Demonstration of the Einstein-Podolsky-Rosen paradox using nondegenerate parametric amplification

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We point out in this paper the possibility of demonstrating the Einstein-Podolsky-Rosen paradox via quadrature phase measurements performed on the two output beams of a nondegenerate parametric amplifier. A technique that might be used to demonstrate the paradox has already been partly developed experimentally.

I. INTRODUCTION

There has been much recent interest in the quantum features displayed by the output fields of a nondegenerate parametric amplifier (or nondegenerate parametric oscillator where the parametric medium is placed in a cavity). In such a device, a pump photon is destroyed and a signal and idler photon pair is created. The correlation of photon number between signal and idler modes is greater than that predicted by standard classical theory, and there is a violation of a classical Cauchy-Schwarz inequality involving intensity correlations. There has been experimental demonstration of these and related features by Burnham and Weinberg,[1] Friberg *et al.*,[2] and Jakeman and Walker.[3] Theoretical quantum analyses have been given by Graham and Haken[4] and McNeil and Gardiner.[5] Reynaud *et al.*[6] have recently predicted a reduction of fluctuations to occur in the spectrum of the signal-idler intensity difference because of the intensity correlation. Such a reduction has now been observed experimentally by Heidmann *et al.*[7]

Graham[8] has pointed out that such photon number correlations in the nondegenerate parametric oscillator might remind one of Einstein-Podolsky-Rosen[9] correlations. Reid and Walls[10] later predicted a violation of Bell's inequality[11] where there is a strong violation of the classical Cauchy-Schwarz inequality involving intensity correlation. There has been a recent two-photon interference experiment by Ou and Mandel[12] demonstrating a violation of Bell's inequality in the output field of a nondegenerate parametric amplifier. Such proposals, as with the Bell inequality experiments of Aspect *et al.*,[13] involve joint photon-counting measurements.

In this paper we are concerned with the demonstration of the Einstein-Podolsky-Rosen (EPR) paradox itself, as distinct from tests of Bell's inequalities. The EPR paradox became famous as an argument for the hypothesis that the quantum-mechanical description of a physical system is "incomplete," and later stimulated Bell to derive his famous inequalities. The paradox concerns the existence of high correlations between observables of two spatially separated subsystems. The paradox occurs where both of two noncommuting observables of one subsystem are highly correlated with observables of the second subsystem. The original version due to EPR was formulated in terms of two spatially separated particles which have highly correlated positions, and momenta. A modified discrete version of the paradox was later presented by Bohm.[14] He considered a pair of spatially separated spin-$\frac{1}{2}$ particles which show high correlation between their various spin components.

The purpose of this paper is to draw attention to the experimental possibility of demonstration of the original EPR paradox in the nondegenerate parametric amplifier (or related systems). Thus we become concerned not only with the signal and idler intensity correlation, but with the signal and idler *phase* correlation as well. Phase correlations in the nondegenerate parametric oscillator have been discussed by Reynaud *et al.*[6] We point out that the paradox may be formulated in terms of the field quadrature phase amplitudes.[15] Such amplitudes are measurable by homodyne detection techniques now well established experimentally. We show that for an ideal parametric amplifier both the conjugate quadrature phase amplitudes of signal and idler are highly correlated and in principle provide an example of the EPR paradox. Such a correlation will manifest itself as a reduction in the fluctuations of the signal and idler quadrature phase amplitude difference. We deduce the level of noise reduction required to imply that the correlations are EPR and that the paradox is demonstrated. We also show that EPR correlations exist between the output beams of a beam splitter with squeezed vacuum and coherent vacuum inputs.

There has been much success recently in producing and detecting squeezed light,[16–20] where the fluctuations in one of the quadrature phase amplitudes are reduced below the coherent-state level. With substantial noise reduction[17] in the quadrature phase amplitude of a single beam now possible, we point out in this paper that the demonstration of the EPR paradox via quadrature phase amplitude measurements on two correlated and spatially separated output beams would seem to be a viable possibility. A technique which could be used to test for such a correlation has been demonstrated by Levenson *et al.*[21] and Schumaker *et al.*[22] A discussion of some of the preliminary results of this paper with reference to a nondegenerate parametric oscillator will be presented elsewhere.[23] A demonstration of the paradox via quadrature phase amplitudes is closely in line with the original version of the paradox put forward by EPR. As far as we know, this original version has not been experimentally realized, though there have been recent theoretical suggestions.[24]

II. THE EPR PARADOX AND QUADRATURE PHASE AMPLITUDES

First, let us summarize the reasoning behind the EPR paradox.[9,11] Einstein, Podolsky, and Rosen make three assumptions. Firstly, they assume quantum mechanics predicts correctly at least the results of the experiment discussed below by them. Secondly, "if without in any way disturbing the system, we can predict with certainty the value of a physical quantity, then there exists an element of physical reality corresponding to this quantity." Thirdly, they assume there is "no action at a distance." Einstein, Podolsky, and Rosen then considered two spatially separated particles which show a maximum correlation between their positions *and* their momenta. Such systems are predicted to occur in quantum mechanics. A measurement of the position of particle 2 implies with certainty the result obtained if the position of particle 1 is measured immediately. Assuming there is no action at a distance, the prediction for the position of particle 1 is made without disturbing the particle. The EPR concept of reality leads them to identify with particle 1 a definite predetermined value for its position. Since the momenta of the two particles are also correlated, one may use a similar argument to ascribe to particle 1 a definite predetermined value for its momentum. These arguments lead EPR to identify with particle 1 a state of definite momentum *and* position. In a quantum-mechanical formalism no state can simultaneously have a definite value of both momentum and position. EPR thus reached the conclusion that quantum mechanics gives only an incomplete description of the state of the particle.

In this paper we consider an analogous situation where one measures, not position and momentum of particles, but the quadrature phase components of two correlated and spatially separated light fields. We consider, in the first instance, two single-mode fields E_A and E_B at positions r_A and r_B, and of frequency ω_a and ω_b, respectively. We note that ω_a may equal ω_b but the fields must be spatially separated. We may write the fields in terms of the boson operators, and the associated quadrature phase amplitude operators, as follows:

$$\begin{aligned}\hat{E}_A &= \lambda(\hat{a}e^{-i\omega_a t}+\hat{a}^\dagger e^{i\omega_a t}) \\ &= \frac{\lambda}{4}[\hat{X}_1\cos(\omega_a t)+\hat{X}_2\sin(\omega_a t)] , \\ \hat{E}_B &= \lambda(\hat{b}e^{-i\omega_b t}+\hat{b}^\dagger e^{i\omega_b t}) \\ &= \frac{\lambda}{4}[\hat{Y}_1\cos(\omega_b t)+\hat{Y}_2\sin(\omega_b t)] .\end{aligned} \tag{2.1}$$

The λ (taken to be equal for each mode) is a constant incorporating spatial factors. We deal with optical frequencies where $(\omega_a-\omega_b)/\omega_a \ll 1$ to a good approximation. The quadrature phase amplitudes are defined

$$\begin{aligned}\hat{X}_\theta &= (\hat{a}e^{-i\theta}+\hat{a}^\dagger e^{i\theta}) , \\ \hat{Y}_\phi &= (\hat{b}e^{-i\phi}+\hat{b}^\dagger e^{i\phi}) ,\end{aligned} \tag{2.2}$$

and we use the notation

$$\hat{X}_1=\hat{X}_0,\quad \hat{X}_2=\hat{X}_{\pi/2},\quad \hat{Y}_1=\hat{Y}_0,\quad \hat{Y}_2=\hat{Y}_{\pi/2} .$$

The conjugate variables $\hat{X}_1$ and $\hat{X}_2$ are noncommuting, with $[\hat{X}_1,\hat{X}_2]=2i$. We write

$$\Delta\hat{X}_1^2\Delta\hat{X}_2^2 \geq 1 . \tag{2.3}$$

Now if the fields are prepared in such a way that the amplitude $\hat{X}_1$ is maximally correlated with $\hat{Y}_2$ (say), and $\hat{X}_2$ is maximally correlated with $\hat{Y}_1$, the EPR reasoning will apply. We next show that, as one example, the output fields of an ideal nondegenerate parametric amplifier exhibit such correlations.

III. PRODUCTION OF THE CORRELATED STATE: THE NONDEGENERATE PARAMETRIC AMPLIFIER

The nondegenerate parametric amplifier is often modeled by the simple interaction Hamiltonian

$$H_I = -\hbar\kappa(\hat{a}^\dagger\hat{b}^\dagger+\hat{a}\hat{b}) . \tag{3.1}$$

Here κ is a nonlinear coupling coefficient proportional to the nonlinear susceptibility of the medium and to the amplitude of the pump field (of frequency $\omega_a+\omega_b$). The pump is assumed to be undepleted and of sufficient intensity that it may be modeled classically. We take κ real for convenience. The modes must be nondegenerate, that is, either different frequencies or different polarizations. We point out that (3.1) is an ideal Hamiltonian. In a realistic situation there will be other effects such as loss which will tend to degrade the correlations predicted from (3.1). Hamiltonians of this type, however, have been successful in predicting experimentally observed quantum effects, such as "squeezing"[17,18] and correlations enabling "quantum nondemolition measurements."[21,22]

From the model (3.1), the quadrature phase amplitudes $\hat{X}_\theta$ after an interaction time $T=L/v$ with the medium are readily found to be

$$\begin{aligned}\hat{X}_1(L) &= \hat{X}_1(0)\cosh r+\hat{Y}_2(0)\sinh r , \\ \hat{X}_2(L) &= \hat{X}_2(0)\cosh r+\hat{Y}_1(0)\sinh r , \\ \hat{Y}_1(L) &= \hat{Y}_1(0)\cosh r+\hat{X}_2(0)\sinh r , \\ \hat{Y}_2(L) &= \hat{Y}_2(0)\cosh r+\hat{X}_1(0)\sinh r ,\end{aligned} \tag{3.2}$$

where $r=\kappa T$. The solutions are a simple model for the situation we envisage of two fields propagating in one direction with speed v through a nonlinear crystal of length L. $\hat{X}_{1(2)}(0)$ and $\hat{Y}_{1(2)}(0)$ are the input amplitudes, and $\hat{X}_{1(2)}(L)$ and $\hat{Y}_{1(2)}(L)$ are the output amplitudes. We denote $\hat{a}$ as the signal field, and $\hat{b}$ as the idler field. Because the fields are nondegenerate, we may spatially separate the output signal and idler amplitudes $\hat{X}_i(L)$ and $\hat{Y}_i(L)$.

The quantum-mechanical Cauchy-Schwarz inequality for the quadrature phase amplitudes is as follows:

$$|\langle \hat{X}_\theta(L)\hat{Y}_\phi(L)\rangle|^2 \le \langle [\hat{X}_\theta(L)]^2\rangle\langle[\hat{Y}_\phi(L)]^2\rangle \ . \quad (3.3)$$

We define the associated quantum-mechanical correlation coefficient

$$C_{\theta\phi} = \frac{\langle X_\theta(L)\hat{Y}_\phi(L)\rangle}{[\langle \hat{X}_\theta(L)^2\rangle\langle \hat{Y}_\phi(L)^2\rangle]^{1/2}} \ . \quad (3.4)$$

We now proceed to calculate the correlation between the quadrature phase amplitude outputs given by (3.2). One has the following relation:

$$\hat{X}_\theta(L) = \hat{X}_1(L)\cos\theta + \hat{X}_2(L)\sin\theta \ ,$$
$$\hat{Y}_\phi(L) = \hat{Y}_1(L)\cos\phi + \hat{Y}_2(L)\sin\phi \ . \quad (3.5)$$

For uncorrelated vacuum inputs with $\langle\hat{a}\rangle = \langle\hat{b}\rangle = \langle\hat{a}^2\rangle = \langle\hat{b}^2\rangle = \cdots = 0$ and $\langle[\hat{X}_1(0)]^2\rangle = \langle[\hat{X}_2(0)]^2\rangle = \langle[\hat{Y}_1(0)]^2\rangle = \langle[\hat{Y}_2(0)]^2\rangle = 1$, one finds

$$\langle \hat{X}_1(L)\hat{Y}_2(L)\rangle = 2\cosh r \sinh r = \langle \hat{X}_2(L)\hat{Y}_1(L)\rangle \ ,$$
$$\langle \hat{X}_1(L)\hat{Y}_1(L) = 0 = \langle \hat{X}_2(L)\hat{Y}_2(L)\rangle = \langle \hat{X}_1(L)\hat{X}_2(L)\rangle \ ,$$
$$\langle [\hat{X}_1(L)]^2\rangle = \cosh^2 r + \sinh^2 r$$
$$= \langle [\hat{X}_2(L)]^2\rangle = \langle [\hat{Y}_1(L)]^2\rangle = \langle [\hat{Y}_2(L)]^2\rangle \ . \quad (3.6)$$

Hence we calculate

$$\langle \hat{X}_\theta(L)\hat{Y}_\phi(L)\rangle = 2\cosh r \sinh r \sin(\theta+\phi) \ ,$$
$$\langle [\hat{X}_\theta(L)]^2\rangle = \langle [\hat{Y}_\phi(L)]^2\rangle = \cosh^2 r + \sinh^2 r \ . \quad (3.7)$$

The correlation coefficient for $\hat{X}_\theta(L)$ and $\hat{Y}_\phi(L)$ is thus

$$C_{\theta\phi} = \tanh(2r)\sin(\theta+\phi) \ . \quad (3.8)$$

The result for $C_{\theta\phi}$ is particularly interesting. We note that for r large, the correlation coefficient becomes

$$C_{\theta\phi} = \sin(\theta+\phi) \ . \quad (3.9)$$

There is a perfect correlation ($|C_{\theta\phi}| = 1$) for $\theta+\phi = \pi/2$. That is, there is perfect correlation between the quadrature phase amplitudes $\hat{X}_1$ and $\hat{Y}_2$, and also $\hat{X}_2$ and $\hat{Y}_1$. Thus we have the situation of the Einstein-Podolsky-Rosen *Gedankenexperiment*, as discussed in Sec. II.

We see how the correlation between output quadratures is very sensitive to the amplification parameter r. In realistic situations r is never infinite, and also there will be other factors, such as loss, downgrading the correlation. We thus ask the question can the *Gedankenexperiment* be properly realized for situations involving less than the maximum correlation?

IV. DEMONSTRATION OF THE EPR PARADOX

The paradox is about the ability to infer an observable of one system from the result of measurement performed on a second system spatially separated from the first. For observables not maximally correlated, there will be an error in making such an inference. It is still possible, however, to obtain a paradox providing the error is small enough, compared to the uncertainty predicted by the Heisenberg uncertainly principle. In this section we calculate the "inference error" and determine how small it has to be in order to have an EPR paradox.

The solutions (3.2) and (3.9) indicate a correlation between the output signal amplitude $\hat{X}_1(L)$ and idler amplitude $\hat{Y}_2(L)$. Let us propose that a measurement of the idler amplitude $\hat{Y}_\phi(L)$ will infer the result for the signal amplitude $\hat{X}_1(L)$.[25–27] Thus one monitors "at a distance" $\hat{X}_1(L)$ by a scaled readout $\hat{X}_1^0(L)$ of $\hat{Y}_\phi(L)$, where

$$\hat{X}_1^0(L) = gY_\phi(L) \ . \quad (4.1)$$

The result of $\hat{X}_1^0(L)$ is the inferred estimate of the signal amplitude $\hat{X}_1(L)$. We have introduced a possible scaling parameter g, which we will adjust to allow for greatest accuracy in the determination of $\hat{X}_1(L)$. The deviation of the scaled readout $\hat{X}_1^0(L)$ from the true signal amplitude $\hat{X}_1(L)$ is determined simply by the difference $\hat{X}_1(L) - \hat{X}_1^0(L)$. Hence the average error $\Delta_{\rm inf}\hat{X}_1(L)$ in our estimate of $\hat{X}_1(L)$ is given by

$$V_1(g,\phi) = \Delta^2_{\rm inf}\hat{X}_1(L) = \langle [\hat{X}_1(L) - \hat{X}_1^0(L)]^2\rangle$$
$$= \langle [\hat{X}_1(L) - g\hat{Y}_\phi(L)]^2\rangle \ . \quad (4.2)$$

Our definition of the best estimate $\hat{X}_1^0(L)$ is such that the scaling factor g and the angle ϕ give a minimum $\Delta^2_{\rm inf}\hat{X}_1(L)$. Setting $\partial V_1(g,\phi)/\partial g = 0$, we deduce that

$$g = \frac{\langle \hat{X}_1(L)\hat{Y}_\phi(L)\rangle}{\langle [\hat{Y}_\phi(L)]^2\rangle} \ . \quad (4.3)$$

In this case the error is minimized with $\phi = \pi/2$ and the corresponding minimum error is

$$[V_1(g,\phi)]_{\rm min} = \langle [\hat{X}_1(L)]^2\rangle - \frac{\langle \hat{X}_1(L)\hat{Y}_2(L)\rangle^2}{\langle [\hat{Y}_2(L)]^2\rangle} \ . \quad (4.4)$$

For the situation of coherent vacuum inputs considered here, the solutions are given by (3.6) and thus the idealized result is

$$[V_1(g,\phi)]_{\rm min} = \Delta^2_{\rm inf}[\hat{X}_1(L)]_{\rm min} = \frac{1}{\cosh(2r)} \quad (4.5)$$

with

$$g = \tanh(2r) \ . \quad (4.6)$$

Clearly the error $\Delta_{\rm inf}\hat{X}_1(L)$ in our estimate of $\hat{X}_1(L)$ becomes negligible with sufficiently large amplification r.

We may compare the results (4.1) and (4.5) with previous calculations by Milburn *et al.*[27] These authors consider, as we do here, the inference of a signal amplitude $\hat{X}_1(L)$ by a measurement of the idler amplitude $\hat{Y}_2(L)$. One would like to calculate the "conditional probability distribution" of the signal variable $\hat{X}_1(L)$, given a particular readout of the idler amplitude $\hat{Y}_2(L)$. Milburn *et al.* calculate by projection operator techniques a density operator for the state of the signal, given a particular result $y_2(L)$ of a measurement of $\hat{Y}_2(L)$ (thus calculating the effect of the idler state reduction). From this reduced density operator they calculate the mean and variance of the conditional signal amplitude $\hat{X}_1(L)_R$, the signal given an idler readout $y_2(L)$. We would hope that our best estimate of $\hat{X}_1(L)$, given a result $y_2(L)$, corresponds to the mean of the conditional distribution, and that the minimum inference error $\Delta^2_{\text{inf}}\hat{X}_1(L)_{\min}$ corresponds to the variance of the conditional distribution. This is indeed the case. The mean and variance of $[\hat{X}_1(L)]_R$ as calculated by Milburn *et al.* is [with vacuum inputs $\langle \hat{X}_1(0)\rangle = \langle \hat{Y}_2(0)\rangle = 0$]

$$\langle [\hat{X}_1(L)]_R \rangle = \tanh(2r) y_2(L) \;, \tag{4.7}$$

$$\Delta^2[\hat{X}_1(L)]_R = \frac{1}{\cosh(2r)} \;. \tag{4.8}$$

The results for $\Delta^2[\hat{X}_1(L)]_R$ in this case are uniform throughout all possible outcomes $y_2(L)$, and hence coincide with our averaged result (4.5). We choose to phrase the paradox in terms of the averaged error $\Delta^2_{\text{inf}}\hat{X}_1(L)$ rather than the conditional error $\Delta^2[\hat{X}_1(L)]_R$ itself because it is measurable experimentally by techniques discussed in Sec. V.

The result (3.9) also indicates a strong correlation between $\hat{Y}_1(L)$ and $\hat{X}_2(L)$, and we could alternatively choose to measure the idler amplitude $\hat{Y}_1(L)$ and infer from this result the signal amplitude $\hat{X}_2(L)$. Using arguments similar to those above, we introduce an estimate $\hat{X}^0_2(L) = \bar{g}\,\hat{Y}_1(L)$ for $\hat{X}_2(L)$, and define an error $\Delta_{\text{inf}}\hat{X}_2(L)$ in the estimate. Choosing $\bar{g}$ to minimize this error, we calculate this error to be

$$\begin{aligned}[V_2(\bar{g},\phi)]_{\min} &= \Delta^2_{\text{inf}}[\hat{X}_2(L)]_{\min} \\ &= \langle [\hat{X}_2(L) - \hat{X}^0_2(L)]^2 \rangle_{\min} \\ &= \langle [\hat{X}_2(L)]^2 \rangle - \frac{\langle \hat{X}_2(L)\hat{Y}_1(L)\rangle^2}{\langle Y_1(L)\rangle^2} \\ &= \frac{1}{\cosh(2r)} \;,\end{aligned} \tag{4.9}$$

where $\bar{g} = \langle \hat{X}_2(L)\hat{Y}_1(L)\rangle / \langle [\hat{Y}_1(L)]^2\rangle = \tanh(2r)$.

We are now able to repeat the EPR argument. A measurement of $Y_2(L)$ of the idler beam simultaneously specifies a value for the amplitude $\hat{X}_1(L)$ of the signal beam, with an error $\Delta_{\text{inf}}\hat{X}_1(L)$. Similarly, a measurement of $\hat{Y}_1(L)$ of the idler beam would simultaneously specify a value for $\hat{X}_2(L)$ of the signal, with an error $\Delta_{\text{inf}}\hat{X}_2(L)$. Thus, since the beams are spatially well separated and assuming there is no action at a distance, the EPR concept of reality would lead one to assign to the signal beam predetermined values for $\hat{X}_1(L)$ [with uncertainty $\Delta_{\text{inf}}\hat{X}_1(L)$] and for $\hat{X}_2(L)$ [with uncertainty $\Delta_{\text{inf}}\hat{X}_2(L)$]. According to quantum mechanics, the signal quadrature phase amplitudes $\hat{X}_1(L)$ and $\hat{X}_2(L)$ are noncommuting operators and cannot both be simultaneously specified with certainty greater than that allowed by the uncertainty principle. Thus (for all quantum-mechanical descriptions of this state)

$$\Delta X_1(L)\Delta\hat{X}_2(L) \geq 1 \;. \tag{4.10}$$

Thus the assignment to the signal beam of a state with values of $\hat{X}_1(L)$ and $\hat{X}_2(L)$ defined to accuracy

$$\Delta_{\text{inf}}\hat{X}_1(L)\Delta_{\text{inf}}\hat{X}_2(L) < 1 \tag{4.11}$$

is in apparent contradiction with quantum-mechanical formalism. In this example we have

$$\Delta_{\text{inf}}\hat{X}_1(L)\Delta_{\text{inf}}\hat{X}_2(L) = 1/\cosh(2r)$$

and hence the paradox.

Experimental demonstration of the paradox thus occurs where

$$V_1(g,\phi)V_2(\bar{g},\bar{\phi}) = \Delta^2_{\text{inf}}\hat{X}_1(L)\Delta^2_{\text{inf}}\hat{X}_2(L) < 1 \;. \tag{4.12}$$

The noise level corresponding to the minimum uncertainty product is that of the input signal vacuum fluctuations $[\Delta^2\hat{X}_1(0) = \Delta^2\hat{X}_2(0) = 1]$. The parameters $g, \bar{g}, \phi$ and $\bar{\phi}$ are arbitrary but in practice are chosen to minimize the variances. The subscripts 1 and 2 are not arbitrary but refer to two conjugate quadrature phase amplitudes of the signal.

The procedure then is to take many measurements and to calculate the averages V_1 and V_2. Because one cannot measure $\hat{X}_1$ and $\hat{X}_2$ at the same time, the measurements of V_1 and V_2 are not made simultaneously. This is the same situation as in the original paradox. However, because we calculate an average error for the situation where the correlation is not maximum, this point may need further discussion (see Appendix A).

V. RELATION TO SQUEEZING AND A POSSIBLE EXPERIMENTAL PROCEDURE

We now discuss the experimental measurement of $V_i(g,\phi)$ and hence demonstration of the inequality (4.12). There is a close relationship to the noise quantity $V_i(g,\phi)$ and to a noise quantity ("four-mode squeezing") discussed by Schumaker[28] which has already been measured in dual homodyne detection experiments by Levenson and co-workers.[21,22]

First, let us consider the general noise quantity

$$V_\theta(g,\phi) = \langle (\hat{X}_\theta \pm g\hat{Y}_\phi)^2 \rangle \;. \tag{5.1}$$

$V_\theta(g,\phi)$ is the fluctuation in the signal and idler quadrature phase amplitude difference. We will denote $V_0(g,\phi)$ and $V_{\pi/2}(g,\phi)$ as $V_1(g,\phi)$ and $V_2(g,\phi)$, respectively, in accordance with the notation (2.2) used for the quadrature operators. Where there is a correlation between $\hat{X}_\theta$ and $\hat{Y}_\phi$, the fluctuation $V_\theta(g,\phi)$ may be considerably reduced for an appropriate choice of ϕ, the $\pm$, and the g.

The experiments to date have been interested primarily in the "squeezing" of the field. This occurs where the variance $V_\theta(g,\phi)$ is less than that observed if the signal and idler are independent coherent fields. We rewrite V_θ as follows:

$$V_\theta(g,\phi) = 1 + g^2 + \langle :\hat{X}^2_\theta: \rangle + g^2\langle :\hat{Y}^2_\phi: \rangle \pm 2g\langle \hat{X}_\theta \hat{Y}_\phi \rangle \;, \tag{5.2}$$

where : : denotes normal ordering. If the signal and idler fields are both (coherent) vacuum fields, then $V_\theta(g,\phi)$ is simply $1+g^2$, that noise arising from the quantum commutation relations. It is usual to normalize the quantity $V_\theta(g,\phi)$ to this "vacuum noise level." Thus we define

$$V_{\rm sq}=\frac{V_\theta(g,\phi)}{1+g^2} \tag{5.3}$$

and squeezing occurs where

$$V_{\rm sq}<1 \ . \tag{5.4}$$

The quantity $V_\theta(g,\phi)$ is directly related to $V_1(g,\phi)$ and $V_2(g,\phi)$, as defined in Eqs. (4.2) and (4.9). We note that the requirement

$$V_\theta(g,\phi)<1 \tag{5.5}$$

is more stringent than that for squeezing. We also point out that the choice of g will be different depending on whether one is minimizing $V_\theta(g,\phi)$ or $V_{\rm sq}$. We point out that this result implies one can demonstrate the paradox only with fields that are quantum, in the sense that they have a singular or negative Glauber-Sudarshan $P(\alpha)$ distribution. This is clear,[29] since a classical field is describable in terms of a well-behaved $P(\alpha)$ which predicts a positive normally ordered variance $:V:$. Hence $V_{\rm sq}>1$, for any choice of angle θ and the result (5.5) is never achieved.

An experimental procedure to measure the noise quantity $V_\theta(g,\phi)$ and to compare it with the (coherent) vacuum noise level has already been demonstrated by Levenson *et al.*[21] and Schumaker *et al.*[22] The correlation between signal and idler quadrature amplitudes was generated in their experiment by four-wave-mixing processes occurring in an optical fiber, cooled to below 2 K. In this paper we have discussed generation of the correlation via a simple model of nondegenerate parametric amplification. Although the details of solutions differ somewhat, the EPR correlations are produced by a number of quadratic Hamiltonians and a general principle of detection would apply.

Figure 1 depicts a possible experimental arrangement. The output signal and idler beams are first well separated spatially in some manner, for example, by a prism or polarizer. The idler and signal quadrature amplitude phases $\hat{Y}_\phi(L)$ and $\hat{X}_\theta(L)$ are each individually homodyne detected. $\hat{Y}_\phi(L)$ is measured by combining the idler field with a local oscillator $E^Y_{\rm LO}$ field at the idler frequency and phase shifted ϕ with respect to the idler. The idler beats with the local oscillator $E^Y_{\rm LO}$ at the idler photodetector and gives rise to a photocurrent which depends on the quadrature operators according to $\hat{i}_Y \sim |E^Y_{\rm LO}|\hat{Y}_\phi(L)$. $\hat{X}_\theta(L)$ is similarly detected by combining the signal field with a local oscillator $E^X_{\rm LO}$, with a relative phase shift θ. The photocurrent from the signal photodetector is $\hat{i}_x \sim |E^X_{\rm LO}|\hat{X}_\theta(L)$. In the parametric amplifier, the local oscillators would be derivatives of the pump field E, converted in frequency and phase shifted in some manner. Since this is hard to do in practice, it may be better to use signal and idler fields of the same frequency but with different polarizations. The pump could be split via beam splitters to provide two local oscillator fields. It is usual[16] to employ a balanced homodyne detection scheme to remove unwanted noise contributions to the photocurrents i_Y, i_X due to the local oscillator. In four-wave mixing, two pumps (local oscillators) are already available. A measurement of this type was made by Schumaker *et al.*,[22] who used copropagating four-wave mixing with two pumps widely separated in frequency.

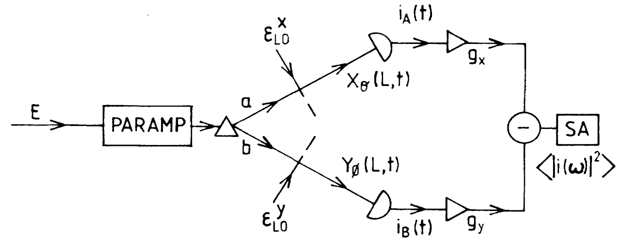

FIG. 1. Schematic diagram of the apparatus used to measure $V_\theta(g,\phi,\omega)$. In the first experiment (as depicted), a and b are the spatially separated but correlated outputs of the parametric amplifier (PARAMP). The homodyne detection enables measurement of the quadrature phase amplitudes $X_\theta(L,t)$ and $Y_\phi(L,t)$. The power spectrum $\langle |i(\omega)|^2\rangle$ of the difference current is measured at the spectrum analyzer. The parameters g_y/g_x and ϕ are chosen to minimize the fluctuations in the power spectrum $\langle |i(\omega)|^2\rangle$. To determine $V_\theta(g,\phi,\omega)$, one must normalize the noise level $\langle |i(\omega)|^2\rangle$ by $\langle |i_x(\omega)|^2\rangle_{\rm coh}$, the vacuum (minimum uncertainty state) noise level of the signal. This is determined in a second experiment. To demonstrate the EPR paradox one must measure both V_1 and V_2 and check $V_1V_2<1$, where the subscripts 1 and 2 refer to two conjugate quadrature phase amplitudes of the signal.

In a realistic traveling-wave situation, we have a frequency band of coupled parametric amplifiers. Best noise reduction measurements are achieved by measuring the spectrum of fluctuations in the photocurrents, so that frequency bands with minimal noise can be isolated. Our idler (signal) output, in fact, is a frequency band of sidemodes $\hat{b}(\omega_b+\omega)$ and $\hat{b}(\omega_b-\omega)$ [and $\hat{a}(\omega_a+\omega)$, $\hat{a}(\omega_a-\omega)$] centered about the local oscillator frequency ω_b (or ω_a). The interaction Hamiltonian is [compare with (3.1)]

$$H=-\hbar\kappa[\hat{a}(\omega)\hat{b}(-\omega)+\hat{a}^\dagger(\omega)\hat{b}^\dagger(-\omega)] \ , \tag{5.6}$$

where ω is the shift in frequency of the sidebands from the signal and idler local oscillator frequencies, ω_a and ω_b. The detectable output Fourier-transformed quadrature operators are[30] (we assume $\omega \ll \omega_a,\omega_b$)

$$\begin{aligned}\hat{X}_\theta(L,\omega)&=\hat{a}(\omega)e^{-i\theta}+\hat{a}^\dagger(-\omega)e^{i\theta} \ ,\\ \hat{Y}_\phi(L,\omega)&=\hat{b}(\omega)e^{-i\phi}+\hat{b}^\dagger(-\omega)e^{i\phi} \ .\end{aligned} \tag{5.7}$$

Their solutions are given in terms of their vacuum inputs $\hat{X}_\theta(0,\omega)$, $\hat{Y}_\phi(0,\omega)$ by the previous result (3.2), but replacing $\hat{X}_i(L)$ with $\hat{X}_i(L,\omega)$ and $\hat{Y}_i(L)$ with $\hat{Y}_i(L,\omega)$ (and also writing the Hermitan conjugate equations). The frequency space quadrature operators $\hat{X}_i(L,\omega)$ themselves are not Hermitian. However, the real and imaginary parts are and correspond to the observable quantities, as discussed by Schumaker.[31] One defines the following operators

$$\mathrm{Re}^*\hat{Y}_\phi(L,\omega)=\frac{\hat{Y}_\phi(L,\omega)+[\hat{Y}_\phi(L,\omega)]^\dagger}{\sqrt{2}}\;,$$
$$\mathrm{Im}^*\hat{Y}_\phi(L,\omega)=\frac{\hat{Y}_\phi(L,\omega)-[\hat{Y}_\phi(L,\omega)]^\dagger}{\sqrt{2}i}\;. \tag{5.8}$$

We note that $\mathrm{Re}^*\hat{Y}_\phi(L,\omega)$ $[\mathrm{Im}^*\hat{Y}_\phi(L,\omega)]$ are $\sqrt{2}$ times the real [imaginary] part of $\hat{Y}_\phi(L,\omega)$. The operators $\mathrm{Re}^*\hat{X}_\theta(L,\omega)$ and $\mathrm{Im}^*\hat{X}_\theta(L,\omega)$ are defined similarly.

It has been shown that[30] the detector currents have Fourier components proportional to the Fourier-transformed quadrature operators:

$$\begin{aligned}\hat{i}_B(\omega)&\sim\int_{-T/2}^{T/2}e^{i\omega t}\hat{i}_B(t)dt\\&\sim|E_{\mathrm{LO}}^Y|^2\int_{-T/2}^{T/2}e^{i\omega t}\hat{Y}_\phi(L,t)dt\\&\sim|E_{\mathrm{LO}}^Y|^2\hat{Y}_\phi(L,\omega)\\&=\frac{|E_{\mathrm{LO}}^Y|^2}{\sqrt{2}}[\mathrm{Re}^*\hat{Y}_\phi(L,\omega)+i\,\mathrm{Im}^*\hat{Y}_\phi(L,\omega)]\end{aligned} \tag{5.9}$$

and

$$\mathrm{Re}^*\hat{Y}_\phi(L,\omega)\sim\sqrt{2}\int_{-T/2}^{T/2}\cos(\omega t)Y_\phi(L,t)dt\;,$$
$$\mathrm{Im}^*\hat{Y}_\phi(L,\omega)\sim\sqrt{2}\int_{-T/2}^{T/2}\sin(\omega t)Y_\phi(L,t)dt\;.$$

Here T is the detection time assumed to be large compared to the field coherence time. One may also write[31]

$$\begin{aligned}\hat{i}_B(t)&\sim|E_{\mathrm{LO}}^Y|\int_{-\infty}^{\infty}e^{-i\omega t}\hat{Y}_\phi(L,\omega)d\omega\\&=\sqrt{2}|E_{\mathrm{LO}}^Y|\int_0^\infty[\mathrm{Re}^*\hat{Y}_\phi(L,\omega)\cos(\omega t)\\&\qquad+\mathrm{Im}^*\hat{Y}_\phi(L,\omega)\sin(\omega t)]d\omega\;.\end{aligned} \tag{5.10}$$

The operator for the modulation signal at the output of a homodyne detector [the frequency ω component of $\hat{i}_B(t)$, say] has the form[31]

$$\mathrm{Re}^*\hat{Y}_\phi(L,\omega)\cos(\omega t)+\mathrm{Im}^*\hat{Y}_\phi(L,\omega)\sin(\omega t)\;.$$

Thus Schumaker points out one might measure the signal $\mathrm{Re}^*\hat{Y}_\phi(L,\omega)$ [or $\mathrm{Im}^*\hat{Y}_\phi(L,\omega)$] by mixing this output of the homodyne detector with a $\cos(\omega t)$ [or $\sin(\omega t)$] wave and examining the resulting zero-frequency output.

We assume the free-field commutation relation for the (input) boson operator $\hat{a}(\omega)$: $[\hat{a}(\omega),\hat{a}^\dagger(\omega)]=1$ and all operators at different frequencies commute. We can thus determine the commutation relations for $\mathrm{Re}^*\hat{X}_\theta(L,\omega)$, $\mathrm{Im}^*\hat{X}_\theta(L,\omega)$, ... We have

$$[\mathrm{Re}^*\hat{X}_1(L,\omega),\mathrm{Re}^*\hat{X}_2(L,\omega)]=2i\;,$$
$$[\mathrm{Im}^*\hat{X}_1(L,\omega),\mathrm{Im}^*\hat{X}_2(L,\omega)]=2i\;. \tag{5.11}$$

The solutions for the outputs in terms of the inputs are readily deduced from the solutions for $\hat{X}_\theta(L,\omega)$, $\hat{Y}_\phi(L,\omega)$ which follow from the Hamiltonian (5.6) as discussed in Sec. III. We have

$$\mathrm{Re}^*\hat{X}_1(L,\omega)=\cosh r\,\mathrm{Re}^*\hat{X}_1(0,\omega)+\sinh r\,\mathrm{Re}^*\hat{Y}_2(0,\omega)\;,$$
$$\mathrm{Re}^*\hat{Y}_2(L,\omega)=\cosh r\,\mathrm{Re}^*\hat{Y}_2(0,\omega)+\sinh r\,\mathrm{Re}^*\hat{X}_1(0,\omega)\;, \tag{5.12}$$

$$\mathrm{Re}^*\hat{X}_2(L,\omega)=\cosh r\,\mathrm{Re}^*\hat{X}_2(0,\omega)+\sinh r\,\mathrm{Re}^*\hat{Y}_1(0,\omega)\;,$$
$$\mathrm{Re}^*\hat{Y}_1(L,\omega)=\cosh r\,\mathrm{Re}^*\hat{Y}_1(0,\omega)+\sinh r\,\mathrm{Re}^*\hat{X}_2(0,\omega)\;,$$

and an identical set of equations for the $\mathrm{Im}^*\hat{X}_i(L,\omega)$ and $\mathrm{Im}^*\hat{Y}_i(L,\omega)$ (simply replace Re* with Im*). The solutions and commutation relations are identical to those discussed earlier [Eq. (3.2)] except that we now have two sets of correlated observables, corresponding to the real and imaginary parts of the $\hat{X}_\theta(L,\omega)$ and $\hat{Y}_\phi(L,\omega)$. This was pointed out by Schumaker *et al.*[22] and Levenson and Shelby.[22] The final solutions for the correlations with vacuum inputs are identical to (3.8), (4.5), and (4.9). Thus the observable homodyne detection signal and idler outputs $\mathrm{Re}^*\hat{Y}_\phi(L,\omega)$ and $\mathrm{Re}^*\hat{X}_\theta(L,\omega)$ [or $\mathrm{Im}^*\hat{Y}_\phi(L,\omega)$ and $\mathrm{Im}^*\hat{X}_\theta(L,\omega)$] are predicted to be correlated in an EPR fashion.

Such a correlation will manifest itself as a reduction of fluctuations in the signal and idler difference current, as observed in the "quantum nondemolition" and "four-mode squeezing" experiments of Levenson and coworkers.[21,22] One can measure directly $\hat{Y}_\phi(L,\omega)$ and $\hat{X}_\theta(L,\omega)$ and subtract to determine the error. This is equivalent to measuring fluctuations in the Fourier component of the difference current. The output currents from the two detectors are individually amplified by factors g_x and g_y. The combined difference current $i_-(t)$ has Fourier component

$$\begin{aligned}\hat{i}_-(\omega)&\sim\int_{-T/2}^{T/2}e^{i\omega t}\hat{i}_-(t)\\&\sim g_x|E_{\mathrm{LO}}^X|\hat{X}_\theta(L,\omega)-g_y|E_{\mathrm{LO}}^Y|\hat{Y}_\phi(L,\omega)\\&=g_x|E_{\mathrm{LO}}^X|\Delta_\theta(g,\omega)\;,\end{aligned} \tag{5.13}$$

where

$$\Delta_\theta(g,\omega)=\hat{X}_\theta(L,\omega)-g\hat{Y}_\phi(L,\omega)$$

and

$$g=\frac{|E_{\mathrm{LO}}^Y|g_y}{|E_{\mathrm{LO}}^X|g_x}\;.$$

One may choose to measure the real or imaginary part, $\mathrm{Re}^*\Delta_\theta(g,\omega)$ or $\mathrm{Im}^*\Delta_\theta(g,\omega)$. We are interested in the noise in this signal. In fact, since we deal with a stationary field we will show that it is sufficient to look at the power spectrum, defined as

$$\langle|i(\omega)|^2\rangle=\lim_{T\to\infty}\frac{1}{2\pi T}\left|\int_{-T/2}^{T/2}dt\,e^{-i\omega t}i(t)\right|^2\;. \tag{5.14}$$

In the limit of T large the time averages become equal to the ensemble averages we use in our theoretical calculations. We write the combined current $i_-(t)$ in terms of its Fourier components as

$$\hat{i}_-(t) \sim \sqrt{2} g_x |E_{\rm LO}^X| \int_0^\infty \{[\mathrm{Re}^*\Delta_\theta(g,\omega)]\cos(\omega t) + [\mathrm{Im}^*\Delta_\theta(g,\omega)]\sin(\omega t)\} d\omega \ . \tag{5.15}$$

Since the components of different frequencies do not correlate, we can write

$$\langle \hat{i}(t)^2 \rangle \sim 2g_x^2 |E_{\rm LO}^X|^2 \int_0^\infty d\omega \{\cos^2(\omega t)\langle [\mathrm{Re}^*\Delta_\theta(g,\omega)]^2\rangle + \sin^2\omega t \langle [\mathrm{Im}^*\Delta_\theta(g,\omega)]^2 \rangle + \cos(\omega t)\sin(\omega t)[\langle \mathrm{Re}^*\Delta_\theta(g,\omega)\mathrm{Im}^*\Delta_\theta(g,\omega)\rangle + \langle \mathrm{Im}^*\Delta_\theta(g,\omega)\mathrm{Re}^*\Delta_\theta(g,\omega)\rangle]\} \ . \tag{5.16a}$$

The time-independent part of the power spectrum is proportional to the average $\{\langle [\mathrm{Re}^*\Delta_\theta(g,\omega)]^2\rangle + \langle [\mathrm{Im}^*\Delta_\theta(g,\omega)]^2\rangle\}/2$. However, for fields of the type we consider, the real and imaginary parts are uncorrelated and have identical correlation properties. In this general case the power spectrum is stationary and we have

$$\langle \hat{i}(t)^2 \rangle \sim 2g_x^2 |E_{\rm LO}^X|^2 \int_0^\infty \langle [\mathrm{Re}^*\Delta_\theta(g,\omega)]^2 \rangle d\omega \ . \tag{5.16b}$$

This result is equivalent to assuming terms such as $\langle [\Delta_\theta(g,\omega)]^2\rangle$ or $\langle [X_\theta(\omega)]^2\rangle$ are zero.[31] This result leads to the standard expression for the power spectrum in terms of quadrature operators:[30]

$$\langle \hat{i}(t)^2 \rangle = 2g_x^2 |E_{\rm LO}^X|^2 \int_0^\infty \langle [\Delta_\theta(g,\omega)\Delta_\theta(g,\omega)^\dagger + \Delta_\theta(g,\omega)^\dagger \Delta_\theta(g,\omega)]/2 \rangle d\omega \ . \tag{5.16c}$$

Thus the time stationary power spectrum of the combined current is

$$\langle |\hat{i}(\omega)|^2 \rangle \sim g_x^2 |E_{\rm LO}^X|^2 \langle |\Delta_\theta(g,\omega)|^2 \rangle = g_x^2 |E_{\rm LO}^X|^2 \langle [\mathrm{Re}^*\Delta_\theta(g,\omega)]^2 \rangle \ , \tag{5.17}$$

where we use the notation[22]

$$\langle |X|^2 \rangle = \langle (XX^\dagger + X^\dagger X)/2 \rangle \ .$$

This noise power spectrum is measurable with a spectrum analyzer as depicted in Fig. 1, and measured in experiments.[16–22] A reduction in the noise level is thus indicative of a correlation between $\hat{X}_\theta(L,\omega)$ and the appropriately deamplified $\hat{Y}_\phi(L,\omega)$ (or the real and imaginary parts of these operators).

The noise power levels must be calibrated relative to the "vacuum" (or shot) noise level of the *signal*. The vacuum noise level is obtained by shining (in a second experiment) coherent light on the detectors.[7,22] The intensities must be the same as the local oscillator intensities ($|E_{\rm LO}^X|^2$ and $|E_{\rm LO}^Y|^2$) in the original experiment. If a balanced homodyne detection scheme is used, the vacuum noise level may be measured by simply removing the correlated fields $\hat{a}$ and $\hat{b}$ from the final beam splitters.[16,18] The input field at each beam splitter is then the (coherent) vacuum, and the power spectrum noise level is the vacuum noise level of the combined signal and idler current. The vacuum (or shot) noise level has been reliably determined in "squeezing" experiments.[16–22] The vacuum noise level of the combined current is

$$\langle |\hat{i}(\omega)|^2 \rangle_{\rm coh} \sim g_x^2 |E_{\rm LO}^X|^2 + g_Y^2 |E_{\rm LO}^Y|^2 \ . \tag{5.18}$$

The noise level corresponding to the vacuum of the *signal* alone is measurable by shining a coherent field on the signal detector and measuring the power spectrum of the signal photocurrent alone.[22] Thus

$$\langle |\hat{i}_X(\omega)|^2 \rangle_{\rm coh} \sim g_x^2 |E_{\rm LO}^X|^2 \ . \tag{5.19}$$

We have assumed 100% detection efficiencies. In a realistic experimental situation, this is not the case and the effect of nonideal detection efficiency would have to be taken into account. We stress that in a quantum-mechanical formulation, this vacuum (or coherent) noise level of the signal is the noise associated with the signal field commutation relations. It represents the minimum uncertainty product for fluctuations in orthogonal quadratures [it represents the 1 on the right-hand side of the inequality (4.12)]. The homodyne detection scheme, which involves beating the quantum field with an intense local oscillator field, magnifies this noise to a macroscopic readout scale. Thus noise levels below this fundamental level can be observed quantitatively.[16–22] It is such a reduction of noise, in the combined current (5.13), which allows us to infer the existence of EPR correlations.

Normalizing to the vacuum noise of the signal, the combined current power level spectrum (5.17) is[22]

$$V_\theta(g,\phi,\omega) = \langle [\mathrm{Re}^*\hat{X}_\theta(L,\omega) - g\,\mathrm{Re}^*\hat{Y}_\phi(L,\omega)]^2 \rangle \ , \tag{5.20}$$

a direct measure of the quantity $V_\theta(g,\phi)$ defined in (4.2) and (5.1). The relative electronic amplification (deamplification) factor g and also the phase ϕ can be adjusted experimentally to give a minimum noise level.

The experiments performed by Schumaker *et al.*[22] have succeeded in a 20% reduction of $V_\theta(g,\phi,\omega)$ below the combined vacuum noise level $V_\theta(g,\phi,\omega) < 1 + g^2$, which they call "four-mode squeezing." Results presented by Levenson and Shelby,[22] in discussion of quantum-nondemolition measurements, are renormalized and indicate a 5% reduction of $V_0(g,\phi,\omega)$ below the vacuum noise level of the signal; i.e., they establish $V_1(g,\phi,\omega) = 0.95$. However, this gives information about the ability to infer only one signal quadrature. We summarize in an appendix the particular correlations corresponding to the experiments of Schumaker *et al.* and Levenson *et al.* and indicate how the inference of the second signal quadrature is more difficult. To achieve an EPR demonstration, two measurements of $V_\theta(g,\phi,\omega)$ are necessary. Both $V_1(g,\phi,\omega)$ and $V_2(\bar{g},\bar{\phi},\omega)$ must be measured and we require that $V_1(g,\phi,\omega)V_2(\bar{g},\bar{\phi},\omega) < 1$.

We have discussed the demonstration of the EPR correlations in terms of spectral noise levels. This is done because the substantial noise reductions ("squeezing") have been achieved experimentally in spectral measurements. There are several points, however, to be made regarding this.

The original paradox as discussed in Secs. II and IV is phrased in terms of instantaneous measurements and ensemble averages. One performs (instantaneous) measurements of quadrature phase amplitudes over many identically prepared systems, thus calculating the ensemble averages $\langle X_\theta(L)Y_\phi(L)\rangle$, $\Delta^2 X_\theta$, etc. The measurement of the spectral quantities, however, involves time averages. The output $\langle |i(\omega)|^2\rangle$ on the power spectrum analyzer already gives the average. It is well known that for stationary, ergodic fields, the time averages equal the ensemble averages [given by solutions (3.2)], provided the detection time T is larger than the coherence time, and the observable spectral correlations are predicted to be of the type discussed in Sec. II.

One of the assumptions in the paradox is that the measurement of the idler $\hat{Y}_1$ does not have any effect on the value of $\hat{X}_1$ in the spatially separated signal field. It is only with the "no action at a distance" assumption that EPR can make the claim that the signal must have preexisted in a state with simultaneously precisely defined $\hat{X}_1$ and $\hat{X}_2$. This assumption is discussed extensively in treatments of Bell's inequality.[11] The assumption is made more solid by experiments of the type performed by Aspect, Dalibard, and Roger,[13] which involve a delayed choice (Fig. 2) of whether the quadrature phase $\hat{Y}_1$ and $\hat{Y}_2$ is measured (one waits until the "photons are in flight"). Then if the measurement of $\hat{Y}_1$ ($\hat{Y}_2$) had any effect on the value $\hat{X}_2$ ($\hat{X}_1$) of the signal state, the effect must violate causality. A finite detection time T means that the phase-shifting local oscillator apparatus must be set for at least that length of time. The exact value of T then becomes important in determining the necessary spatial separation of signal and idler, if one wishes to ensure any action at a distance means violation of causality.

VI. BEAM SPLITTERS WITH SQUEEZED INPUTS

We consider the correlated output state produced by an interaction describable by the following Hamiltonian:

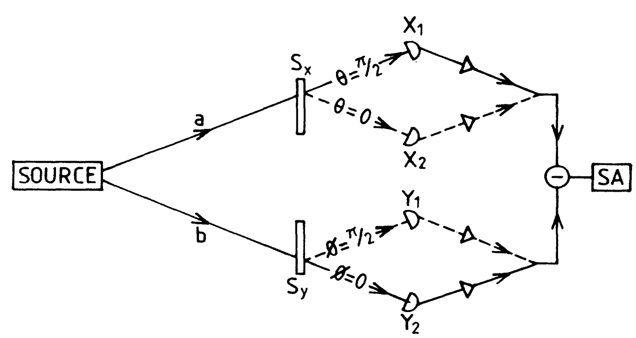

FIG. 2. A possible delayed-choice EPR experiment, after the experiment of Aspect, Dalibard, and Roger (Ref. 13). Time-varying analyzers (local oscillators measuring X_1 or X_2, Y_1 or Y_2) are provided by switching devices.

$$H_I=\hbar\kappa\hat{a}^\dagger\hat{b}+h\kappa^*\hat{a}\hat{b}^\dagger . \tag{6.1}$$

The solutions for the quadrature phases are ($\theta=|\kappa|L$ where κ is taken real)

$$\begin{aligned}
\hat{X}_1(L)&=\hat{X}_1(0)\cos\theta+\hat{Y}_2(0)\sin\theta ,\\
\hat{X}_2(L)&=\hat{X}_2(0)\cos\theta-\hat{Y}_1(0)\sin\theta ,\\
\hat{Y}_1(L)&=\hat{Y}_1(0)\cos\theta+\hat{X}_2(0)\sin\theta ,\\
\hat{Y}_2(L)&=\hat{Y}_2(0)\cos\theta-\hat{X}_1(0)\sin\theta .
\end{aligned}\tag{6.2}$$

Alternatively, one could consider the following transformation of the type given by a simple beam splitter (κ imaginary):

$$\begin{aligned}
\hat{a}(L)&=\hat{a}(0)\cos\theta+\hat{b}(0)\sin\theta ,\\
\hat{b}(L)&=\hat{b}(0)\cos\theta-\hat{a}(0)\sin\theta .
\end{aligned}\tag{6.3}$$

These have solutions for quadrature phase amplitudes

$$\begin{aligned}
X_{1(2)}(L)&=\hat{X}_{1(2)}(0)\cos\theta+\hat{Y}_{1(2)}(0)\sin\theta ,\\
\hat{Y}_{1(2)}(L)&=\hat{Y}_{1(2)}(0)\cos\theta-\hat{X}_{1(2)}(0)\sin\theta .
\end{aligned}\tag{6.4}$$

The transformations provide a simple model for any device which couples the two modes while conserving total photon number—beam splitters, frequency converters, Faraday rotators.

Calculations show, for the transformations (6.2),

$$\begin{aligned}
\langle[\hat{X}_1(L)]^2\rangle&=\langle[\hat{X}_1(0)]^2\rangle\cos^2\theta+\langle[\hat{Y}_2(0)]^2\rangle\sin^2\theta ,\\
\langle[\hat{Y}_2(L)]^2\rangle&=\langle[\hat{Y}_2(0)]^2\rangle\cos^2\theta+\langle[\hat{X}_1(0)]^2\rangle\sin^2\theta ,\\
\langle\hat{X}_1(L)\hat{Y}_2(L)\rangle&=-\langle\hat{X}_1(0)^2\rangle\cos\theta\sin\theta\\
&\quad+\langle\hat{Y}_2(0)^2\rangle\cos\theta\sin\theta .
\end{aligned}\tag{6.5}$$

[The solutions for the transformation (6.4) are obtained from (6.5) by replacing $\hat{Y}_2(0)$ and $\hat{Y}_2(L)$ with $\hat{Y}_1(0)$ and $\hat{Y}_1(L)$, respectively. Thus the results will be similar and will not be given separately.]

Thus for usual coherent vacuum inputs ($\langle[\hat{X}_1(0)]^2\rangle=\langle[\hat{Y}_2(0)]^2\rangle=1$), there is no correlation between $\hat{X}_1(L)$ and $\hat{Y}_2(L)$. However, a correlation is possible with squeezed vacuum inputs. We consider a situation where the input for mode $\hat{a}$ is a coherent vacuum, but the input for $\hat{b}$ is a squeezed vacuum such that $\langle[\hat{Y}_2(0)]^2\rangle=\epsilon$ ($\epsilon<1$) and $\langle\hat{Y}_1(0)^2\rangle=\epsilon^{-1}=\eta$ ($\eta>1$). Thus

$$\begin{aligned}
\langle[\hat{X}_1(L)]^2\rangle&=\cos^2\theta+\epsilon\sin^2\theta ,\\
\langle[\hat{Y}_2(L)]^2\rangle&=\sin^2\theta+\epsilon\cos^2\theta ,\\
\langle\hat{X}_1(L)\hat{Y}_2(L)\rangle&=-\cos\theta\sin\theta(1-\epsilon) .
\end{aligned}\tag{6.6}$$

The inference error quantity as defined by (4.4) is

$$V_1=\frac{\epsilon}{\sin^2\theta+\epsilon\cos^2\theta} , \tag{6.7}$$

which for ϵ small can clearly satisfy the requirement $V_1<1$.

Similarly, we consider the quantity (4.9). We find that

$$V_2 = \frac{\eta}{\eta\cos^2\theta + \sin^2\theta} . \tag{6.8}$$

The error product is (we suppose we have a squeezed minimum uncertainty state where $\eta = 1/\epsilon$)

$$\Delta^2_{\rm inf}\hat{X}(L)\Delta^2_{\rm inf}\hat{X}_2(L) = V_1 V_2 = \frac{1}{1+\frac{\sin^2 2\theta}{4}(\eta - 2 + 1/\eta)} . \tag{6.9}$$

Since $\eta > 1$, we see the possibility of a demonstration of the EPR paradox

$$\Delta^2_{\rm inf}\hat{X}_1(L)\Delta^2_{\rm inf}\hat{X}_2(L) < 1 ,$$

provided $\theta \neq 0, \pi/2, \ldots$.

A similar result has been pointed out by Paul,[32] who considered a photon number state impinging on a beam splitter, and showed the correlations in photon number and in phase between the two output beams to be like those of the EPR paradox. The formulation in terms of the two conjugate quadrature amplitudes $\hat{X}_1$, $\hat{X}_2$ is advantageous, since reduced fluctuations in these quantities have now been measured, and squeezed minimum uncertainty states have been produced experimentally.

In fact, in a recent experiment Xiao and co-workers[33] used light produced by combining a squeezed vacuum field with a strong coherent field across a beam splitter. Their experiment was concerned with the use of a squeezed vacuum to improve precision in measurement of phase modulation in an interferometer. However, one could imagine an amended arrangement depicted in Fig. 3, designed to measure the quadrature correlation quantities V_1 or V_2. The input $\hat{b}$ is the squeezed vacuum obtained from the output of an optical parametric oscillator (OPO). The input $\hat{a}$ comprises a strong coherent field (local oscillator) at zero frequency and a coherent vacuum

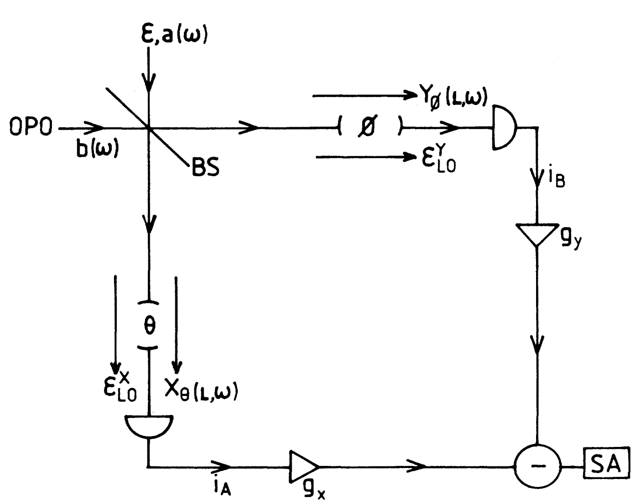

FIG. 3. Schematic diagram of a possible apparatus used to measure quadrature phase amplitude correlations between the outputs of a beam splitter (BS). The input field $b(\omega)$ is a squeezed vacuum, while $a(\omega)$ is a coherent vacuum.

at sideband frequencies. One might use phase-shifting cavities as designed by Shelby *et al.*,[17] to phase shift the strong field with respect to the vacuum sidebands and thus measure the particular spectral quadrature phase amplitudes $X_\theta(L,\omega)$ and $Y_\phi(L,\omega)$ defined by Eqs. (5.7)–(5.11). The correlations for these operators are those of (6.2) and (6.5) but replacing $X_j(L)$ with $X_j(L,\omega)$, $X_j(0)$ with $X_j(0,\omega)$, $Y_j(L)$ with $Y_j(L,\omega)$ and $Y_j(0)$ with $Y_j(0,\omega)$.

VII. CONCLUSION

We have pointed out that the correlation in quadrature phase of the two output beams of the nondegenerate parametric amplifier is such as to provide an example of the original EPR paradox. This correlation manifests itself as a reduction in fluctuations of the signal and idler quadrature phase amplitude difference. If the fluctuations are below a certain level (corresponding to the vacuum fluctuations of one of the beams) then the EPR paradox becomes demonstrable. The quadrature phase amplitudes are measurable by homodyne detection techniques. We have discussed a possible experimental procedure which provides evidence of EPR correlations between spectral quadrature phase operators. Such an experiment is closely related to four-mode squeezing experiments already performed.

The EPR paradox is a paradox about quantum mechanics. If quantum mechanics is correct, then there exist correlations between spatially separated subsystems which (provided there is no action at a distance) imply that quantum mechanics needs to be completed, at least according to the EPR concept of reality. The paradox questions our understanding of quantum mechanics as an ultimate theory. Thus experiments which confirm (in agreement with quantum mechanics) the existence of the paradoxical correlations would seem in themselves significant. Previous demonstrations of the paradox have been in connection with tests of Bell's inequality. They are thus concerned with the discrete version of the paradox presented by Bohm, and the experiments measure the correlation of spin between spatially separated photons (or protons). The optical amplitude version as suggested here is interesting in that it is more closely in line with the original version of the paradox which, as far as we know, has not been experimentally demonstrated.

Of course it is not so straightforward in practice to measure sufficient correlation that the paradox by evidenced. (In fact, a field which is completely describable by standard classical electromagnetic theory cannot predict such correlations.) The spin-correlation experiments, for example, are troubled by detector inefficiencies which tend to reduce the measured correlation. The homodyne measurement is very efficient and the quantum noise levels are readable macroscopically. The preparation though of the state with sufficient quadrature phase correlation is not trivial. Loss, for example, downgrades the correlation. However, with the recent laboratory success in the detection of fluctuations below the quantum minimum uncertainty state noise level, the measurement of EPR correlations as discussed in this paper would appear to be possible in the near future. In view of the significant noise reduction possible with parametric oscillators operating below threshold, the nondegenerate parametric oscillator would seem to be a particularly good candidate for such an EPR experiment. Calculations predicting EPR correlations in the external fields of the nondegenerate parametric oscillator below threshold are presented in another paper.[23]

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APPENDIX A

For the original paradox, we have 100% correlation between suitable signal and idler observables. Hence once a "particle" has been emitted, we can determine with 100% certainty either the ("position") $\hat{X}_1$ or the ("momentum") $\hat{X}_2$ depending on our choice, by making measurements on the idler. Experience tells us that our estimate of $\hat{X}_1$ will *always be* precise, or if we choose to measure $\hat{X}_2$, our estimate of $\hat{X}_2$ will *always be* precise. This is true for each and every possible state—i.e., regardless of the value of $\hat{X}_1$ or $\hat{X}_2$, which would be obtained on measurement. Thus for any given particle emitted, the EPR concept of reality allows EPR to infer definite values of $\hat{X}_1$ and $\hat{X}_2$ existing, simultaneously, for that state.

For the situation where the correlation is not maximum we can only talk of estimating $\hat{X}_1$, or $\hat{X}_2$, with a certain precision. There may be a small probability that the deviation in our estimate is actually quite large. We can make many measurements of the error and build up a picture of its distribution, and determine the average error which we call $\Delta_{\text{inf}}\hat{X}_1$. Similarly, we can then determine the average error $\Delta_{\text{inf}}\hat{X}_2$ in the inference of $\hat{X}_2$. Now the EPR reasoning allows EPR to deduce that the signal is described by a state for which the average error in X_1 is $\Delta^2_{\text{inf}}\hat{X}_1$ and the average error in $\hat{X}_2$ is $\Delta^2_{\text{inf}}\hat{X}_2$. This might raise the following objection. Let us denote all possible emitted states by ψ_i and the probability of their being emitted by P_i. The simultaneous errors are $\Delta_{\text{inf}}\hat{X}_{1i}$ and $\Delta_{\text{inf}}\hat{X}_{2i}$. We can measure either $\Delta_{\text{inf}}\hat{X}_{1i}$ or $\Delta_{\text{inf}}\hat{X}_{2i}$ but not both. Now the average inference error in $\hat{X}_1$ will be given by

$$\Delta^2_{\text{inf}}\hat{X}_1=\sum_i P_i\Delta^2_{\text{inf}}\hat{X}_{1i}=\langle \Delta^2_{\text{inf}}\hat{X}_{1i}\rangle \equiv V_1$$

and similarly for the average inference error in $\hat{X}_2$, and these are the quantities we can measure. But what is relevant from the point of view of the uncertainty principle and hence of the paradox is the value of the simultaneous products $(\Delta_{\text{inf}}\hat{X}_{1i})(\Delta_{\text{inf}}\hat{X}_{2i})$. It might be possible to have

$$(\Delta_{\text{inf}}\hat{X}_{1i})(\Delta_{\text{inf}}\hat{X}_{2i})>1$$

for all emitted states, even though the averages $\Delta_{\text{inf}}\hat{X}_1$, $\Delta_{\text{inf}}\hat{X}_2$ are small. In this case, there is no paradox. This is not possible, however, since the Cauchy-Schwarz inequality states

$$\langle (\Delta_{\text{inf}}\hat{X}_{1i})(\Delta_{\text{inf}}\hat{X}_{2i})\rangle^2 \le \langle \Delta^2_{\text{inf}}\hat{X}_{1i}\rangle \langle \Delta^2_{\text{inf}}\hat{X}_{2i}\rangle = V_1V_2 \;.$$

Here we note

$$\langle (\Delta_{\text{inf}}\hat{X}_{1i})(\Delta_{\text{inf}}\hat{X}_{2i})\rangle = \sum_i P_i\Delta_{\text{inf}}\hat{X}_{1i}\Delta_{\text{inf}}\hat{X}_{2i} \;.$$

Thus if $V_1V_2<1$, we must have $(\Delta_{\text{inf}}\hat{X}_{1i})(\Delta_{\text{inf}}\hat{X}_{2i})$ for at least some of the emitted states ψ_i. Thus, with these comments, the EPR reasoning can be extended to give a paradoxical situation even in the case where the correlation is not perfect. It suffices to measure averaged errors of inference for two conjugate observables of the signal.

APPENDIX B

We now discuss the EPR predictions for the experiments of Levenson *et al.*[21] and Schumaker *et al.*[22] The theory is explained by Levenson and Shelby.[22] The solutions for the output frequency space quadrature operators are

$$\hat{X}_2(L,\omega)=\hat{X}_2(0,\omega)+2r_x\hat{X}_1(0,\omega)+4\gamma r_xr_y\hat{Y}_1(0,\omega) \;,$$
$$\hat{X}_1(L,\omega)=\hat{X}_1(0,\omega) \;,$$
$$\hat{Y}_2(L,\omega)=\hat{Y}_2(0,\omega)+2r_y\hat{Y}_1(0,\omega)+4\gamma r_xr_y\hat{X}_1(0,\omega) \;,$$
$$\hat{Y}_1(L,\omega)=\hat{Y}_1(0,\omega) \;,$$

where r_x and r_y, respectively, are proportional to the $\chi^{(3)}$ nonlinear susceptibility for signal and idler modes and the length of the fiber, and γ is a coefficient relating to the decorrelation of polarization of the fields as they propagate through the fiber. While these quadrature operators permit the inference of the amplitude modulations (X_1 and Y_1) of each wave from a measurement made on the other, no cross correlation develops for the input phase modulation quadrature operators.

Here we are interested in calculating the errors in inferring the signal amplitude modulation $\hat{X}_1(L,\omega)$, and in inferring the signal phase modulation $\hat{X}_2(L,\omega)$. The signal amplitude $\hat{X}_1(L,\omega)$ is determined by measurement of the idler $\hat{Y}_2(L,\omega)$. Thus we calculate

$$V_{X_1}=\langle \{\text{Re}^*[\hat{X}_1(L,\omega)-g\hat{Y}_2(L,\omega)]\}^2\rangle = \langle |\hat{X}_1(L,\omega)-g\hat{Y}_2(L,\omega)|^2\rangle$$

which is minimum for

$$g=\frac{|\langle \hat{X}_1^\dagger(L,\omega)\hat{Y}_2(L,\omega)\rangle|^2}{\langle \hat{Y}_2^\dagger(L,\omega)\hat{Y}_2(L,\omega)\rangle} \;.$$

The minimum value is

$$V_{X_1}=\langle \hat{X}_1^\dagger(L,\omega)\hat{X}_1(L,\omega)\rangle - \frac{|\langle X_1^\dagger(L,\omega)\hat{Y}_2(L,\omega)\rangle|^2}{\langle \hat{Y}_2^\dagger(L,\omega)\hat{Y}_2(L,\omega)\rangle} \;.$$

The solutions indicate that (with vacuum inputs)

$$V_{X_1}=1-\frac{16\gamma^2 r_x r_y}{1+4r_y^2+16\gamma^2 r_x r_y} .$$

In fact, as explained in Ref. 22, because of two-mode squeezing a better value of V_{X_1} may be achieved by choosing ϕ slightly different to $\pi/2$. Similarly we calculate

$$V_{X_2}(g)=\langle \hat{X}_2(L,\omega)-gY_1(L,\omega)|^2\rangle ,$$

which has a minimum value

$$V_{X_2}=\langle \hat{X}_2^\dagger(L,\omega)\hat{X}_2(L,\omega)\rangle-\frac{|\langle \hat{X}_2^\dagger(L,\omega)\hat{Y}_1(L,\omega)\rangle|^2}{\langle \hat{Y}_1^\dagger(L,\omega)\hat{Y}_1(L,\omega)\rangle}$$

$$=1+4r_x^2$$

The error in estimating amplitude modulation is predicted to go below the vacuum (standard quantum) limit. This is not the case with the estimation of signal phase modulation. The product $V_{X_1}V_{X_2}$, however, is predicted to go below the vacuum noise level ($V_{X_1}V_{X_2}<1$) and thus the system may exhibit EPR correlation. Experimentally Levenson *et al.*[22] obtained a value of $V_{X_1}=0.95$. The phase modulation of the signal, however, was not measured. The noise V_{X_2} is predicted to be greater than that of V_{X_1}. [The quantity

$$V_{Y_2}(g)=\langle |\hat{Y}_2(L,\omega)-g\hat{X}_1(L,\omega)|^2\rangle$$

was measured by Levenson *et al.* (one normalizes results to the vacuum noise level of the idler) and found to be significantly greater than 1, as expected.]

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