

## Multisqueezing of mechanical states and back-action evasion measurements

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A technique is proposed to improve the squeezing of the mechanical noise in a back-action evasion measurement. The scheme is based on the use of a parametric multipump system and, in the limit of many pump components, approaches a stroboscopic measurement. A simple model of the back-action evasion effect for this scheme is developed. Experimental evidence of a significant increase in the squeezing with respect to the two-pump scheme and in accordance with the model is reported.

### I. INTRODUCTION

New measurement strategies, called quantum non-demolition (QND), have been introduced for the detection of displacements of macroscopic systems in a quantum regime.<sup>1-3</sup> The research on these strategies and their implementation on real electromechanical transducers has been motivated mainly by the development of gravitational-wave antennae in order to overcome the limitation on the sensitivity imposed by the Heisenberg principle. Furthermore, the study of these techniques is an interesting topic in itself, due to the possibility of testing in detail the Copenhagen interpretation of the measurement process in quantum mechanics.<sup>4</sup> In a classical scenario such techniques allow one to overcome the classical sensitivity limit imposed by the amplifier<sup>5</sup> and therefore they are also known as back-action evasion (BAE) strategies. A proposed BAE scheme consists of the measurement of only one component of the complex amplitude of the mechanical oscillator which models the electromechanical transducer. Here we show how the concept of the BAE measurement scheme as introduced in Ref. 3 may be generalized. The advantages of this proposed generalization are explained with a simple model and experimental evidence for the so-called multisqueezing effect is reported, according to the theoretical analysis. A discussion of the potential uses of this line of research completes the paper.

### II. MULTISQUEEZING EFFECT IN GENERALIZED BAE MEASUREMENTS

A general picture of an electromechanical transducer is obtained considering two oscillators, one mechanical and the other electrical, coupled through an interaction system as in Fig. 1. The Hamiltonian for the system consists of the Hamiltonians of the two free oscillators, the interaction term and the external generalized force term expressed as

$$H = \frac{p^2}{2m} + \frac{m\omega_1^2 x^2}{2} + \frac{\pi^2}{2L} + \frac{L\omega_2^2 q^2}{2} + H_i - xF(t) - qV(t), \quad (1)$$

where  $(x, p)$  and  $(q, \pi)$  are the coordinates and momenta of the two oscillators.  $F$  and  $V$  are the generalized forces.  $m$  and  $\omega_1$  are the mass and frequency of the mechanical oscillator and  $L$  and  $\omega_2$  are the inductance and frequency of the electrical oscillator. Finally,  $H_i$  is the interaction Hamiltonian. If  $H_i = 0$  the two systems are uncoupled and the relative temporal evolutions will be uncorrelated. The interaction between the two oscillators allows us to obtain information about the mechanical quantities via the electrical ones. Moreover, this interaction perturbs the same quantities: such a perturbation is what we mean by back action, both in the quantum as well as in the classical regime. In order to show more explicitly this effect we assume that the interaction Hamiltonian  $H_i$  has the form

$$H_i = E(t)xq, \quad (2)$$

where  $E = E(t)$  is the time-dependent electric field. By doing so we restrict our analysis to systems in which the

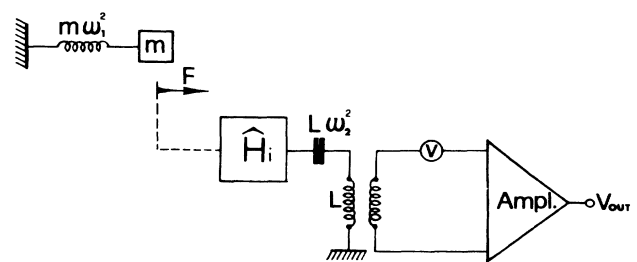


FIG. 1. General picture for an electromechanical transducer consisting of one mechanical oscillator and one electrical oscillator coupled through an interaction Hamiltonian  $H_i$ .

mechanical displacement is measured by means of the charge variation in the electrical oscillator, as depicted in Fig. 1. Other equivalent schemes involving Hamiltonians which contain other pairs of mechanical and electrical quantities such as the momentum and the magnetic flux can be analyzed with the same formalism by simply performing a canonical transformation on the Hamiltonian variables. The important point is that the interaction Hamiltonian is linearly dependent on both mechanical and electrical variables by means of a time-dependent electric field.

The sensitivity of an electromechanical transducer to impulsive forces is evaluated through the burst noise temperature  $T_b$  defined as the energy in units of the Boltzmann constant  $k_B$ , which would be deposited in the mechanical oscillator from an impulse which gives a signal to noise ratio of 1 in the filtered output.<sup>6-8</sup> The evaluation of the sensitivity dependence on the particular  $E(t)$  chosen, may be quantified by introducing a merit factor  $r$  such that the burst noise temperature  $T_b$  can be written as

$$T_b = 2 \frac{\omega_1}{\omega_2} T_a \frac{1}{r}, \quad (3)$$

where  $T_a$  is the amplifier noise temperature. For instance the minimum detectable energy in a measurement time  $\Delta t$  by using a parametric up-converter, when only an electric field at  $\omega_2 - \omega_1$  frequency is used, is<sup>9</sup>

$$E_b = k_B T_b = \frac{k_B T \omega_1 \Delta t}{Q_1} + k_B T_a \frac{\omega_1}{\omega_2} \left[ \beta \omega_1 Q_2 \Delta t + \frac{1}{\beta \omega_1 Q_2 \Delta t} \right], \quad (4)$$

where  $Q_1$ ,  $Q_2$  are the quality factors of the mechanical and the electric oscillators,  $T$  is the thermodynamical temperature of the system, and  $\beta = E_0^2 / mL\omega_1^2\omega_2^2$  is the electromechanical coupling of the system. The first term in (4) represents the energy fluctuation due to the Brownian motion, the second is the back-action contribution and the last one is the wide-band contribution of the amplifier noise. By optimizing the measurement time  $\Delta t$  the minimum burst noise temperature is that of Eq. (3) where

$$r = \left[ 1 + \frac{T}{T_a} \frac{1}{\beta Q_1} \right]^{-1/2}, \quad (5)$$

which is limited to the unity also if the Brownian noise is negligible ( $T/Q_1 \rightarrow 0$ ). On the other hand a BAE measurement corresponds to  $r \gg 1$ . It has been suggested<sup>3</sup> that a BAE effect is obtained through a continuous measurement of the complex amplitude of the mechanical oscillator by choosing for the electric field

$$E(t) = \frac{E_0}{2} [\cos(\omega_2 - \omega_1)t + \cos(\omega_2 + \omega_1)t], \quad (6)$$

i.e., by pumping with a superposition of two sinusoidal signals whose frequencies are the sum and the difference between the electrical and the mechanical frequencies. Indeed the minimum detectable energy in a measurement

time  $\Delta t$  in this last situation is<sup>9</sup>

$$E_b = k_B T_b = \frac{k_B T \omega_1 \Delta t}{Q_1} + \frac{k_B T_a}{2\sqrt{2}Q_2} \left[ \frac{\beta \omega_2 \Delta t}{16\sqrt{2}} + \frac{16\sqrt{2}}{\beta \omega_2 \Delta t} \right], \quad (7)$$

which corresponds, by optimizing the measurement time  $\Delta t$ , to a merit factor

$$r = \left[ \frac{1}{8} \left( \frac{\omega_2}{\omega_1} \right)^2 \frac{1}{Q_2^2} + 8 \frac{T}{T_a} \frac{\omega_2}{\omega_1} \frac{1}{\beta Q_1 Q_2} \right]^{-1/2} \quad (8)$$

which may be greater than the unity, in the limit of negligible Brownian noise, provided that  $Q_2 \gg \omega_2/\omega_1$ , and allows to beat the quantum limit by using a quantum limited amplifier. This configuration may be considered as a superposition of two transduction mechanisms already well known in parametric amplifier engineering.<sup>8,10</sup> Indeed the physical interpretation of this scheme is quite simple if we think of the properties of a parametric amplifier and an inverting converter. By referring to Fig. 2, we see that the output electrical noise, which is responsible for the back action, has a Lorentzian spectrum centered at the frequency  $\omega_2$  with a bandwidth related to the electrical quality factor of the electrical circuit. The electrical noise that lies in a band extending from  $\omega_2 - \omega_1$  to  $\omega_2 + \omega_1$  is converted simultaneously by the two pumps into a mechanical force. The parametric process of one pump is phase inverting and the other phase preserving so the coherent superposition is canceled in one mechanical phase and added in the other. The electrical noise which lies outside the band previously considered is converted with the same phase by both pumps. Thus, the cancellation mechanism of the noise in one phase is not effective. The squeezing will be strongly dependent on the amplitude of uncorrelated noise near  $\omega_2 - 2\omega_1$  and  $\omega_2 + 2\omega_1$ , and therefore on the quality factor of the electrical mode. A possible way to cancel these contributions is obtained by increasing the noise correlation region in the frequency domain by means of two other pumps, at

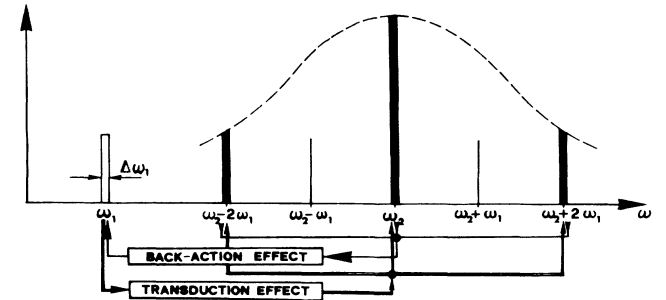


FIG. 2. Spectrum of the noises in a BAE parametric transducer. The mechanical noise is up-converted to the frequency  $\omega_2$  (transduction effect) through the two pumps. By the same mechanism the electrical noise at frequency  $\omega_2$  is down-converted to the mechanical frequency (back-action effect). In this situation the back-action force is squeezed in one mechanical phase. The bandwidths of the converted noises are related to the quality factors of the respective oscillators.

frequencies  $\omega_2 - 3\omega_1$  and  $\omega_2 + 3\omega_1$ . The same procedure can be repeated and we observe a full correlation of the noise within a bandwidth equal to  $6\omega_1$ . A natural generalization of such a scheme is the stroboscopic measurement of  $X_1$  realized with the interaction Hamiltonian<sup>11</sup>

$$H_i = E(t)X_1q = E_0(-1)^n \delta(t - n\pi/\omega_1)xq. \quad (9)$$

The Fourier representation for the time-dependent electric field  $E(t)$  is

$$E(t) = \sum_{n=-\infty}^{+\infty} c_{2n+1} e^{i(2n+1)\omega_1 t}, \quad (10)$$

where

$$c_{2n+1} = E_0 \tau \frac{\omega_1}{\pi} \frac{\sin[(2n+1)\omega_1 \tau/2]}{(2n+1)\omega_1 \tau/2}, \quad (11)$$

and  $\tau$  is the width of the pulse height  $E_0$  which simulates, for  $\tau \ll \omega^{-1}$ , a  $\delta$  function in a realistic scenario. The relationships (10) and (11) show the presence of only odd harmonics in the pump spectrum that, eventually, may be upconverted around an electrical mode with frequency  $\omega_2$ . The standard BAE measurement using two pumps is the zeroth-order approximation of the expansion (10), and the proposed four pumping scheme is the first-order one. A calculation of the ratio of the variances  $\sigma_1^2$  and  $\sigma_2^2$  on the two quadrature phases of the electrical signal may be done by assuming that the mechanical oscillator is sensitive to the down converted electrical noise only in the resonance region, i.e., around  $\omega_1$  with a width  $\Delta\omega_1 = \omega_1/2Q_1$ . In this hypothesis, as well as  $\Delta\omega_1 \ll \Delta\omega_2$ , we obtain, for a pump with  $2n$  components, a squeezing factor

$$\rho_n^2 = \frac{\sigma_2^2}{\sigma_1^2} = \frac{\arctan \left[ \frac{\Delta\omega_1}{\Delta\omega_2} \frac{1}{1+4n^2\omega_1^2/\Delta\omega_2^2} \right]}{\arctan \left[ \frac{\Delta\omega_1}{\Delta\omega_2} \right] + 2 \sum_{k=1}^{n-1} \arctan \left[ \frac{\Delta\omega_1}{\Delta\omega_2} \frac{1}{1+4k^2\omega_1^2/\Delta\omega_2^2} \right] + \arctan \left[ \frac{\Delta\omega_1}{\Delta\omega_2} \frac{1}{1+4n^2\omega_1^2/\Delta\omega_2^2} \right]}. \quad (12)$$

If  $\omega_1/\Delta\omega_2 \gg 1$  the formula (12) simplifies to

$$\rho_n \simeq \frac{1}{4n} \frac{\omega_2}{\omega_1} \frac{1}{Q_2}. \quad (13)$$

The use of  $2n$ -mode pumping is equivalent to the use of a standard BAE 2-mode pumping having an effective electrical quality factor  $Q_2' = nQ_2$ .

In the opposite approximation, i.e.,  $\omega_1/\Delta\omega_2 \ll 1$ , the formula (12) can be simplified again as

$$\rho_n \simeq \frac{1}{n}. \quad (14)$$

This last configuration is interesting because it corresponds to a low or a null electrical quality factor ( $Q_2 \ll \omega_2/2\omega_1$ ). This is a new feature of this scheme. In the first situation the burst noise temperature is of the order of

$$T_{bn} \simeq 2T_a \frac{1}{Q_2} = 2T_a \frac{\omega_1}{\omega_2} \left[ \frac{\omega_2}{\omega_1} \frac{1}{nQ_2} \right], \quad (15)$$

and the ultimate sensitivity is bounded by the electrical quality factor (i.e., by the squeezing factor  $(\omega_2/\omega_1)(1/nQ_2)$ ). In the last situation there is no dependence on the electrical quality factor and we obtain, for the burst noise temperature, the value

$$T_{bn} \simeq 2T_a \frac{\omega_1}{\omega_2} \frac{1}{n}, \quad (16)$$

and this does not give any limit on the sensitivity when the ratio  $\omega_1/\omega_2$  is low. This peculiarity of a multipump-

ing scheme seems highly interesting in view of the development of a high-sensitivity transducer and has been tested experimentally.

### III. EXPERIMENTAL EVIDENCE FOR THE MULTISQUEEZING EFFECT

We have shown in the previous paragraph that a particular configuration of a multipumping scheme exists for which there are no requirements on the electrical quality factor. It is possible, by using a multipumping system, to cancel the noise contribution on the lateral bands around the signal peak of a parametric transducer. This means that by adding a third pump at  $\omega_2 \pm 3\omega_1$  in a conventional BAE scheme it is possible to squeeze the noise that lies respectively around  $\omega_2 \pm 2\omega_1$ . This consideration suggests the kind of measurement required: first we have to produce a mechanical squeezed state by down-converting the electrical noise at  $\omega_2$  with two pumps in a standard BAE scheme, then we degrade the squeezing by adding noise around  $\omega_2 \pm 2\omega_1$  and finally we restore the squeezing with a third pump at  $\omega_2 \pm 3\omega_1$ , respectively, with a careful check, of course, of the phase-sensitive transduction.

In order to test experimentally the multisqueezing effect we have used a mechanical transducer consisting of a three-plate capacitor in a push-pull configuration (Fig. 3). This transducer consists of a single piece of aluminum alloy Al 6061 in which a small cylindrical mass is the resonating element. The resonant frequency is fixed by means of springs which connect the small mass to a relatively big support in order to decouple the mechanical mode from the outside noise. The springs are obtained

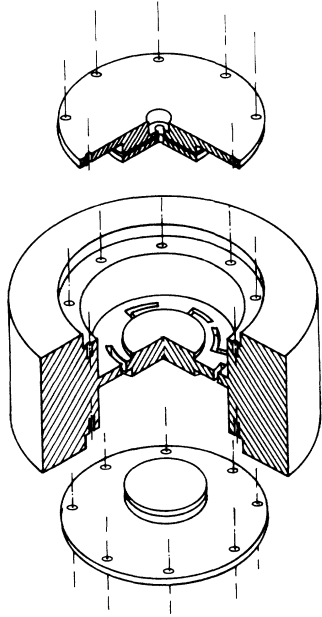


FIG. 3. Exploded view of the electromechanical transducer used for the measurements.

by cutting with a particular geometry a diaphragm which connects the moving mass and the support. All the diaphragm, the mass, and the support are machined from a single piece of aluminum alloy. The transducer is suspended in a vacuum chamber. The mechanical quality factor for the vibrational mode under consideration, at a frequency of  $\nu_m = 1744.2$  Hz, is  $Q_1 = 1.8 \times 10^3$ , which is enough for our goal and we can avoid the complications of working at a low temperature. Moreover, the value of the mechanical quality factor is a good compromise between the need for a sensible mechanical amplification of the down-converted signal and the tuning requirements of the two-pump frequencies to the mechanical one, which is more difficult as the mechanical quality factor increase. Of course one of the disadvantages of a room temperature setup is the frequency drift of the mechanical mode which requires continuous check for the tuning during the period of measurements.

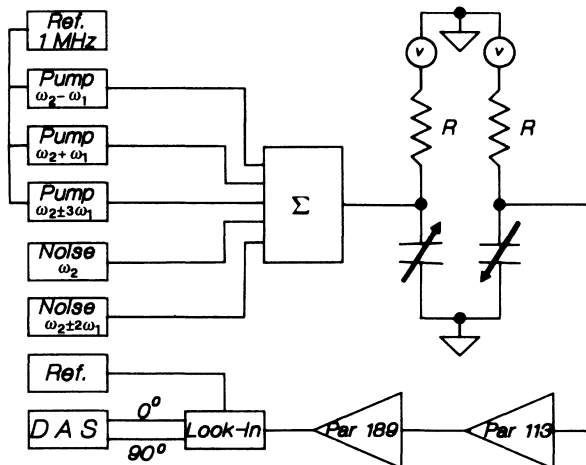


FIG. 4. Scheme of the experimental setup for the measurements.

The two faces of the transducer are inserted in two different electrical circuits (Fig. 4). The excitation circuit consists of a set of synthesizers both as pumps and noise sources whose outputs are summed by means of an active analogic adder. All the pumps are phase locked to a high-stability oscillator at 1 MHz. The relative amplitude factor of the injected noise simulates the electrical quality factor, in order to test the multipumping effect as expressed by Eqs. (14) and (16).

The detection circuit consists of a standard chain with a polarization circuit, a low noise preamplifier, a selective amplifier, and a lock-in driven with a reference signal at the mechanical frequency. The data are digitally recorded by using a standard analog-to-digital converter and sent to a microcomputer both for the on-line control of the correct working of the apparatus and for the further off-line signal analysis and processing. After several trials we realized that the best working situation consists in the use of the synthesizers as controlled noise sources. The available synthesizers allow the possibility of programming a frequency sweep in a given time around a fixed frequency. This choice was made to reduce the direct pickup between the excitation and the detection circuits that prevented us from using wideband high-power noise sources. The well-known Fourier expansion allows us to simulate a wideband noise by means of sinusoidal signal whose frequency slowly varies within a given range.

We have performed measurements in various configurations, with both monochromatic signals and frequency-swept signals. A typical measurement was performed as follows.

- (i) The relative phase between the two primary pumps is adjusted in order to squeeze the back-action force due to the swept signal around a chosen frequency  $\omega_2$ .
- (ii) A swept signal at  $\omega_2 \pm 2\omega_1$  frequency is added and the phase of the third pump (at  $\omega_2 \pm 3\omega_1$ ) is adjusted in order to squeeze the force due to this second noise.

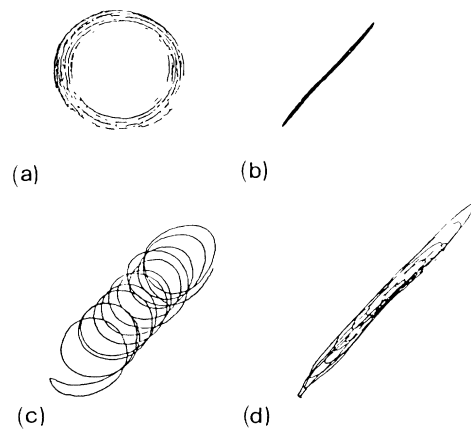


FIG. 5. Measured forces acting on the mechanical oscillators in four different configurations: (a) only one pump is effective, no squeezing; (b) squeezing of the swept noise around  $\omega_2$  due to the two-pump scheme; (c) demolition of the squeezing due to injected additional noise near the frequency  $\omega_2 \pm 2\omega_1$ ; (d) restoring of the squeezing due to the third pump with an optimized phase.

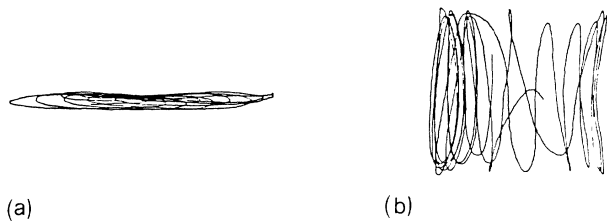


FIG. 6. Proof of the phase sensitivity for the restoring of the squeezing with a third pump: (a) maximum squeezing; (b) squeezing obtained by changing the phase by  $180^\circ$  with respect to (a).

In Figs. 5(a)–5(d) the two lock-in quadrature outputs for four different configurations are shown. In the first situation [Fig. 5(a)] we use a swept noise at  $\omega_2$  and only one pump; there is, of course, no squeezing. By adding the second pump [Fig. 5(b)] the noise is squeezed. The squeezing is not complete when a second source of swept noise at  $\omega_2 \pm 2\omega_1$  [Fig. 5(c)] is added. The squeezing is restored by adding a third pump and by changing the phase of this last pump with respect to the previous two [Fig. 5(d)].

In Figs. 6(a) and 6(b) a comparison between the maximum squeezing and the situation obtainable by means of a change of phase respectively equal to  $180^\circ$  is reported. In the second situation a mixed squeezing is obtained due to the fact that the noise at  $\omega_2$  is squeezed in one phase while the noise at  $\omega_2 \pm 2\omega_1$  is squeezed in the other phase.

In Fig. 7 the simultaneous monitoring of the displacements  $X_1$  and  $X_2$  due to the back-action force is reported.

The result is obtained when a “noise” signal with a frequency which is slowly varied around the central one  $\omega_2$  is applied at the excitation circuit. Being that the output “noise” bandwidth  $\Delta\omega_N \ll 2\omega_1$ , a proper choice of the pump phase should allow perfect squeezing. The departure from this expected behavior is mainly due to the detuning of the two pumps. More precisely, if the two pumps are at the frequencies  $\omega_+$  and  $\omega_-$  the tuning condition  $\omega_+ - \omega_- = 2\omega_1$  is not achieved. The two down-converted back-action forces are not perfectly centered with respect to the mechanical frequency so they do not have the same amplitude in the relevant bandwidth, which generates a dependence of the squeezing factor on the frequency, which we call dispersive squeezing. To investigate the dependence of the dispersive squeezing from the detuning effect we have performed measurements with different bandwidths and the results are shown in Figs. 8(a)–8(d). The outer ellipses correspond to the center frequency, for which the gain is maximum due to the selectivity of the mechanical oscillator. The inner ellipses correspond to frequencies far from the central frequency and we observe that the amplitude of the principal axis decreases. Both a decrease in the eccentricity of the ellipses (amplitude dispersion) as well as a precession of the principal axis (phase dispersion) are evident as the sweeping bandwidth is increased from 8(a) to 8(d). This is in qualitative agreement with our hypothesis, although the last figure, 8(d), is affected by the lock-in filtering. However, the lock-in filtering does not affect our considerations because it works in the same way on the two phases. It is possible to estimate simply that in order to neglect the dispersive squeezing the detuning of the two

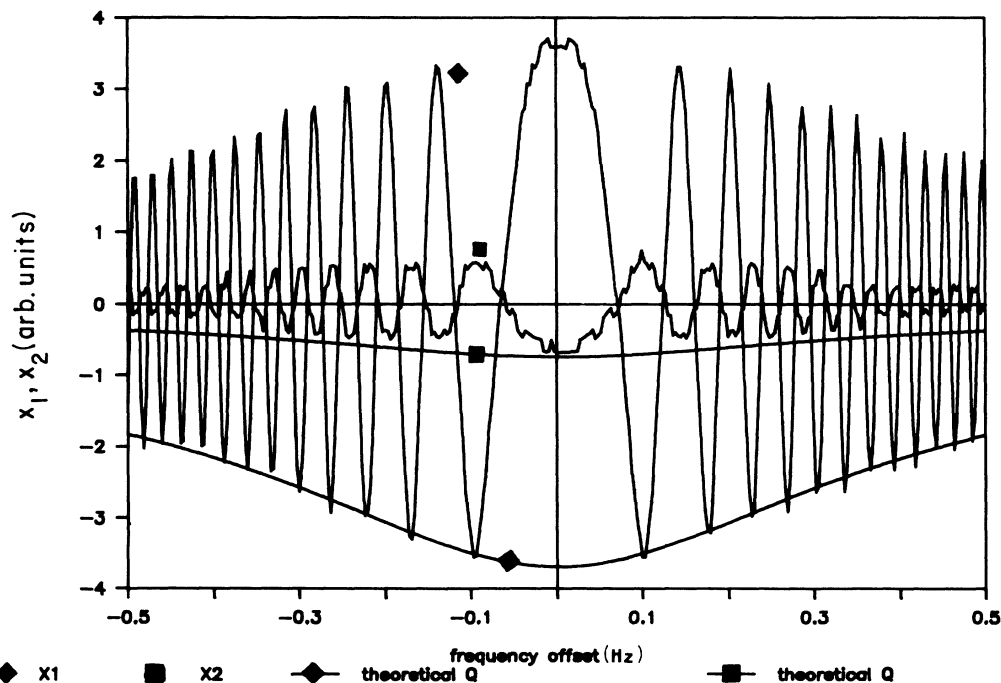


FIG. 7. Back-action forces acting on the two quadrature phases when a swept signal around  $\omega_2$  is applied. On the  $x$  axis the offset from the central frequency is shown while on the  $y$  axis the two lock-in outputs  $X_1$  and  $X_2$  in arbitrary units are reported. The continuous curves represent the best fit of the theoretical attenuation due to the mechanical filtering.

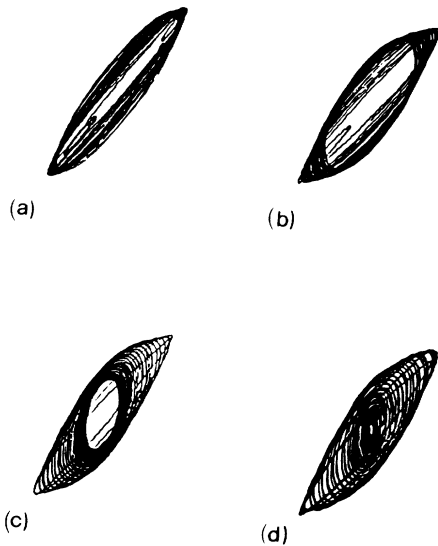


FIG. 8. The detuning of the two pumps is evidenced by measurements with increasing values for the equivalent bandwidth of the noise as appears in (a)–(d). The dispersive squeezing is shown both by a decrease in the eccentricity of the ellipses as well as a precession of the principal axis as the sweeping bandwidth is increased.

pumps  $\delta\omega = \omega_+ + \omega_- - 2\omega_1$  has to satisfy the relationship  $\delta\omega \ll \Delta\omega_1$ .

This effect may be a serious drawback when operating with very high mechanical quality factors as those obtainable at liquid-helium temperature, although in this last configuration the frequency drift of the mechanical mode is small. Unless a very high-stability pumping system and a stable mechanical frequency are obtained an automatic adjustment of the pump frequencies is needed to avoid dispersive squeezing.

#### IV. CONCLUSIONS

We have given a theoretical proof and experimental evidence for the new concept of a multisqueezing strategy. The model developed to understand the behavior of

this system is quite simple and can be expressed in terms of well-known concepts. It is clear that a new scheme of transduction of sensitivity comparable to that of the current and standard BAE schemes may be developed. In fact, once a high parametric gain has been chosen, the absence of stringent requirements on the electrical quality factor makes the mechanical quality factor the only limitation to the maximum obtainable squeezing, apart from amplitude and phase noise of the pumps. We want to point out, however, that the simultaneous use of more pumps implies a decrease in the maximum value of the electric field for each pump, originating therefore in a lower electromechanical coupling factor. Thus, the optimal sampling time is increased and in these optimized conditions the contribution of the Brownian noise increases too. This problem may be overcome by detecting a signal not only at  $\omega_2$  but also at  $\omega_2 \pm 2\omega_1$ ,  $\omega_2 \pm 4\omega_1$ , . . . , etc. and performing a sum, analog as well digital, of all the signals. A detailed calculation of the sensitivity of a transducer based on this idea is in progress and we will refer to it in a future publication. Further, by using a fully multipumping scheme, i.e., a stroboscopic one with a square signal, it will be possible to overcome the pump-phase noise that is the present practical limitation of these devices.

Finally, we observe the close analogy between the concepts developed here and similar concepts in quantum optics, namely in the production of squeezed states of light.<sup>12</sup> In fact it has been shown theoretically<sup>13</sup> that a four-mode wave mixing gives an improvement of the squeezing of light. Such a scheme has been shown to work properly<sup>14</sup> via nonlinear interactions in an optical fiber and from this point of view our work may be considered as the mechanical counterpart of these nonlinear-quantum-optics developments.

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<sup>1</sup>V. B. Braginskij and Yu. Vorontsov, Usp. Fiz. Nauk **114**, 41 (1974) [Sov. Phys.—Usp. **17**, 644 (1975)].

<sup>2</sup>V. B. Braginskij, Yu. Vorontsov, and F. Khalili, Zh. Eksp. Teor. Fiz. **73**, 1340 (1977) [Sov. Phys.—JETP **46**, 705 (1977)].

<sup>3</sup>C. M. Caves, K. S. Thorne, R. W. P. Drever, V. D. Sandberg, and M. Zimmermann, Rev. Mod. Phys. **52**, 341 (1980).

<sup>4</sup>*Quantum Theory and Measurement*, edited by J. A. Wheeler and W. H. Zurek (Princeton University Press, Princeton, 1983).

<sup>5</sup>R. Giffard, Phys. Rev. D **14**, 2478 (1976).

<sup>6</sup>M. F. Bocko and W. W. Johnson, Phys. Rev. Lett. **47**, 1184 (1981); **48**, 1371 (1982).

<sup>7</sup>M. F. Bocko, F. Bordoni, F. Fuligni, and W. W. Johnson, in *Physical Systems and 1/f Noise* (Elsevier, Amsterdam, 1986), p. 47.

<sup>8</sup>F. Bordoni, in *Proceedings of the International Workshop on Gravitational Waves Signal Analysis and Processing, Amalfi,*

1988 (World Scientific, Singapore, in press).

<sup>9</sup>F. Bordoni, S. De Panfilis, F. Fuligni, V. Iafolla, and S. Nozzoli, in *Proceedings of the Fourth Marcell Grossman Meeting on General Relativity*, edited by R. Ruffini (North-Holland, Amsterdam, 1986), p. 523.

<sup>10</sup>J. C. Decroly, L. Laurent, J. C. Lienard, G. Marechal, and J. Vorsbeitchik, *Parametric Amplifiers* (MacMillan, London, 1973).

<sup>11</sup>R. Onofrio, in *Proceedings of the International Workshop on Gravitational Waves Signal Analysis and Processing, Amalfi, 1988* (World Scientific, Singapore, in press).

<sup>12</sup>D. F. Walls, Nature (London) **306**, 141 (1983).

<sup>13</sup>B. L. Schumaker, J. Opt. Soc. Am. A **2**, 92 (1985); Phys. Rep. **125**, 318 (1986).

<sup>14</sup>B. L. Schumaker, S. H. Perlmutter, R. M. Schelby, and M. D. Leveson, Phys. Rev. Lett. **58**, 357 (1987).