Radiative effects in the theory of beam propagation at nonlinear interfaces

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A theory is presented that describes radiative effects in beam propagation at interfaces separating two (or three) self-focusing dielectric media. The nonlinear interface is formed by two media, both of which are nonlinear, so that in both media the nonlinear wave packet representing the self-focused channel (optical beam) may be described by a soliton solution of the nonlinear Schrödinger equation (NLSE), the interface being a steplike inhomogeneity. Assuming the perturbation to be small, we apply the perturbation theory for solitons based on the inverse scattering technique and study adiabatic and radiative effects stipulated by the soliton scattering. In the adiabatic approximation the scattering is described by equations for the soliton parameters that correspond to a motion equation for a classical particle in an effective potential. The reflection coefficient of the beam (the NLSE soliton) is related to a radiation during the scattering, and calculated in the Born approximation of the perturbation theory for the cases of a single and two nonlinear interfaces. An analytical comparison with the scattering of a linear wave packet is carried out. In particular, it is demonstrated that the nonlinear reflection coefficient may be sufficiently smaller than the linear one. We predict also the nonmonotonic dependence of the single-interface reflection coefficient versus the beam power, and analytically describe the nonlinear resonant scattering by two interfaces.

I. INTRODUCTION

The scattering of light beams by interfaces between two dielectric media is the important subject of modern optics and has been intensively investigated in recent years. In particular, the complete results have been obtained for the case of two linear media, when an angle of incidence is close to that for total internal reflection.[1]

Beginning with the well-known paper by Kaplan,[2] the nonlinear case of the light-beam scattering in which one of the dielectric media is supposed to be nonlinear, i.e., its refractive index depends on the intensity (a Kerr-type nonlinearity), has been investigated. For this case a plane-wave theory explaining some effects was constructed (see, e.g., Ref. 2) and some computer experiments taking into account an input Gaussian beam were carried out.[3] The first experimental observation of the nonlinear interface effects was reported in the paper by Smith *et al.*[4] The experiments showed the existence of the switch from the total internal reflection of the input beam to its partial transmission at some threshold intensity. Experimental data on beam reflectivity showed the existence of hystereses (or jumps) in the form of the reflectivity versus input intensity. There is, as of now, no generally accepted explanation of this fact. Some data from experiments showed good quantitative agreement with the plane-wave theory and some did not (see, e.g., discussions in Ref. 3). Additionally, for the case of the interface between linear and nonlinear Kerr dielectric media, some particular solutions, including the nonlinear surface waves, have been obtained in a number of papers (see, e.g., Refs. 5–8).

In recent papers,[9,10] the theory which describes the reflection and transmission properties of nonlinear wave packets at an oblique angle to the interface separating two nonlinear dielectric media was presented. The light beam (self-focused channel) was represented as a soliton of the nonlinear Schrödinger equation (NLSE) and the equivalent particle theory for the soluton was developed. In particular, the effective potential for the beam scattering was obtained and it was demonstrated that the analytical theory was in good agreement with numerical simulations of the problem.

However, the authors of Refs. 9 and 10 consider the problem in the adiabatic approximation for solitons (when the dynamics of an initial beam may be described by simple equations for its parameters, e.g., its coordinate) that admits a simple mechanical interpretation as a particle motion in an effective potential relief stipulated by the interface. The spectral density of radiation generated during such a scattering was not analyzed, because the similar effects are beyond the particle theory of beam propagation.

The purpose of the paper is the study of radiative effects accompanying the nonlinear beam scattering by an interface between two (or three) nonlinear dielectric media. The similar effects are important for calculation of the nonlinear reflection coefficient, which is determined by reflected wave packets only. We also study the scattering in the adiabatic approximation and demonstrate that the minimum point of the effective potential for the beam corresponds to the exact surface wave along the interface, which is a generalization of the solution obtained by Tomlinson.[5]

For our theoretical analysis we take the well-known model. In this model, a spatially localized optical beam is described by the NLSE for an envelope of the electric field, and the light beam corresponds to a soliton solution of this nonlinear equation. The medium inhomogeneities induced by interfaces may be considered as perturbations in the NLSE. We use the perturbation theory for solitons[11-14] to describe the influence of interfaces on beam propagation. In the framework of this approach a soliton scattering by an inhomogeneity can be described in the so-called adiabatic approximation as a classical particle motion in the effective potential. Taking into account radiative effects described by the next order of the perturbation theory permits us to calculate such an important characteristic as the reflection coefficient.[13] The analysis of the nonlinear reflection coefficient of the optical beam in the region of the validity of the NLSE is the main result of the present paper. We demonstrate that the reflection coefficient of a nonlinear wave packet (the NLSE soliton) can be considerably less than that of a linear wave packet. This result could be very important from the viewpoint of the application of nonlinear interfaces in various optical devices, e.g., optical limiters, optical switchers, etc.

Besides the analysis of a single interface, we consider analytically the beam scattering by two interfaces and study nonlinear interference effects. In particular, we show that all interference effects disappear in the nonlinear case when the width of the beam is less than (or comparable to) the distance between two interfaces. The most interesting result is the nonlinear interference. The latter follows from the condition that the distance of the beam motion between interfaces is proportional to the wavelength of the internal nonlinear oscillations of the beam. There is no analog of this effect in linear theory.

The paper is organized as follows. In Sec. II we briefly describe the model of interaction of an optical beam with a nonlinear interface and obtain the perturbed NLSE. When the interface is absent the initial beam is a simple one-soliton solution of the NLSE. Inhomogeneities will affect the soliton dynamics. In Sec. III we present general results of the perturbation theory for solitons paying more attention to the calculation of radiative effects and, in particular, to the reflection coefficient of the beam.

In Sec. IV we analyze the beam scattering by a single interface. In particular, the results of Refs. 9 and 10 follow directly from the adiabatic approximation of the perturbation theory for solitons, and the nonlinear reflection coefficient is defined by the radiation propagating in the backward direction from the interface. We obtain the general formula for the reflection coefficient of a soliton and compare it with the reflection coefficient of a linear wave packet that has the same envelope. As a result, the nonlinear reflection coefficient can be considerably less than that for the linear case.

In Sec. V we analyze the case of two nonlinear interfaces separating three Kerr nonlinear optical media. In this case, the interference phenomena are of the most interest. We also calculate the nonlinear reflection coefficient and analyze the influence of Kerr nonlinearity on the interference phenomena. In Sec. VI we present the validity conditions of our approximations. Sec. VII concludes the paper.

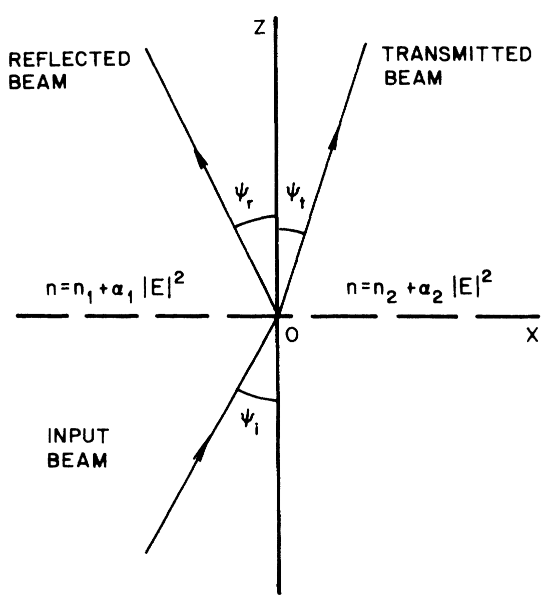

FIG. 1. Interface configuration and coordinate system for a single interface.

II. FORMULATION OF THE PROBLEM

Let us consider the propagation of a collimated beam of light at two adjoining nonlinear dielectric media. The geometry is sketched in Fig. 1, which shows the beam incident at a small angle to the interface separating the two neighboring nonlinear dielectric media. The light channel will propagate close to the z axis and will be bounded in the transverse x dimension. We anticipate that the light channel may be scattered by the interface, i.e., it has the reflected and transmitted parts after the scattering (see Fig. 1). Two optical media differ by refractive indices; we assume them to be of the Kerr type, i.e., to depend on the electric field E as follows:

$$n(x,|E|^2)=\begin{cases} n_1+\alpha_1|E|^2, & x<0 \\ n_2+\alpha_2|E|^2, & x>0 \,. \end{cases} \tag{1}$$

The propagation of the transverse electric wave in the x-z plane of optical medium is described by the scalar wave equation

$$\frac{\partial^2 E}{\partial z^2}+\frac{\partial^2 E}{\partial x^2}=-nk_0^2 E \,, \tag{2}$$

where k_0 is the free space wave number. Supposing that the phase varies fast along the interface,

$$E(x,z)=F(x,z)\exp(i\beta k_0 z) \,,$$

we obtain the following propagation equation for the slowly varying envelope $F(x,z)$ of the optical field:

$$2i\beta k_0\frac{\partial F}{\partial z}+\frac{\partial^2 F}{\partial x^2}-k_0^2(\beta^2-n)F=0 \,.$$

Making the change of variables $x'=k_0x$, $z'=k_0z$ and dropping the primes we finally obtain the following equation:

$$2i\beta\frac{\partial F}{\partial z}+\frac{\partial^2 F}{\partial x^2}-(\beta^2-n)F=0\ . \tag{3}$$

Each optical medium has its own equation (3) with different refractive indices of the form (1). Making the transformations

$$F(x,z)=\sqrt{2/\alpha_1}\Psi(x,t)e^{-i(\beta^2-n_1)t}\ ,\quad t=z/2\beta\ , \tag{4}$$

we reduce the initial Eq. (3) to the dimensionless equation which coincides with the well-known NLSE,

$$i\frac{\partial\Psi}{\partial t}+\frac{\partial^2\Psi}{\partial x^2}+2|\Psi|^2\Psi=U(x,|\Psi|^2)\Psi\ . \tag{5}$$

Two equations for each medium can be considered as a single one, but with the steplike potential

$$U(x,|\Psi|^2)=\begin{cases}0, & x<0\\ \Delta-A|\Psi|^2, & x>0\ ,\end{cases} \tag{6}$$

where $A=2(\alpha_2/\alpha_1-1)$, $\Delta\equiv n_1-n_2$. Equation (5) is written as a usual NSLE, however, it is necessary to remember that the time t plays the role of the coordinate z along the interface in our problem. The potential U has a more complicated form as considered by us earlier for other problems,[13,15] since there is a jump of not only the linear but also the nonlinear term in the perturbed NLSE.

At $U=0$ Eq. (5) has an exact solution in the form of a NLSE soliton

$$\Psi_s(x,t)=2i\eta\frac{\exp[-2i\xi x+i\delta(t)]}{\cosh\{2\eta[x-X_0(t)]\}}\ , \tag{7}$$

where 2η and 4ξ are its amplitude and velocity, respectively, $\delta(t)=4(\xi^2-\eta^2)t$ is the phase, and $X_0(t)=-4\xi t$ is the coordinate. In this paper we will consider a nonlinear input beam as a NLSE soliton. In the previous variables the solution may be present in the following form:

$$E(x,z)=\frac{2i\eta(2/\alpha_1)^{1/2}}{\cosh\{2\eta k_0[x-(2\xi/\beta)z]\}}\times\exp\left[-2i\xi k_0 x-\frac{ik_0 z}{2\beta}[4(\xi^2-\eta^2)+(\beta^2+n_1)]\right]\ . \tag{8}$$

The velocity of the soliton envelope $2\xi/\beta$ appears to be proportional to the sine of the beam angle of incidence ψ_i, $\sin\psi_i=2\xi/\beta$, and its amplitude, to its power,

$$P=\int_{-\infty}^{\infty}|E|^2dx=\frac{2}{\alpha_1}\int_{-\infty}^{\infty}|\Psi|^2dx=\frac{2}{\alpha_1 k_0}N\ , \tag{9}$$

where $N=4\eta$ is the value of the integral motion of the NLSE, which has a meaning of a number of photons bound in the soliton as a bound state.

Therefore, the nonlinear light-beam scattering by an interface between two media is reduced to motion of the NLSE soliton in the potential [Eq. (6)]. We will assume that the transmitted beam is also a soliton, i.e., the scattering is small. The new beam has other parameters, so that a portion of the incident energy in the form of linear wave packets is emitted in the backward direction (the reflected light beam, Fig. 1). We define the reflection coefficient as a ratio of the incident beam power P_i to the power of the reflected beam P_r,

$$R=\frac{P_r}{P_i}=\frac{N_r}{N_i}\ . \tag{10}$$

The transmission coefficient T is

$$T=1-R\ . \tag{11}$$

If the media differ weakly from each other, i.e., the conditions Δ, $A\ll1$ are valid, the potential (6) can be considered as a small perturbation affecting to an initial soliton of the NLSE. The perturbation theory, used under such a condition, permits us to investigate a number of radiative effects accompanying the light beam scattering by the interface, and also to calculate and analyze the reflection coefficient (10) in the nonlinear case.

III. FORMALISM OF THE SOLITON PERTURBATION THEORY

If in the perturbed NLSE of the form

$$i\frac{\partial\Psi}{\partial t}+\frac{\partial^2\Psi}{\partial x^2}+2|\Psi|^2\Psi=\epsilon R(\Psi) \tag{12}$$

the parameter ϵ turns out to be small ($\epsilon\ll1$), the soliton scattering can be studied by means of the perturbation theory. In this paper we use the soliton perturbation theory based on the inverse scattering technique (IST) (see, e.g., Refs. 11–15).

The slow change of soliton parameters can be described in the framework of so-called adiabatic approximation, assuming that the beam shape is still defined by the expression (7) but its parameters are changed (particle approximation). The corresponding equations for the NLSE soliton parameters can be obtained both by means of the regular perturbation theory, if we keep terms of the order of ϵ, and as a result of the Hamiltonian approach based on the Hamiltonian for the perturbed equation (12).[15]

The next step of the soliton perturbation theory is to calculate the first correction to the adiabatic shape of the soliton solution; it consists of terms of two types: the first type is the localized functions moving with the soliton (the distortion of its form), and the second type is two dispersive wave packets propagating in both directions from the interface (radiation). The reflected wave packet defines the soliton reflection coefficient from the inhomogeneity. The IST gives us the corresponding expression for the spectral density of emitted quasiparticles N (see, e.g., Ref. 13),

$$n_{\rm rad}(\lambda,t)\approx\frac{1}{\pi}|b(\lambda,t)|^2,\quad |b(\lambda_1 t)|^2\ll 1 \tag{13}$$

$b(\lambda,t)$ being the so-called Jost coefficient. The spectral parameter λ appearing in the IST is connected with the wave number $k(\lambda)$ and frequency $\omega(\lambda)$ of generated linear waves by the relation $\omega(\lambda)=k^2(\lambda)=4\lambda^2$. The influence of a perturbation leads to a change of the IST spectral parameters including the Jost coefficient $b(\lambda,t)$, which at $\epsilon\ll 1$ can be written in the form of the equation (see, e.g., Ref. 13)

$$\frac{\partial b(\lambda,t)}{\partial t}=4i\lambda^2 b(\lambda,t)+\epsilon\int_{-\infty}^{\infty}dx[R(\Psi)\Phi_1^{(1)}(x,t;\lambda)\Phi_2^{(2)}(x,t;\lambda)-R^*(\Psi)\Phi_2^{(1)}(x,t;\lambda)\times\Phi_1^{(2)}(x,t;\lambda)]\ ,$$

where $\Phi_{1,2}^{(1)}(x,t;\lambda)$ and $\Phi_{1,2}^{(2)}(x,t;\lambda)$ are the components of the Jost functions. For the one-soliton solution this equation takes the form[12,14]

$$\frac{\partial b(\lambda,t)}{\partial t}=4i\lambda^2 b(\lambda,t)+\frac{\epsilon}{[(\lambda-\xi)^2+\eta^2]}\left[\eta^2\int_{-\infty}^{\infty}dx\,R(\Psi_s)\frac{e^{-2i\lambda x+4i\xi x+2i\delta}}{\cosh^2 Z}-\int_{-\infty}^{\infty}dx\,R^*(\Psi_s)e^{-2i\lambda x}(\lambda-\xi-i\eta\tanh Z)^2\right]\ , \tag{14}$$

where $Z=2\eta(x-X_0)$. If before scattering the beam corresponds to a pure soliton (7), the initial condition for Eq. (14) should be taken in the form $b(\lambda,t=-\infty)=0$. Having integrated this equation, one can find the radiative density after the scattering with the help of formula (13), where it is necessary to take the limit $t\to+\infty$.

In this paper we consider the scattering of a fast soliton, i.e., we calculate the radiation in the so-called Born approximation. It means that in Eq. (14) the additional dependence of the soliton parameters in time, connected with their change during propagation at the inhomogeneity, may not be taken into account. Then the reflected beam power (the emission in the backward direction) appears to be proportional to ϵ^2. We should note that all $\lambda<0$ correspond to the reflected beam and $\lambda>0$ to the transmitted one. The reflection coefficient (10) should be written now as[13]

$$R=\frac{N_r}{N}=\frac{1}{4\eta}\int_0^{\infty}n_{\rm rad}(-\lambda)d\lambda\ , \tag{15}$$

and will be used for calculation of the radiative effects below.

IV. BEAM SCATTERING BY AN INTERFACE

A. Adiabatic approximation

We start to analyze the scattering of a nonlinear light beam (a NLSE soliton) by an interface between two nonlinear media. The perturbation in force should be chosen for a soliton in the form (6) and the parameters Δ, A are to be taken small ($\Delta, A\ll 1$). The adiabatic approximation gives us the following equations for a slow change of soliton parameters (cf. Ref. 15):

$$\frac{d\eta}{dt}=0\ , \tag{16}$$

$$\frac{dX_0}{dt}=-4\xi\ , \tag{17}$$

$$\frac{d\xi}{dt}=\frac{\Delta}{2}\frac{\eta}{\cosh^2(2\eta X_0)}-\frac{A\eta^3}{\cosh^4(2\eta X_0)}\ , \tag{18}$$

$$\frac{d\delta}{dt}=4(\xi^2-\eta^2)+\frac{\Delta}{2}[1+\tanh(2\eta X_0)]-3\eta^2A[\tfrac{2}{3}+\tanh(2\eta X_0)-\tfrac{1}{3}\tanh^3(2\eta X_0)]\ . \tag{19}$$

Equation (16) does not mean that the shape of a soliton scattered by the interface is not changed, since there are also localized corrections not described by our adiabatic approximation.

Equations (17) and (18) can be considered separately from others and transformed into a single equation

$$\frac{d^2X_0}{dt^2}=-\frac{\partial W}{\partial X_0}\ ,$$

which is that for a classical particle motion in the external potential

$$W(s)=\Delta(1+\tanh s)-2\eta^2A(\tfrac{2}{3}+\tanh s-\tfrac{1}{3}\tanh^3 s)\ ,\quad s\equiv 2\eta X_0\ . \tag{20}$$

The analog "soliton as classical particle" is well known, and the potential (20) coincides with the accuracy of constants with potentials obtained in Refs. 9 and 10. The general form of the potential is determined by the only parameter $\gamma\equiv\Delta/2\eta^2A$. For $0<\gamma<1$ it has a minimum and a maximum at points $\tanh s_0=\pm\sqrt{1-\gamma}$ (see Fig. 2). Their existence means the possibility of the capture of a soliton by the interface, i.e., the transformation of an input self-focused channel into a trapped stationary nonlinear surface wave (NSW), and a minimum, which for $\Delta>0$ is at the point $s_0>0$, that corresponds to a stable one. The condition $\gamma<1$ gives us the critical amplitude $\eta_{\rm cr}=(\Delta/2A)^{1/2}$ above which the existence of NSW is possible. It is important to note that, except the threshold power $\eta_{\rm cr}$, for the existence of surface wave there must also be a critical angle $\xi_{\rm cr}$ below which an input beam will be captured by the interface due to radiative losses of beam intensity during its propagation. These values cannot be determined within the adiabatic theory and radiative corrections must be taken into account.

The scattering of a beam in the framework of the adiabatic approximation will now be studied for the case $\Delta > 0$. The case $\Delta < 0$ corresponds to the particle moving from the right medium to the left or to the turning over the potential. Various shapes of the potential (20) for $\Delta > 0$ are shown in Fig. 2. For large γ (or small η^2) the maximum value of this potential is at $s = +\infty$

$$W(+\infty) = 2\Delta \left[1 - \frac{2}{3\gamma}\right] .$$

It decreases with the growth of η^2. At $\gamma = 1$ the local maximum at point $-s_0$ appears,

$$W_{\max} = \frac{\Delta}{3\gamma}(1-\sqrt{1-\gamma})^2(2\sqrt{1-\gamma}+1) , \tag{21}$$

and for $\gamma < \frac{3}{4}$ it becomes larger than $W(+\infty)$. The condition of transmission of a beam through the interface may be determined in terms of the soliton parameter ξ^2, i.e., in terms of the incident angle

$$8\xi^2 > \max W = \max\{W(+\infty), W_{\max}\} .$$

The refractive angle ψ_t (see Fig. 1) is determined by the simple expression

$$\sin\psi_t \equiv \frac{2}{\beta}\xi ,$$
$$\xi^2 = \xi_0^2 - \frac{1}{8}W(+\infty) = \xi_0^2 - \frac{\Delta}{4} + \tfrac{1}{3}A\eta^2 . \tag{22}$$

Let us consider this process for a fixed incident angle, i.e., fixed $\xi_0 \equiv \beta \sin\psi_i / 2$. The change of the incident power η means the change of the form of the potential (20). For $\xi_0^2 > \Delta/4$ the beam is always transmitted by the effective potential and in the opposite case there is a threshold power, above which the beam is transmitted to the new medium. The latter is determined either by the condition $8\xi_0^2 = W(+\infty)$ for $\Delta/36 < \xi_0^2 < \Delta/4$, i.e., the result is

$$\frac{A\eta_{thr}^2}{\Delta} = \frac{3}{4}\left[1 - \frac{4\xi_0^2}{\Delta}\right] , \tag{23}$$

or by the condition $8\xi_0^2 = W_{\max}$ for $\xi_0^2 < \Delta/36$. In this case we have

$$\frac{A\eta_{thr}^2}{\Delta} = \tfrac{4}{3}\{(-12\zeta^2+12\zeta+1) + [(-12\zeta^2+12\zeta+1)^2 - \tfrac{32}{3}(2\zeta+1)]^{1/2}\}^{-1} \tag{24}$$

where $\zeta \equiv 4\xi_0^2/\Delta$. In the latter case the transmitted beam in the right medium can appear only at an angle larger than some definite value $\xi_*^2 = \xi_0^2 - \frac{1}{8}W(+\infty, \eta_{thr})$ (see Fig. 3). The refractive angle is determined by Eq. (22). Its dependence on the beam intensity for the fixed incident angle ξ_0 is depicted in Fig. 3. The similar curve as in Fig. 3(b) was obtained in Ref. 3 for the case of the interface between linear and nonlinear media and the Gaussian input beam.

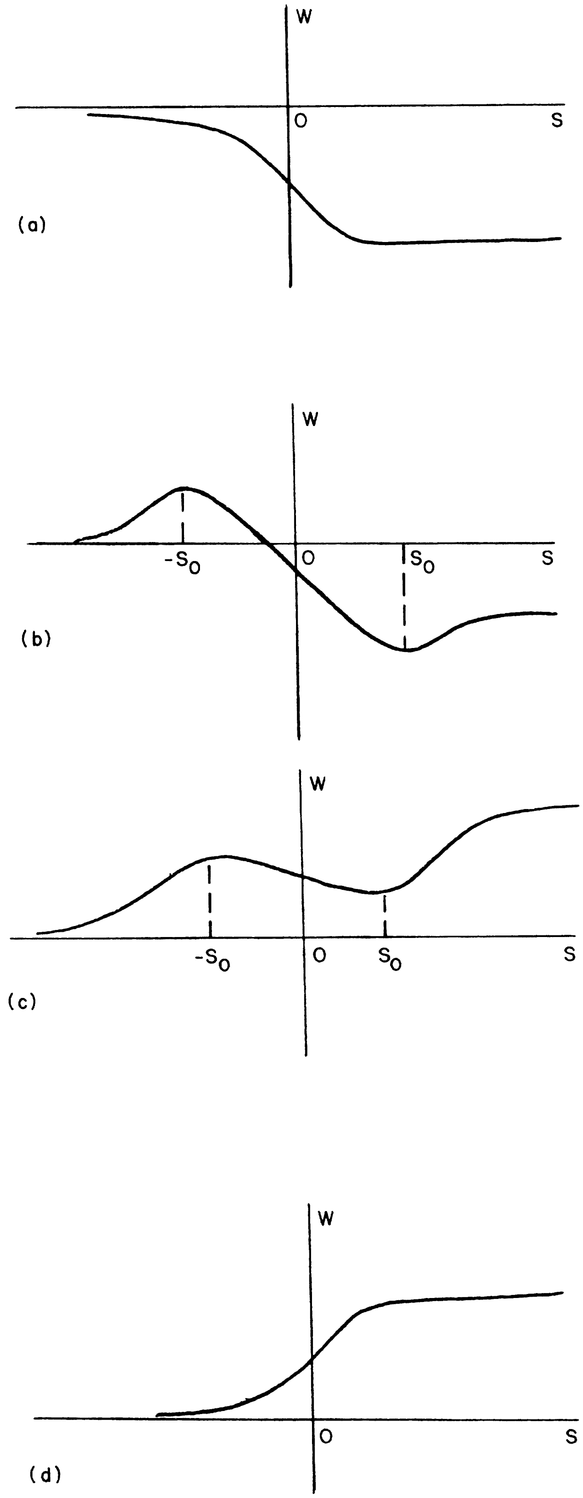

FIG. 2. Shapes of the effective potential (20) for various values of the parameter $\gamma \equiv \Delta/2\eta^2 A$: (a) $\gamma < 0$, (b) $0 < \gamma < \frac{2}{3}$, (c) $\frac{2}{3} < \gamma < 1$, (d) $\gamma > 1$.

The same problem may be considered for the fixed intensity that means the fixed shape of the potential (20) and varying incident angle, i.e., ξ. This also leads to the existence of the threshold value $\xi_{\rm thr}$ above which the beam is transmitted to the new medium. The result is

$$\xi_{\rm thr}^2=\begin{cases}\dfrac{\Delta}{4}\left[1-\dfrac{2}{3\gamma}\right], & \gamma>\frac{3}{4}\\[2mm] \dfrac{\Delta}{24\gamma}(1-\sqrt{1-\gamma})^2(2\sqrt{1-\gamma}+1), & \gamma<\frac{3}{4}\,.\end{cases} \tag{25}$$

The dependence of the refractive angle ($\sim\xi$) on the incident angle ($\sim\xi_0$) is determined by expression (22). In the latter case the transmitted beam may also appear at an angle above some critical value.

The fact that the transmitted beam angle is equal to zero for $\xi<\xi_{\rm thr}$ (or $\eta<\eta_{\rm thr}$) does not mean that there is no field in the right medium at all. In the adiabatic theory we can predict only the propagation of the main self-focused channel—a beam of the solitonlike form. When this main beam is reflected by the interface, a portion of its energy is transmitted by it, but when $\Delta, A\sim\epsilon\ll 1$ as it is in the adiabatic approach, its power is of the order of ϵ^2 and turns out to be small quantity. These effects will be described in Sec. IV C as radiative effects.

Finally, the condition that the soliton is fast can be formulated. It is valid if the kinetic energy of a soliton like a classical particle ($\sim\xi_0^2$) is much more than the maximum of the effective potential energy [Eq. (20)],

$$8\xi_0^2\gg \max W=\max[W(+\infty),W_{\max}]\,. \tag{26}$$

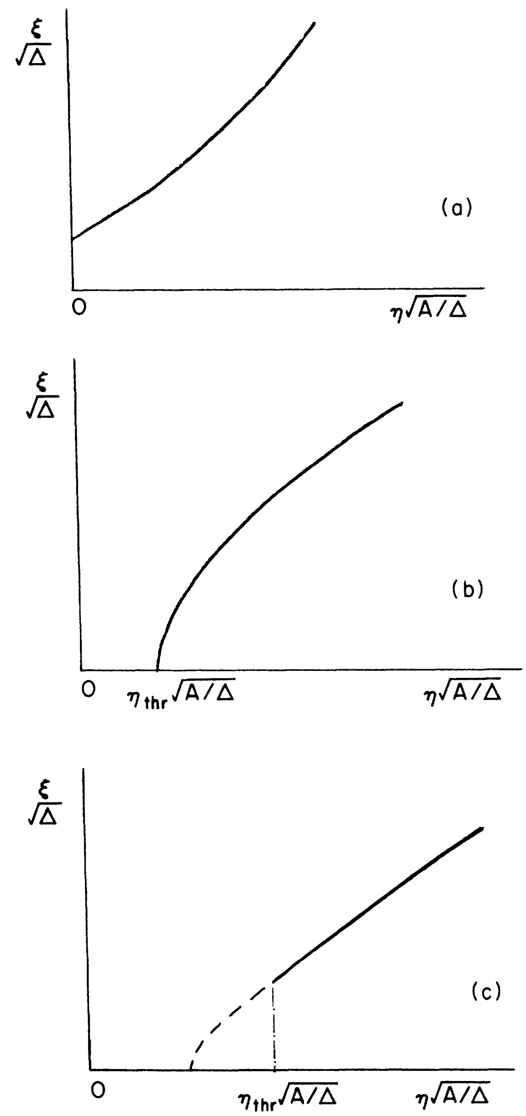

FIG. 3. Angle of a transmitted beam as a function of its input power in the framework of an adiabatic approach for various values of the input beam angle: (a) $\xi_0^2>\Delta/4$, (b) $\Delta/36<\xi_0^2<\Delta/4$, (c) $\xi_0^2<\Delta/36$.

B. Nonlinear surface waves

In this section we consider a special type of self-focused channel—a nonlinear surface wave (NSW). These waves are well known and have been investigated both experimentally and theoretically, especially for the case of an interface separating linear and nonlinear media.[5] They propagate along the interface and, thus, the incident angle of such a wave is equal to zero. The method of computing of such waves is well known.[5] First, we must solve Eq. (5) for the envelope of this wave for two media taking into consideration the solutions of the form (7) with $\xi=0$. For arbitrary Δ and $A>-2$ they have the forms

$$\Psi(x,t)=\begin{cases}2i\eta_1\dfrac{\exp(4i\eta_1^2t)}{\cosh[2\eta_1(x-X_1)]}, & x<0\\[2mm] 2i\eta_2\sqrt{2/(2+A)}\dfrac{\exp(4i\eta_2^2t-i\Delta t)}{\cosh[2\eta_2(x-X_2)]}, & x>0\,.\end{cases} \tag{27}$$

The four parameters η_1, η_2, X_1, and X_2 are not independent because they should be connected by the condition of equality of the internal frequency in the channel which gives us

$$\eta_2^2=\eta_1^2+\frac{\Delta}{4} \tag{28}$$

and the continuity of Ψ and Ψ_x for NSW at the interface,

$$2\eta_1\,\mathrm{sech}(2\eta_1X_1)=\sqrt{2/(2+A)}(4\eta_1^2+\Delta)^{1/2}\times\mathrm{sech}[(4\eta_1^2+\Delta)^{1/2}X_2]\,,$$

$$4\eta_1^2\tanh(2\eta_1X_1)\mathrm{sech}(2\eta_1X_1)=\sqrt{2/(2+A)}(4\eta_1^2+\Delta)\times\tanh[(4\eta_1^2+\Delta)^{1/2}X_2]\times\mathrm{sech}[(4\eta_1^2+\Delta)^{1/2}X_2]\,.$$

Solving the latter conditions for X_1 and X_2, we can find that

$$X_1=\pm\frac{1}{2\eta_1}\operatorname{arctanh}\left[\left[1-\frac{\Delta}{2\eta_1^2 A}\right]^{1/2}\right],$$
$$X_2=\pm\frac{1}{(4\eta_1^2+\Delta)^{1/2}}\operatorname{arctanh}\left[\left[\frac{2(2\eta_1^2 A-\Delta)}{A(4\eta_1^2+\Delta)}\right]^{1/2}\right]. \tag{29}$$

In order to have real solutions we must demand

$$0<\frac{2\eta_1^2 A-\Delta}{2\eta_1^2 A}<1,\quad 0<\frac{2(2\eta_1^2 A-\Delta)}{A(4\eta_1^2+\Delta)}<1 .$$

Besides this, the following conditions should be taken into consideration. First, $\eta_2^2>0$ gives us

$$4\eta_1^2+\Delta>0 .$$

Second, in order to have the proper sign in the nonlinear term in NLSE, we must take $A>-2$. The combination of all these conditions implies conditions on η_1 and interface parameters, namely

$$0<\frac{\Delta}{2\eta_1^2 A}<1 . \tag{30}$$

Therefore, NSW can exist only when its power is more than some critical $(\eta_1)_{cr}=\sqrt{\Delta/2A}$.

We can see that there are two types of such waves with peaks at either X_1 or X_2. One of them is stable and one is not. The stability analysis may be made as follows. One needs to present small deviations from the exact functions (27). This leads to addendum to the effective coordinate $X_1-X_2=y$. The simple, but rather cumbersome calculations give an effective Hamiltonian of the system as a function of the effective coordinate y. Stable solutions correspond to small oscillations near the stable points. As a result, the solution with a maximum at X_1 is stable for $A<0$. The shape of the stable NSW is presented in Fig. 4. The condition (30) and the values of equilibrium points are in complete agreement with the results obtained above in the framework of the perturbation theory, when Δ and A are small. Indeed, for Δ, $A\ll 1$ from Eq. (29) we have

$$\tanh X_2\approx-\tanh X_1=\left[1-\frac{\Delta}{2\eta_1^2 A}\right]^{1/2},$$

the latter being the relation for the extrema of the effective potential (20).

C. Reflection coefficient

We now calculate radiative effects accompanying the scattering of the nonlinear light beam by an interface between two dielectric media. Substituting the perturbation of the form (6) in Eq. (14) and integrating the latter, one can obtain the emitted density as follows:

$$n_{\rm rad}(\lambda)=\frac{\pi}{2^8\xi^4}\left[\Delta-\frac{A\eta^2}{3}\left[1+\frac{(\lambda^2+\eta^2-\xi^2)^2}{4\eta^2\xi^2}\right]\times\left[1+\frac{4\xi(4\xi-\lambda)}{[(\lambda-\xi)^2+\eta^2]}\right]\right]^2\times\operatorname{sech}^2\left[\frac{\pi}{4\eta\xi}(\lambda^2+\eta^2-\xi^2)\right]. \tag{31}$$

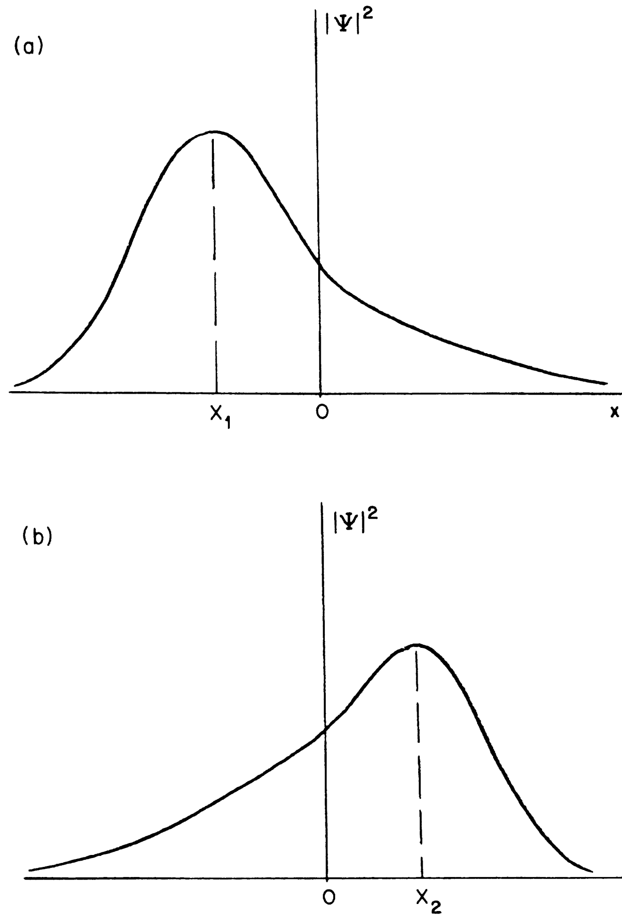

FIG. 4. Shapes of a nonlinear surface wave for the cases: (a) $A<0$, (b) $A>0$.

Using formula (15), it is easy to calculate the reflection coefficient of the nonlinear light beam from the interface. Making the change of variables $y=\lambda/\xi$, $\alpha=\eta/\xi$, we finally obtain

$$R=\frac{\pi}{2^{10}\alpha\xi^4}\int_0^\infty dy\left[\Delta-\frac{A\eta^2}{3}\left[1+\frac{(y^2+\alpha^2-1)^2}{4\alpha^2}\right]\times\left[1+\frac{4(4+y)}{[(y+1)^2+\alpha^2]}\right]\right]^2\times\operatorname{sech}^2\left[\frac{\pi}{4\alpha}(y^2+\alpha^2-1)\right]. \tag{32}$$

We may also calculate the asymptotic expansion of Eq. (32) for the fixed incident angle and for

$$\alpha\ll 1,\ \alpha^3\left[\frac{A\xi}{\Delta}\right]^2\ll 1$$

that corresponds to the so-called light soliton[13] or a small power beam. The result is

$$R\approx R_0\left[(1+\alpha^2)-\frac{16}{3}\frac{A\xi^2\alpha^2}{\Delta}(1+\tfrac{2}{15}\alpha^2)+\frac{A^2\xi^4\alpha^4}{\Delta^2}\left[\frac{384}{5}+\frac{2149}{4}\alpha^2\right]\right], \tag{33}$$

where $R_0 \equiv \Delta^2/2^8\xi^4$. When the nonlinear jump is absent ($A=0$), this expression can be compared with that for the scattering of the linear wave packet.

In the limit $\eta \to 0$ the soliton delocalizes and corresponds to a plane wave with a wave number $k = k_* = -2\xi$. The reflection coefficient of a linear wave with a wave number k from a steplike potential $U = \Delta\theta(x)$ has the form

$$R_0(k) = \left[\frac{k-(k^2-\Delta)^{1/2}}{k+(k^2-\Delta)^{1/2}} \right]^2 . \quad (34)$$

In the limit $k^2 \gg \Delta$ that corresponds to Eq. (26) it takes the form

$$R_0(k) = \Delta^2/16k^4 . \quad (35)$$

A narrow linear wave packet

$$\Psi(x,t) = \int_{-\infty}^{\infty} dk\, A\left[\frac{k+2\xi}{\eta} \right] e^{ikx - ik^2 t} , \quad (36)$$

where

$$A(z) = \frac{i}{2\cosh(\pi z/4)} \quad (37)$$

is to correspond to the soliton [Eq. (7)] with finite but small η (see Ref. 13).

It is easy to calculate the reflection coefficient of such a wave packet from the potential $U = \Delta\theta(x)$ with the help of Eqs. (34) and (35). According to the well-known formula of linear scattering

$$R = \frac{\pi}{2\eta} \int_{-\infty}^{\infty} dk\, R_0(k) |A(k)|^2 , \quad (38)$$

where $R_0(k)$ corresponds to the scattering of the linear wave and is determined by Eq. (35). For $\alpha \ll \sqrt{\Delta}/\xi \ll 1$

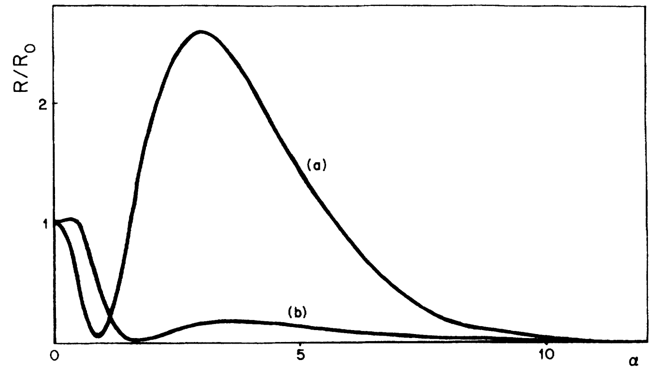

FIG. 6. The same as the curve in Fig. 5 (solid line) but for $A \neq 0$, with (a) $A\xi^2/\Delta = 0.5$ and (b) $A\xi^2/\Delta = 0.15$.

the coefficient (38) has the asymptotic form

$$R \approx R_0(1 + \tfrac{10}{3}\alpha^2) , \quad (39)$$

where $R_0 \equiv R_0(2\xi) = \Delta^2/2^8\xi^4$. Let us compare this result with the reflection coefficient of the nonlinear beam (33) for the case $A=0$. As it follows from Eqs. (39) and (33) at $A=0$, the reflection coefficient of a soliton with small amplitude is less than that of the linear wave packet (see Fig. 5).

It follows from Eq. (33) that there are two types of curves $R(\alpha)$ for small α and $A \neq 0$, because increasing the reflection coefficient with the growth of α changes to decreasing (see Fig. 6).

The case of a "heavy" soliton (larger power beam) is characterized by exponentially small emission due to a large bound energy of a soliton. The asymptotic expansion follows from Eq. (32) for $\alpha \gg 1$,

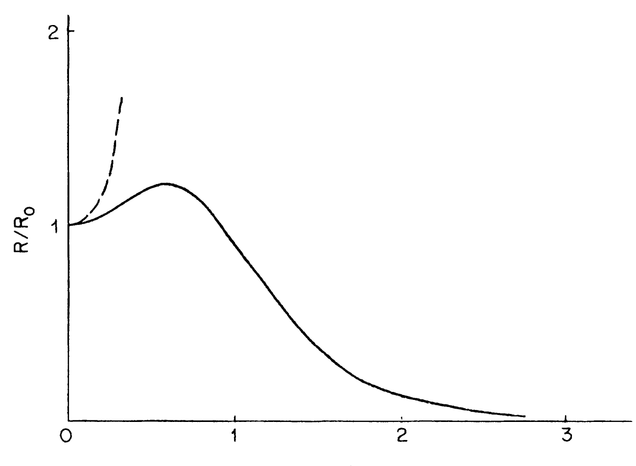

FIG. 5. Dependence of the beam reflection coefficient on the parameter $\alpha \equiv \eta/\xi$ in the case of a single interface with $A=0$ (solid line), R_0 being the reflection coefficient of a linear plane wave with the wave number $k_* = -2\xi$. The dashed line is the same for a linear wave packet [Eq. (36)], see Eq. (39) (arbitrary units).

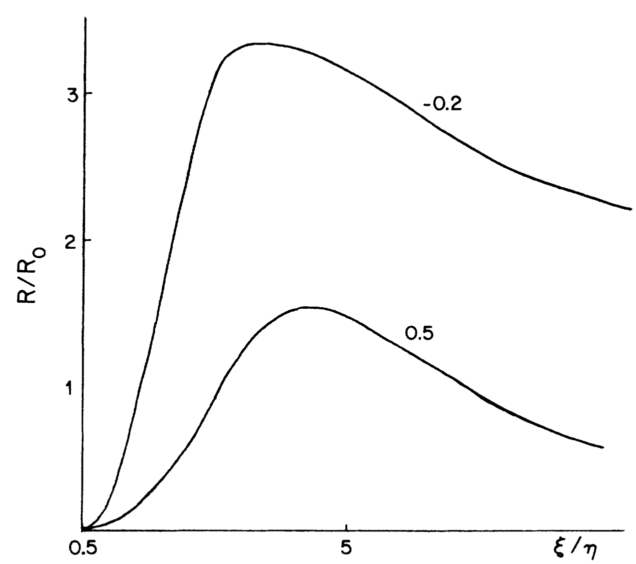

FIG. 7. The same as in Fig. 5 (solid line) but as a function of the input angle of the beam with fixed power. The values of the parameter $A\eta^2/\Delta$ are indicated near the curves.

$$R \approx \frac{\pi\sqrt{2}}{2^9 \xi^4 \alpha^{1/2}} e^{-\pi\alpha/2} \left[\Delta - \frac{A}{12}\alpha^4 \xi^2 \right]^2 . \tag{40}$$

The general dependence of $R(\alpha)$ for $A \neq 0$ is plotted in Fig. 6. Its characteristic feature appears in the existence of a rather strong minimum which corresponds to a practically transparent region for propagation of a beam at the interface. This effect may be useful for applications of nonlinear interfaces in various optical devices.

In Fig. 7 we present the reflection coefficient R as a function of ξ, i.e., the incident angle at fixed η. The similar maxima may be observed too.

V. BEAM SCATTERING BY TWO INTERFACES

The problem of nonlinear beam scattering can be also considered for two interfaces (see Fig. 8) which have different refractive indices,

$$n(x,|E|^2) = \begin{cases} n_1 + \alpha_1 |E|^2, & x < 0 \\ n_2 + \alpha_2 |E|^2, & 0 < x < a \\ n_3 + \alpha_3 |E|^2, & x > a . \end{cases} \tag{41}$$

As far as we know, previous papers have dealt only with nonlinear waves propagating along the interface, as well various bifurcations and stability of such surface waves (see, e.g., Refs. 5–8 and 16). Scattering by interfaces [Eq. 41] has been investigated in Ref. 10 numerically but interference effects have not been discussed. We pay attention to these very interference effects and study the dependence of the reflection coefficient on the effective distance between interfaces.

Repeating the reduction of the problem to the NLSE, we obtain the perturbing potential U in the form

$$U(x,|\Psi|^2) = \begin{cases} (n_1 - n_2) - 2(\alpha_2/\alpha_1 - 1)|\Psi|^2, & 0 < x < a \\ (n_1 - n_3) - 2(\alpha_3/\alpha_1 - 1)|\Psi|^2, & x > a . \end{cases} \tag{42}$$

This potential has two jumps at points $x = 0$ and $x = a$. In the particular case of an optical layer situated in the medium with the refractive index $n = n_1 + \alpha_1 |E|^2$ ($n_3 = n_1$, $\alpha_3 = \alpha_1$), the potential is not equal to zero only for $0 < x < a$.

Using the perturbation theory based on the IST for the NLSE soliton and the potential (42), we obtain the radiation $[n_{\rm rad}^{(2)}(\lambda)]$ density after the nonlinear beam scattering by two interfaces (cf. results of Ref. 13),

$$\begin{aligned} n_{\rm rad}^{(2)}(\lambda) = {} & [n_{\rm rad}(\lambda)]_1 + [n_{\rm rad}(\lambda)]_2 \\ & + 2\{[n_{\rm rad}(\lambda)]_1 [n_{\rm rad}(\lambda)]_2\}^{1/2} \\ & \times \cos\left[\frac{a}{\xi}[(\lambda - \xi)^2 + \eta^2] \right] . \end{aligned} \tag{43}$$

The first and the second terms in Eq. (43) describe independent scattering of light beam by each interface.

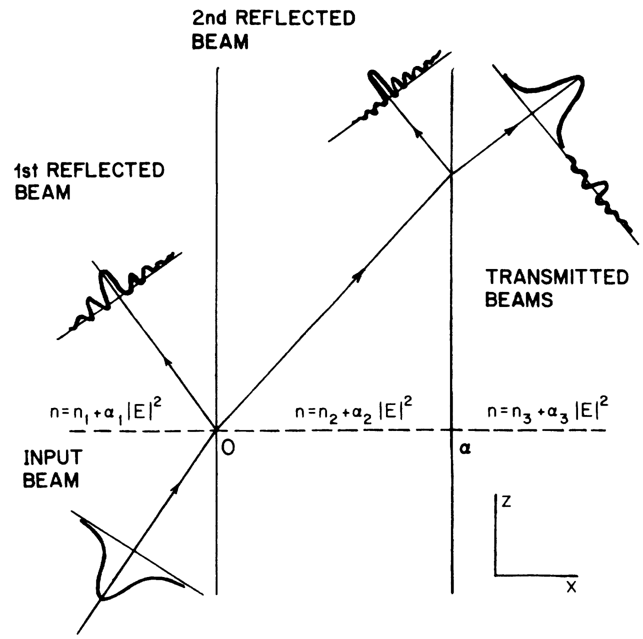

FIG. 8. Interface configuration and coordinate system for two interfaces.

They are determined by formula (31) where we must substitute

$$\begin{aligned} & \Delta \equiv \Delta_i, \quad A \equiv A_i, \quad i = 1,2 , \\ & \Delta_1 \equiv n_1 - n_2, \quad A_1 \equiv 2(\alpha_2/\alpha_1 - 1) , \\ & \Delta_2 \equiv n_2 - n_3, \quad A_2 \equiv 2(\alpha_3/\alpha_1 - \alpha_2/\alpha_1) . \end{aligned} \tag{44}$$

For $a \to 0$ the result [Eq. (43)] transforms into a density of emission generated due to soliton scattering by a single interface, which is characterized by the perturbative potential with the values of jumps

$$\Delta = \Delta_1 + \Delta_2, \quad A = A_1 + A_2 = 2(\alpha_3/\alpha_1 - 1) .$$

For $a \to \infty$ the scattering by two interfaces appears to be independent. With that the emission densities are summed up and $n_{\rm rad}^{(2)} = (n_{\rm rad})_1 + (n_{\rm rad})_2$. The third term in Eq. (43) describes interference effects, appearing due to the interaction of emission from two interfaces. Thus, during scattering by two interfaces the nonlinear light beam shows both corpuscular and wave properties.

In the particular case of a layer in a homogeneous medium we have $\Delta_2 = -\Delta_1$ and $A_2 = -A_1$, consequently, $(n_{\rm rad})_1 = (n_{\rm rad})_2$, so that

$$n_{\rm rad}^{(2)}(\lambda) = 4[n_{\rm rad}(\lambda)]_1 \sin^2\left[\frac{a}{2\xi}[(\lambda - \xi)^2 + \eta^2] \right] . \tag{45}$$

Furthermore we calculate the reflection coefficient (15) of a beam scattered by two interfaces. Integrating Eq. (43) over all $\lambda < 0$ and making the change of variables $\alpha = \eta/\xi$, $y = \lambda/\xi$, we can represent the final result as

$$R^{(2)} = R_1 + R_2 + \frac{\pi}{2^9 \alpha \xi^4} \int_0^\infty dy \frac{G_1(y,\alpha) G_2(y,\alpha) \cos\{d[(y+1)^2 + \alpha^2]\}}{\cosh^2\left[\frac{\pi}{4\alpha}(y^2 + \alpha^2 - 1) \right]} , \tag{46}$$

$$G_i(y,\alpha)=\Delta_i-\frac{A_i\eta^2}{3}\left[1+\frac{(y^2+\alpha^2-1)^2}{4\alpha^2}\right]\left[1+\frac{4(4+y)}{[(y+1)^2+\alpha^2]}\right],\quad i=1,2 \tag{47}$$

$$d\equiv a\xi\ . \tag{48}$$

The first two terms in Eq. (46) describe independent scattering by each interface, and they are determined by the formula (32) where one must substitute the parameters (44). The third term in Eq. (46) describes the interference effect. The asymptote of expression (46) for the fixed incident angle can easily be obtained under the conditions $\alpha \ll 1$, $\alpha^2 d \ll 1$, $\alpha^3(A_i\xi^2/\Delta_i)\ll 1$. It is

$$R^{(2)}=R_1+R_2+\frac{\cos(4d)}{2^7\xi^4}[\Delta_1\Delta_2F_1(4\alpha d)+A_1A_2\alpha^4\xi^4F_2(4\alpha d)-(\Delta_1A_2+A_2A_1)\alpha^2\xi^2\times F_{12}(4\alpha d)]\ , \tag{49}$$

where

$$F_1(x)=\frac{x}{\sinh x}\ , \tag{50a}$$

$$F_{12}(x)=\frac{2}{\sinh^3 x}(\sinh 2x-2x)\ , \tag{50b}$$

$$F_2(x)=\frac{16}{\sinh^5 x}[4x\sinh^2 x+6x-3\sinh 2x]\ . \tag{50c}$$

The expressions for R_1 and R_2 in Eq. (49) are determined by the asymptotics [Eq. (33)] where in order to keep the proper accuracy we should leave only the terms of the order of 1. Therefore, the reflection coefficient of the nonlinear beam scattered by two interfaces oscillates as a function of the parameter $d\equiv a\xi$. The resonant relation follows from the condition that the time of soliton motion between interfaces ($\sim a/\xi$) is proportional to the inverse frequency of generated waves ($\sim\xi^{-2}$) (cf. Ref. 13).

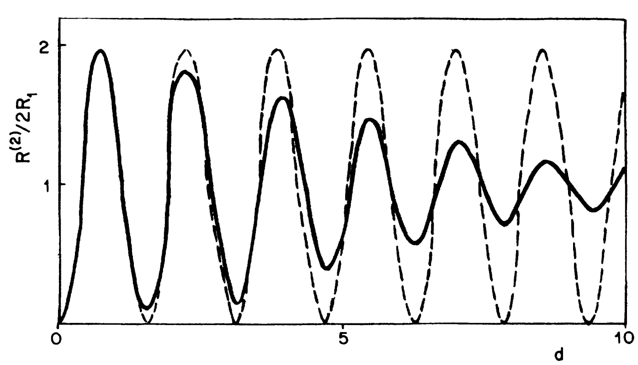

FIG. 9. The beam reflection coefficient vs the parameter $d\equiv a\xi$ in the case of an optical layer with $A=0$ and $\alpha=0.1$. The function R_1 is the reflection coefficient for a single interface. The dashed line is for the scattering of the linear wave (arb. units).

Now we consider the particular case of scattering by a layer in homogeneous medium. In this case the reflection coefficient is simplified and take the form

$$R^{(2)}=2R_1-\frac{\cos(4d)}{2^7\xi^4}[\Delta_1^2F_1(4\alpha d)+A_1^2\alpha^4\xi^4F_2(4\alpha d)-2\Delta_1A_1\alpha^2\xi^2F_{12}(4\alpha d)]\ , \tag{51}$$

where the functions F_1, F_2, and F_{12} are determined in Eq. (50). We should note that the reflection coefficients from each interface are equal, and for $a\to\infty$ the result [Eq. (51)] tends to zero.

In the linear limit ($\eta\to 0$) from Eq. (51) we obtain the reflection coefficient of a monochromatic plane wave with a wave number $k=2\xi$ from the steplike potential $U=\Delta[\theta(x)-\theta(x-a)]$. For $k^2\gg\Delta$ it is

$$R_0^{(2)}=4R_0\sin^2(ak)\ , \tag{52}$$

where $R_0=\Delta^2/16k^4$ is the reflection coefficient of this wave from a single interface [see Eqs. (34) and (35)]. Unlike the linear case (52), the oscillations of the reflection coefficient (51) of the nonlinear case disappear with the increase of the distance between interfaces.

The other interesting case is when the jump of the nonlinear term in Eq. (42) can be neglected, i.e., $A\eta^2\ll\Delta$. Then the reflection coefficient of the beam scattered by the optical layer has the form

$$R^{(2)}=2R_0\left[1-\frac{4\alpha d}{\sinh(4\alpha d)}\cos(4d)\right]\ . \tag{53}$$

In the limit $\alpha\gg 1$ the reflection coefficients [Eqs. (46)–(48)] have the following asymptotic expansion:

$$R^{(2)}=R_1+R_2+2\sqrt{R_1R_2}\frac{\cos[\alpha^2 d+\frac{1}{2}\arctan\gamma]}{(1+\gamma^2)^{1/4}}\ , \tag{54}$$

where $\gamma=2a\alpha\xi/\pi$ and R_1 and R_2 are the coefficients of

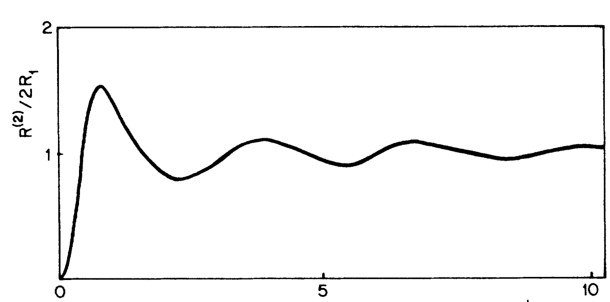

FIG. 10. The same as in Fig. 9, but for $\alpha=1$.

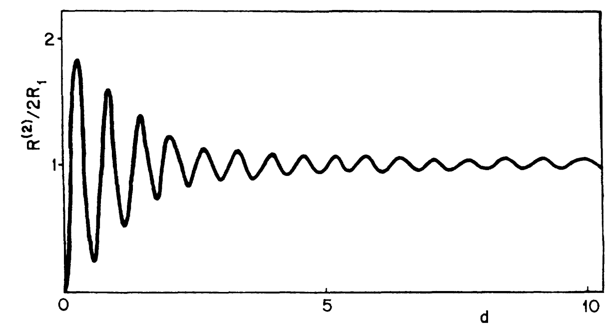

FIG. 11. The same as in Fig. 9, but for $\alpha=3$.

the independent reflection by the first and the second interfaces, respectively. They are determined by formula (40), where we should substitute Eq. (44). In this case the reflection coefficient is exponentially small as a function of the parameter α and oscillates as a function of the parameter $\alpha^2 d \equiv \eta^2 a/\xi$. If one considers this reflection coefficient as a function of the parameter d, as it was in Eq. (51), then the frequency of oscillations turns out to be larger in the case $\alpha \gg 1$. The resonant relation is the following: time of the beam propagation between the interfaces ($\sim \alpha/\xi$) should be proportional to the period of internal oscillations of a soliton [Eq. (7)] which is now proportional to η^{-2} (cf. results of Ref. 13).

The reflection coefficient due to the propagation of the nonlinear beam through a layer versus the parameter $d=a\xi$ is depicted in Figs. 9–11 for various values of α.

VI. VALIDITY OF THE EMPLOYED APPROXIMATIONS

We now discuss briefly the validity of the employed approximations. First of all, the NLSE is obtained from more general field equations. Therefore, it is valid when $\partial^2 F/\partial z^2 \ll \beta k_0 \partial F/\partial z$. The latter has two inequalities $\eta \ll \beta$ and $\xi \ll \beta$ so that the incident angle must be small, $(2\xi/\beta)=\sin\psi_i \ll 1$. Secondly, we assume that a nonlinear interface is formed by two nonlinear media, but most often the real systems consist of linear plus nonlinear media. This condition is surely convenient since we can expect that in both media the localized beam is described by a solitonlike solution of the NLSE. In this end, we need $\Delta, A \ll 1$. As a result we do not have the case with strongly different parameters, e.g., if one medium is linear then another medium must be slightly nonlinear in the model. This limitation is very strong, but some of our results give the similar predictions as numerical ones for the above limit case (e.g., cf. Fig. 3 and results of Ref. 3).

At last, for calculation of radiative effects and the reflection coefficient in Secs. IV and V we use the Born approximation when the incident angle is not so small. This leads to the condition $\xi^2 \gg A\eta^2, \Delta$.

VII. CONCLUSIONS

In conclusion, by means of the perturbation theory for solitons we have investigated the adiabatic and radiative effects accompanying the scattering of optical beams by nonlinear interfaces. In the framework of the well-known approach, the dynamics of the electric field in the self-focusing dielectric medium is described by the nonlinear parabolic equation for its envelope, i.e., the nonlinear Schrödinger equation. The solution of this equation in the form of the nonlinear self-focused channel is a soliton solution, and an interface may be considered as a perturbation. In the framework of the Born approximation of the soliton perturbation theory we have calculated the soliton reflection coefficient stipulated by the interface. As a result, the reflection coefficient is a monotonic function versus the beam power and, in particular, exponentially decreases with the growth of the intensity ($\alpha \gg 1$). Besides, when the interface parameters are changed, at some finite value of the nonlinearity ($\alpha \sim 1$) there are regions where the reflection coefficient is considerably less in comparison with that of the scattering of linear wave packets.

For the case of two interfaces we have analyzed the influence of the Kerr nonlinearity on interference phenomena and have shown that the interference disappears when the characteristic beam width is considerably less than the distance between the nonlinear interfaces. In the case of a weak nonlinearity ($\alpha \ll 1$) our formulas generalize the well-known results characterizing the interference of a linear wave scattered by the optically inhomogeneous layer. In the case of a strong nonlinearity ($\alpha \gg 1$) the other type of interference is possible, the resonant condition for it follows from the relation of the distance between interfaces and the frequency of internal oscillations of the nonlinear beam. There is no analog of such scattering in the linear theory.

Described effects may be useful for building of all-optical integrated devices using interfaces, e.g., optical limiters, bistable switchers, upper and lower threshold devices, etc.

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