PHYSICAL REVIEW A VOLUME 40, NUMBER 12 DECEMBER 15, 1989

k-photon Jaynes-Cummings model with coherent atomic preparation: Squeezing and coherence

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(Received 16 June 1989)

The k-photon Jaynes-Cummings model is studied with respect to the properties of the radiation mode. In particular, the generation of coherence and squeezing in the one- and two-photon model is examined for the case that the field is initially in the vacuum state and the atom is prepared in a coherent superposition of states. Coherence is found in the one-photon model, and squeezing appears in both the one- and the two-photon model, the maximum noise reduction being 25% and 45%, respectively. In the one-photon model coherent squeezed radiation is generated, whereas the two-photon model yields a squeezed vacuum. For an initial atomic preparation close to the ground state, this squeezed vacuum may be interpreted as the result of two-photon interaction between the quantized cavity field and a classical atomic current.

I. INTRODUCTION

The feasibility of performing experiments with highly excited Rydberg atoms in a high-Q cavity (for example, cf. Refs. 1–3) has rendered it possible to observe a variety of interesting and fundamental quantum-optical effects predicted theoretically (for example, cf. Refs. 4 and 5). In particular, the practical relevance of the (one-photon) Jaynes-Cummings model has been established. Besides effects such as the atomic decay and revival behavior, the question has been raised as to what kinds of states of the radiation field may be generated within a high-Q cavity resonantly interacting with single atoms.

Based on the Jaynes-Cummings model, it was shown that the generation of coherent light is a nontrivial problem since the density matrices of both the field and the atom remain diagonal for all times if they are initially diagonal.[6] Thus it seems more natural that a sub-Poissonian field can be generated.[6,7] The problems are similar in the case of squeezed-light generation. For producing a squeezed field in the cavity there are two possibilities. When the field has initially off-diagonal elements of the photon density matrix (e.g., in the case of a coherent state) squeezing may occur by injecting into the cavity an atom incoherently prepared in the ground or excited state.[8] Provided that the cavity field has initially no off-diagonal elements, which seems to be more natural, squeezing may be generated if the atom is initially in a coherent superposition of the ground and excited state.[9] It should be emphasized that, not only the generation of coherent, sub-Poissonian or squeezed cavity fields is complicated, but also the experimental proof of these properties is nontrivial since standard methods of photodetection cannot be applied within the cavity. An alternative way is to determine the photon statistics of the field by injecting into the cavity test atoms, which are initially prepared in the ground or excited state, and studying their ionization behavior when they leave the cavity.[3,4] Moreover, for probing coherence properties of the field, ionization measurements must be performed by using test atoms which are initially prepared in a coherent superposition of the excited and ground state.[10] In order to test the cavity field with respect to squeezing, additional measurements with coherently prepared two-photon resonant atoms are necessary.[10] It is worth noting that the dynamics of an atom interacting with the cavity mode may drastically be changed when the atom is initially prepared in a coherent superposition of states, as it can be seen from the results given in Ref. 10 for the general case of the k-photon Jaynes-Cummings model and recent results for the one-photon model.[11]

Stimulated by the progress in experimentally proving fundamental results of the one-photon Jaynes-Cummings model, the k-photon model has become of increasing interest.[10,12,13] For example, it has been found that in the two-photon model the revivals may be more complete than in the one-photon case.[10] Furthermore, the generation of squeezed light in the k-photon model has been predicted for the case when the field starts from a coherent state[12,13] or (in the two-photon model) from a binomial state.[14]

Since the probe of coherence and squeezing of the cavity field is closely related to the dynamics of one- and two-photon resonant test atoms prepared coherently,[10] it is, vice versa, of interest whether such kinds of atoms may produce a coherent and/or squeezed cavity field, which, for its part, starts from an incoherent state (thermal or vacuum field). It should be emphasized that this assumption about the initial field preparation seems to be more realistic than the assumption of a coherent initial state.

In the present paper we study the problem of resonant interaction between a single atom and a single-mode cavity field on the basis of the k-photon Jaynes-Cummings model. Clearly, this problem is far from describing a maser regime, which requires the successive injection of many atoms inside the cavity during the lifetime of the cavity field in order to generate a macroscopic-type field. Nevertheless, both kinds of problems are related to each other because the interaction of each injected atom with the cavity field follows the Jaynes-Cummings model.[6,7] At each stage, the initial field is given by the field generated by all the preceding atoms. In particular, it is reasonable to assume that in the case of the first atom the cavity field is initially a thermal one.

Our paper is subdivided as follows. The general solution of the k-photon Jaynes-Cummings model with arbitrary initial conditions is presented in Sec. II. In Sec. III general results for squeezing and coherence in the k-photon model are given. Special emphasis on the cases with $k=1,2$ is put in Secs. IV and V. In Sec. IV it is shown that coherence as well as squeezing can be generated in the one-photon case when the field is initially in the vacuum state and the atom is initially coherently prepared. Section V is devoted to the field properties in the case of a two-photon resonant atom. It is shown that under the same initial conditions as in the one-photon case, the two-photon model cannot give rise to coherence, but it yields squeezing in the sense of a squeezed vacuum. The attainable noise reduction of 45% is substantially larger than that of 25% in the one-photon model. A summary and some conclusions are given in Sec. VI.

II. GENERAL SOLUTION OF THE k-PHOTON JAYNES-CUMMINGS MODEL

The effective Hamiltonian in the k-photon Jaynes-Cummings model may be written as follows:

$$H^{(k)}=H_0+H_I^{(k)} , \tag{2.1}$$

$$H_0=\hbar\omega_1 A_{11}+\hbar\omega_2 A_{22}+\hbar\omega a^\dagger a , \tag{2.2}$$

$$H_I^{(k)}=-\hbar\lambda^{(k)}[(a^\dagger)^k A_{12}+A_{21}a^k] , \tag{2.3}$$

where $A_{ij}=|i\rangle\langle j|$, $i,j=1,2$ are the atomic flip operators for the (effective) two-level system of transition frequency $\omega_{21}=\omega_2-\omega_1$, and a and $a^\dagger$, respectively, are the photon destruction and creation operators of the cavity field mode of frequency ω. The k-photon coupling constant is denoted by $\lambda^{(k)}$. The temporal evolution of the density operator $\rho^{(k)}$ may be represented as

$$\rho^{(k)}(t)=U^{(k)}(t,t')\rho^{(k)}(t')[U^{(k)}(t,t')]^\dagger , \tag{2.4}$$

where the time evolution operator U satisfies the equation of motion

$$i\hbar\frac{d}{dt}U^{(k)}(t,t')=H^{(k)}U^{(k)}(t,t'), \quad U^{(k)}(t',t')=1 . \tag{2.5}$$

Performing the calculations in the representation defined by means of the eigenkets of H_0, viz.,

$$H_0|i,n\rangle=\hbar(\omega_i+n\omega)|i,n\rangle , \tag{2.6}$$

from Eqs. (2.1)–(2.3), and (2.5), we derive

$$\left[\frac{d}{dt}+i(\omega_1+m\omega)\right]U^{(k)}_{1m,in}(t,t')=\frac{i}{2}\Omega_m^{(k)}U^{(k)}_{2(m-k),in}(t,t') , \tag{2.7}$$

$$\left[\frac{d}{dt}+i[\omega_2+(m-k)\omega]\right]U^{(k)}_{2(m-k),in}(t,t') =\frac{i}{2}\Omega_m^{(k)}U^{(k)}_{1m,in}(t,t') , \tag{2.8}$$

$$U^{(k)}_{1m,in}(t',t')=\delta_{1i}\delta_{mn} , \tag{2.9}$$

$$U^{(k)}_{2(m-k),in}(t',t')=\delta_{2i}\delta_{(m-k)n} , \tag{2.10}$$

where the Rabi frequencies $\Omega_m^{(k)}$ are defined by the relation

$$\Omega_m^{(k)}=2\lambda^{(k)}\{m(m-1)\cdots[m-(k-1)]\}^{1/2} . \tag{2.11}$$

Equations (2.7)–(2.10) may be solved by standard methods. Straightforward calculation yields the following expressions for the nonvanishing matrix elements of the time evolution operator:

$$U^{(k)}_{1n,1n}(t,t')=\exp[-i(\omega_1+n\omega+\tfrac{1}{2}\delta^{(k)})(t-t')] \times\left[\cos[\tfrac{1}{2}\Delta_n^{(k)}(t-t')] +i\frac{\delta^{(k)}}{\Delta_n^{(k)}}\sin[\tfrac{1}{2}\Delta_n^{(k)}(t-t')]\right] , \tag{2.12}$$

$$U^{(k)}_{2n,2n}(t,t')=\exp[-i(\omega_2+n\omega-\tfrac{1}{2}\delta^{(k)})(t-t')] \times\left[\cos[\tfrac{1}{2}\Delta_{n+k}^{(k)}(t-t')] -i\frac{\delta^{(k)}}{\Delta_{n+k}^{(k)}}\sin[\tfrac{1}{2}\Delta_{n+k}^{(k)}(t-t')]\right] , \tag{2.13}$$

$$U^{(k)}_{1n,2(n-k)}(t,t')=\exp[-i(\omega_1+n\omega+\tfrac{1}{2}\delta^{(k)})(t-t')] \times i\frac{\Omega_n^{(k)}}{\Delta_n^{(k)}}\sin[\tfrac{1}{2}\Delta_n^{(k)}(t-t')] , \tag{2.14}$$

$$U^{(k)}_{2n,1(n+k)}(t,t')=\exp[-i(\omega_2+n\omega-\tfrac{1}{2}\delta^{(k)})(t-t')] \times i\frac{\Omega_{n+k}^{(k)}}{\Delta_{n+k}^{(k)}}\sin[\tfrac{1}{2}\Delta_{n+k}^{(k)}(t-t')] , \tag{2.15}$$

where

$$\delta^{(k)}=\omega_{21}-k\omega \tag{2.16}$$

is the detuning of the atomic transition frequency from the k-photon resonance, and

$$\Delta_n^{(k)}=[(\Omega_n^{(k)})^2+(\delta^{(k)})^2]^{1/2} . \tag{2.17}$$

Now, the density-matrix elements at time t may be expressed in terms of the matrix elements of the time evolution operator and the density-matrix elements at time t'. Making use of Eq. (2.4) we may write

$$\rho^{(k)}_{in,jm}(t)=\sum_{i',j'}\sum_{n',m'}U^{(k)}_{in,i'n'}(t,t')[U^{(k)}_{jm,j'm'}(t,t')]^*\times\rho^{(k)}_{i'n',j'm'}(t')\ , \qquad (2.18)$$

the only nonvanishing matrix elements $U^{(k)}_{in,i'n'}(t,t')$ being given in Eqs. (2.12)–(2.15).

In what follows we are mainly interested in the properties of the cavity mode at time t. For this reason we calculate the (reduced) photon density matrix

$$\rho^{(k)}_{nm}(t)=\sum_i\rho^{(k)}_{in,im}(t)=\rho^{(k)}_{1n,1m}(t)+\rho^{(k)}_{2n,2m}(t)\ . \qquad (2.19)$$

Combining Eqs. (2.18), (2.12)–(2.15), and (2.19) yields

$$\begin{aligned}\rho^{(k)}_{nm}(t)=&U^{(k)}_{1n,1n}(t,t')[U^{(k)}_{1m,1m}(t,t')]^*\rho^{(k)}_{1n,1m}(t')+U^{(k)}_{2n,1(n+k)}(t,t')[U^{(k)}_{2m,1(m+k)}(t,t')]^*\rho^{(k)}_{1(n+k),1(m+k)}(t')\\&+U^{(k)}_{2n,2n}(t,t')[U^{(k)}_{2m,2m}(t,t')]^*\rho^{(k)}_{2n,2m}(t')+U^{(k)}_{1n,2(n-k)}(t,t')[U^{(k)}_{1m,2(m-k)}(t,t')]^*\rho^{(k)}_{2(n-k),2(m-k)}(t')\\&+U^{(k)}_{1n,1n}(t,t')[U^{(k)}_{1m,2(m-k)}(t,t')]^*\rho^{(k)}_{1n,2(m-k)}(t')+U^{(k)}_{2n,1(n+k)}(t,t')[U^{(k)}_{2m,2m}(t,t')]^*\rho^{(k)}_{1(n+k),2m}(t')\\&+U^{(k)}_{2n,2n}(t,t')[U^{(k)}_{2m,1(m+k)}(t,t')]^*\rho^{(k)}_{2n,1(m+k)}(t')+U^{(k)}_{1n,2(n-k)}(t,t')[U^{(k)}_{1m,1m}(t,t')]^*\rho^{(k)}_{2(n-k),1m}(t')\ . \qquad (2.20)\end{aligned}$$

Identifying the time t' with the initial time t_0 when the atom enters the cavity and the interaction process starts,[2,3] we may factor the density matrix at time t_0 as follows:

$$\rho^{(k)}_{in,jm}(t')=\rho_{nm}(t')\sigma_{ij}(t'),\quad t'=t_0\ , \qquad (2.21)$$

where $\rho_{nm}(t_0)$ and $\sigma_{ij}(t_0)$, respectively, are the density matrix elements of the free-cavity field and the free atom.

Before proceeding to study some field properties in detail we note that in a way analogous to that leading to Eq. (2.20) the (reduced) atomic density matrix at time t may be derived. Explicit formulas are given in Ref. 10.

III. SQUEEZING AND A COHERENCE OF THE CAVITY MODE

Squeezing is a phase-dependent reduction of the noise of the electric field strength (or, equivalently, of one of the field quadratures) below the vacuum level (for example, see the reviews on squeezing in Refs. 15 and 16). In the case under consideration the operator of the electric field strength of the high-Q cavity mode may be defined as

$$E=ig(a-a^\dagger)\ , \qquad (3.1)$$

where $g=g(\mathbf{r})$ describes the spatial structure of the cavity mode.

The quantitative condition for squeezing may be formulated by means of the normally ordered variance $\langle:(\Delta E)^2:\rangle$. The cavity field shows squeezing if the condition

$$\langle:(\Delta E)^2:\rangle<0 \qquad (3.2)$$

is fulfilled for appropriately chosen phase of the field. It is readily seen that the inferior limit is equal to $-g^2$. Clearly, in this case the vacuum noise (given by g^2) is completely suppressed. It should be noted that the standard observation method for squeezing, which is based on homodyne detection (cf. Refs. 17–21), cannot be applied inside the cavity. As shown in Ref. 10, squeezing of the cavity field may be probed by means of coherently prepared test atoms which are in one- and two-photon resonance with the cavity mode.

Expressing the normally ordered variance $\langle:(\Delta E)^2:\rangle$ in terms of the photon density-matrix elements of the cavity mode, we derive

$$\langle:[\Delta E(t)]^2:\rangle=2g^2\langle N(t)\rangle-4g^2\left[\sum_{n=0}^{\infty}\mathrm{Re}[i(n+1)^{1/2}\rho_{n(n+1)}(t)]\right]^2-2g^2\sum_{n=0}^{\infty}\mathrm{Re}\{[(n+1)(n+2)]^{1/2}\rho_{n(n+2)}(t)\}\ , \qquad (3.3)$$

where

$$\langle N(t)\rangle=\sum_{n=0}^{\infty}n\rho_{nn}(t) \qquad (3.4)$$

is the mean photon number. In the k-photon Jaynes-Cummings model under study the photon density-matrix elements ar given in Eq. (2.20) [together with Eq. (2.21)]. For producing a squeezed-cavity mode, from Eq. (3.3) it is seen that the photon density matrix must exhibit nonvanishing off-diagonal elements $\rho_{n(n+1)}$ and/or $\rho_{n(n+2)}$. An inspection of Eq. (2.20) shows that there are different possibilities of generating them.

For example, if one starts with an atom initially prepared in the ground (or excited) state, the initial field must be prepared in such a way that it already exhibits off-diagonal matrix elements of the type required. This way of generating squeezing was studied in several works on the basis of a coherent initial state[8,12,13] or a binomial state.[14]

However, it seems to be more natural to assume that the cavity field starts from a thermal field. In particular, by sufficiently cooling, the field may (approximately) be assumed to be in the vacuum state.[22] In these cases squeezing may be obtained in the one- and two-photon Jaynes-Cummings model provided that the atom is initially (that is, before it enters the cavity) prepared in a coherent superposition of its states, so that $\sigma_{12}(t_0)\neq0$ is valid.

Another problem of interest concerns the generation of coherence of the cavity field. Expressing the mean value of the electric field strength in terms of the photon density-matrix elements, we obtain

$$\langle E(t)\rangle = -2g\sum_{n=0}^{\infty}\mathrm{Re}[i(n+1)^{1/2}\rho_{n(n+1)}(t)] . \quad (3.5)$$

A quantitative measure for the coherence may be defined by the fraction of the number of coherent photons $\langle N\rangle_{\mathrm{coh}}$,

$$\langle N\rangle_{\mathrm{coh}} = |\langle a\rangle|^2 = \left|\sum_{n=0}^{\infty}(n+1)^{1/2}\rho_{n(n+1)}(t)\right|^2 , \quad (3.6)$$

to the total photon number $\langle N\rangle$ defined in Eq. (3.4).

From Eq. (3.5), together with Eq. (2.20), it is clearly seen that, similar to the problem of generating squeezing, in the case when the photon density-matrix elements are initially diagonal, coherence can only be produced by initially preparing the atom, which is in one-photon resonance with the cavity mode, in a coherent superposition of states.

An experimental scheme that allows an initial preparation of the atomic off-diagonal density-matrix elements is proposed in Ref. 23. It can readily be proved that in the case of coherent preparation the atomic density matrix may be written in the following form:

$$\sigma_{22}(t_0) = \sin^2\chi , \quad (3.7)$$

$$\sigma_{11}(t_0) = 1-\sigma_{22}(t_0) , \quad (3.8)$$

$$\sigma_{12}(t_0) = \tfrac{1}{2}\exp(-i\varphi)\sin 2\chi , \quad (3.9)$$

$$\sigma_{21}(t_0) = \sigma_{12}^*(t_0) , \quad (3.10)$$

($0\le\chi\le\pi/2$, $0\le\varphi\le 2\pi$). The simplest case of an initial cavity field with diagonal photon density matrix is the (low-temperature) vacuum limit; viz.,

$$\rho_{nm}(t_0) = \delta_{nm}\delta_{n0} . \quad (3.11)$$

Moreover, assuming that the detuning of the atomic transition frequency from the k-photon resonance may be neglected ($\delta^{(k)}\approx 0$), from Eqs. (2.20), (2.21), and (3.7)–(3.11), together with Eqs. (2.12)–(2.17), we obtain for the nonvanishing density-matrix elements of the photon density matrix at time t the following simple result:

$$\rho_{kk}^{(k)}(t) = \sigma_{22}(t_0)\sin^2[\tfrac{1}{2}\Omega_k^{(k)}(t-t_0)] , \quad (3.12)$$

$$\rho_{00}^{(k)}(t) = 1-\rho_{kk}^{(k)}(t) , \quad (3.13)$$

$$\rho_{0k}^{(k)}(t) = -i\sigma_{12}(t_0)\exp[ik\omega(t-t_0)]\sin[\tfrac{1}{2}\Omega_k^{(k)}(t-t_0)] , \quad (3.14)$$

$$\rho_{k0}^{(k)}(t) = [\rho_{0k}^{(k)}(t)]^* . \quad (3.15)$$

IV. ONE-PHOTON RESONANCE: COHERENCE AND SQUEEZING

We now turn to the problem of coherence and squeezing in the one-photon Jaynes-Cummings model ($k=1$) for the case when the cavity field starts from the vacuum and the atom starts from a coherent superposition of states. From Eqs. (3.4)–(3.6), together with Eqs. (3.12)–(3.15) and (3.7)–(3.10), we derive

$$\langle E(t)\rangle = -g\sin 2\chi\sin[\tfrac{1}{2}\Omega_1^{(1)}(t-t_0)]\cos[\omega(t-t_0)-\varphi] , \quad (4.1)$$

$$\langle N(t)\rangle = \sin^2\chi\sin^2[\tfrac{1}{2}\Omega_1^{(1)}(t-t_0)] , \quad (4.2)$$

$$\frac{\langle N(t)\rangle_{\mathrm{coh}}}{\langle N(t)\rangle} = \frac{|\sigma_{12}(t_0)|^2}{\sigma_{22}(t_0)} = \cos^2\chi . \quad (4.3)$$

Combining Eqs. (4.1) and (4.2) we see that the coherent field amplitude as a function on time attains its maximum values when the mean photon number also attains its maximum values:

$$|\langle E(t)\rangle|^2 = 4g^2\cos^2\chi\langle N(t)\rangle . \quad (4.4)$$

Further, from Eq. (4.1) we find that, dependent on the atomic preparation, the maximum coherent field amplitude can be obtained in the case $\chi=\pi/4$, which corresponds to the coherent atomic initial preparation $|\sigma_{12}(t_0)|=\sigma_{22}(t_0)=\sigma_{11}(t_0)=\frac{1}{2}$ [cf. Eqs. (3.7)–(3.10)]. On the other hand, Eq. (4.3) shows that the smaller the value of χ, the larger the fraction of the number of coherent photons to the total photon number becomes. In particular, this fraction tends to unity as the value of $\cos\chi$ goes to unity, that is, as the coherent atomic initial preparation becomes close to the ground-state preparation.

Making use of Eqs. (3.7)–(3.15) and Eq. (4.2), we may rewrite Eqs. (3.12)–(3.15) as follows:

$$\rho_{00}(t) = 1-\langle N(t)\rangle , \quad (4.5)$$

$$\rho_{11}(t) = \langle N(t)\rangle , \quad (4.6)$$

$$\rho_{01}(t) = -i\kappa\exp\{i[\omega(t-t_0)-\varphi]\}\cos\chi[\langle N(t)\rangle]^{1/2} , \quad (4.7)$$

$$\rho_{10}(t) = \rho_{01}^*(t) , \quad (4.8)$$

where

$$\kappa = \mathrm{sgn}\{\sin[\tfrac{1}{2}\Omega_1^{(1)}(t-t_0)]\} . \quad (4.9)$$

As known, the density matrix of a coherent state may be written as[24]

$$\rho_{nm}^{\mathrm{coh}} = (n!m!)^{-1/2}\alpha^n(\alpha^*)^m\exp(-|\alpha|^2) . \quad (4.10)$$

In the case when the field is sufficiently weak so that we may confine ourselves to diagonal and off-diagonal density-matrix elements up to terms of the order $|\alpha|^2$ and $|\alpha|$, respectively, from Eq. (4.10) we derive

$$\rho_{00}^{\mathrm{coh}} = 1-|\alpha|^2 , \quad (4.11)$$

$$\rho_{11}^{\mathrm{coh}} = |\alpha|^2 , \quad (4.12)$$

$$\rho_{01}^{\mathrm{coh}} = \alpha^* = |\alpha|\exp(-i\varphi_\alpha) , \quad (4.13)$$

$$\rho_{10}^{\mathrm{coh}} = (\rho_{01}^{\mathrm{coh}})^* . \quad (4.14)$$

Comparing Eqs. (4.11)–(4.14) with Eqs. (4.5)–(4.8) and remembering Eq. (4.2) we see that in the case $|\cos\chi|\approx 1$,

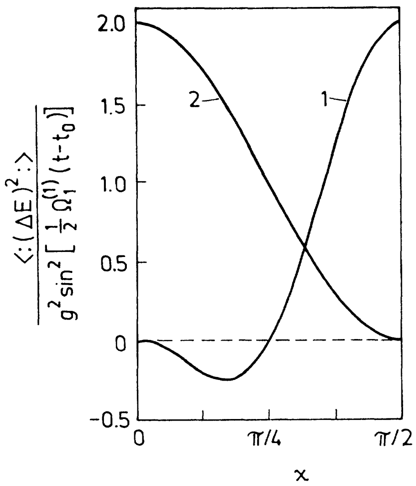

FIG. 1. The field noise $\langle :(\Delta E)^2: \rangle$ is shown as a function of the atomic preparation angle χ. The field quadratures with $\phi=2\pi n$ (curve 1) and $\phi=2\pi n+\frac{1}{2}\pi$ (curve 2) are given. Maximum squeezing with $\langle :(\Delta E)^2: \rangle=-0.25g^2$ occurs when the photon number attains its maximum value for $\Omega_1^{(1)}(t-t_0)=\pi$.

and hence $\langle N(t)\rangle \ll 1$, the cavity field tends to a coherent state. Clearly, in this case the coherent field amplitude is far from its maximum value.

Let us now briefly turn to the problem of squeezing. Combining Eq. (3.3) and Eqs. (3.12)–(3.15) for the case $k=1$ yields

$$\langle :[\Delta E(t)]^2: \rangle = 2g^2\langle N(t)\rangle(1-2\cos^2\phi\cos^2\chi) \ , \qquad (4.15)$$

where the mean photon number $\langle N(t)\rangle$ is given in Eq. (4.2). Note that in Eq. (4.15) the phase ϕ defined by $\phi=\omega(t-t_0)-\varphi$ includes the rapid time dependence, which, of course, is meaningless when slowly varying quantities are of interest. From an inspection of Eq. (4.15) we see that squeezing requires the condition $\cos^2\phi\cos^2\chi>\frac{1}{2}$ to be fulfilled. Moreover, we find that in dependence on time the effect of squeezing behaves like the mean photon number. The influence of the coherent atomic initial preparation on the noise of the electric field strength of the cavity mode is shown in Fig. 1, from which the maximum noise reduction is seen to be 25%. This result was also found in Ref. 9. Note that the choice of $\cos^2\phi=0,1$ in Fig. 1 corresponds to the consideration of the two field quadratures.

V. TWO-PHOTON RESONANCE: SQUEEZED VACUUM

As it is readily seen from Eqs. (3.5) and (3.14), in the case of two-photon resonance ($k=2$), the coherent amplitude of the electric field strength of the cavity mode vanishes:

$$\langle E(t)\rangle = 0 \ . \qquad (5.1)$$

By combining Eqs. (3.3), (3.4), (3.12)–(3.15), and (3.7)–(3.10), the normally ordered variance of the electric field strength may be represented in the following form:

$$\langle :[\Delta E(t)]^2: \rangle = 2g^2\{\langle N(t) - \kappa[\langle N(t)\rangle]^{1/2}\cos\chi\sin(2\phi+\varphi)\} \ , \qquad (5.2)$$

where the mean photon number $\langle N(t)\rangle$ is given by the relation

$$\langle N(t)\rangle = 2\sin^2\chi\sin^2[\tfrac{1}{2}\Omega_2^{(2)}(t-t_0)] \qquad (5.3)$$

and

$$\kappa=\mathrm{sgn}\{\sin[\tfrac{1}{2}\Omega_2^{(2)}(t-t_0)]\} \ . \qquad (5.4)$$

From an inspection of Eq. (5.2) it is seen that as long as $\kappa=+1$ is valid, the optimum condition for squeezing is $\sin(2\phi+\varphi)=1$. In the opposite case, when the relation $\sin(2\phi+\varphi)=-1$ is valid, the strongest increase of noise is expected. Note that these two cases correspond to the two field quadratures, the phases of which just differ in $\Delta\phi=\pi/2$. Since for fixed value of the phase $2\phi+\varphi$ the change of the sign of the second term in the brackets in Eq. (5.2) can also be induced by κ, we may restrict ourselves to the case $\sin(2\phi+\varphi)=1$,

$$\langle :[\Delta E(t)]^2: \rangle = 2g^2\{\langle N(t)\rangle - \kappa[\langle N(t)\rangle]^{1/2}\cos\chi\} \ . \qquad (5.5)$$

Equation (5.5) shows that, in general, the maximum noise reduction of the electric field strength does not coincide in point of time with the maximum photon number of the field, which is in contrast to the case of one-photon resonance.

In Fig. 2(a) the maximum squeezing effect is shown as a function of the coherent atomic initial preparation χ. For comparison, the corresponding photon number is shown in Fig. 2(b) [from which, together with Eq. (5.3), the interaction time needed may be determined]. We see that for small values of χ satisfying the inequality $\chi<\chi_0$ ($\chi_0=\mathrm{arc\,cot}\sqrt{8}\approx 0.34$), maximum squeezing is observed at the time $\frac{1}{2}\Omega_2^{(2)}(t-t_0)=\pi/2$ for which the mean number of cavity photons becomes equal to the maximum

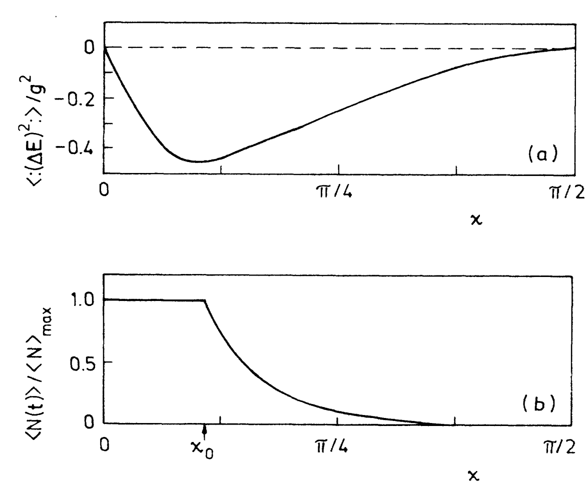

FIG. 2. (a) Maximum squeezing effect is given as a function of the atomic preparation. (b) The corresponding mean photon number.

photon number $\langle N(t)\rangle=\langle N\rangle_{\max}=2\sin^2\chi$. In other words, the atom completely delivers its energy to the cavity field. The situation is changed when the value of χ becomes larger than the value of χ_0 ($\chi>\chi_0$). Now, with increasing value of χ we observe a decrease of the mean number of cavity photons for which maximum squeezing appears. In these cases the energy of the atom is partly left in the cavity. Figure 2(a) reveals that, dependent on χ, the attainable maximum squeezing effect is $\langle:(\Delta E)^2:\rangle\approx-0.45\,g^2$. It is worth noting that this noise reduction of about 45% is obtained for the small mean photon number $\langle N(t)\rangle=\langle N\rangle_{\max}=2\sigma_{22}(t_0)\approx0.174$ [cf. Eqs. (3.4), (3.12) and (5.13)]. Thus a substantial squeezing effect may be obtained in cases when the atom is coherently prepared close to its ground state.

In Figs. 3 and 4 the mean photon number and the noise behavior of the electric field strength of the cavity field are shown to be dependent on the interaction time for a coherent atomic initial preparation with $\chi<\chi_0$ and $\chi>\chi_0$, respectively ($\chi=0.3$ in Fig. 3 and $\chi=\pi/4$ in Fig. 4). In addition to the results discussed above, we see that in the case when the inequality $\chi<\chi_0$ is valid, the noise reduction observed on the time scale $0\le\frac{1}{2}\Omega_2^2(t-t_0)\le\pi$ is followed by a noise enhancement on the time scale $\pi\le\frac{1}{2}\Omega_2^{(2)}(t-t_0)\le2\pi$ (the maximum noise reduction and enhancement are observed when maximum photon numbers are built up). Note that the temporal evolution of the noise of the electric field strength is periodic (period $\tau=4\pi/\Omega_2^{(2)}$). In the case when the inequality $\chi>\chi_0$ is valid, the situation becomes more complicated. As seen, on the time scale $0\le\frac{1}{2}\Omega_2^{(2)}(t-t_0)\le\pi$ there are now two ranges of noise reduction which are symmetric to a center range of noise enhancement. In this case, maximum noise enhancement corresponds to maximum mean photon number, whereas (maximum) noise reduction corresponds to smaller photon numbers. On the time scale $\pi\le\frac{1}{2}\Omega_2^{(2)}(t-t_0)\le2\pi$, again noise enhancement is observed. As mentioned above, the cases $0\le\frac{1}{2}\Omega_2^{(2)}(t-t_0)\le\pi$ and $\pi\le\frac{1}{2}\Omega_2^{(2)}(t-t_0)\le2\pi$ ($\kappa=+1$ and $\kappa=-1$, respectively) also describe the noise behavior of the two field quadratures [cf. Eqs. (5.2) and (5.4)].

Comparing the results found for the case of one-photon resonance with those for the case of two-photon resonance, we find that in the latter case the range of χ values giving rise to squeezing is wider and the strength of squeezing is more substantial than in the first case.

Combining Eq. (5.3) and Eqs. (3.7)–(3.10), and (3.12)–(3.15) for the case $k=2$, we may represent the photon density matrix in the following way:

$$\rho_{00}(t)=1-\tfrac{1}{2}\langle N(t)\rangle\ , \tag{5.6}$$

$$\rho_{22}(t)=\tfrac{1}{2}\langle N(t)\rangle\ , \tag{5.7}$$

$$\rho_{02}(t)=-i\kappa\exp\{i[2\omega(t-t_0)-\varphi]\}\cos\chi[\tfrac{1}{2}\langle N(t)\rangle]^{1/2}\ , \tag{5.8}$$

$$\rho_{20}(t)=\rho_{02}^*(t)\ , \tag{5.9}$$

κ being given in Eq. (5.4). Let us now compare this density matrix with the density matrix

$$\rho_{nm}^{\rm SV}=\begin{cases}(-1)^{(n+m)/2}\dfrac{(n!m!)^{1/2}}{(n/2)!(m/2)!}\dfrac{1}{|\mu|}\left[\dfrac{\nu}{2\mu}\right]^{n/2}\left[\dfrac{\nu^*}{2\mu^*}\right]^{m/2}, & \text{for } n \text{ and } m \text{ even}\\ 0, \text{ otherwise}\end{cases} \tag{5.10}$$

of a squeezed vacuum state of the type

$$|0\rangle_{\rm SV}=\exp\{-\tfrac{1}{2}[\xi(a^\dagger)^2-\xi^*a^2]\}|0\rangle\ , \tag{5.11}$$

where the relations $\mu=\cosh|\xi|$ and $\nu=(\xi/|\xi|)\sinh|\xi|$

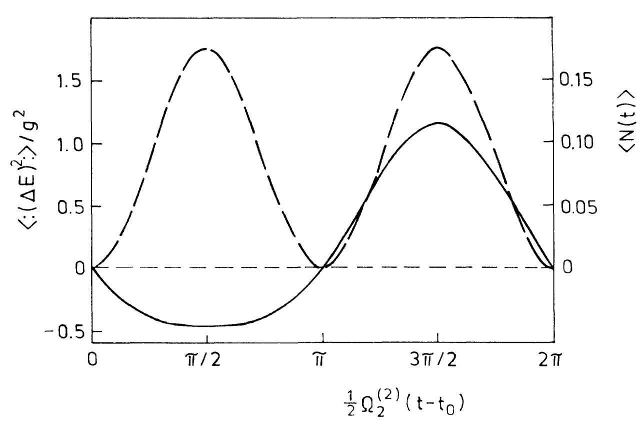

FIG. 3. Noise of the electric field strength (solid line) and the mean photon number (dashed line) are given as functions of time for the coherent atomic initial preparation $\chi=0.3<\chi_0$.

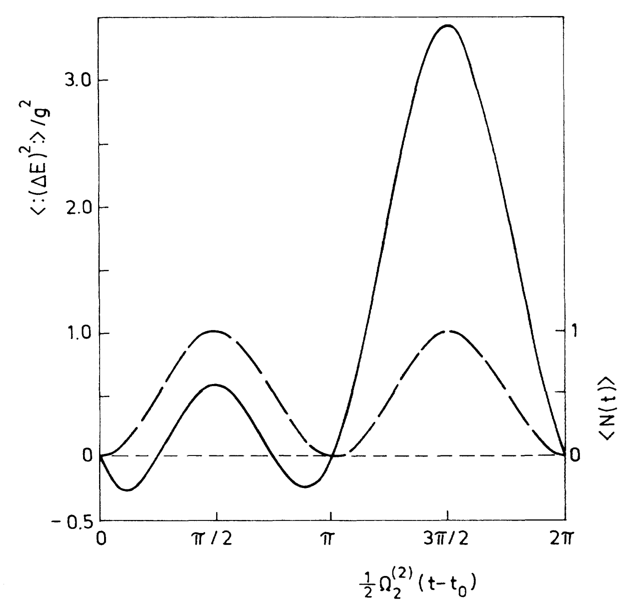

FIG. 4. Noise of the electric field strength (solid line) and the mean photon number (dashed line) are given as functions of time for the coherent atomic initial preparation $\chi=\pi/4>\chi_0$.

are valid, $|\nu|^2$ being the mean photon number (cf. Refs. 15, 16, and 25). Considering a sufficiently weak field ($|\nu|^2 \ll 1$) enables us to restrict ourselves (approximately) to the diagonal density matrix elements proportional to $|\nu|^2$ and to the off-diagonal elements proportional to $|\nu|$. In this case, from Eq. (5.10) we obtain

$$\rho_{00}^{SV}=1-\tfrac{1}{2}|\nu|^2 \ , \tag{5.12}$$

$$\rho_{22}^{SV}=\tfrac{1}{2}|\nu|^2 \ , \tag{5.13}$$

$$\rho_{02}^{SV}=-\frac{1}{\sqrt{2}}|\nu|\exp(-i\varphi_\nu) \ , \tag{5.14}$$

$$\rho_{20}^{SV}=(\rho_{02}^{SV})^* \ . \tag{5.15}$$

Comparing Eqs. (5.12)–(5.15) with Eqs. (5.6)–(5.9) reveals that the state of the field in the two-photon Jaynes-Cummings model with coherent atomic initial preparation and initial vacuum field tends to the squeezed vacuum state given in Eq. (5.11) if $|\cos\chi|\approx 1$, or in other words, if the atomic initial preparation is sufficiently close to the ground-state preparation. In particular, in the case of maximum squeezing (cf. Fig. 2) we have $\cos\chi\approx 0.955$ and according to the interaction time, $\langle N(t)\rangle=\langle N\rangle_{\max}\approx 0.175$, so that the field state produced somewhat differs from the squeezed vacuum state given in Eq. (5.11). Clearly, with increasing mean number of photons the photon density matrix in the two-photon Jaynes-Cummings model significantly deviates from the density matrix according to Eq. (5.10), which in general, cannot be reduced to the few elements considered in Eqs. (5.12)–(5 15).

The agreement of the squeezed vacuum as given in Eqs. (5.12)–(5.15) (for $|\nu|^2 \ll 1$) with the field in the two-photon Jaynes-Cummings model with a coherent atomic initial preparation close to the ground state ($|\cos\chi|\approx 1$) and initial vacuum cavity field suggests that under these conditions, the interaction between cavity field and atom may be described semiclassically by treating the atom as a classical time-dependent polarization current. In such a picture, in the Hamiltonian given in Eq. (2.1) together with Eqs. (2.2) and (2.3) (for $k=2$) we must omit the unperturbed atomic Hamiltonian and substitute in the interaction term for the atomic operators A_{12} and A_{21} appropriately chosen time-dependent density-matrix elements $\sigma_{21}(t)$ and $\sigma_{12}(t)$, respectively,

$$\tilde{H}^{(2)}=\hbar\omega a^\dagger a-\hbar\lambda^{(2)}[(a^\dagger)^2\sigma_{21}(t)+a^2\sigma_{12}(t)] \ . \tag{5.16}$$

Starting from the photon vacuum we may represent the state vector of the cavity field at time t as follows:

$$|\psi(t)\rangle=\exp[-i\omega a^\dagger a(t-t_0)]\tilde{U}(t,t_0)|0\rangle \ , \tag{5.17}$$

where

$$\tilde{U}(t,t_0)=T\exp\left[i\lambda^{(2)}\int_{t_0}^{t}d\tau[(a^\dagger)^2e^{2i\omega\tau}\sigma_{21}(\tau)+a^2e^{-2i\omega\tau}\sigma_{12}(\tau)]\right] \ . \tag{5.18}$$

Now, $\sigma_{21}(t)$ and $\sigma_{12}(t)$ are chosen to be the correct solutions of the two-photon Jaynes-Cummings model $\sigma_{ij}(t)=\sum_n\rho_{in,jn}^{(2)}(t)$. Making use of Eq. (2.18) together with Eqs. (2.12)–(2.15) (note that $\omega_{21}=2\omega$ is valid) and the initial conditions given in Eqs. (3.7)–(3.11) we derive

$$\sigma_{21}(t)=e^{-i\omega_{21}(t-t_0)}\sin\chi\cos\chi e^{i\varphi}\cos[\tfrac{1}{2}\Omega_2^{(2)}(t-t_0)] \ . \tag{5.19}$$

Hence Eq. (5.18) may be rewritten as

$$\tilde{U}(t,t_0)=\exp\{-\tfrac{1}{2}[\xi(t-t_0)(a^\dagger)^2-\xi^*(t-t_0)a^2]\} \ , \tag{5.20}$$

where

$$\xi(t)=-i\sqrt{2}\sin\chi\cos\chi e^{i\varphi}\sin[\tfrac{1}{2}\Omega_2^{(2)}(t-t_0)] \ . \tag{5.21}$$

Thus the initial vacuum state evolves (in the semiclassical approximation) into a squeezed vacuum state of the type defined in Eq. (5.11). For sufficiently weak field ($\xi\approx\nu$, $|\nu|^2\ll 1$) we therefore obtain for the relevant photon density-matrix elements [cf. Eqs. (5.12)–(5.15)]

$$\rho_{00}^{SV}(t)=1-\sin^2\chi\cos^2\chi\sin^2[\tfrac{1}{2}\Omega_2^{(2)}(t-t_0)] \ , \tag{5.22}$$

$$\rho_{22}^{SV}(t)=\sin^2\chi\cos^2\chi\sin^2[\tfrac{1}{2}\Omega_2^{(2)}(t-t_0)] \ , \tag{5.23}$$

$$\rho_{02}^{SV}(t)=-i\sin\chi\cos\chi e^{i[2\omega(t-t_0)-\varphi]}\sin[\tfrac{1}{2}\Omega_2^{(2)}(t-t_0)] \ , \tag{5.24}$$

$$\rho_{20}^{SV}(t)=[\rho_{02}^{SV}(t)]^* \ . \tag{5.25}$$

Comparing the semiclassical result given in Eqs. (5.22)–(5.25) with the exact result [cf. Eqs. (5.6)–(5.9) together with Eq. (5.3)] we find agreement if (provided the field starts from the vacuum) the coherent atomic initial preparation is close to the ground-state preparation $|\cos\chi|\approx 1$. This result may be understood from the argument that in this case (over the whole time range) the interaction between cavity field and atom is mainly determined by the polarization of the atom proportional to $\sigma_{21}(t)$ and $\sigma_{12}(t)$, whereas (over the whole time range) the upper atomic quantum state is hardly excited by quantum transitions. Its population probability therefore remains small compared with the polarization $\sigma_{22}(t)\ll|\sigma_{12}(t)|\ll 1$, the relation $\sigma_{22}(t)=|\sigma_{12}(t)|^2$ is approximately valid. This situation is similar to the case of a classical treatment of the field mode, where the condition $\langle a^\dagger(t)a(t)\rangle\approx|\langle a(t)\rangle|^2$ must be fulfilled. In the present case, such a classical treatment of the field is impossible, which is reflected by the squeezing effect. An approximate classical description of the atom (for $|\cos\chi|\approx 1$) yields $\langle a^\dagger(t)a(t)\rangle=|\xi(t)|^2$ and $\langle a(t)\rangle=0$, supporting the impossibility of a classical treatment of the field.

VI. SUMMARY AND CONCLUSIONS

We have studied the field generated in the k-photon Jaynes-Cummings model under the (realistic) assumption that the initial field state is the vacuum. To produce coherence and squeezing we have assumed that the atom is initially prepared in a coherent superposition of states. An experimental setup for observing effects of coherent atomic superposition was proposed by Krause, Scully, and Walther.[23]

Under the conditions mentioned, a coherent field amplitude can be generated in the case of one-photon resonance ($k=1$). In order to obtain a coherent state the initial coherent atomic preparation must be close to the ground state. A squeezing effect with the maximum of 25% noise reduction can be achieved, which is in agreement with the result found by Knight.[9]

In the case of two-photon resonance ($k=2$) squeezing may be observed for a larger range of coherent atomic initial preparations than in the one-photon case. A maximum effect of 45% noise reduction is achievable. Since the coherent field amplitude is equal to zero the field shows the features of a squeezed vacuum. A squeezed vacuum state [Eq. (5.11)] is (approximately) generated in the case of a coherent atomic initial preparation sufficiently close to the ground state. We have shown that, although the Jaynes-Cummings model is a fully quantum-theoretical one, the generation of this squeezed vacuum can be described by treating the atom as a classical current.

To our knowledge experiments concerning the two- (or even k-) photon Jaynes-Cummings model have not been performed. However, since for highly excited Rydberg atoms both the dipole moments are large and the states are dense there may be some hope that sufficiently large coupling constants for a two-photon transition in the sense of our model may be achievable.

For the case of a Rydberg maser cavity the assumption of an initial vacuum state of the field might seem to be somewhat far from the reality. However, it is possible to perform experiments at cavity temperatures $T \lesssim 0.5$ K.[3] Moreover, it should be feasible to realize initial cavity fields with a mean thermal photon number of 6.8×10^{-3}.[22] Clearly, in such a case the field can be expected to be sufficiently close to the vacuum so that the results of the present paper may be regarded as a suitable approximation. A rough estimation shows that for an initial thermal field with mean photon number of 6.8×10^{-3} the predicted squeezing effect of about 45% might be reduced to a value not smaller than 43%.

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