoons at intensities around 2×10^{16} W/cm². As an alternative, however, these authors also considered the possibility that the pulsations could be caused by the ponderative force or by ion waves.

The model based on unstable ablation flow becomes unsatisfactory, however, when the two-dimensional simulations of unstable critical surface bubbles and corrugations by Estabrook¹¹ are taken into account. Estabrook has explained that for the process to be important "the density gradient must be long enough so that at least one wavelength of light can be trapped. . . . We have tried using steeper gradients and bubbles do not occur because there is not enough plasma in the low density region for large amplitude ion compression to occur." In particular, although bubbles occurred for the plasma density scale lengths $L \approx 7\lambda_0$ at the critical, no such bubbles were observed for $L \approx 2\lambda_0$ (here λ_0 is the wavelength of the pump). Thus the scale of $3\lambda_0$ suggested in Refs. 9 and 10 as necessary to produce 15-psec duration bursts would appear to be too short for the production of such bubbles and corrugations. Furthermore, the major limitation of the simulations was that they did not include plasma flow which would have a stabilizing effect on the hydrodynamic instabilities since it pushes the unstable modes out of the unstable region.

Time-resolved $\frac{3}{2}\omega_0$ emission was also studied by Lin *et al.*¹² Planar targets were irradiated by 250-psec duration Nd-laser pulses at $(0.3-2) \times 10^{14}$ W/cm² and viewed at 90° to the beam axis. The $\frac{3}{2}\omega_0$ emission was observed to be strongly modulated on a time scale of ≈ 20 psec and appeared to originate from unstable filaments in the region of the plasma where $n < n_c/4$ (n_c is the critical density). It was argued that transient $n_c/4$ density surfaces provided temporary regions where the two-plasmon decay instability could operate, which then generated the $\frac{3}{2}\omega_0$ harmonic.

The observations of temporal pulsations have not been restricted to plasmas generated by 1- μm neodymium

lasers, but also extend to results taken using CO_2 lasers and various harmonics of the Nd laser ($\frac{1}{2}$ and $\frac{1}{3}$ μm). For example, Jaanimagi, Enright, and Richardson,¹³ while recording emission of the 10th and 11th harmonic of a CO_2 laser emitted from plasmas created by nanosecond duration pulses at $\approx 10^{14}$ W/cm^2 observed that the emission occurred in bursts lasting about 100 psec. After eliminating any role for temporal structure on the laser pulse itself, they attributed this behavior to rapid variations in the resonant electrostatic field involved in harmonic generation possibly due to rapid variations in the density profile, although they did not suggest an origin for such variations.

The second harmonic of a Nd laser was used as the pump in Ref. 14 for an investigation of time-resolved transmission and backscatterer from foil targets irradiated by 0.9-nsec duration pulses at intensities between 10^{12} and 5×10^{14} W/cm^2 . The transmitted light was spatially resolved and revealed pulsations of emission occurring at different points across the target and lasting only about 10 psec. This was explained as being due to the formation of unstable filaments. The backscattered light, which was both temporally and spectrally resolved, showed fast temporal bursts on the time scale of 5–20 psec with the emission in each burst covering a broadband of frequencies (similar to our Fig. 7 of Ref. 2). The authors offered an explanation of the $1\omega_0$ pulsations suggesting that they were due to beating between Brillouin backscattered light originating from different plasma densities with, as a result, slightly different frequencies.

Neither of these explanations seem entirely satisfactory to us, particularly in the light of our own observations. The question of filamentation will be dealt with in a following paper where we will present evidence that suggests it does not occur in our experiments. The second explanation involving beating of stimulated Brillouin scattering (SBS) waves is also hard to accept since it seems unreasonable to suggest that only a very restricted range of frequencies would be generated by SBS. Even if this were the case when only a very small region of the plasma was viewed as in Ref. 14, it is far less likely in our experiments² where a scattering screen was used to spatially average the emission from the hole plasma and yet bursts of emission in frequency space were still observed. Furthermore, it is difficult to accept that the scattering densities and hence the beat frequency would remain unchanged throughout the laser pulse since, for one thing, the density scale length in the underdense region would be expected to increase considerably in time changing the threshold conditions for SBS and affecting the backscattering spectrum. In general, an increase in SBS bandwidth should occur both near the pulse peak and as the density scale length increases with time suggesting that the beat frequency would be rather sensitive to experimental parameters, e.g., pulse duration. This is not consistent with the experimental data.

The third harmonic of a Nd laser was used in Ref. 15 as the pump for experiments to investigate $\omega_0/2$ emission. The laser pulse duration was 800 psec and the laser intensity between 5×10^{13} and 10^{15} W/cm^2 . Pulsations of the emission less than the resolution of the recording sys-

tem (20 psec) and separated by 50–90 psec were observed. The same explanation as used in Ref. 9 and 10 was suggested. There is, however, an inconsistency wavelength and this would predict that the burst duration and separation should be three times shorter than was observed in Refs. 9 and 10, yet this does not correlate with the data.

Adding our own observations to these, it seems very reasonable to conclude that virtually all emission pulsates and that the nature of these pulsations is virtually independent of the parameters of the experiment. A wide range of intensities (varying by three orders of magnitude) and pulse durations (≈ 10 psec \rightarrow 1 nsec) produced essentially similar pulsations as did a variety of target materials including CH, and W,¹⁴ Al,^{13,14} glass,^{1,2,5,10} Pb,^{1,2} and a wide range of polymers and ^{238}U .⁹ Furthermore, different laser wavelengths produce similar effects and emissions ($1\omega_0$, $2\omega_0$, $\frac{3}{2}\omega_0$, and $\omega_0/2$) created by very different physical processes all seem to pulsate. Therefore our aim has to be to find a mechanism that can give rise to such pulsations lasting typically 10 psec and separated by 30–50 psec that is independent or only weakly dependent on parameters such as those listed above.

Before introducing our ideas, we continue with a summary of the theories that exist that predict pulsation phenomena.

III. SUMMARY OF EXISTING THEORIES PREDICTING TEMPORAL MODULATION OF HARMONIC EMISSION

Erokhin, Moiseev, and Mukhin¹⁶ were the first to show that when resonance absorption dominates the second-harmonic emission may oscillate as the electron temperature in the plasma increases. This is a direct consequence of the phase-matching conditions of an inhomogeneous plasma for the $2\omega_0$ production mechanism, the coalescence of a plasmon with a photon, which occurs at a distance from the critical surface given by

$$d_{2\omega_0} \approx 9(v_e/c)^2 L, \quad (1)$$

where v_e is the electron thermal velocity c the speed of light, and L the plasma density scale length near the critical surface. As the temperature rises during the temporal evolution of the laser pulse, and $d_{2\omega_0}$ increases, the $2\omega_0$ emission point passes the standing-wave structure of the fundamental electromagnetic wave and the second-harmonic energy oscillates accordingly. Andreev *et al.*¹⁷ have investigated these effects numerically and also extended the analysis to third-harmonic generation. However, from their results it is clear that long scale lengths around critical ($L \geq 5\lambda_0$) and very high thermal velocities ($v_e \geq 0.1c$) are necessary for oscillations to occur. These conditions are not relevant to the plasmas created in our experiments.^{1,2}

Reference 17 also considered the effect of temperature on the second-harmonic emission when it arises from the intense resonant field structures created inside cavitons (density wells). Again, oscillations can occur because of

the temperature-dependent dispersion relationship for plasma waves. This time the code showed significant oscillations with temperature even for steep profiles $L = \lambda_0$, provided $v_e \geq 0.1c$. In order for cavitons to occur, however, it is necessary for the flow to be subsonic and that is unlikely to occur in the underdense region of our plasmas created by short, high-intensity pulses since the sharp leading edge of the pulse drives a heat wave into the plasma, thus providing uniform heating of the ablated mass. It was numerically shown in Ref. 7 that when subsonic flow *does* exist then, for the case of 3×10^{14} W/cm², 1.06- μ m radiation obliquely incident on a homogeneous plasma ($T_e = 1.25$ keV), the characteristic period of the second-harmonic oscillations, and the out-of-phase reflected fundamental, was $\tau \approx (k_0 c_s)^{-1} \approx 1$ psec where $k_0 = 2\pi/\lambda_0$ is the pump wave number. On the other hand, when the flow was supersonic no such oscillations were found.

Several papers have already appeared in the literature that have attempted to explain the temporal modulation of the backscattered heating radiation in terms of an instability of stimulated Brillouin scattering, where the SBS is saturated by a nonlinear phase shift of the ion acoustic wave involved in the scattering process.

Kurin and Permitin¹⁸ have considered scattering by SBS off an initially homogeneous slab of plasma, under the assumption of subsonic flow. The scattered ion wave was allowed to develop a nonlinear phase shift due to two mechanisms: (a) the displacement of plasma from the wave interaction region by the pondermotive force and (b) the inherent nonlinear coupling between the velocity of the density perturbations and the perturbation amplitude. In this case, saturation of the three wave interaction, which leads to a steady state, resulted from a competition between the pump and a detuning of the three waves induced by the nonlinear phase shift. The nonlinear phase shift was characterized by the parameter

$$\alpha = \alpha_0 = \frac{n_c E_0}{n_0 E_p}, \quad (2)$$

where n_0 is the electron density in the slab, E_0 is the amplitude of the incident radiation, and E_p is the characteristic plasma field given by $E_p = (16\pi n_0 T_e)^{1/2}$ where T_e is the electron temperature. The analysis showed that the nonlinear phase shift becomes important for $\alpha \geq 1$.

At high pump intensities, other nonlinear effects become important, such as trapping of particles and the excitation of harmonics of the sound wave. In the latter case, when there is a nonzero dispersion of the ion sound wave, the harmonics that are excited affect the phase of the fundamental ion wave, causing it to deviate from the optimum value for the interaction, thus limiting the scattering level. This problem has also been treated in Ref. 19. In this case the nonlinear shift was characterized by

$$\alpha = \alpha_2 = \frac{n_c E_0}{6k_0^2 \lambda_D^2 n_0 E_p}, \quad (3)$$

where λ_D is the Debye length. Once again, the nonlinear phase shift was important for $\alpha \geq 1$. Therefore, since the

wave number of the ion-acoustic wave $k_2 \approx 2k_0$, i.e., $\alpha \propto k_2^{-1}$, it is the longer-wavelength ion sound waves that undergo the greatest nonlinear phase shift due to harmonic generation. When the steady-state solution of this problem was discussed, it was shown that for $\alpha \gg 1$ the amplitude of the pump was almost uniform across the slab, which led to excitation of perturbations which were out of phase with the steady-state solution. Thus no steady-state SBS can exist when the nonlinear phase shift is substantial.

The self-modulation that arises from this unstable solution was further investigated by Kurin,¹⁹ who solved a set of coupled equations for the temporal and spatial evolution for the three waves involved, using numerical methods. The important parameters were the slab thickness L_0 , the critical interaction length Γ^{-1} , and the dimensionless pump amplitude Q , the last two of which were defined as follows:

$$\Gamma = \frac{k_0 n_0 E_0}{n_c E_p}, \quad (4)$$

$$Q = (c_s/c)^{1/2} \Gamma L_0. \quad (5)$$

The critical interaction length defines the propagation distance of the waves over which the phase mismatch changes by π . The behavior of the reflectivity and the spectrum of the reflected signal were investigated as a function of the pump amplitude Q for the case of zero damping and fixed n_0 and T_e . It was found that as Q increased, the process of scattering passed from a periodic regime through a quasiperiodic regime up to a completely stochastic regime of stimulated scattering. Then, when damping was steadily increased, all other parameters now being fixed, the sequence was reversed and the temporal evolution changed from stochastic to quasiperiodic to become eventually periodic again. Such self-modulations of SBS are equivalent to self-excited oscillations in a non-conservative system.²⁰

If for a given set of parameters k_0 , n_0 , T_e , and L_0 characterized by a single parameter $P \equiv 4\alpha c_s/c \Gamma L_0$, the threshold for the onset of self-modulation $\alpha \approx 1$ [which occurs when $Q = Q_{\text{mod}} = 3(c_s/c)^{3/2}/P$] is below the threshold for the scattering $Q_{\text{th}} = 0.5\pi(c_s/c)^{1/2}$, the self-modulation is "softly" excited²⁰ with the period

$$\tau \approx \frac{L_0}{c_s Q}. \quad (6)$$

For a fixed value of n_0/n_c , τ scales as $\tau \propto T_e^{1/4} (I_0 \lambda_0^2)^{-1/2}$ while for fixed incident wavelength, pump intensity and electron temperature, the period scales as $\tau \propto (n_0/n_c)^{-1/2}$.

In contrast to these predictions, however, we do not see any clear dependence of τ in a wide range (approximately three orders of magnitude) of intensity I_0 . Moreover, the conditions of our experiments are such that we expect the plasma to flow supersonically in the underdense region, in contrast to the assumption of subsonic flow in these models. Kurin and Permitin²¹ have developed a theory of the problem for supersonic flow. In the linear approximation it has been shown that the

transfer of energy from the supersonic flow to the ion-acoustic wave (which in this case has negative energy) results in explosive amplification²² of all three waves involved in the scattering process. Introducing a nonlinear phase shift removes the mathematical singularity from the problem and causes the scattering to saturate. In contrast to the subsonic case, which is characterized by the presence of an infinite set of bifurcation points on the line $R=0$ in the (R, α) plane, here the reflectivity from the slab was found to be multivalued within some range of the pump amplitude, but became single valued when the pump exceeded some critical value corresponding to $\alpha \cong 1$. One may again speculate about the stability of such a steady solution but, even if it is unstable, the time scale of the resulting self-modulation would again be $\tau \cong L_0/c_s Q$ as it was in the subsonic case.

Each of the models considered above assumed no reflections from the boundaries of the expanding plasma slab, on the basis that the boundaries were smooth on the scale of the wavelengths of the interacting waves. This is appropriate in only those situations where the plasma inhomogeneities occur on a gradual scale and where the linear phase-matching conditions are approximately satisfied. If, however, even a small amount of energy of the daughter waves is reflected off the boundaries (e.g., due to a discontinuity in the density profile at these points), then the feedback, which is thus introduced, turns the problem into one that is qualitatively quite different from those described above. Baumgartel and Sauer²³ have investigated stimulated Brillouin scattering from a supersonically expanding slab of plasma, where this feedback mechanism was introduced by allowing the scattered wave to reflect off both boundaries. Whereas for supersonic flow the SBS is absolute, and for supersonic flow the SBS is convective, here the introduction of feedback into the supersonic problem changes the character of the instability back to absolute, provided the pump intensity exceeds a certain threshold. Then, for certain phase jump conditions across the two boundaries, the scattered radiation oscillates with a period τ which is long compared with the period of the acoustic wave. τ is given by

$$\tau \approx \frac{\pi L (1 - n_0/n_c)^{1/2}}{8(1+M)c_s}, \quad (7)$$

where M is the Mach number of the flow and it has been assumed that $\omega_{ia} \approx 2k_0 c_s$. The authors claim that τ scales with wavelength, leading to values of $\tau \approx 10$ and 100 psec for Nd and CO₂ laser light, respectively, where $n_0 = 0.3n_c$, ≤ 2 , $c_s/c = 0.001$, and $L = 25\lambda_0$ has been assumed. However, this is true only if one assumes that $k_0 L$ is same for both 1 and 10- μ m plasmas. In fact, Eq. (7) predicts $\tau \propto L$, but this contradicts our experimental data and apparently others' data as well.

Randall and Albritton²⁴ have combined aspects of Refs. 19 and 23 and have presented a numerical simulation of the supersonic problem which includes both reflection boundaries and a nonlinear phase shift due to the generation of harmonics of the ion-acoustic wave. Their results clearly indicate oscillations in the reflected

wave while their numerically obtained spectra are remarkably similar to those obtained experimentally by Turner and Goldman.²⁵ The period of the oscillations that were illustrated [$\approx 3(k_0 c_s)^{-1}$], however, is much too short to explain the time-resolved data reported previously in the literature as well as our results, and, moreover, would display the same intensity and scale-length dependence as the analytic solutions previously discussed, which contradicts our observations.

Finally, we note that numerical analysis of Pattinkangas and Salomaa²⁶ of the so-called double stimulated Brillouin scattering²⁷ (DSBS) reveal a periodic behavior for the scattered radiation. However, according to their results the interval between the bursts varies linearly with the pump intensity, in contrast to the experimental observations.

IV. MODEL LEADING TO PHASE MODULATION AND PULSATION OF HARMONIC EMISSION

As an alternative to the various different explanations given above, we now wish to present a model that incorporates all of the features necessary to explain the large body of experimental evidence now available on the temporal and spectral properties of the emission. In accordance with our conclusions from Refs. 1 and 2, where we showed that phase modulation of the emission could lead to all the observed features of both time-integrated and time-resolved $1\omega_0$ and $2\omega_0$ emission spectra, we concentrate on a situation where the plasma contains two reflecting surfaces, one of which is the usual critical density surface, and the other which is assumed to be located in the underdense region of the plasma where $n < n_c/4$. The reflectivity of the underdense region switches on and off in time, for reasons that will be explained later, and hence the reflection point moves cyclically between this region and the critical density surface. As the reflection point moves, the phase of the reflected light itself varies rapidly leading to frequency sweeping of the back-reflected light. Furthermore, since the underdense reflection point lies below the quarter critical density, as its reflectivity switches on and off this leads to the pump intensity for processes occurring at higher densities (n_c or $n_c/4$, for example) to pulsate. The time scale of the bursts of reflection from the low-density region is, thereby, imprinted on all mechanisms that occur only at higher density. As a result as was shown in Ref. 1, cyclic variations in pump intensity lead to phase modulation of second-harmonic emission, as is observed experimentally,² through the action of the pondermotive force.

To account for pulsation of the reflectivity of the low-density plasma we invoke a model where the reflection is generated by SBS possibly seeded either by reflection from the critical surface or by long wavelength ion waves which propagate down the density profile in the supersonically expanding plasma.²⁸ The growth of the ion density fluctuations results in the appearance of a nonlinear phase mismatch term that forces the instability to saturate. The hydrodynamic properties of the plasma which results in a steady increase in the extent of the underdense corona with time, and hence an increase in the size

of the scattering region, in combination with this nonlinear phase mismatch causes the reflectivity to pulsate. The pulsations are driven not only by the constructive and destructive interference of the back-scattered wave due to changes in the interaction length but also by changes in the ion wave amplitude that respond to the relative phasing of the incident and reflected waves.

Although we have now introduced SBS into the problem and hence presuppose the existence of ion waves resulting from that instability, we point out that the frequency of these waves is too small for them to directly modulate the time-integrated spectra in the manner that is observed. In our model, therefore, SBS leads primarily to the temporal modulation of the intensity of the light reaching the high-density regions by providing the second reflection point for the incident wave. Note that since the ion-acoustic waves are of such low frequency, little broadening of the spectra would occur due to the SBS itself.

This explanation has a simple analogy that permits some additional modeling to be performed, namely, it produces a similar effect on the phase of the emitted wave as is obtained by reflecting light off an étalon with a time-dependent thickness. It is conceivable that étalon effects may occur in the plasma itself due to the growth and decay of cavitons, as is often observed in simulations and have been seen in experiments.²⁹ In our case we use the étalon analogy to demonstrate that phase modulation and frequency sweeping of the reflected light does in fact result from such a structure and show that the calculations mimic features of the experimental data.

V. THEORY

Let us consider a laser-produced plasma expanding into vacuum. We assume that the heating radiation undergoes stimulated Brillouin scattering within some region $x_1(t) < x < x_2(t)$ in the underdense plasma and that the plasma flow through this region is supersonic. For the sake of simplicity we begin with several additional assumptions which are listed below.

(i) The electron density within $x_1 < x_2$ is homogeneous and has a value $n_0 < n_c$.

(ii) The scattering region flows with constant velocity $u > c_s$.

(iii) The linear size of this region $L_0 = x_2 - x_1$ is constant.

(iv) Within the scattering region the frequencies and the linear phases of the interacting heating (ω_0, \mathbf{k}_0) , back-scattered (ω_1, \mathbf{k}_1) and ion-acoustic $(\omega_2 = k_2 u - k_2 c_s, \mathbf{k}_2)$ waves are perfectly matched, i.e., $\omega_0 + \omega_2 = \omega_1$, with the linear phase mismatch given by $\delta \mathbf{k}_L = \mathbf{k}_0 + \mathbf{k}_2 - \mathbf{k}_1 = 0$.

(v) We assume $k_2^2 \lambda_D^2 \ll 1$. Such an approximation is acceptable for high-intensity heating when $\Gamma(L_{HD}/k_2)^{1/2} \gg 1$ where

$$L_{HD} = \max[n(dn/dx)^{-1}, u(du/dx)^{-1}]$$

characterizes the actual inhomogeneity of the underdense plasma and Γ^{-1} is the previously introduced critical interaction length of the convective instability [see Eq. (4)].

(vi) We finally assume that $\delta k_L \gg \Gamma$ outside the plasma layer (x_1, x_2) .

In a steady state the dimensionless amplitudes of the three interacting waves are then described by the following set of equations:¹⁹

$$\begin{aligned} \frac{da_0}{d\xi} &= -a_1 a_2, \\ \frac{da_1}{d\xi} &= -a_0 a_2^*, \\ (1-M) \frac{da_2}{d\xi} &= a_0 a_1^* + i \delta k_{NL} \text{Over} \Gamma a_2, \end{aligned} \quad (8)$$

where

$$E = eE_0 \left[a_0 \exp(i\omega_0 t + ik_0 x) + \left(\frac{c_0 \omega_1}{c_1 \omega_0} \right)^{1/2} a_1 \exp(i\omega_1 t + ik_1 x) + \text{c.c.} \right]$$

is the net electric field of the heating and backscattered waves, E_0 is the amplitude of the heating wave, c_0 and c_1 are the corresponding group velocities,

$$\delta n = 2in_0 \frac{E_0}{E_p} [a_2 \exp(i\omega_2 t + ik_2 x) + \text{c.c.}]$$

is the density perturbation associated with the ion-acoustic wave, $\xi = \Gamma x$ is the dimensionless spatial variable, $M = u/c_s$ is the Mach number of the flow, and δk_{NL} is the nonlinear wave-number shift of an arbitrary origin. The nonlinear phase shift is then given by δk_{NL} multiplied by the distance of propagation of the wave. It is this, generally introduced, δk_{NL} that removes the mathematical singularity from the steady state problem which is of explosive character^{21,22} and that brings the system to saturation allowing one to investigate the steady-state solution.

Now, we will replace the three complex amplitudes a_0 , a_1 , and a_2 by a set of six real functions $A_{0,1,2}$ and $\phi_{0,1,2}$ by the following transformation:

$$\begin{aligned} a_{0,1} &= (M-1)^{1/2} A_{0,1}(\xi) e^{i[\Delta\Omega_{0,1}\tau + \phi_{0,1}(\xi)]}, \\ a_2 &= A_2(\xi) e^{i[\Delta\Omega_2\tau + \phi_2(\xi)]}, \end{aligned} \quad (9)$$

where $\tau = c_s \Gamma t$ is the dimensionless temporal variable and

$$\Delta\Omega_0 = 0, \quad \Delta\Omega_1 + \Delta\Omega_2 = 0.$$

Inserting Eqs. (9) into Eqs. (8) obtains

$$\frac{dA_m}{d\xi} = -A_j A_k \cos\phi, \quad m, j, k = 0, 1, 2, \quad m \neq j \neq k \quad (10)$$

and

$$\frac{d\phi}{d\xi} + \tan\phi \frac{d}{d\xi} \ln(A_0 A_1 A_2) = \frac{\delta k_{NL}}{\Gamma(M=1)} + \Psi, \quad (11)$$

where $\phi = \phi_1 + \phi_2$ and Ψ is an arbitrary constant which is defined by the requirement that $\sin\phi \rightarrow 0$ when $A_{1,2} \rightarrow 0$ (see Ref. 21). The boundary conditions read as follows:

$$\begin{aligned}
A_0(\xi=\Gamma L_0) &= (M-1)^{1/2}, \\
A_0^2(\xi=0) - Q_1 &= A_{10}^2 = A_1^2(\xi=0), \\
A_0^2(\xi=0) - Q_2 &= A_{20}^2 = A_2^2(\xi=0),
\end{aligned} \tag{12}$$

where A_{10} and A_{20} are the "seed" amplitudes of the secondary waves and $Q_{1,2}$ are the two integrals of the system (10)

$$\begin{aligned}
Q_1 &= A_0^2(\xi) - A_1^2(\xi) = \text{const}, \\
Q_2 &= A_0^2(\xi) - A_2^2(\xi) = \text{const}.
\end{aligned} \tag{13}$$

In what follows we will treat Eq. (10) as a system describing parametric generation in a weak coupling limit ($\delta k_{NL} > \Gamma$). Assuming that the heating wave is intense and that $A_0(\xi) \cong \text{const}$, and following the standard procedure (see, e.g., Ref. 30), one obtains the following expression for the gain of the output wave which has been normalized to A_{20} , the amplitude of the ion seed ion-acoustic wave:

$$g_1 = G \left[\frac{\Gamma}{\delta k_{NL}} \right]^2 \sin^2 \delta k_{NL} L_0, \tag{14}$$

where G is a constant dependent on the phase relation between the pump (heating) wave and the "seeds." For example, if $A_{20}^2 \gg A_{10}^2$, then $G \rightarrow 1$ (independently of the phase relation between the seed and the pump since the generated field A_1 will adjust its own phase to provide maximum gain $g_1 \equiv A_1^2(\xi=\Gamma L_0)/A_{20}^2$). In fact, the absorption results of Perry, Luther-Davies, and Dragila²⁸ indicate that there is a narrow region near the critical density which acts as a good absorber even in the conditions that favor neither resonance absorption nor parametric processes. The data obtained at that time thus seemed to be consistent with absorption induced by the presence of a long wavelength [$k_2 \lambda_D \approx 0.04-0.2$ (see, for example, Ref. 31)] ion-acoustic waves. These waves, in turn, may provide a good seed for the SBS. In such a situation, the coefficient of reflectivity R for SBS back-scattering, behaves like

$$R \propto \left[\frac{k_0}{\delta k_{NL} L_0} \right]^2 \sin^2 \delta k_{NL} L_0. \tag{15}$$

Let us now drop assumptions (i) and (iii), which restrict the above equation to a homogeneous plasma having a scattering region which is constant in length, and consider an inhomogeneous plasma where the scattering takes place in a bounded region of size L_{HD} . Since L_{HD} is, in reality, a slow function of time compared to the time scale of the ion-acoustic oscillations, and is given by $L_{HD} \simeq c_s t$, then one can replace L_0 in Eq. (15) by $c_s \tau$. This results in the temporal oscillations of the coefficient reflectivity:

$$R \propto \sin^2(\delta k_{NL} c_s t)$$

with an oscillation period τ given by

$$\tau = \frac{\tau}{\delta k_{NL} c_s}. \tag{16}$$

Finally, it remains to evaluate the nonlinear wave number shift δk_{NL} . According to Kurin and Permitin¹⁸ and Kurin,¹⁹ δk_{NL} can be expressed as

$$\delta k_{NL} = 4\alpha A_2^2 \Gamma, \tag{17}$$

where either $\alpha = \alpha_0$ if produced by displacement of the plasma by the ponderomotive force associated with the heating and back-scattered waves, or $\alpha = \alpha_2 \gg \alpha_0$ if δk_{NL} results from generation of higher harmonics of the ion-acoustic wave. There is, however, a competitive process of ion trapping that also causes a nonlinear wave number shift:³²

$$\frac{\delta k_{NL}^{(i)}}{k_2} \cong 1.6 \left[\frac{e\Phi_0}{T_e} \right]^{1/2} F \left[\frac{\omega_2}{k_2 v_i}, \frac{T_e}{T_i} \right], \tag{18}$$

where e is the electron charge, Φ_0 is the electrostatic potential associated with the density perturbations ($e\Phi_0/T_e = \delta n/n_0$), T_i is the ion temperature, and v_i is the ion thermal velocity. Assuming that the ions have a Maxwellian velocity distribution and that the ion waves are weakly damped, the introduced function becomes³²

$$F \left[z, \frac{T_e}{T_i} \right] = \frac{c_s z(z^2-1) \exp(-z^2/2)}{2\pi[(z^2-1)(1+k_2^2 \lambda_D^2) T_e/T_i - 1] v_g}, \tag{19}$$

where v_g is the group velocity of the ion wave. For comparison, if determined by generation of harmonics, δk_{NL} becomes [from Eq. (17) after setting $\alpha = \alpha_2$]

$$\frac{\delta k_{NL}^{(h)}}{k_2} = \frac{1}{12k_0^2 \lambda_D^2} \left[\frac{\delta n}{n_0} \right]^2. \tag{20}$$

Let us now evaluate δk_{NL} for some realistic parameters.

If we assume that the nonlinear phase shift is due to particle trapping then from Fig. 2 of Ref. 32, one obtains $\delta k_{NL}^{(i)} \cong 6 \times 10^{-2} k_2$ under the assumptions of $k_2 \cong k_0$, $T_e/T_i \approx 10$, and $e\Phi_0/T_e = 0.05$. Assuming further that $c_s = 1 \times 10^7$ cm/sec, one obtains from the separation between the bursts of the backscattered light $\tau^{(i)} \approx 40$ psec, which means that the corresponding half-width of the bursts themselves would be ≈ 20 psec. On the other hand, if one assumes that generation of harmonics is the cause of δk_{NL} , then, for the same plasma parameters, Eq. (20) gives

$$\frac{\delta k_{NL}^{(h)}}{k_2} \approx \frac{2.5 \times 10^4}{k_0^2 \lambda_D^2}.$$

Therefore, for $T_e \approx 1$ keV and $n_0/n_c = 0.1$, one obtains $\delta k_{NL}^{(h)}/k_2 \approx 1.6 \times 10^{-3}$, which is well below the corresponding value due to ion trapping. As can be seen by comparing Eqs. (18) and (20), $\delta k_{NL}^{(h)}$ is much more sensitive to the amplitude of the density fluctuations associated with the ion-acoustic wave. However, since it is the ion trapping in particular that imposes the upper limit on the amplitude of the ion waves $\delta n/n_0$, such that $\delta n/n_0 < 0.2$ (e.g., see Ref. 33), then for the considered set of realistic plasma parameters, we always have $\delta k_{NL}^{(i)} > \delta k_{NL}^{(h)}$. If the dephasing length $\simeq \delta k_{NL}^{-1}$ is short relative to the average value of L_{HD} during the laser

pulse, then the back-scattered radiation is emitted in a series of many short bursts. The effect of the dephasing on shortening in the interaction length to a value that is well below L_{HD} was, in fact, experimentally observed in Ref. 34. We note that for the modulation effect to occur the size of the “parametric resonator” has to be $L_0 \cong n/\delta k_{NL}$ (at the end of the laser pulse) where n is the total number of bursts. Since $\delta k_{NL} \ll k_2$ and since one also has to satisfy the linear wave number matching ($\delta k_L = 0$) within the “resonator,” such an effect can occur only in that region of the underdense plasma where the plasma is either weakly inhomogeneous or rare, i.e., well below the critical where $k_2 \cong 2k_0$ (k_0 is the vacuum wave number of the incident radiation). However, it does not mean that SBS does not occur at higher densities as well. It only means that the modulation of the backscattered radiation originates from the low-density plasma.

Finally, the separation between the bursts, $\tau^{(t)}$, and consequently the duration of the bursts, $\tau_b^{(t)}$, both scale as

$$\tau^{(t)}, \tau_b^{(t)} \propto \lambda_0 T_e^{-1/2}, \quad (21)$$

i.e., both would be 10 times longer for a CO₂ laser-produced plasma than for a Nd-laser-produced plasma, for otherwise identical conditions. Also, most importantly, both are only very weakly dependent on the intensity of the heating radiation through T_e (for our plasmas³⁵ $T_e \propto I_0^{0.1}$). This is in agreement with our observations, since variation of the laser radiation by two orders of magnitude had no significant effect on the burst duration and separation. Notice that since the intensity of the scattered wave behaves as $\sin^2(\delta k_{NL} L_0)$, the ratio of the burst duration to the burst separation is 1:2. Dependence on the target material is also very weak and is brought in by the speed of sound $c_s \propto (Z/A)^{1/2}$, where Z is the charge and A is the atomic weight. As a result of these weak dependences of the plasma and laser parameters, the mechanism appears able to explain the observed temporal modulations of the back-scattered heating radiation.

Summarizing, therefore, the basic physics involves SBS in a supersonically expanding plasma. A mismatch of the three interacting waves then arises due to an ion trapping induced nonlinear wave number shift of the ion-acoustic wave. The same mechanism is responsible for saturation of the otherwise explosive instability. As in the experiment, the theoretically obtained burst duration and the burst separation is virtually insensitive to the plasma and radiation parameters, with the exception of the wavelength of the heating radiation. This result is consistent with the only available data from an experiment which used 0.5- μm radiation,¹⁴ which the burst duration and separation was typically shorter than those observed in our and others' experiments, as well as the emission bursts observed in CO₂ experiments¹³ and also the more recent CO₂ experiments of Baryanov *et al.*³⁶ However, the results of Ref. 15, although obtained under relatively similar experimental conditions as those of Ref. 14, reveal slower temporal modulation than one would expect from the linear dependence of the time scale of the modulation on the wavelength of the heating radiation.

The presence of SBS can, however, have further conse-

quences and result in temporal modulation of the other harmonics. If SBS occurs at densities below $n_c/4$, then all harmonics of the heating radiation starting from $\omega_0/2$ have to reveal corresponding temporal modulation driven by the modulation of the pump. Since harmonic generation is a nonlinear process, one may expect that the duration of the bursts will be shorter than those of the fundamental, while their *separation* remains unchanged. These conclusions are consistent with observations of a wide range of harmonics, from $\omega_0/2$ to $11\omega_0$, in agreement with our scaling law (21).

The model also predicts that phase modulation of the emitted $1\omega_0$ wave will occur as a result of the reflection point moving back and forth between the critical density surface and the region where SBS is taking place. Although this can be seen qualitatively from the model, complex computer codes are necessary before it can be quantitatively investigated and this has not been done during this work. The phase will be further affected by the action of the ponderomotive force on the density profile, although this mechanism also requires a complete fluid and radiation code before its effect can be adequately determined.

For the SBS mechanism just considered, the separation between the bursts is given by Eq. (16). Since the nonlinear phase-shifting mechanism is assumed to reach saturation, δk_{NL} is considered to be constant, and therefore Eq. (16) leads to the desired dependence of τ on the various plasma parameters.

The effect of oscillating reflection point has a simple analogy with the reflection properties of an étalon with time-dependent parameters, e.g., thickness. Modeling the plasma reflection properties by such an étalon effect mimics the effect of the presence of cavitons in the plasma which are expanding and/or collapsing. Therefore the temporal behavior of this mechanism is also associated with the plasma dynamics.

Consider a one-dimensional, stepwise homogeneous plasma, consisting of three regions of uniform density, $n_1 > n_c$ ($x \leq c$), $n_2 < n_c$ ($0 < x \leq a$), and $n_3 < n_c$ ($a < x \leq b$), as depicted in Fig. 1. Since the overdense region is semi-infinite in extent then, if $n_2 = n_3 = 0$, total reflection would occur at the $x = 0$ interface. For the more general case of $n_3 \neq 0$, however, which we will now consider (still assuming $n_2 = 0$ for simplicity), the incident radiation is partially reflected from each of the three interfaces at $x = 0$, $x = a$, and $x = a + b$. The amplitude and also the phase of the reflected wave depend on how near, or how far, the system is from resonance.

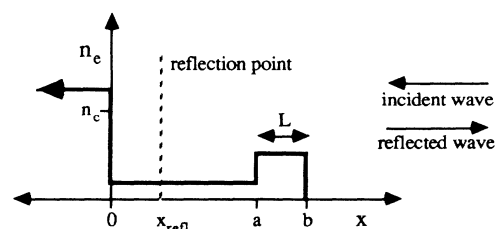


FIG. 1. Electron density profile of a stepwise homogeneous plasma designed to model the oscillatory motion of the reflection region.

If we now allow a to become a function of time, $a(t) = a_0 + v_0 t$ and we keep the width b of the density step constant, then each time a moves a distance of $\lambda_0/2$, the system will pass through a resonance, resulting in a peak in the reflectivity. These peaks will therefore be separated in time by

$$\tau_{\text{etalon}} = \frac{\pi}{k_0 v_0 \sqrt{\epsilon_2}}. \quad (22)$$

$$R = \frac{k_0(1 - \beta e^{-2ik_0 a})(1 + \alpha e^{2ik_3 b}) - k_1(1 + \beta e^{-2ik_0 a})(1 - \alpha e^{2ik_3 b})}{k_1(1 + \beta e^{-2ik_0 a})(\alpha - e^{2ik_3 b}) - k_0(1 - \beta e^{-2ik_0 a})(\alpha + e^{2ik_3 b})} \quad (23)$$

where

$$\alpha = \frac{k_3 - k_0}{k_0 + k_3}, \quad \beta = \frac{ik_0 - ik_1}{ik_0 + ik_1}.$$

Here $k = k_0 \sqrt{\epsilon}$, where $\epsilon = 1 - n/n_c - i\nu/\omega$, and the subscripts 1 and 3 denote the regions $x \leq 0$ and $a < x \leq b$, respectively, as before. To obtain a feel for the temporal behavior of the reflected fundamental wave, we have evaluated the above expressions numerically. In this analysis, approximately 30% absorption was modeled by means of the effective collision frequency ν , which was made proportional to the density.

Notice that the only substantial difference between this equation and Eq. (16) is in the replacement of δk_{NL} by $k_0 \sqrt{\epsilon_2}$, and Eq. (22), therefore, displays the same dependence on the laser wavelength and electron temperature as previously [see Eq. (21)].

Assuming that all field quantities are normalized to the amplitude of the incident wave then, for the case $n_2 = 0$, the complex amplitude of the reflected wave can be written as follows:

Figures 2(a) and 2(b) show the time evolution of the phase ϕ of the reflected wave and the coefficient of reflectivity, respectively, obtained from a moving density profile where the input parameters were $n_1 = 1.1n_c$, $n_2 = 0$, $n_3 = 0.55n_c$, $b = \lambda_0$, $a_0 = 0$, and $v_0 = 1.5 \times 10^6$ cm/sec with respect to the velocity of the critical surface $x = 0$ (see Fig. 1). Figure 2(c) depicts the quantity $d\phi/dt$ (plotted on the x axis), which represents the "instantaneous frequency" (see Ref. 1), and this graph therefore represents the theoretical time-resolved spectrum of the reflected fundamental which would be obtained in the absence of any mechanism other than the one under consideration. Finally, in Fig. 2(d) we show the theoretical time-integrated spectrum which has been calculated from the Fourier transform of the electric field

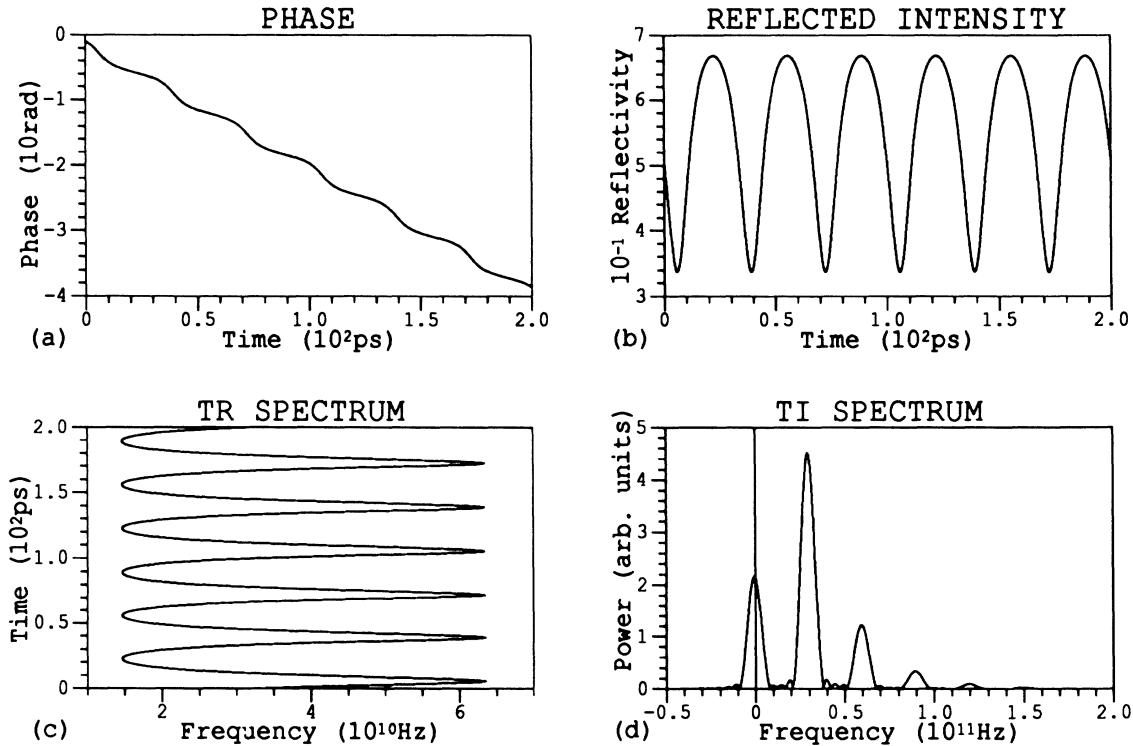


FIG. 2. Simulated time evolution of (a) the phase of the fundamental wave reflected from the density profile of Fig. 1, (b) the coefficient of (intensity) reflectivity, (c) the time derivative of the phase, (d) the corresponding time-integrated spectrum which would be obtained assuming a spectrometer slit width of $100 \mu\text{m}$.

$E(t) \propto \sqrt{I_R(t)} e^{i\phi(t)}$ of the reflected wave. Here $I_R(t)$ represents the temporal envelope of the intensity of the reflected $1\omega_0$ wave. In the spectra, $f > 0$ represents a blue shift, with respect to the purely Doppler-shifted fundamental.

The numerically obtained reflection coefficient, the time-resolved spectrum, and the time-integrated spectrum all display features which are in qualitative agreement with experiment. The detailed shape of the peaks (or troughs) in the reflectivity and the time-resolved spectrum is a complex function of the listed parameters. The dips are sharpest for certain combinations of n_3 , and b , which result in the strongest resonances. These are also the conditions which produce the broadest spectra. Where these stronger resonances occur is difficult to see from the analytic expressions. The simulations showed, however, that variation of the bump density n_3 and its width b over a fairly wide range produced relatively minor differences in the sharpness of the peaks, except when n_3 was $< 0.2n_c$ in which case the peak widths were almost equal to that of the troughs. On the other hand, the modulation depth and the breadth of the spectra decreases as the bump density was lowered, so that the strong effects were not observed for $n_3 < 0.3n_c$. Increasing ν/ω , the effective collision frequency, had little effect except to lower the average level of the reflectivity. The ratio of peak to trough widths remained unchanged by quite a large variation in ν/ω . Obviously, the model is very crude and so somewhat arbitrary, but the basic features in Fig. 2 are qualitatively in agreement with the experiment.

VI. CONCLUSIONS

We have suggested a mechanism that can lead to temporal modulation of the phase and the amplitude of plasma emission at a variety of frequencies. Its basis is the existence of two reflection points for the fundamental wave in the plasma, one corresponding to the normal critical density surface and the other located in the low-density region. The reflectivity in the latter region is produced by SBS, which saturates due to the growth of a nonlinear phase-mismatch term proportional to the ion wave amplitude. Plasma expansion then leads to pulsation of the reflectivity on a time scale $\tau \approx \pi/(\delta k_{NL} c_s)$

where δk_{NL} is the nonlinear wave number mismatch. It has been shown that the most dominant source of this mismatch is ion trapping and that this results in a reflectivity time scale that scales only very weakly with most experimental parameters, in accordance with the experimental observations. Its major sensitivity is to laser wavelength where a linear dependence is expected. This is in reasonable agreement with the observation that the pulsations are about ten times slower for CO_2 radiation^{13,36} than is observed using Nd lasers and observation of faster pulsations for $0.5\text{-}\mu\text{m}$ heating radiation.¹⁴ However, the data obtained in the $0.35\text{-}\mu\text{m}$ experiment¹⁵ under very similar experimental conditions as in Ref. 14 reveal pulsations that are slower than prediction following the linear scaling law.

The mechanism leads directly to phase modulation of the reflected light as the reflection point moves back and forward between the low-density region and the critical surface. This accounts for the frequency sweeping observed in our experiments.^{1,2} Additionally, the reflectivity pulsations from the low-density region result in the intensity of the laser light at higher densities being modulated in time, thereby imprinting the same modulation characteristics on all processes occurring at those densities (resonance absorption, harmonic generation, two-plasmon decay instability, Raman scattering, etc.). This explains the reason for similar bursts in emission produced by a wide range of different physical processes each of which, however, is sensitive to the amplitude of the pump.

Phase modulation of second-harmonic emission as we have also observed experimentally is not a direct consequence of this model, but we have shown in Ref. 1 that a time varying pump intensity can produce phase modulation of the second-harmonic because the plasma density profile then undergoes rapid cyclic changes through the action of the ponderomotive force. Phase modulation is thus produced indirectly.

In conclusion, therefore, we consider that the model presented here provides a very good explanation of most of the observations of burst phenomena reported in the literature. To fully simulate all the physics involved would take a quite massive computational effort, which is outside the scope of our resources. However, such a simulation might be essential if the physics of short-pulse laser-produced plasmas is to be fully understood.

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