Radiative properties of atoms near a conducting plane: An old problem in a new light

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We examine the level shifts and radiation rates of an atom near an infinite, perfectly conducting plane. Following the work of Dalibard, Dupont-Roc, and Cohen-Tannoudji [J. Phys. (Paris) **43**, 1617 (1982); **45**, 637 (1984)], we distinguish between the effects of radiation reaction and vacuum fluctuation. This separation provides some new physical insight into the nature of "cavity" modifications of atomic properties. In particular, we are able to identify the Casimir interaction as entirely due to the vacuum fluctuations.

I. INTRODUCTION

The radiative properties of atoms are modified in cavities, because the electromagnetic field surrounding the atom is modified by the boundaries of the cavity. The presence of the cavity affects both the natural lifetimes and the energies of atomic levels. Although these effects have been known theoretically for many years they have only recently begun to be demonstrated in the laboratory. With the use of lasers it is now possible to study highly excited states of atoms which have large polarizabilities and therefore are strongly coupled to the electromagnetic field. Furthermore, the excited atoms can be detected with high efficiency, so that experiments can be performed at the single atom level. These developments have made it possible to observe cavity effects in the laboratory[1] and have led to the growth of a new field called "cavity quantum electrodynamics."

Quantum electrodynamics (QED) provides a quantitative theory of the radiative properties of atoms in confined space, although a clear and simple physical picture does not always emerge from the full QED description.[2–15] From one point of view, the cavity effects may be regarded as modifications of the vacuum field distribution and mode density associated with the fluctuations of the quantized radiation field. On the other hand, the same effects can often be described by considering the reaction of the instantaneous atomic dipole to its own radiation field reflected from the cavity walls. In the second viewpoint the physical mechanism is the fluctuation of the atomic dipole and it does not seem necessary to consider the field fluctuations directly.

Both mechanisms, an atom driven by fluctuations of the electromagnetic environment, and an atom reacting to its own field, yield identical answers for many of the radiative properties. In fact, we know from the work of Ackerhalt, Knight, and Eberly,[12] Senitzky,[13] and Milonni, Ackerhalt, and Smith,[14] that "radiation reaction" and "vacuum fluctuations" are in a sense two sides of the same coin. They have shown that within standard QED, the extent to which each mechanism contributes to the total effect can be chosen at will merely by changing the ordering of the matter and radiation operators in the interaction Hamiltonian.[15]

Following a suggestion by Fain,[16] Dalibard, Dupont-Roc, and Cohen-Tannoudji[17,18] (henceforth called DDC) have discussed an extension of these ideas. They consider the rate of variation of an arbitrary atomic observable G due to the coupling of the electron momentum with the vector potential of the vacuum $\mathbf{A}$,

$$\left(\frac{dG}{dt}\right)_{\text{coupling}} = e\mathbf{N}\cdot\mathbf{A}\ , \tag{1}$$

where $\mathbf{N}$ is the relevant atomic operator. $\mathbf{A}$ is the sum of a vacuum field $\mathbf{A}_v$ and a field generated by the atom $\mathbf{A}_a$. Although $\mathbf{A}$ commutes with N, $\mathbf{A}_v$ and $\mathbf{A}_a$ individually do not. Consequently, the contribution of each field to the total rate seems to depend on the ordering of $\mathbf{N}$ and $\mathbf{A}$. However, DDC show that the rate of change of G separates unambiguously into a vacuum fluctuation part (VF) and a self-reaction part (SR) when each part is required separately to be Hermitian:

$$\begin{aligned}\left(\frac{dG}{dt}\right)_{\text{VF}} &= \tfrac{1}{2}(\mathbf{N}\cdot\mathbf{A}_v + \mathbf{A}_v\cdot\mathbf{N})\ ,\\ \left(\frac{dG}{dt}\right)_{\text{SR}} &= \tfrac{1}{2}(\mathbf{N}\cdot\mathbf{A}_a\cdot\mathbf{A}_a\cdot\mathbf{N})\ .\end{aligned} \tag{2}$$

Of course, each part *must* be separately Hermitian if it is to have separate physical meaning.

A generalization of this idea allows them to separate the two parts even when the atom-radiation coupling is more complicated than Eq. (1). This approach seems to provide a simple physical basis (Hermiticity) for determining which radiative effects are due to vacuum fluctuations and which to self-reaction.

DDC then proceed to solve Eq. (2) by perturbation theory taken to second order in atom-field coupling. In particular, they give expressions for the separate contributions from vacuum fluctuation and self-reaction to the energy-level shifts and radiation rates of an atom. Their results are formally identical with an application of the fluctuation-dissipation theorem both to the atom coupled with a fluctuating electromagnetic vacuum and to the electromagnetic field coupled with a fluctuating atom. Of course, their perturbative solution cannot describe situations such as an atom interacting with a single mode of the electromagnetic field,[19] but Eq. (2) itself is quite general.

In this work we calculate the changes in the energy and spontaneous decay rate when an atom is in the vicinity of an infinite plane conducting surface (a prototype for a cavity). We refer to the region on one side of the surface as semiconfined space. This is not a new problem, but by separating the vacuum fluctuating and self-reaction parts according to the scheme of DDC, we are able to gain some new physical insight into the cavity-induced effects.

This work is an extension of the free-space results of DDC to the case of a rudimentary cavity. On three occasions, Eq. (22), (29), and (37), we take a result of DDC as a starting point for our discussion. The reader is referred to Sec. 4 of Ref. 17 for the derivation of these results.

We begin with a discussion in Sec. II of the statistical functions which are used within the DDC framework to describe the atom and the vacuum field in semiconfined space. In Secs. III and IV we examine the effect of the conducting boundary on the spontaneous decay rates and energy levels of the atom. Finally, we compare our results with the work of other authors and draw some conclusions about the physical origins of cavity-induced effects.

II. STATISTICAL FUNCTIONS IN SEMICONFINED SPACE

In this section we calculate the basic statistical functions which will be used to describe the vacuum and the atom in semiconfined space. The conducting boundary that defines the space will be called the mirror.

A. Field modes

Let the normal to the conducting surface define the z axis. We distinguish two types of plane-wave modes for the field in front of the surface. These we call E modes ($E_z=0$) and M modes ($B_z=0$) by analogy with the TE and TM modes of a waveguide. The wave vector of a mode is $\mathbf{k}$ and its components normal and parallel to the mirror are $\mathbf{k}_z$ and $\boldsymbol{\kappa}$. The position vector is $\mathbf{r}$ and its components normal and parallel to the mirror are $\mathbf{z}$ and $\boldsymbol{\rho}$. The notation and coordinate system are illustrated in Fig. 1. The field operators for E and M modes are

$$\begin{aligned}\mathbf{A}^E(\mathbf{k})&=\left[\frac{\hbar}{\epsilon_0\omega V}\right]^{1/2}\sin(k_z z)\hat{\boldsymbol{\kappa}}\times\hat{\mathbf{z}}a_{\mathbf{k}}^E e^{i(\kappa\rho-\omega t)}+\text{H.c.}\ ,\\ \mathbf{A}^M(\mathbf{k})&=\left[\frac{\hbar}{\epsilon_0\omega V}\right]^{1/2}\left[\frac{\kappa}{k}\cos(k_z z)\hat{\mathbf{z}}-i\frac{k_z}{k}\sin(k_z z)\hat{\boldsymbol{\kappa}}\right]a_{\mathbf{k}}^M e^{i(\kappa\rho-\omega t)}+\text{H.c.}\ ,\end{aligned}\tag{3}$$

in which $\hbar$, ϵ_0, and ω have the usual meanings, V is the normalization volume, $a_{\mathbf{k}}^E$ and $a_{\mathbf{k}}^M$ are the annihilation operators for the E and M modes having wave vector $\mathbf{k}$ and H.c. denotes the Hermitian conjugate.

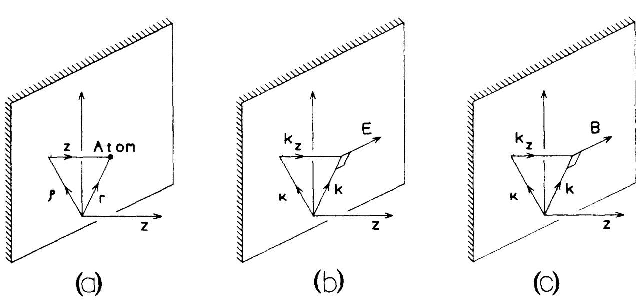

FIG. 1. This figure defines the notation and coordinate system. (a) The cylindrical position coordinates of the atom. (b) The electric field, the wave vector, and its components in an E mode. (c) The magnetic field, the wave vector, and its components in an M mode.

B. Correlation and susceptibility functions for the field

Following DDC we define the correlation functions for the vacuum field,

$$C_\mu^F(\mathbf{r},\tau)=\tfrac{1}{2}\sum_{\lambda\,(=E,M)}\langle 0|[A_\mu^\lambda(\mathbf{r},t),A_\mu^\lambda(\mathbf{r},t-\tau)]_+|0\rangle\ .\tag{4}$$

The subscript plus (minus) indicates anticommutation (commutation). A_μ is a Cartesian component of the vacuum field in the presence of the mirror but not yet perturbed by an atom. In general, the correlation function is a tensor $C_{\mu\nu}$, but in this case all the off-diagonal elements are zero. Similarly the susceptibility of the vacuum field is defined as

$$\chi_\mu^F(\mathbf{r},\tau)=\frac{i}{\hbar}\sum_{\lambda\,(=E,M)}\langle 0|[A_\mu^\lambda(\mathbf{r},t),A_\mu^\lambda(\mathbf{r},t-\tau)]_-|0\rangle\Theta(\tau)\ ,\tag{5}$$

in which $\Theta(\tau)$ is the Heaviside step function.

1. Transverse fields

From Eqs. (3) and (4) it follows that for E modes

$$C_x^{F(E)}(\mathbf{r},\tau)=\frac{\hbar}{2\epsilon_9}\int\frac{d^3k}{4\pi^3}\left[\frac{1}{\omega}(\sin^2 k_z z)(\sin^2\phi)\times(e^{-i\omega\tau}+e^{i\omega\tau})\right] \quad (6)$$

and for the M modes

$$C_x^{F(M)}(\mathbf{r},\tau)=\frac{\hbar}{2\epsilon_0}\int\frac{d^3k}{4\pi^3}\left[\frac{1}{\omega}(\sin^2 k_z z)(\cos^2\theta)\times(\cos^2\phi)(e^{-i\omega\tau}+e^{i\omega\tau})\right], \quad (7)$$

where ϕ is the azimuthal angle and θ is the polar angle of $\mathbf{k}$. After integrating over all orientations of $\mathbf{k}$ and adding E and M modes we obtain

$$C_x^F(\mathbf{r},\tau)=\frac{\hbar}{8\pi^2\epsilon_0 c^3}\int_{-\infty}^{+\infty}d\omega\, e^{i\omega\tau}|\omega|\left[\frac{2}{3}-\frac{\sin(\omega T)}{\omega T}-\frac{\cos(\omega T)}{(\omega T)^2}+\frac{\sin(\omega T)}{(\omega T)^3}\right], \quad (8)$$

where T stands for $2z/c$, the time taken for light to propagate to the mirror and back. The first term in Eq. (8) is just the free-space correlation function, therefore the change in correlation function of the transverse field due to the presence of the mirror is

$$C_x^{\text{cav}}(\mathbf{r},\tau)=\frac{\hbar}{8\pi^2\epsilon_0 c^3}\int_{-\infty}^{+\infty}d\omega\, e^{i\omega\tau}|\omega|\left[-\frac{\sin(\omega T)}{\omega T}-\frac{\cos(\omega T)}{(\omega T)^2}+\frac{\sin(\omega T)}{(\omega T)^3}\right]. \quad (9)$$

For the susceptibility of the transverse field a nearly identical calculation yields

$$\chi_x^{\text{cav}}(\mathbf{r},\tau)=\frac{-i}{4\pi^2\epsilon_0 c^3}\int_{-\infty}^{+\infty}d\omega\, e^{i\omega\tau}\omega\Theta(\tau)\left[-\frac{\sin(\omega T)}{\omega T}-\frac{\cos(\omega T)}{(\omega T)^2}+\frac{\sin(\omega T)}{(\omega T)^3}\right]. \quad (10)$$

From the symmetry of the problem it is evident that $C_y=C_x$ and $\chi_y=\chi_x$.

2. Perpendicular fields

Only the M modes contribute to the normal component of the field. From Eqs. (3) and (4) it follows that

$$C_z^{F(M)}(\mathbf{r},\tau)=\frac{\hbar}{2\epsilon_0}\int\frac{d^3k}{4\pi^3}\left[\frac{1}{\omega}(\cos^2 k_z z)(\sin^2\theta)\times(e^{-i\omega\tau}+e^{i\omega\tau})\right] \quad (11)$$

and hence

$$C_z^F(\mathbf{r},\tau)=\frac{\hbar}{4\pi^2\epsilon_0 c^3}\int_{-\infty}^{+\infty}d\omega\, e^{i\omega\tau}|\omega|\left[\frac{1}{3}-\frac{\cos(\omega T)}{(\omega T)^2}+\frac{\sin(\omega T)}{(\omega T)^3}\right]. \quad (12)$$

After subtracting the free-space part we obtain

$$C_z^{\text{cav}}(\mathbf{r},\tau)=\frac{\hbar}{4\pi^2\epsilon_0 c^3}\int_{-\infty}^{+\infty}d\omega\, e^{i\omega\tau}|\omega|\left[-\frac{\cos(\omega T)}{(\omega T)^2}+\frac{\sin(\omega T)}{(\omega T)^3}\right]. \quad (13)$$

Similarly, the change in susceptibility of the perpendicular field due to the mirror is

$$\chi_z^{\text{cav}}(\mathbf{r},\tau)=\frac{-i}{2\pi^2\epsilon_0 c^3}\int_{-\infty}^{+\infty}d\omega\, e^{i\omega\tau}\omega\Theta(\tau)\left[-\frac{\cos(\omega T)}{(\omega T)^2}+\frac{\sin(\omega T)}{(\omega T)^3}\right]. \quad (14)$$

3. Fourier transforms of the field statistical functions

We define the transform $\hat{f}(\omega)$ of $f(\tau)$ through the relation

$$f(\tau)=\int_{-\infty}^{+\infty}d\omega\, e^{i\omega\tau}\hat{f}(\omega)\ . \quad (15)$$

This is consistent with the convention of DDC. The transforms of the correlation functions [Eqs. (9) and (13)] are obtained by inspection:

$$\hat{C}_x^{\text{cav}}(\omega)=\frac{\hbar}{8\pi^2\epsilon_0 c^3}|\omega|\left[-\frac{\sin(\omega T)}{\omega T}-\frac{\cos(\omega T)}{(\omega T)^2}+\frac{\sin(\omega T)}{(\omega T)^3}\right],$$
$$\hat{C}_z^{\text{cav}}(\omega)=\frac{\hbar}{4\pi^2\epsilon_0 c^3}|\omega|\left[-\frac{\cos(\omega T)}{(\omega T)^2}+\frac{\sin(\omega T)}{(\omega T)^3}\right]. \quad (16)$$

We have dropped the argument $\mathbf{r}$ in order to simplify the notation but of course there remains a position dependence through the quantity $T\equiv 2z/c$. The susceptibilities are not so easy to transform because of the presence of $\Theta(\tau)$ within the integrals. Appendix A shows how we obtain the results

$$\hat{\chi}_x^{\text{cav}}(\omega)=\frac{1}{8\pi^2\epsilon_0 c^3}\frac{1}{T}\left[\left[\frac{\sin(\omega T)}{\omega T}+\frac{\cos(\omega T)}{(\omega T)^2}-\frac{1}{(\omega T)^2}-e^{-i\omega T}\right]+i\left[\frac{\cos(\omega T)}{\omega T}-\frac{\sin(\omega T)}{(\omega T)^2}\right]\right],$$
$$\hat{\chi}_z^{\text{cav}}(\omega)=\frac{1}{4\pi^2\epsilon_0 c^3}\frac{1}{T}\left[\left[\frac{\sin(\omega T)}{\omega T}+\frac{\cos(\omega T)}{(\omega T)^2}-\frac{1}{(\omega T)^2}\right]+i\left[\frac{\cos(\omega T)}{\omega T}-\frac{\sin(\omega T)}{(\omega T)^2}\right]\right]. \tag{17}$$

C. Correlation and susceptibility functions for the atom

Following DDC we now define the correlation functions for an atom in state a,

$$C_\mu^a(\tau)=\frac{1}{2}\left[\frac{e}{m}\right]^2\langle a|[p_\mu(t),p_\mu(t-\tau)]_+|a\rangle , \tag{18}$$

where $p_\mu(t)$ is a Cartesian component of the electron momentum operator. Once again we are interested only in the diagonal elements of the more general tensor. This operator evolves only under the internal atomic interactions; the atom has yet to be coupled to the vacuum. Similarly, the susceptibility of the atom is defined as

$$\chi_\mu^a(\tau)=\frac{i}{\hbar}\left[\frac{e}{m}\right]^2\langle a|[p_\mu(t),p_\mu(t-\tau)]_-|a\rangle\Theta(\tau) . \tag{19}$$

We will be interested in the Fourier transforms of these statistical functions which are given by DDC:

$$\hat{C}_\mu^a(\omega)=\frac{1}{2}\left[\frac{e}{m}\right]^2\sum_b|\langle b|p_\mu|a\rangle|^2[\delta(\omega-\omega_{ab})+\delta(\omega+\omega_{ab})] \tag{20}$$

and

$$\hat{\chi}_\mu^a(\omega)=\frac{-1}{2\hbar}\left[\frac{e}{m}\right]^2\sum_b|\langle b|p_\mu|a\rangle|^2\left\{\frac{1}{\pi}\left[\mathrm{P}\left[\frac{1}{\omega+\omega_{ab}}\right]-\mathrm{P}\left[\frac{1}{\omega-\omega_{ab}}\right]\right]+i[\delta(\omega+\omega_{ab})-\delta(\omega-\omega_{ab})]\right\} , \tag{21}$$

where δ is the Dirac delta function, $\hbar\omega_{ab}$ is the energy interval E_a-E_b, and P indicates that the principal part is to be taken in subsequent integration.

The key results of this section are Eqs. (16), (17), (20), and (21), the correlation functions and the susceptibilities of the vacuum and the atom. Following DDC we will now calculate the decay rates and shifts of the atomic levels in the context of perturbation theory.

III. SPONTANEOUS DECAY RATE

The power radiated from an atom consists of two parts, P_{VF} due to vacuum fluctuations and P_{SR} due to self-reaction, as outlined in Eq. (2). In the coupled reservoir model of DDC these can be written as (i)

$$P_{\mathrm{SR}}=-2\pi\sum_\mu\int_{-\infty}^{+\infty}d\omega\,\omega\hat{C}_\mu^a(\omega)\mathrm{Im}[\hat{\chi}_\mu^F(\omega)]$$

and (ii) (22)

$$P_{\mathrm{VF}}=2\pi\sum_\mu\int_{-\infty}^{+\infty}d\omega\,\omega\hat{C}_\mu^F(\omega)\mathrm{Im}[\hat{\chi}_\mu^a(\omega)] .$$

P_{SR} and P_{VF} can be positive or negative, enhancing or suppressing radiative decay in confined space.

A. Self-reaction-induced decay rate

The modification of P_{SR} due to the presence of the mirror is obtained by subtracting the free-space value. Hence

$$P_{\mathrm{SR}}^{\text{cav}}=-2\pi\sum_\mu\int_{-\infty}^{+\infty}d\omega\,\omega\hat{C}_\mu^a(\omega)\mathrm{Im}[\hat{\chi}_\mu^{\text{cav}}(\omega)] . \tag{23}$$

With the help of Eqs. (17) and (20) we find

$$P_{\mathrm{SR}}^{\text{cav}}=\frac{e^2}{4\pi\epsilon_0 m^2c^3}\sum_b\omega_{ab}^2\left[|\langle b|p_\rho|a\rangle|^2\left[-\frac{\sin(\omega_{ab}T)}{\omega_{ab}T}-\frac{\cos(\omega_{ab}T)}{(\omega_{ab}T)^2}+\frac{\sin(\omega_{ab}T)}{(\omega_{ab}T)^3}\right]+2|\langle b|p_z|a\rangle|^2\left[-\frac{\cos(\omega_{ab}T)}{(\omega_{ab}T)^2}+\frac{\sin(\omega_{ab}T)}{(\omega_{ab}T)^3}\right]\right] , \tag{24}$$

where $|\langle b|p_\rho|a\rangle|^2=|\langle b|p_x|a\rangle|^2+|\langle b|p_y|a\rangle|^2$.

B. Vacuum-induced decay rate

The mirror-induced change in P_{VF} is

$$P_{\text{VF}}^{\text{cav}}=2\pi\sum_{\mu}\int_{-\infty}^{+\infty}d\omega\omega\hat{C}_{\mu}^{\text{cav}}(\omega)\text{Im}[\hat{\chi}_{\mu}^{a}(\omega)] \; . \tag{25}$$

With the help of Eqs. (16) and (21) we find

$$P_{\text{VF}}^{\text{cav}}=\frac{e^2}{4\pi\epsilon_0 m^2c^3}\sum_b \omega_{ab}|\omega_{ab}|\left[|\langle b|p_\rho|a\rangle|^2\left[-\frac{\sin(\omega_{ab}T)}{\omega_{ab}T}-\frac{\cos(\omega_{ab}T)}{(\omega_{ab}T)^2}+\frac{\sin(\omega_{ab}T)}{(\omega_{ab}T)^3}\right]\right.$$
$$\left.+2|\langle b|p_z|a\rangle|^2\left[-\frac{\cos(\omega_{ab}T)}{(\omega_{ab}T)^2}+\frac{\sin(\omega_{ab}T)}{(\omega_{ab}T)^3}\right]\right] \; . \tag{26}$$

Note that Eq. (26) is identical to Eq. (24) except for the replacement of ω_{ab} by $|\omega_{ab}|$.

C. Total decay rate

The total power radiated from the atom in semiconfined space is $P=P^0+P_{\text{VF}}^{\text{cav}}+P_{\text{SR}}^{\text{cav}}$, where P^0 is the power that would be radiated in free space,

$$P^0=\frac{e^2}{6\pi\epsilon_0 m^2c^3}\sum_b \omega_{ab}(\omega_{ab}+|\omega_{ab}|)\times(|\langle b|p_\rho|a\rangle|^2+|\langle b|p_z|a\rangle|^2) \; . \tag{27}$$

For ground-state atoms it is reassuring to note that each term in $P_{\text{VF}}^{\text{cav}}$ is canceled exactly by the corresponding term in $P_{\text{SR}}^{\text{cav}}$ because ω_{ab} is negative for every b. This cancellation also ensures more generally that there are no upward radiative transitions from any initial state. For downward transitions from an excited state, P acquires equal contributions from $P_{\text{VF}}^{\text{cav}}$ and from $P_{\text{SR}}^{\text{cav}}$ regardless of the distance from the mirror. This is the same kind of conspiracy that DDC found in the free-space case and which is contained in the term $(\omega_{ab}+|\omega_{ab}|)$ in Eq. (27). It seems to tell us that the vacuum fluctuations and the self-reaction are equally important in radiative decay, both in free space and in the presence of a boundary.

At distances far from the mirror ($\omega_{ab}T\gg 1$), the modifications become negligible and the radiated power approaches the free-space value as one would expect.

Close to the mirror ($\omega_{ab}T\ll 1$) the power radiated by a dipole parallel to the surface goes quadratically to zero [$P\propto(\omega_{ab}T)^2$] while a perpendicular dipole radiates at twice the free-space rate. This behavior has a simple explanation. Close to the mirror the field A_ρ goes linearly to zero because it must satisfy the boundary condition $A_\rho=0$ in each mode. This applies both to the vacuum field and to the field generated by the atom. Hence the decay rate goes quadratically to zero. In a similar way the reflection at the boundary doubles the perpendicular component of both the vacuum field and the field produced by atomic fluctuations. This generates four times the power density of decay radiation, but only in half the free-space volume. Hence the radiated power doubles.

IV. ATOMIC ENERGY LEVEL SHIFTS

It is convenient to work in the Coulomb gauge where the Hamiltonian describing the external interactions of the atom in the dipole approximation is

$$H=\frac{e^2}{2m}A^2-\frac{e}{m}\mathbf{A}\cdot\mathbf{p}+V_{\text{stat}} \; . \tag{28}$$

In this section we will separate H into vacuum-fluctuation and self-reaction parts and will examine the level shifts (calculated to order e^2) associated with each. Evidently the A^2 term is to be associated with the vacuum, while the "instantaneous" Coulomb interaction energy V_{stat} is a self-reaction term. This assignment is not only intuitively reasonable, it is also required if the separation into vacuum-fluctuation and self-reaction contributions is to be gauge invariant.[20] The $\mathbf{A}\cdot\mathbf{p}$ term will be decomposed into a vacuum-fluctuation part and a self-reaction part as discussed by DDC.

A. Self-reaction-induced level shifts

The self-reaction part of the level shift due to H can be written following DDC as

$$\Delta_{\text{SR}}=\langle a|V_{\text{stat}}|a\rangle-\pi\sum_{\mu}\int_{-\infty}^{+\infty}d\omega[\hat{C}_{\mu}^{a}(\omega)]^*\hat{\chi}_{\mu}^{F}(\omega) \; . \tag{29}$$

Here we are only interested in the mirror modification of Δ_{SR} which we obtain by subtracting the free-space part from χ^F:

$$\Delta_{\text{SR}}^{\text{cav}}=\langle a|V_{\text{stat}}|a\rangle-\pi\sum_{\mu}\int_{-\infty}^{+\infty}d\omega[\hat{C}_{\mu}^{a}(\omega)]^*\hat{\chi}_{\mu}^{\text{cav}}(\omega) \; . \tag{30}$$

1. Electrostatic term

The first term in Eq. (30) is the London–van der Waals interaction of the instantaneous electric dipole with its image:[21]

$$\langle a|V_{\text{stat}}|a\rangle=-\frac{e^2}{4\pi\epsilon_0}\frac{\langle a|\rho^2+2z^2|a\rangle}{16z^3} \; . \tag{31}$$

Here we are making the approximation that the size of the atom is much less than z, the distance to the mirror. It will prove useful to rewrite this in terms of T ($\equiv 2z/c$) and $\mathbf{p}$, the electron momentum,

$$\langle a|V_{\rm stat}|a\rangle = -\frac{e^2}{8\pi\epsilon_0 m^2c^3}\frac{1}{T}\sum_b \frac{1}{(\omega_{ab}T)^2}(|\langle b|p_\rho|a\rangle|^2+|\langle b|\sqrt{2}p_z|a\rangle|^2)\ . \tag{32}$$

2. *Self-reaction contribution to the* A·p *term*

The second term in Eq. (30) is readily evaluated with the help of Eqs. (17) and (20). The result is

$$\frac{e^2}{8\pi\epsilon_0 m^2c^3}\frac{1}{T}\sum_b \left[|\langle b|p_\rho|a\rangle|^2\left[\frac{1}{(\omega_{ab}T)^2}-\frac{\cos(\omega_{ab}T)}{(\omega_{ab}T)^2}-\frac{\sin(\omega_{ab}T)}{(\omega_{ab}T)}+\cos(\omega_{ab}T)\right]\right.$$
$$\left.+|\langle b|\sqrt{2}p_z|a\rangle|^2\left[\frac{1}{(\omega_{ab}T)^2}-\frac{\cos(\omega_{ab}T)}{(\omega_{ab}T)^2}-\frac{\sin(\omega_{ab}T)}{(\omega_{ab}T)}\right]\right]\ . \tag{33}$$

3. *Total self-reaction-induced level shift*

The sum of Eqs. (32) and (33) yields the total levels shift due to self-reaction,

$$\Delta_{\rm SR}^{\rm cav} = \frac{e^2}{8\pi\epsilon_0 m^2c^3}\frac{1}{T}\sum_b \left[|\langle b|p_\rho|a\rangle|^2\left[-\frac{\cos(\omega_{ab}T)}{(\omega_{ab}T)^2}-\frac{\sin(\omega_{ab}T)}{(\omega_{ab}T)}+\cos(\omega_{ab}T)\right]\right.$$
$$\left.+|\langle b|\sqrt{2}p_z|a\rangle|^2\left[-\frac{\cos(\omega_{ab}T)}{(\omega_{ab}T)^2}-\frac{\sin(\omega_{ab}T)}{(\omega_{ab}T)}\right]\right]\ . \tag{34a}$$

For future use it is convenient also to write this result in the form

$$\Delta_{\rm SR}^{\rm cav} = \sum_b \Delta_{\rm SR}^{\rm cav}(b)\ . \tag{34b}$$

In the limit of small $\omega_{ab}T$ this shift is dominated by the $1/T^3$ (i.e., $1/z^3$) terms and is just the London–van der Waals interaction

$$\Delta_{\rm SR}^{\rm cav}|_{\omega_{ab}T=0} = \langle a|V_{\rm stat}|a\rangle\ . \tag{35}$$

At large distances the shift becomes

$$\Delta_{\rm SR}^{\rm cav}|_{\omega_{ab}T=\infty} = \frac{e^2}{8\pi\epsilon_0 m^2c^3}\frac{1}{T}\sum_b |\langle b|p_\rho|a\rangle|^2\cos(\omega_{ab}T)\ , \tag{36}$$

which is precisely the interaction energy of a classical electric dipole of amplitude $|\langle b|p_\rho|a\rangle|/m\omega_{ab}$ with its own reflected far field.

B. Vacuum-induced level shifts

The vacuum-induced part of the level shift due to H can be written following DDC as

$$\Delta_{\rm VF} = \frac{e^2}{2m}\langle 0|A^2|0\rangle - \pi\sum_\mu \int_{-\infty}^{+\infty} d\omega\, \hat{C}_\mu^F(\omega)^*\hat{\chi}_\mu^a(\omega)\ . \tag{37}$$

Once again we are interested in the modification of the energy due to the mirror which we obtain by subtracting the free-space part from Eq. (37),

$$\Delta_{\rm VF}^{\rm cav} = \frac{e^2}{2m}\sum_\mu C_\mu^{\rm cav}(\mathbf{r},\tau=0) - \pi\sum_\mu \int_{-\infty}^{+\infty} d\omega [\hat{C}_\mu^{\rm cav}(\omega)]^*\hat{\chi}_\mu^a(\omega)\ . \tag{38}$$

1. *The* A^2 *term*

The first term of Eq. (38) represents that part of the interaction $e^2A^2/2m$ due to the presence of the mirror. It is readily evaluated using Eqs. (9) and (13). The result is

$$\frac{e^2}{2m}\sum_\mu C_\mu^{\rm cav}(\mathbf{r},\tau=0) = \frac{e^2\hbar}{4\pi^2\epsilon_0 mc^3}\frac{1}{T^2}\ , \tag{39}$$

where, as usual, $T\equiv 2z/c$. This contribution to the level shift comes entirely from A_z. For future reference it is useful to rewrite this result using a version of the Thomas-Reiche-Kuhn sum rule

$$\hbar = -\frac{1}{m}\sum_b |\langle b|\sqrt{2}p_z|a\rangle|^2\frac{\omega_{ab}}{|\omega_{ab}|^2} \tag{40}$$

to obtain

$$\frac{e^2}{2m}\sum_\mu C_\mu^{\rm cav}(\mathbf{r},\tau=0) = -\frac{e^2}{4\pi^2\epsilon_0 m^2c^3}\frac{1}{T^2}\sum_b |\langle b|\sqrt{2}p_z|a\rangle|^2\frac{\omega_{ab}}{|\omega_{ab}|^2}\ . \tag{41}$$

2. *Vacuum-fluctuation contribution to the* A·p *term*

Now we evaluate the second term in Eq. (38) with the help of Eqs. (16) and (21). The contribution from the x and y components of the field is

$$\frac{e^2}{16\pi^2\epsilon_0 m^2c^3}\sum_b\int_{-\infty}^{+\infty}d\omega|\omega|\left[-\frac{\sin(\omega T)}{\omega T}-\frac{\cos(\omega T)}{(\omega T)^2}+\frac{\sin(\omega T)}{(\omega T)^3}\right]|\langle b|p_\rho|a\rangle|^2\left[\mathrm{P}\left[\frac{1}{\omega+\omega_{ab}}\right]-\mathrm{P}\left[\frac{1}{\omega-\omega_{ab}}\right]\right] \tag{42a}$$

and from the z component

$$\frac{e^2}{16\pi^2\epsilon_0 m^2c^3}\sum_b\int_{-\infty}^{+\infty}d\omega|\omega|\left[-\frac{\cos(\omega T)}{(\omega T)^2}+\frac{\sin(\omega T)}{(\omega T)^3}\right]|\langle b|\sqrt{2}p_z|a\rangle|^2\left[\mathrm{P}\left[\frac{1}{\omega+\omega_{ab}}\right]-\mathrm{P}\left[\frac{1}{\omega-\omega_{ab}}\right]\right]. \tag{42b}$$

Although we are able to evaluate these integrals analytically as described in Appendix B, they have a rather inelegant form. The results for the transverse (ρ) and normal (z) contributions are, respectively,

$$\frac{e^2}{4\pi^2\epsilon_0 m^2c^3}\frac{1}{T}\sum_b|\langle b|p_\rho|a\rangle|^2\frac{\omega_{ab}}{|\omega_{ab}|}\left[\frac{1+g(|\omega_{ab}|T)}{|\omega_{ab}|T}+\frac{f(|\omega_{ab}|T)}{(|\omega_{ab}|T)^2}-f(|\omega_{ab}|T)\right.$$
$$\left.+\frac{\pi}{2}\left[-\frac{\cos(\omega_{ab}T)}{(\omega_{ab}T)^2}-\frac{\sin(\omega_{ab}T)}{\omega_{ab}T}+\cos(\omega_{ab}T)\right]\right], \tag{43a}$$

and

$$\frac{e^2}{4\pi^2\epsilon_0 m^2c^3}\frac{1}{T}\sum_b|\langle b|\sqrt{2}p_z|a\rangle|^2\frac{\omega_{ab}}{|\omega_{ab}|}\left[\frac{1+g(|\omega_{ab}|T)}{|\omega_{ab}|T}+\frac{f(|\omega_{ab}|T)}{(|\omega_{ab}|T)^2}+\frac{\pi}{2}\left[-\frac{\cos(\omega_{ab}T)}{(\omega_{ab}T)^2}-\frac{\sin(\omega_{ab}T)}{\omega_{ab}T}\right]\right], \tag{43b}$$

where the auxiliary functions f and g are defined in Appendix B.

3. *Total vacuum-induced level shift*

When we sum Eqs. (41), (43a), and (43b) to find the total vacuum-induced level shift, we see that the first term in Eq. (43b) is canceled by the contribution from Eq. (41) and that some of the terms in Eqs. (43a) and (43b) correspond exactly to terms in the self-reaction shift given in Eq. (34a). Thus the total vacuum-induced shift can be written as

$$\Delta_{\mathrm{VF}}^{\mathrm{cav}}=\sum_b\frac{\omega_{ab}}{|\omega_{ab}|}\Delta_{\mathrm{SR}}^{\mathrm{cav}}(b)+\frac{e^2}{4\pi^2\epsilon_0 m^2c^3}\frac{1}{T}\sum_b\frac{\omega_{ab}}{|\omega_{ab}|}\left[|\langle b|p_\rho|a\rangle|^2\left[\frac{1+g(|\omega_{ab}|T)}{|\omega_{ab}|T}+\frac{f(|\omega_{ab}|T)}{(|\omega_{ab}|T)^2}-f(|\omega_{ab}|T)\right]\right.$$
$$\left.+|\langle b|\sqrt{2}p_z|a\rangle|^2\left[\frac{g(|\omega_{ab}|T)}{|\omega_{ab}|T}+\frac{f(|\omega_{ab}|T)}{(|\omega_{ab}|T)^2}\right]\right]. \tag{44}$$

C. Total level shift

The total level shift is $\Delta^{\mathrm{cav}}=\Delta_{\mathrm{SR}}^{\mathrm{cav}}+\Delta_{\mathrm{VF}}^{\mathrm{cav}}$. Close to the mirror, when $\omega_{ab}T\ll 1$, $\Delta_{\mathrm{VF}}^{\mathrm{cav}}$ diverges no faster than $1/T^2$, as shown in Appendix C, whereas $\Delta_{\mathrm{SR}}^{\mathrm{cav}}$ diverges as $1/T^3$ [see Eq. (35)]. Therefore the total level shift near the mirror is dominated by the London–van der Waals interaction and is due entirely to self-reaction.

At a large distance from the mirror ($\omega_{ab}T\gg 1$), $\Delta_{\mathrm{FV}}^{\mathrm{cav}}$ is dominated by the first term of Eq. (44), which has the $1/T$ form shown in Eq. (36). This corresponds, as we remarked in connection with the self-reaction shift, to the atom interacting with the reflected far field of its own radiation pattern. Thus the total shift of level a due to lower-lying states comes half from the self-reaction and half from the vacuum fluctuation, in complete accord with our conclusions about the spontaneous-emission rate. Similarly the shifts due to states lying above state a cancel as do the spontaneous-emission rates. This cancellation means that the energy of a ground-state atom far from the mirror has no such "far-field" shift. This is a reasonable result because the ground state does not radiate. In this case the shift is due entirely to the vacuum fluctuations and is contained in the second and third parts of Eq. (44).

The large-distance limit of Eq. (44) is discussed in Appendix C. We find that the large-distance level shift for ground-state atoms is

$$(\Delta_{\mathrm{SR}}^{\mathrm{cav}}+\Delta_{\mathrm{VF}}^{\mathrm{cav}})|_{\omega_{ab}T\to\infty}=\frac{e^2}{4\pi\epsilon_0 m^2c^3}\frac{1}{T}\sum_b\frac{\omega_{ab}}{|\omega_{ab}|}|\langle b|\mathbf{p}|a\rangle|^2\frac{4}{(|\omega|_{ab}T)^3}. \tag{45}$$

This result is precisely the Casimir-Polder shift,[2] first obtained in 1948. It is usually written in terms of the static scalar polarizability α_{stat} defined as

$$\alpha_{\mathrm{stat}}=-\sum_b\frac{2|\langle b|e\mathbf{r}|a\rangle|^2}{3\hbar\omega_{ab}}, \tag{46}$$

whence

$$(\Delta_{\mathrm{SR}}^{\mathrm{cav}}+\Delta_{\mathrm{VF}}^{\mathrm{cav}})|_{\omega_{ab}T\to\infty}=-\frac{1}{4\pi\epsilon_0}\frac{3\hbar c}{8\pi z^4}\alpha_{\mathrm{stat}}. \tag{47}$$

This result leads us to accept the following physical picture of the Casimir-Polder shift.[6] The fluctuating vacuum induces an ac Stark shift in the atom which, in free space, is Bethe's famous self-energy contribution[23] to the Lamb shift. When a boundary is imposed on the vacuum, the spectrum of fluctuations is changed as indicated by Eqs. (9) and (13), and the corresponding correction to the Lamb shift is the Casimir-Polder shift. The change in $|A|^2$ is significantly only for those wavelengths that are long compared with the distance to the boundary, in which case $|A_\rho|^2$ becomes zero while $|A_z|^2$ is doubled. For these frequencies we crudely take the change in the field to be equal to the unperturbed field. The atomic level shift, ignoring factors of order unity, is then just

$$\Delta \sim \int_0^{1/2z} dk\, k^2 \alpha(k) E_k^2 \;, \tag{48}$$

where $\alpha(k)$ is the electric polarizability. At large distances, where $1/T$ is less than all the atomic resonance frequencies, $\alpha(k)$ is effectively constant. Then

$$\Delta \sim \frac{\hbar c}{\epsilon_0}\alpha_{\text{stat}} \int_0^{1/2z} dk\, k^3 \sim \frac{\hbar c}{\epsilon_0 z^4}\alpha_{\text{stat}} \;, \tag{49}$$

which has all the essential features of the more exact result in Eq. (47).

The Casimir-Polder force has frequently been described as a "retarded London–van der Waals force." This now seems inappropriate since the Casimir-Polder force is entirely due to vacuum fluctuations while the London–van der Waals force is purely the result of self-reaction.

V. SUMMARY AND CONCLUSIONS

Several authors have already analyzed this problem from various points of view and there seems to be general agreement as to the net values of the decay rates and level shifts. In particular, we agree completely with the results given recently by Barton.[4] Perhaps the only significant deviation is in the short-range level shift found by Barut and Dowling[9] which includes an extra London–van der Waals energy for excited states. We do not find such a term and believe that it is erroneous.

What is new about our work is that we have attempted to separate the self-reaction and vacuum-fluctuation mirror effects subject to the physical constraint that each interaction must be separately Hermitian. As pointed out by DDC, this provides a unique prescription for separating the effects.

We find that vacuum fluctuation and self-reaction contribute equally to the decay rates regardless of distance from the mirror. Close to the mirror, the level shifts are dominated by the $1/z^3$ London–van der Waals self-reaction. Far from the mirror the level shifts of excited states are dominated by the $1/z$ interaction with reflected spontaneous radiation to which the two mechanisms contribute equally. For ground states the spontaneous radiation is absent and the leading level shift at large distances is then the $1/z^4$ Casimir-Polder interaction which is entirely due to the vacuum fluctuations.

From the experimental point of view it would be interesting to observe the Casimir-Polder energy of an atom near a conducting boundary. Perhaps the most appealing arrangement[24] is an atom in its ground state at the center of a spherical cavity where there is no London–van der Waals interaction and the (nonzero) level shift is entirely due to the Casimir-Polder energy. Unfortunately spectroscopic methods necessarily involve an excited state whose long-range resonant self-interaction will tend to dominate the shift of the interval, as observed, for example, by Heinzen *et al.*[1]

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APPENDIX A: FOURIER SPECTRUM OF THE FIELD SUSCEPTIBILITY

The mirror modifications to the field susceptibility are given in Eqs. (10) and (14). They have the form

$$\chi_\mu^{\text{cav}}(\tau) = \int_{-\infty}^{+\infty} d\omega\, e^{i\omega\tau} g(\omega)\Theta(\tau) \;, \tag{A1}$$

We define the spectrum of the susceptibility as

$$\hat{\chi}_\mu^{\text{cav}}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\tau\, e^{-i\omega\tau}\chi_\mu^{\text{cav}}(\tau) \;. \tag{A2}$$

This is just the inverse of the relation given in Eq. (15). Taking Eqs. (A1) and (A2) together we find that

$$\hat{\chi}_\mu^{\text{cav}}(\omega) = \tfrac{1}{2}g(\omega) + \frac{i}{2\pi}\text{P}\int_{-\infty}^{+\infty}\frac{d\omega'}{\omega'-\omega}g(\omega') \;, \tag{A3}$$

in which P indicates the principal value of the integral. The relevant integrals are

$$\begin{aligned}
\frac{i}{2\pi}\text{P}\int_{-\infty}^{+\infty}\frac{d\omega'}{\omega'-\omega}\frac{\omega'\sin(\omega'T)}{\omega'T} &= \frac{i}{2T}\cos(\omega T) \;,\\
\frac{i}{2\pi}\text{P}\int_{-\infty}^{+\infty}\frac{d\omega'}{\omega'-\omega}\frac{\omega'\cos(\omega'T)}{(\omega'T)^2} &= -\frac{i}{2T}\frac{\sin(\omega T)}{\omega T} \;,\\
\frac{i}{2\pi}\text{P}\int_{-\infty}^{+\infty}\frac{d\omega'}{\omega'-\omega}\frac{\omega'\sin(\omega'T)}{(\omega'T)^3} &= \frac{i}{2T}\frac{\cos(\omega T)-1}{(\omega T)^2} \;.
\end{aligned} \tag{A4}$$

The use of Eqs. (A3) and (A4) takes us immediately from the time-dependent susceptibilities given in Eqs. (10) and (14) to the frequency-dependent functions given in Eq. (17).

APPENDIX B: INTEGRATION OF EQS. (42b) AND (42a)

1. The basic integrals

The functions $f(a)$ and $g(a)$ are defined as in Abramowitz and Stegun,[22] Secs. 5.2.6 and 5.2.7,

$$\begin{aligned}
\int_0^\infty dx\frac{\sin x}{x+a} &= f(a) \;,\\
\int_0^\infty dx\frac{\cos x}{x+a} &= g(a) \;.
\end{aligned} \tag{B1}$$

Since

$$\int_{-\infty}^{\infty} dx \frac{\sin x}{x+a} = \pi \cos a , \quad \int_{-\infty}^{\infty} dx \frac{\cos x}{x+a} = \pi \sin a , \tag{B2}$$

it follows that

$$\int_0^{\infty} dx \frac{\sin x}{x-a} = -f(a) + \pi \cos a , \quad \int_0^{\infty} dx \frac{\cos x}{x+a} = g(a) - \pi \sin a . \tag{B3}$$

2. Integration of Eq. (42b)

The integral to be performed is Eq. (42b) (see Sec. IV B 2). For simplicity we drop the constants in front of the integral, the summation, the squared matrix element and the principal value symbols. In the remaining integral, which we denote by I, we substitute $x=\omega T$ and $a=|\omega_{ab}|T$. This gives

$$I = \frac{2}{T} \frac{\omega_{ab}}{|\omega_{ab}|} \int_0^{\infty} dx \left[-\frac{\cos x}{x} + \frac{\sin x}{x^2} \right] \left[\frac{1}{x+a} - \frac{1}{x-a} \right] . \tag{B4}$$

Now we make the substitutions

$$\frac{1}{x} \left[\frac{1}{x+a} - \frac{1}{x-a} \right] = \frac{2}{ax} - \frac{1}{a} \left[\frac{1}{x+a} + \frac{1}{x-a} \right] , \quad \frac{1}{x^2} \left[\frac{1}{x+a} - \frac{1}{x-a} \right] = -\frac{2}{ax^2} + \frac{1}{a^2} \left[\frac{1}{x+a} - \frac{1}{x-a} \right] , \tag{B5}$$

and the integral becomes

$$I = \frac{2}{T} \frac{\omega_{ab}}{|\omega_{ab}|} \int_0^{\infty} dx \left[\frac{2}{a} j_1(x) + \frac{\cos x}{a} \left[\frac{1}{x+a} + \frac{1}{x-a} \right] + \frac{\sin x}{a^2} \left[\frac{1}{x+a} - \frac{1}{x-a} \right] \right] , \tag{B6}$$

where $j_1(x)$ is the spherical Bessel function. With the help of Eqs. (B1) and (B3) we find

$$I = \frac{4}{T} \frac{\omega_{ab}}{|\omega_{ab}|} \left[\frac{1+g(a)}{a} + \frac{f(a)}{a^2} + \frac{\pi}{2} \left[-\frac{\cos a}{a^2} - \frac{\sin a}{a} \right] \right] . \tag{B7}$$

3. Integration of Eq. (42a)

In this case the integral analogous to (B4) is

$$I = \frac{2}{T} \frac{\omega_{ab}}{|\omega_{ab}|} \int_0^{\infty} dx \left[-\sin x - \frac{\cos x}{x} + \frac{\sin x}{x^2} \right] \times \left[\frac{1}{x+a} - \frac{1}{x-a} \right] . \tag{B8}$$

The new terms are given by Eqs. (B1) and (B3) with the result

$$I = \frac{4}{T} \frac{\omega_{ab}}{|\omega_{ab}|} \left[\frac{1+g(a)}{a} + \frac{f(a)}{a^2} - f(a) + \frac{\pi}{2} \left[-\frac{\cos a}{a^2} - \frac{\cos a}{a} + \cos a \right] \right] . \tag{B9}$$

Equations (B7) and (B9) lead immediately to the results in Eqs. (43).

APPENDIX C: SHORT- AND LONG-DISTANCE LIMITS OF $\Delta_{\rm VF}^{\rm cav}$

Equation (44) gives the energy shift $\Delta_{\rm VF}^{\rm cav}$ due to the vacuum fluctuation. Here we determine the asymptotic behavior of $\Delta_{\rm VF}^{\rm cav}$ at large distance ($\omega_{ab}T \Longrightarrow \infty$) and small distance ($\omega_{ab}T \Longrightarrow 0$).

1. Short distance

As $x \Longrightarrow 0$

$$f(x) \Longrightarrow \frac{\pi}{2} + (\gamma - 1 + \ln x)x + \left[\frac{\pi}{4} \right] x^2 + O(x^3) , \quad g(x) \Longrightarrow -(\gamma + \ln x) + \left[\frac{\pi}{2} \right] x + O(x^2) , \quad \left[-\frac{\cos x}{x^2} - \frac{\sin x}{x} \right] \Longrightarrow -\frac{1}{x^2} - \frac{1}{2} + O(x^2) . \tag{C1}$$

Therefore

$$\frac{g(x)}{x} + \frac{f(x)}{x^2} + \frac{\pi}{2} \left[-\frac{\cos x}{x^2} - \frac{\sin x}{x} \right] \Longrightarrow -\frac{1}{x} + \frac{\pi}{2} + O(x) \tag{C2}$$

and

$$\frac{1+g(x)}{x} + \frac{f(x)}{x^2} - f(x) + \frac{\pi}{2} \left[-\frac{\cos x}{x^2} - \frac{\sin x}{x} + \cos x \right] \Longrightarrow \frac{\pi}{2} + O(x) . \tag{C3}$$

Hence the short-distance limit of Eq. (44) is

$$\Delta_{\rm VF}^{\rm cav} = \frac{e^2}{4\pi^2 \epsilon_0 m^2 c^3} \frac{1}{T} \sum_b \frac{\omega_{ab}}{|\omega_{ab}|} \left[|\langle b|p_\rho|a\rangle|^2 \left[\frac{\pi}{2} + O(\omega_{ab}T) \right] + |\langle b|\sqrt{2}p_z|a\rangle|^2 \left[-\frac{1}{|\omega_{ab}|T} + \frac{\pi}{2} + O(\omega_{ab}T) \right] \right] . \tag{C4}$$

2. Long distance

As $x \Longrightarrow \infty$

$$f(x) \Longrightarrow \frac{1}{x}\left[1-\frac{2!}{x^2}+\frac{4!}{x^3}-\cdots\right], \quad g(x) \Longrightarrow \frac{1}{x^2}\left[1-\frac{3!}{x^2}+\frac{5!}{x^4}-\cdots\right], \tag{C5}$$

therefore

$$\left[\frac{1+g(|\omega_{ab}|T)}{|\omega_{ab}|T}+\frac{f(|\omega_{ab}|T)}{(|\omega_{ab}|T)^2}-f(|\omega_{ab}|T)\right] \Longrightarrow \frac{4}{(|\omega_{ab}|T)^3} \tag{C6}$$

and

$$\left[\frac{g(|\omega_{ab}|T)}{|\omega_{ab}|T}+\frac{f(|\omega_{ab}|T)}{(|\omega_{ab}|T)^2}\right] \Longrightarrow \frac{2}{(|\omega_{ab}|T)^3} . \tag{C7}$$

Equations (C6) and (C7) lead us immediately from Eq. (44) to Eq. (45).

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