Plasma heating by two laser fields in the presence of a strong magnetic field

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The heating of plasma electrons by two laser fields in the presence of a uniform strong static magnetic field via the inverse bremsstrahlung process is considered. A kinetic equation is derived, and the change in kinetic energy of the Landau electrons is calculated. For laser radiation propagating parallel to the magnetic field and incident on cold electrons, it is found that the heating rate increases as the laser frequencies approach the electron cyclotron frequency.

I. INTRODUCTION

Attention is focused currently on the problem of plasma heating (in θ pinch or solenoidal magnetic fields) using laser radiation regarding nuclear hot fusion.[1-4] In particular, it has been shown[4] that the correct and efficient way to achieve rapid energy absorption and a large heating rate is to illuminate the plasma with two laser fields, namely, a strong (pumping) field and weak (probing) field, respectively, in contrast to the mechanisms considered previously. Although the electron cyclotron frequency in these experiments is much smaller than the laser frequencies, the magnetic field probably has little effect on the rate of absorption of laser energy by the electrons but has a major effect on particle confinement. However, a resonance condition, where the laser frequency is equal to the electron cyclotron frequency, may be approached by increasing the magnetic field strength or by using longer-wavelength lasers. Since intense submillimeter lasers are now available,[5] it is therefore important to consider the cyclotron resonance absorption of these radiations.

The inverse bremsstrahlung process is now believed to play a role in the heating of a plasma by two laser radiations. During this process, a plasma electron gains energy from the two laser fields by absorbing laser photons during a collision with a nucleus. We consider here the inverse bremsstrahlung absorption of two laser fields, and include the effects of a strong (quantizing) magnetic field.

The laser beams are treated as classical plane electromagnetic waves in the dipole approximation. The plasma electrons are described by the solution to the Schrödinger equation for an electron in the laser fields and a uniform static magnetic field. Here, contrary to the method described in Ref. 2, we will make use of unitary transformation method to eliminate the laser field dependences of the kinetic energy term. To be specific, by using a unitary transformation the problem of an electron in the three external fields will be reduced to the simple problem of an electron in the presence only of a magnetic field.

II. FORMALISM

In this section we set up a procedure to solve quantum-mechanical problems with the time-dependent Hamiltonian. To begin with we write down the Schrödinger equation for an electron in the two laser fields in the presence of a strong magnetic field along the $\hat{z}$ direction, namely,

$$-\frac{\hbar^2}{2m}\left[\mathbf{P}-\frac{e}{c}\mathbf{A}(t)-\frac{e}{c}\mathbf{A}_0\right]^2\psi(\mathbf{r},t)=-i\hbar\frac{\partial\psi}{\partial t}(\mathbf{r},t)\ , \quad (1)$$

where $\mathbf{A}(t)$ is the total vector potential of the two lasers and $\mathbf{A}_0$ is the vector potential of the magnetic field. We now perform a unitary transformation in Eq. (1), namely,[6]

$$\psi(\mathbf{r},t)=U\psi'(\mathbf{r},t)\ ,$$

where

$$U=e^{i\beta(t)\cdot\mathbf{p}/\hbar}e^{i\alpha(t)\cdot\mathbf{r}/\hbar}\ . \quad (2)$$

The function $\boldsymbol{\beta}(t)$ produces a translation in space and the function $\boldsymbol{\alpha}(t)$ produces a translation in momentum. Under a unitary transformation using the above operator U, the Schrödinger equation for ψ' will have a modified Hamiltonian. Since the functions $\boldsymbol{\beta}(t)$ and $\boldsymbol{\alpha}(t)$ are arbitrary, we can use them to cancel unwanted terms in the modified Schrödinger equation to transform the time-dependent problem into a problem of a particle in the presence only of a static magnetic field.

By substituting the expression for ψ in the Schrödinger equation (1) we obtain the equation for ψ':

$$i\hbar\frac{\partial\psi'}{\partial t}=\tilde{H}\psi\ , \quad (3)$$

where

$$\tilde{H}=\frac{1}{2m}\left|\mathbf{p}+\boldsymbol{\alpha}-\frac{e}{2c}\mathbf{B}\times(\mathbf{r}-\boldsymbol{\beta})-\frac{e}{c}\mathbf{A}(t)\right|^2+\frac{\partial\boldsymbol{\alpha}}{\partial t}\cdot\mathbf{r}+\frac{\partial\boldsymbol{\beta}}{\partial t}\cdot\mathbf{p}\ .$$

The components of the vector functions $\boldsymbol{\beta}(t)$ and $\boldsymbol{\alpha}(t)$ are chosen to cancel the terms of $\tilde{H}$ which are time dependent and linear in $\mathbf{r}$ or $\mathbf{p}$. Assuming the case of linear polarization for the two laser beams the following relations result:[6]

$$\boldsymbol{\beta}=(\beta_x(t),\beta_y(t),0)\ ,$$

$$\boldsymbol{\alpha}=(\alpha_x(t),\alpha_y(t),0)\ ,$$

$$\beta_x(t)=-\frac{eE_{01}}{m(\omega_c^2-\omega_1^2)}\cos\omega_1 t-\frac{eE_{02}}{m(\omega_c^2-\omega_2^2)}\cos\omega_2 t\ ,$$

$$\beta_y(t)=\frac{eE_{01}\omega_c}{m\omega_1(\omega_c^2-\omega_1^2)}\sin\omega_1 t+\frac{eE_{02}\omega_c}{m\omega_2(\omega_c^2-\omega_2^2)}\cos\omega_2 t\ ,$$

$$\alpha_x(t)=-\frac{eE_{01}\omega_c^2}{2\omega_1(\omega_c^2-\omega_1^2)}\sin\omega_1 t-\frac{eE_{02}\omega_c^2}{2\omega_2(\omega_c^2-\omega_2^2)}\cos\omega_2 t\ ,$$

$$\alpha_y(t)=-\frac{eE_{01}\omega_c}{2(\omega_c^2-\omega_1^2)}\cos\omega_1 t-\frac{eE_{02}\omega_c}{2(\omega_c^2-\omega_2^2)}\cos\omega_2 t\ .$$

With this choice for $\boldsymbol{\beta}(t)$ and $\boldsymbol{\alpha}(t)$, the modified Hamiltonian becomes

$$\tilde{H}=\frac{1}{2m}\left|\mathbf{p}-\frac{e}{2c}\mathbf{B}\times\mathbf{r}\right|^2\ ,$$

which is the Hamiltonian of an electron in a static magnetic field $\mathbf{B}$. The solution of Eq. (3) with the Hamiltonian $\tilde{H}$ is well known and is given by the Landau wave function.[7]

Therefore, under U the problem of an electron in the presence of the two laser fields and a static magnetic field is reduced to the one of an electron in the presence only of the magnetic field with the original wave function ψ given by

$$\psi_\nu(\mathbf{r},t)=\frac{1}{L}e^{i\boldsymbol{\beta}\cdot\mathbf{p}/\hbar}e^{i\boldsymbol{\alpha}\cdot\mathbf{r}/\hbar}\times e^{ip_x x/\hbar}e^{ip_z z/\hbar}e^{(-i/\hbar)\epsilon_\nu t}\chi_n(\xi-\xi_0)\ , \tag{4}$$

where

$$\epsilon_\nu=\frac{p_z^2}{2m}+\hbar\omega_c(n+\tfrac{1}{2})\ ,$$

$$\nu\equiv(p_x,p_z,n)\ ,$$

$$\chi_n(\xi-\xi_0)=\frac{1}{(\sqrt{\pi}n!2^n r_c)^{1/2}}e^{-(1/2)(\xi-\xi_0)^{1/2}}H_n(\xi-\xi_0)\ ,$$

with

$$\xi=y/r_c,\quad \xi_0=y_0/r_c\ ,$$

$$r_c=(\hbar/m\omega_c)^{1/2},\quad y_0=p_x/m\omega_c\ .$$

In Eq. (4) $\chi_n(\xi)$ is the harmonic-oscillator wave function.

III. TRANSITION PROBABILITY

Treating the nuclear potential $V(\mathbf{r})$ as a perturbation, the probability amplitude for the transition from the initial state i with quantum number $\nu\equiv(p_x,p_z,n)$ to the final state f with quantum number $\nu'\equiv(p_x',p_z',n')$ is

$$a(\nu\to\nu')=(-i/\hbar)\int\int_{-\tau/2}^{+\tau/2}d^3r\,dt\,\psi_{\nu'}^{*}V(\mathbf{r})\psi_\nu\ . \tag{5}$$

We write the Coulomb potential in the form

$$V(\mathbf{r})=-4\pi Ze^2\hbar^2\sum_{\mathbf{q}}\mathbf{q}^{-2}e^{i\mathbf{q}\cdot(\mathbf{r}+\mathbf{r}_\alpha)/\hbar}\ , \tag{6}$$

where $\mathbf{r}_\alpha$ is the position of the nucleus. Substituting Eqs. (4) and (6) into Eq. (5) and performing the integrations over x and z, we obtain

$$a(\nu\to\nu')=2iZe^2(2\pi\hbar)^3\sum_{\mathbf{q}}q^{-2}\exp[(i\mathbf{q}\cdot\mathbf{r}_\alpha)/\hbar]\delta(p_x'-p_x-q_x)\delta(p_z'-p_z-q_z)I(y)\times\int_{-\tau/2}^{\tau/2}dt\,\exp\left[-i\frac{\lambda_1}{\hbar\omega_1}\cos\omega_1 t-i\frac{\lambda_2}{\hbar\omega_2}\cos\omega_2 t+\left[\frac{i}{\hbar}\right](\varepsilon_{\nu'}-\varepsilon_\nu)t\right]\ , \tag{7}$$

where

$$I(y)=\int dy\,\chi_{n'}(\xi-\xi_0)e^{iq_y y/\hbar}\chi_n(\xi-\xi_0)$$

and

$$\lambda_i=e\mathbf{E}_{0i}\cdot\Delta\mathbf{p}\omega_i/m(\omega_i^2-\omega_c^2),\quad i=1,2\ .$$

is the field parameter.

Integrals in the y variable similar to the one in Eq. (7) may be found elsewhere.[8] The result is

$$I(y)=F(n,n',\rho)\ ,$$

where

$$F(n,n',\rho)=\delta_{n',n}e^{-\rho/2}L_{n'}(\rho)+\Theta(n-n')\left[\frac{n'!}{n!}\right]^{1/2}e^{i(n-n')\phi}e^{-\rho/2}\rho^{(n-n')/2}L_{n'}^{n-n'}(\rho)+\Theta(n'-n)\left[\frac{n!}{n'!}\right]^{1/2}e^{i(n'-n)\phi}e^{-\rho/2}\rho^{(n'-n)/2}L_{n}^{n'-n}(\rho)\ ,$$

$$\phi=\tan^{-1}(q_y/q_x)\ , \tag{8}$$

$$\rho=\hbar^2(q_x^2+q_y^2)/2m\hbar\omega_c\ , \tag{9}$$

and $L_n^{n'-n}(\rho)$ is the Laguerre polynomial.

The integration over t in Eq. (7) may be performed after expanding the exponentials $\exp[(-i\lambda_i/\hbar\omega_i)\cos\omega_i t]$ in the form

$$e^{-i(\lambda_1/\hbar\omega_1)\cos\omega_1 t}=\sum_{l=-\infty}^{+\infty}(-i)^l J_l\left[\frac{\lambda_1}{\hbar\omega_1}\right]e^{-il\hbar\omega_1 t}$$

$$e^{-(\lambda_2/\hbar\omega_2)\cos\omega_2 t}=\sum_{m=-\infty}^{+\infty}(-i)^m J_m\left[\frac{\lambda_2}{\hbar\omega_2}\right]e^{-im\hbar\omega_2 t}\ .$$

Then Eq. (7) may be written

$$a(\nu\to\nu')=2iZe^2(2\pi\hbar)^4\sum_{\mathbf{q}}q^{-2}\exp(i\mathbf{q}\cdot\mathbf{r}_\alpha/\hbar)\delta(p_x'-p_x-q_x)\delta(p_z'-p_z-q_z)F(n,n',\rho)\sum_{m=-\infty}^{+\infty}(-i)^m$$
$$\times\sum_{l=-\infty}^{+\infty}(-i)^l J_l(\lambda_1/\hbar\omega_1)J_m(\lambda_2/\hbar\omega_2)\delta(\varepsilon_{\nu'}-\varepsilon_\nu-l\hbar\omega_1-m\hbar\omega_2)\ . \tag{10}$$

In Eq. (10) $J_j(x)$ is the Bessel function of order j and argument x.

Equation (10) is now squared to obtain the transition probability per unit time. We assume that the nuclei are randomly distributed in space. Then the sums over the positions of the uncorrelated nuclei are[9]

$$\sum_\alpha\sum_{\alpha'}\exp[i(\mathbf{q}\cdot\mathbf{r}_\alpha-\mathbf{q}'\cdot\mathbf{r}_{\alpha'})/\hbar]=N_i\delta_{\mathbf{q},\mathbf{q}'}\ ,$$

where N_i is the ion density. The transition probability per unit time, summed over the nuclei, is

$$|a(\nu\to\nu')|^2/\tau=\sum_{l=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty}T(l,m;\nu\to\nu')\ ,$$

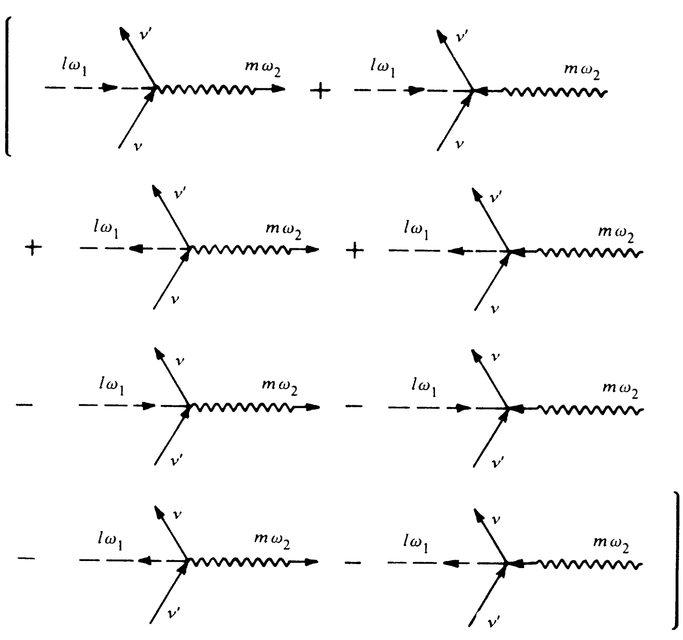
$$T(l,m;\nu\to\nu')=4Z^2e^4N_i(2\pi\hbar)^3\sum_{q_y}q_0^{-4}J_l^2(\lambda_1/\hbar\omega_1)J_m^2(\lambda_2/\hbar\omega_2)|F(n,n',\rho_0)|^2\delta(\varepsilon_{\nu'}-\varepsilon_\nu-l\hbar\omega_1-m\hbar\omega_2)\ , \tag{11}$$

where $\mathbf{q}_0=(p_x'-p_x,q_y,p_z'-p_z)$ and ρ_0 is given by Eq. (9) with $\mathbf{q}_0$ instead of $\mathbf{q}$. It followed from the δ function of Eq. (11) that the transitions are induced between Landau levels n and n' with the absorption $(l,m>0)$ or emission $(l,m<0)$ of $|l|$ and $|m|$ photons of the two laser fields.

IV. KINETIC EQUATION

The change in $N_e(\nu')$, the number of electrons in state ν', may be written schematically as Eq. (12),

$$\frac{\partial N_e(\nu')}{\partial t}=\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\sum_{\nu}$$



, (12)

where the third sum is over the quantum numbers of the initial state. As in Ref. 2, we convert the schematic equation (12) to a mathematical equation by substituting the transition probability (11). For example, the third term in Eq. (12) becomes

$$T(-l,-m;\nu\to\nu')N_e(\nu)[1-N_e(\nu')] ,$$

where $N_e(\nu)$ is the square of the matrix element of the fermion destruction operator and $[1-N_e(\nu')]$ is the square of the matrix element of the fermion creation operator. These factors appear in the transition probability when the electrons are treated using second quantized theory rather than the first quantized theory used in Sec. III. From Eq. (11), it may be shown that $T(l,m;\nu\to\nu')=T(-l,-m;\nu'\to\nu)$. Thus Eq. (12) may be written

$$\frac{\partial N_e(\nu')}{\partial t}=\sum_{\substack{l=-\infty\\ l\neq 0}}^{+\infty}\sum_{\substack{m=-\infty\\ m\neq 0}}^{+\infty} T(l,m;\nu\to\nu')[N_e(\nu)-N_e(\nu')] . \tag{13}$$

We now take the classical limit of Eq. (13) letting

$$\hbar\to 0,\quad n\to\infty ,$$

such that

$$\hbar\omega_c(n+\tfrac{1}{2})\to\tfrac{1}{2}mv_\perp^2 .$$

Taking into account the degeneracy in p_x given by

$$(2\pi\hbar)^{-1}\int dp_x\psi^*\psi=m\omega_c/2\pi\hbar ,$$

the sum over the quantum numbers of Landau state ν is

$$\sum_\nu=\frac{m\omega_c}{2\pi\hbar}\sum_{n=0}^{\infty}\sum_{p_z} .$$

Letting the sums over n and p_z become integrals and using $d_u=(m/\hbar\omega_c)v_\perp dv_\perp$, we obtain in the classical limit

$$\sum_\nu\to\left[\frac{m}{2\pi\hbar}\right]^3\int d^3v .$$

A Maxwellian distribution is now assumed for the plasma electrons. The classical limit of Eq. (13) is

$$\frac{\partial f_e(\mathbf{v}')}{\partial t}=4z^2e^4N_iN_em^3(m/2\pi k_BT)^{3/2}\int d^3v(e^{-mv^2/2k_BT}-e^{-mv'^2/2k_BT})\times\sum_{q_y}q_0^{-4}|F(n,n',\rho_0)|^2\sum_{\substack{l=-\infty\\ l!=0}}^{+\infty}\sum_{\substack{m=-\infty\\ m\neq 0}}^{+\infty}J_l^2(\lambda_1/\hbar\omega_1)J_m^2(\lambda_2/\hbar\omega_2)\times\delta[\tfrac{1}{2}m(v')^2-\tfrac{1}{2}mv^2-l\hbar\omega_1-m\hbar\omega_2] . \tag{14}$$

Where $f_e(\mathbf{v})$ is the electron distribution function. Equation (14) is the kinetic equation for the plasma electrons.

V. HEATING RATE

We proceed now to evaluate the sums and integrals in the kinetic equation (14) from which the heating rate in evaluated. From the beginning we have assumed the laser fields to be linearly polarized plane waves

$$\mathbf{E}_i=E_{0i}\mathbf{e}_x\sin\omega_i t\quad |i=1,2)$$

so that the field parameters $\lambda_i=eE_{0i}q_{0\perp}\omega_i/m(\omega_i^2-\omega_c^2)$ appearing in the arguments of the Bessel functions in Eq. (14) depend on the laser field strengths E_{0i} $(i=1,2)$, the laser frequencies ω_i, and the electron cyclotron frequency ω_c. The case $\omega_c\ll\omega_i$ is essentially the problem considered in a previous paper.[4] We consider here only the interesting case $\omega_c=\omega_i$. We also consider the case where one of the two laser fields, say, $i=1$, is a weak laser field and laser $i=2$ is the strong pumping field. Then $\lambda_2\gg\hbar\omega_2$ and the argument of the Bessel function $J_m(\lambda_2/\hbar\omega_2)$ is large. For large values of argument, the Bessel function j_m is small, except when the order m is equal to the argument. The sum over m in Eq. (14) may be written approximately[4]

$$\sum_{\substack{m=-\infty\\ m\neq 0}}^{+\infty} J_m^2(\lambda_2/\hbar\omega_2)\delta(\tilde{\Omega}-m\hbar\omega_2)=\tfrac{1}{2}[\delta(\tilde{\Omega}-\lambda_2)+\delta(\tilde{\Omega}+\lambda_2)] ,$$

where $\tilde{\Omega}\equiv\Omega-l\hbar\omega_1$, $\Omega\equiv\tfrac{1}{2}mv'^2-\tfrac{1}{2}mv^2$. The factor $\tfrac{1}{2}$ may be verified by integrating both sites of the equation over Ω. The kinetic equation (14) becomes

$$\frac{\partial f_e(\mathbf{v}')}{\partial t}=2Z^2e^4N_iN_em^3(m/2\pi k_BT)^{3/2}\exp[-m(v')^2/2k_BT]G\ ,$$

$$G\equiv\sum_{\substack{l=-\infty\\ l\neq 0}}^{+\infty}J_l^2(\lambda_1/\hbar\omega_1)\int d^3v\sum_{q_y}q_0^{-4}|F(n,n',\rho_0)|^2[(e^{(\lambda_2+l\hbar\omega_1)/k_BT}-1)\delta(\Omega-l\hbar\omega_1-\lambda_2)$$
$$+(e^{(-\lambda_2+l\hbar\omega_1)/k_BT}-1)\delta(\Omega-l\hbar\omega_1+\lambda_2)]\ . \tag{15}$$

The first δ function corresponds to the absorption and the second to the emission of $\lambda_2/\hbar\omega_2$ photons of strong laser field. Since $\lambda_2\gg\hbar\omega_2$, only multiphoton processes are significant. We assume that the electron temperature is low. Then $k_BT\ll\lambda_2$ and the emission term in Eq. (15) is negligible compared to the absorption term. Equation (15) becomes

$$\frac{\partial f_e(\mathbf{v}')}{\partial t}=2Z^2e^4N_iN_em^3(m/2\pi h_BT)^{3/2}\sum_{\substack{l=-\infty\\ l\neq 0}}^{+\infty}J_l^2(\lambda_1/\hbar\omega_1)\int d^3v\sum_{q_y}q_0^{-4}|F(n,n',\rho_0)|^2\exp(-mv^2/2k_BT)$$
$$\times\delta(\Omega-l\hbar\omega_1-\lambda_2)\ . \tag{16}$$

As for laser 1, the weak laser field, $\lambda_1\ll\hbar\omega_1$ and the Bessel function J_l^2 appearing in Eq. (16) may be written approximately[2]

$$J_l^2(\lambda_1/\hbar\omega_1)\cong\frac{1}{(l!)^2}\left[\frac{1}{2}\frac{\lambda_1}{\hbar\omega_1}\right]^{2|l|}$$

and, consequently, only the $l=\pm1$ terms should be retained; i.e., in the weak-field regime of laser 1 only single-photon processes are significant. Also, in evaluating the heating rate from Eq. (16) we shall retain only the $l=+1$ term (i.e., we neglect photoemission processes). Proceeding further, in the limit of low temperature, the Maxwellian distribution

$$N_e(m/2\pi k_BT)^{3/2}\exp[-(mv^2/2k_BT)]$$

reduces to the δ function $\delta(\mathbf{v})$. Before using this δ function to evaluate the integration over velocity we notice from the δ function of Eq. (16) that $v'\gg v$ for the intense field case. Then, $(\mathbf{v}'-\mathbf{v})$. $\mathbf{e}_x$ appearing in the expression of the weak-field parameter, namely,

$$\lambda_1=eE_{01}(\mathbf{v}'-\mathbf{v})\cdot\mathbf{e}_x/(\omega_i^2-\omega_c^2)$$

is written approximately as $\mathbf{v}'\cdot\mathbf{e}_x$. Equation (16) then becomes after integrating over velocity

$$\frac{\partial f_e(\mathbf{v}')}{\partial t}=2Z^2e^4N_im^3\sum_{q_y}(q')^{-4}|F(n,n',\rho')|^2\left[\frac{1}{2}\frac{eE_{01}\mathbf{v}'\cdot\mathbf{e}_x}{\hbar\omega_1}\right]^2\delta\left[m\frac{v'^2}{2}-eE_{02}\omega_2q'_\perp\Big/(\omega_2^2-\omega_c^2)\right]\ , \tag{17}$$

where $\mathbf{q}'=(p'_x,q_y,p'_z)$ and ρ' is given by Eq. (9) with $\mathbf{q}'$ instead of $\mathbf{q}$.

The expression for $F(n',n,\rho')$ in Eq. (8) is very complicated for arbitrary $\mathbf{q}$ because, in general, both the $n\rightarrow n$ and $n\rightarrow n'$ $(n\neq n')$ transitions are possible. For simplicity we shall consider only transitions between neighboring Landau levels $n'=n+1$ and also consider only upward $n'=n+1$ electron transitions. Which is valid if we assume that the electron concentration in the plasma is such that only the $n=0$ Landau level is fully occupied. Under the foregoing considerations, Eq. (8) reduces to

$$F(n,n',\rho')=\left[\frac{n!}{n'!}\right]^{1/2}e^{i(n'-n)\phi}e^{-\rho'/2}$$
$$\times(\rho')^{(n'-n)/2}L_n^{n'-n}(\rho'),\quad n'>n\ . \tag{18}$$

If we now make use of the properties of the Laguerre polynomials,[10] Eq. (18) can be drastically simplified in the case when $\rho'\ll1$ $(\rho'=\hbar q_\perp'^2/2m\omega_c)$. Under these conditions,

$$L_n^r(\rho')\approx\frac{(n+r)!}{n!r!}-\frac{(n+r)!}{(n-1)!(r+1)!}\rho'\ .$$

With the use of these results for $L_n^r(\rho')$ and taking into account the situation where $n=0$, Eq. (18) reduces to

$$F(0,1,\rho')\cong e^{i\phi}(\rho')^{1/2}\ . \tag{19}$$

The condition $\rho'\ll1$ can easily be obtained by increasing the magnetic field strength $(\rho'\propto1/H_0)$. Equation (17) then becomes

$$\frac{\partial f_e(\mathbf{v}')}{\partial t}=2Z^2e^4N_im^3\sum_{q_y}\rho'(q')^{-4}\left[\frac{eE_{01}\mathbf{v}'\cdot\mathbf{e}_x}{2\hbar\omega_1}\right]^2$$
$$\times\delta\left[\frac{m(v')^2}{2}-\frac{eE_{02}\omega_2q'_\perp}{(\omega_2^2-\omega_c^2)}\right]\ . \tag{20}$$

Finally the rate of change of the average kinetic energy of the Landau electrons is written as

$$\frac{d\langle\epsilon\rangle}{dt}=\int d^3v'\frac{m(\mathbf{v}')^2}{2}\frac{\partial f_e(\mathbf{v}')}{\partial t}$$

$$=\frac{E_{01}^2Z^2e^6N_im^4}{4\hbar^2\omega_1^2}$$

$$\times\int d^3v'(v')^4$$

$$\times\sum_{q_y}\rho'(q')^{-4}\delta\left[\frac{m(v')^2}{2}-\frac{eE_{02}\omega_2q_1'}{(\omega_2^2-\omega_c^2)}\right]. \quad (21)$$

Assuming now for the sake of the simplicity that the electrons are moving parallel to the x direction, which is actually the direction of maximum absorption of the strong field photons ($\mathbf{v}'\cdot\mathbf{e}_x$ is maximum), the sum over q_y and the integral over v' in Eq. (21) can easily be carried out to give for the heating rate the final expression

$$\frac{d\langle\varepsilon\rangle}{dt}=\frac{2^6(\delta\pi)^{5/2}}{3c^{5/2}m^2}\frac{Z^2\alpha^2e^5N_i\omega_2^3I_1I_2^{3/2}}{(\omega_1^2-\omega_c^2)^2(\omega_2^2-\omega_c^2)^3}, \quad (22)$$

where I_1 and I_2 are the weak and strong laser intensities, respectively, and α is the fine-structure constant.

VI. DISCUSSION AND CONCLUSIONS

Equation (22) is the plasma heating rate in the presence of two laser fields under a strong dc magnetic field. Upon comparison with a previous calculation[4] in which no magnetic field was assumed, we find that Eq. (22) shows laser-cyclotron resonance factors. In other words, the presence of the strong magnetic field introduces a new channel for absorption of the laser energies by the electrons. It thus follows that the heating rate can be made very large whenever the laser frequencies equal the cyclotron frequency of the plasma electrons. Absorption far from resonance ($\omega_c \ll \omega_i$) is essentially the problem considered in a previous paper.[4]

In closing, it has been proposed in this paper that plasma be heated to thermonuclear temperature by the rapid absorption of energy from two laser fields in the additional presence of a strong (quantizing) magnetic field. We have shown that the joint action of the two laser beams plus the magnetic field results in a very large heating rate whenever $\omega_i=\omega_c$ (the resonance condition), in contrast to the case where no magnetic field is present. This shows that the plasma heating by two laser fields plus a strong magnetic field may be one of the most efficient mechanisms for the heating of a plasma by external fields.

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