

Coherence in two-photon down-conversion induced by a laser

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We discuss the situation in which idler beams from two parametric down-converter crystals are allowed to interfere. We show that, when two mutually coherent signal beams derived from a common laser are injected into the down-converters, the two idler beams can become mutually coherent also. Moreover, the resulting interference pattern can, in principle, have 100% visibility when the number of injected photons per unit down-converter bandwidth is large. This is just the condition for stimulated down-conversion to dominate over spontaneous down-conversion.

I. INTRODUCTION

Although signal and idler photons produced in the process of spontaneous parametric down-conversion are strongly correlated in time,¹ there is no phase relationship between the signal and idler fields $\hat{E}_s^{(+)}$ and $\hat{E}_i^{(+)}$. The theory of down-conversion has been treated with various degrees of approximation,²⁻¹⁸ and it is easy to show that, in general,

$$\langle \hat{E}_s^{(-)} \hat{E}_i^{(+)} \rangle = 0, \tag{1}$$

so that signal and idler fields do not interfere. Entanglement of the down-converted field with the vacuum in the sense of Horne, Shimony, and Zeilinger¹⁹ allows the field to carry information about the pump phase,^{17,18} as has recently been demonstrated experimentally,²⁰ and makes it possible for $\langle \hat{E}_s^{(+)} \hat{E}_i^{(+)} \rangle$ to be nonzero. Yet Eq. (1) still holds, and signal and idler are still mutually incoherent. The situation changes, however, when a coherent reference beam is injected into the crystal, say at a frequency close to and in the direction of the signal field. This has the effect of inducing stimulated down-conversions, so that signal and idler fields becomes mutually coherent.

In the simplest possible two-mode model of down-conversion, the parametric interaction \hat{H}_I in the interaction picture is written

$$\hat{H}_I = (i\hbar g \hat{a}_s^\dagger \hat{a}_i^\dagger V + hc). \tag{2}$$

Here g is the (real) mode coupling constant, V is the complex amplitude of the classical pump field, and \hat{a}_s and \hat{a}_i are photon annihilation operators for the signal and idler modes. It then follows that the state $|\psi(t)\rangle$ of the down-converted field after a time t in the interaction picture is related to the initial state $|\psi(0)\rangle$ by

$$|\psi(t)\rangle = e^{-i\hat{H}_I t/\hbar} |\psi(0)\rangle = [1 + gt(\hat{a}_s^\dagger \hat{a}_i^\dagger V - hc)] |\psi(0)\rangle + \dots, \tag{3}$$

where it is assumed that t is short compared with the average time interval between down-conversions. If the initial state $|\psi(0)\rangle$ is the vacuum $|\psi_{vac}\rangle_s |\psi_{vac}\rangle_i$ for both signal and idler, we find from Eq. (3) that

$$\langle \psi(t) | \hat{a}_s^\dagger \hat{a}_i | \psi(t) \rangle = 0. \tag{4}$$

On the other hand, if the initial state is the coherent state $|v\rangle_s$ for the signal and the vacuum state $|\psi_{vac}\rangle_i$ for the idler, then we readily obtain from Eq. (3)

$$|\psi(t)\rangle = |v\rangle_s |\psi_{vac}\rangle_i + gtV\hat{a}_s^\dagger |v\rangle_s |1\rangle_i + \dots, \tag{5}$$

so that

$$\langle \psi(t) | \hat{a}_s^\dagger \hat{a}_i | \psi(t) \rangle = gtVv^*2, \tag{6}$$

which is nonzero in general. The coherent signal field has therefore induced coherence between signal and idler.

II. PRINCIPLE OF COHERENCE INDUCED IN TWO DOWN-CONVERTERS

In the following we analyze the closely related, but somewhat more complicated situation, illustrated in Fig. 1. Two nonlinear crystals NL1 and NL2 are both optically pumped by coherent waves V_1, V_2 at the same pump frequency ω_0 . Down-converted signal and idler beams emerge in slightly different directions from each crystal. We allow the two idler beams to come together and interfere, as shown. Needless to say, in the absence of any other injected signal, the two idler beams are mutually incoherent, and do not give rise to an interference

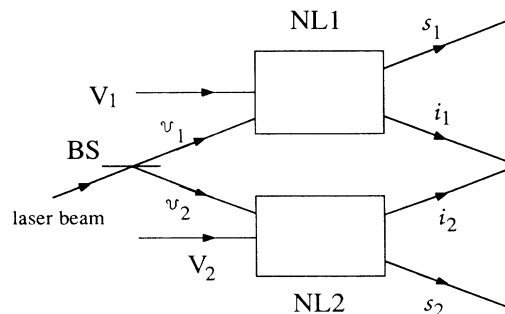


FIG. 1. Outline of the situation being treated.

pattern. However, when two monochromatic signal beams of complex amplitudes v_1 and v_2 at the signal frequency ω_s are injected, which may be derived from a common laser source by use of the beam splitter BS, as illustrated in Fig. 1, they give rise to coherence between the two idler beams, as we shall show.

We may consider the quantum state $|\psi(t)\rangle$ of the down-converted light to be a direct product of the states produced by the two crystals, so that from Eq. (5) we write

$$|\psi(t)\rangle = (|v_1\rangle_{s1} |\psi_{\text{vac}}\rangle_{i1} + g_1 t V_1 \hat{a}_{s1}^\dagger |v_1\rangle_{s1} |1\rangle_{i1}) \otimes (|v_2\rangle_{s2} |\psi_{\text{vac}}\rangle_{i2} + g_2 t V_2 \hat{a}_{s2}^\dagger |v_2\rangle_{s2} |1\rangle_{i2}). \quad (7)$$

It follows that

$$\langle \psi(t) | \hat{a}_{i1}^\dagger \hat{a}_{i2} | \psi(t) \rangle = (g_1 t)(g_2 t) V_1^* V_2 v_1 v_2^*, \quad (8)$$

whereas

$$\langle \psi(t) | \hat{a}_{i1}^\dagger \hat{a}_{i1} | \psi(t) \rangle = (g_1 t)^2 |V_1|^2 (|v_1|^2 + 1), \quad (9)$$

$$\langle \psi(t) | \hat{a}_{i2}^\dagger \hat{a}_{i2} | \psi(t) \rangle = (g_2 t)^2 |V_2|^2 (|v_2|^2 + 1). \quad (10)$$

Hence, if $g_1 = g = g_2$, $|V_1| = |V| = |V_2|$, $|v_1| = |v| = |v_2|$, we have for the interference pattern

$$\begin{aligned} \langle \psi(t) | (\hat{a}_{i1}^\dagger + \hat{a}_{i2}^\dagger) (\hat{a}_{i1} + \hat{a}_{i2}) | \psi(t) \rangle \\ = 2(g t)^2 |V|^2 [|v|^2 + 1 + |v|^2 \cos(\theta_2 - \theta_1 + \phi_1 - \phi_2)], \end{aligned} \quad (11)$$

$$|\psi(t)\rangle = \left[1 + \eta V \delta \omega \sum_{[\omega']_s} \sum_{[\omega'']_i} \phi(\omega', \omega'') \frac{\sin \frac{1}{2}(\omega' + \omega'' - \omega_0)t}{\frac{1}{2}(\omega' + \omega'' - \omega_0)} e^{1/2i(\omega' + \omega'' - \omega_0)t} \hat{a}_s^\dagger(\omega') \hat{a}_i^\dagger(\omega'') + \text{H.c.} \right] |\psi(0)\rangle + \dots \quad (13)$$

Here $\phi(\omega', \omega'')$ is a spectral weight function for the signal modes ω' and idler modes ω'' , which are assumed to be distinct and nonoverlapping, and $\delta\omega$ is the mode spacing. As $\delta\omega \rightarrow 0$ sums over discrete modes turn into integrals, with

$$2\pi\delta\omega \sum_{[\omega]_s} |\phi(\omega, \omega_0 - \omega)|^2 \rightarrow 2\pi \int d\omega |\phi(\omega, \omega_0 - \omega)|^2 = 1. \quad (14)$$

$$|\psi_1(t)\rangle_1 = M_1 |\{v_1\}\rangle_{s1} |\psi_{\text{vac}}\rangle_{i1}$$

$$+ \eta_1 V_1 \delta\omega \sum_{\omega'} \sum_{\omega''} \phi_1(\omega', \omega'') \frac{\sin \frac{1}{2}(\omega' + \omega'' - \omega_0)t}{\frac{1}{2}(\omega' + \omega'' - \omega_0)} e^{(1/2i)(\omega' + \omega'' - \omega_0)t} \hat{a}_{s1}^\dagger(\omega') |\{v_1\}\rangle_{s1} |\omega''\rangle_{i1}, \quad (16)$$

and similarly for $|\psi_2(t)\rangle_2$. Here $|\{v_1\}\rangle_{s1}$ is the multimode coherent state of the signal field. $|\omega''\rangle_{i1}$ represents a state of one idler photon of frequency ω'' , and M_1 and M_2 , representing the amplitudes of the initial state, are very close to unity.

We shall represent the two emerging idler fields $\hat{E}_{i1}^{(+)}$ and $\hat{E}_{i2}^{(+)}$ by the mode expansions

$$\hat{E}_{i1}^{(+)}(t) = \left[\frac{\delta\omega}{2\pi} \right]^{1/2} \sum_{\omega} \hat{a}_{i1}(\omega) e^{-i\omega(t-\tau_1)}, \quad \hat{E}_{i2}^{(+)}(t) = \left[\frac{\delta\omega}{2\pi} \right]^{1/2} \sum_{\omega} \hat{a}_{i2}(\omega) e^{-i\omega(t-\tau_2)}, \quad (17)$$

where we have written

$$\begin{aligned} V_1 &= |V| e^{i\theta_1}, \\ V_2 &= |V| e^{i\theta_2}, \\ v_1 &= |v| e^{i\phi_1}, \\ v_2 &= |v| e^{i\phi_2}. \end{aligned} \quad (12)$$

When $|v|^2 \gg 1$, the interference of the idler beams exhibits a 100% depth of modulation or visibility. Once again the coherent signals have induced coherence between the two idlers through the process of stimulated emission.

However, this conclusion is based on a simple monochromatic two-mode model of down-conversion, which is known to be inadequate and misleading for some purposes. We shall therefore repeat the calculation using a more realistic model of the process.

III. MULTIMODE TREATMENT

We shall make use of the formalism for the down-conversion process developed previously.¹⁸ The state $|\psi(t)\rangle$ of the down-converted light at time t is related to its initial state $|\psi(0)\rangle$ by

V is the pump wave amplitude expressed in units such that $|V|^2$ gives the pump intensity in photons per second, and η is a dimensionless number such that $|\eta|^2 |V|^2$ gives the rate of down-conversion in photons per second.

In the present problem we again express the state $|\psi(t)\rangle$ of the down-converted light as a direct product

$$|\psi(t)\rangle = |\psi_1(t)\rangle_1 |\psi_2(t)\rangle_2, \quad (15)$$

with

in which τ_1 and τ_2 are propagation times from crystals NL1 and NL2 to the detector in the interference plane.

We now use Eqs. (15) to (17) to calculate

$$\Gamma_{nm} \equiv \langle \psi(t) | \hat{E}_{in}^{(-)}(t) \hat{E}_{im}^{(+)}(t) | \psi(t) \rangle, \quad (n, m = 1, 2). \quad (18)$$

We then find

$$\begin{aligned} \Gamma_{12} = & \left[\frac{\delta\omega}{2\pi} \right]^3 M_1 \eta_1^* V_1^* \sum_{\omega'_1} \sum_{\omega''_1} \phi_1^*(\omega'_1, \omega''_1) \frac{\sin \frac{1}{2}(\omega'_1 + \omega''_1 - \omega_0)t}{\frac{1}{2}(\omega'_1 + \omega''_1 - \omega_0)} e^{-(1/2)i(\omega'_1 + \omega''_1 - \omega_0)t} e^{i\omega''_1(t - \tau_1)} \langle \{v_1\} | \hat{a}_{s2}(\omega'_1) | \{v_1\} \rangle_{s1} \\ & \times M_2^* \eta_2 V_2 \sum_{\omega'_2} \sum_{\omega''_2} \phi_2(\omega'_2, \omega''_2) \frac{\sin \frac{1}{2}(\omega'_2 + \omega''_2 - \omega_0)t}{\frac{1}{2}(\omega'_2 + \omega''_2 - \omega_0)} \\ & \times e^{(1/2)i(\omega'_2 + \omega''_2 - \omega_0)t} e^{-i\omega''_2(t - \tau_1)} \langle \{v_2\} | \hat{a}_{s2}^\dagger(\omega'_2) | \{v_2\} \rangle_{s2}. \end{aligned} \quad (19)$$

We now replace ω'_1 and ω''_1 by putting

$$\omega'_1 + \omega''_1 - \omega_0 = \Omega_1,$$

$$\omega'_2 + \omega''_2 - \omega_0 = \Omega_2,$$

and consider the long-time limit when t is much larger than any reciprocal frequency widths (although still small compared with the average time interval between down-conversions). We convert $(\delta\omega/2\pi) \sum_{\Omega}$ to an integral when $\delta\omega \rightarrow 0$, and note that for long t the dominant contributions come from small values of Ω , so that we may replace $\phi_1(\omega'_1, \omega_0 - \omega'_1 + \Omega_1)$ by $\phi_1(\omega', \omega_0 - \omega')$ and $\phi_2(\omega'_2, \omega_0 - \omega'_2 + \Omega_2)$ by $\phi_2(\omega'_2, \omega_0 - \omega'_2)$ to a good approxi-

mation. Then the Ω_1 and Ω_2 integrals reduce to

$$\begin{aligned} \frac{1}{2\pi} \int d\Omega_1 \frac{\sin \frac{1}{2}\Omega_1 t}{\frac{1}{2}\Omega_1} e^{i\Omega_1(1/2t - \tau_1)} &= 1, \\ \frac{1}{2\pi} \int d\Omega_2 \frac{\sin \frac{1}{2}\Omega_2 t}{\frac{1}{2}\Omega_2} e^{-i\Omega_2(1/2t - \tau_2)} &= 1. \end{aligned} \quad (20)$$

Finally, on recalling that

$$\begin{aligned} \langle \{v_1\} | \hat{a}_{s1}(\omega'_1) | \{v_1\} \rangle &= v_1(\omega'_1), \\ \langle \{v_2\} | \hat{a}_{s2}^\dagger(\omega'_2) | \{v_2\} \rangle &= v_2^*(\omega'_2), \end{aligned} \quad (21)$$

we have from Eqs. (19)–(21)

$$\Gamma_{12} = 2\pi\delta\omega M_1 M_2^* \eta_1^* V_1^* \eta_2 V_2 \sum_{\omega'_1} \phi_1^*(\omega'_1, \omega_0 - \omega'_1) v_1(\omega'_1) e^{i(\omega_0 - \omega'_1)(t - \tau_1)} \sum_{\omega'_2} \phi_2(\omega'_2, \omega_0 - \omega'_2) v_2^*(\omega'_2) e^{-i(\omega_0 - \omega'_2)(t - \tau_2)}. \quad (22)$$

In practice the spectral functions $\phi_1(\omega, \omega_0 - \omega)$ and $\phi_2(\omega, \omega_0 - \omega)$ characterizing the down-conversions are very broad functions compared with $v_1(\omega)$ and $v_2(\omega)$, which describe the injected laser beam. If the laser beam is centered on frequency ω_s , we are justified in replacing $\phi_1^*(\omega'_1, \omega_0 - \omega'_1)$ by $\phi_1^*(\omega_s, \omega_0 - \omega_s)$ and $\phi_2(\omega'_2, \omega_0 - \omega'_2)$ by $\phi_2(\omega_s, \omega_0 - \omega_s)$ under the summation. On converting sums to integrals by writing in analogy with Eq. (17),

$$\begin{aligned} \left[\frac{\delta\omega}{2\pi} \right]^{1/2} \sum_{\omega} v_1(\omega) e^{-i\omega\tau} &\equiv \mathcal{R} \mathcal{W}(\tau), \\ \left[\frac{\delta\omega}{2\pi} \right]^{1/2} \sum_{\omega} v_2(\omega) e^{-i\omega\tau} &\equiv \mathcal{T} \mathcal{W}(\tau), \end{aligned} \quad (23)$$

where $\mathcal{W}(\tau)$ is the complex amplitude of the laser field, and assuming that both coherent signal fields are derived from the same laser by a beam splitter of complex reflectivity \mathcal{R} and transmissivity \mathcal{T} , we obtain

$$\begin{aligned} \Gamma_{12} = & (2\pi)^2 M_1 M_2^* \eta_1^* V_1^* \eta_2 V_2 \phi_1^*(\omega_s, \omega_0 - \omega_s) \\ & \times \phi_2(\omega_s, \omega_0 - \omega_s) \mathcal{R} \mathcal{T}^* \mathcal{W}(t - \tau_1) \mathcal{W}^*(t - \tau_2) \\ & \times e^{i\omega_s(\tau_2 - \tau_1)}. \end{aligned} \quad (24)$$

Now if the laser beam has a long coherence time, which is much longer than τ_1, τ_2 , we may approximate $\mathcal{W}(t - \tau_1)$ and $\mathcal{W}^*(t - \tau_2)$ by writing

$$\begin{aligned} \mathcal{W}(t - \tau_1) &\approx \mathcal{W}(t) e^{i\omega_s \tau_1}, \\ \mathcal{W}^*(t - \tau_2) &\approx \mathcal{W}^*(t) e^{-i\omega_s \tau_2}. \end{aligned} \quad (25)$$

Then Eq. (24) finally reduces to

$$\begin{aligned} \Gamma_{12} = & (2\pi)^2 \mathcal{R} \mathcal{T}^* M_1 M_2^* \eta_1^* V_1^* \eta_2 V_2 \phi_1^*(\omega_s, \omega_0 - \omega_s) \\ & \times \phi_2(\omega_s, \omega_0 - \omega_s) |\mathcal{W}(t)|^2 e^{i(\omega_0 - \omega_s)(\tau_2 - \tau_1)}. \end{aligned} \quad (26)$$

Here $|\mathcal{W}(t)|^2$ is the intensity of the laser beam expressed in photons per second.

We now proceed to calculate Γ_{11} and Γ_{22} in a similar manner. From Eqs. (15) to (17) we find

$$\Gamma_{11} = \left(\frac{\delta\omega}{2\pi} \right)^3 |\eta_1 V_1|^2 \sum_{\omega'_1} \sum_{\omega''_1} \phi_1^*(\omega'_1, \omega''_1) \frac{\sin \frac{1}{2}(\omega'_1 + \omega''_1 - \omega_0)t}{\frac{1}{2}(\omega'_1 + \omega''_1 - \omega_0)} e^{-(1/2)t(\omega'_1 + \omega''_1 - \omega_0)} e^{i\omega''_1(t - \tau_1)} \\ \times \sum_{\omega'_2} \sum_{\omega''_2} \phi_1(\omega'_2, \omega''_2) \frac{\sin \frac{1}{2}(\omega'_2 + \omega''_2 - \omega_0)t}{\frac{1}{2}(\omega'_2 + \omega''_2 - \omega_0)} e^{(1/2)t(\omega'_2 + \omega''_2 - \omega_0)} e^{-i\omega''_2(t - \tau_1)} [v_1(v'_1)v_1^*(\omega'_2) + \delta_{\omega'_1\omega'_2}]. \quad (27)$$

After going to the long-time limit and using Eqs. (20), as before, together with the definitions (23) and the quasimonochromatic approximations (25), we arrive at

$$\Gamma_{11} = |\eta_1 V_1|^2 \left[2\pi\delta\omega \sum_{\omega} |\phi_1(\omega, \omega_0 - \omega)|^2 + (2\pi)^2 |\phi_1(\omega_s, \omega_0 - \omega_s)|^2 |\mathcal{R}|^2 |\mathcal{W}(t)|^2 \right] \\ = |\eta_1 V_1|^2 [1 + (2\pi)^2 |\mathcal{R}|^2 |\phi_1(\omega_s, \omega_0 - \omega_s)|^2 |\mathcal{W}(t)|^2]. \quad (28)$$

The last line follows from the previous one with the help of Eq. (14). In a similar manner we obtain

$$\Gamma_{22} = |\eta_2 V_2|^2 [1 + (2\pi)^2 |\mathcal{T}|^2 |\phi_2(\omega_s, \omega_0 - \omega_s)|^2 |\mathcal{W}(t)|^2]. \quad (29)$$

We may now combine Eqs. (26), (28), and (29) to arrive at

$$\langle (\hat{E}_{i1}^{(-)} + \hat{E}_{i2}^{(-)}) (\hat{E}_{i1}^{(+)} + \hat{E}_{i2}^{(+)}) \rangle \\ = |\eta V|^2 [2 + (2\pi)^2 |\phi(\omega_s, \omega_0 - \omega_s)|^2 |\mathcal{W}(t)|^2 \\ \times \{1 + 2|\mathcal{R}\mathcal{T}| \cos[(\omega_0 - \omega_s)(\tau_2 - \tau_1) + \chi]\}], \quad (30)$$

where

$$\chi \equiv \arg\phi_2(\omega_s, \omega_0 - \omega_s) - \arg\phi_1(\omega_s, \omega_0 - \omega_s) \\ + \arg(\eta_2 V_2 \eta_1^* V_1^*) + \arg(\mathcal{R}\mathcal{T}^*) + \arg(M_1 M_2^*), \quad (31)$$

and we have assumed that

$$|\eta_1 V_1| = |\eta_2 V_2| = |\eta V|, \\ |\phi_1(\omega_s, \omega_0 - \omega_s)| = |\phi(\omega_s, \omega_0 - \omega_s)| \\ = |\phi_2(\omega_s, \omega_0 - \omega_s)|, \quad (32) \\ |M_1 M_2| \approx 1,$$

in order to simplify the answer.

IV. DISCUSSION

From Eq. (30) we see that by virtue of the cosine modulation at the idler frequency $\omega_0 - \omega_s$, there is an interference pattern that depends on the optical path difference $c(\tau_2 - \tau_1)$. However, the visibility of this pat-

tern depends strongly on the magnitude of the term $(2\pi)^2 |\phi(\omega_s, \omega_0 - \omega_s)|^2 |\mathcal{W}(t)|^2$. If

$$(2\pi)^2 |\phi(\omega_s, \omega_0 - \omega_s)|^2 |\mathcal{W}(t)|^2 \gg 1, \quad (33)$$

then the visibility is given by $2|\mathcal{R}\mathcal{T}|$ and can be 100% when $|\mathcal{R}| = 1\sqrt{2} = |\mathcal{T}|$. On the other hand, when the same term is very small the visibility is very small also.

Let us then examine the order of magnitude of this term. From Eq. (14) it follows that there are about as many nonvanishing contributions to the sum on the left as the number of times that $\delta\omega$ can be divided into the total bandwidth $\Delta\omega$ of the spontaneously down-converted signal light. Hence $2\pi |\phi(\omega_s, \omega_0 - \omega_s)|^2$ must have the order of magnitude $1/\Delta\omega$. It then follows that the critical term

$$(2\pi)^2 |\phi(\omega_s, \omega_0 - \omega_s)|^2 |\mathcal{W}(t)|^2 \sim 2\pi |\mathcal{W}(t)|^2 / \Delta\omega \quad (34)$$

depends on the ratio of the injected signal intensity in photons per second, divided by the bandwidth of the spontaneous down-converted signal light. The critical parameter is therefore the number of injected photons per unit down-converted bandwidth, which is essentially the same parameter that expresses the ratio of the stimulated to the spontaneous emission probability. It is only when the stimulated emission dominates, that the injected signal beams induce coherence between the two idlers.

In principle, this condition for high visibility should be relatively easy to achieve. But in practice the bandwidth $\Delta\omega$ of the spontaneously down-converted light is so large compared with the laser bandwidth that correct alignment may be very difficult.

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