Radiation guiding in channeling beam x-ray laser by Bragg reflection coupling

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The effects of radiation guiding on an electron-beam channeling x-ray laser are investigated. An essential feature of this laser is distributed feedback induced by Bragg reflections. Using the Maxwell-Bloch scheme including the transverse Laplacian it is shown that reflection guiding and absorption guiding can avoid the diffraction of the amplified radiation out of the amplification medium for a beam radius in the range of 100–1000 Å. This opens the possibility of considering a channeling x-ray laser with high current density (10^5 A/cm^2), low emittance, but with very small total current (10 μA), similar to the beam used in scanning electron microscopy. On the other hand, gain guiding can avoid radiation diffraction in such small beam systems, but only with a further increase in the requirements on the beam current density. It is noted that planar superlattice crystals with a periodic structure of the order of 100 Å transverse to the beam propagation can be useful for radiation guiding.

I. INTRODUCTION

A relativistic electron beam propagating through planar or axial channels in a crystal free of imperfections may populate bound transverse energy eigenstates.[1] Spontaneous dipolar transitions between these discrete eigenstates have been shown experimentally to yield narrow-width, highly polarized, and intense x-ray radiation which is strongly forward peaked.[2] Previous estimates suggest that for using the channeling mechanism as a coherent x-ray source in a one-pass amplification scheme even modest gains may require very high current density in the range of 10^7-10^8 A/cm^2.[2,3-5]

An efficient scheme to significantly reduce the gain requirements for a channeling x-ray laser was proposed based on the concept of a distributed feedback laser (DFB), which is supplied by multiple Bragg reflections of the radiation.[6] The advantages of using DFB lasers includes the high degree of spectral selectivity with low gain per pass without the need for cavity mirrors. The radiation tunability is obtained by adjusting the electron-beam energy so that the Doppler up-shifted radiation can be tuned onto a line in the DFB model spectrum near the Bragg reflection frequency. These mirrorlike structures have the possibility to reduce high-beam-current-density requirements to the range 10^4-10^5 A/cm^2. High-current-density beams (10^5-10^6 A/cm^2, dc) with very low emittance have been developed for scanning electron microscopy.[7] In such systems the electron-beam source is a field emission type with very small beam radius (10 Å) and thus with very low total current. The total current is limited by the requirement that the beam, which is dc, should not damage the cathode. The beam emittance is about 10^{-8} rad cm, which is about 3 orders of magnitude lower than that for the best conventional electron sources. The beam expands after emission, but is routinely refocused to 10^6 A/cm^2 with a magnetic lens. For present purposes, the beam may be pulsed, in which case it should be possible to increase the current without damaging the cathode. Presumably this can be done without increasing the emittance; however, this remains to be documented in the laboratory. We assume that with a pulsed emitter the current can be increased by a factor of 10^3 (from 30 nA dc to 30 μA in a 10-nsec pulse) so that a current density of 10^5 A/cm^2 can be produced over a 1000-Å beam radius. For such a small beam radius one must consider a mechanism to avoid radiation diffraction out of the amplification medium. For a system of length L, the Fesnel spatial guiding condition is $F_0=\pi a^2/\lambda L \gg 1$, where λ is the radiation wavelength and a is the beam radius. For a system with $L=0.1$ cm, $a=1000$ Å, and $\lambda=3$ Å, then $F_0<1$ and radiation guiding cannot be maintained. In this paper we identify an efficient scheme for radiation guiding for such small radius beams.

We consider the channeling DFB scheme using the Maxwell-Bloch equations including the effects of radiation reflection, absorption, and diffraction. In Ref. 6 only a one-dimensional effect along the amplification direction is considered. In this paper we extend the results to a three-dimensional treatment including diffraction effects as guiding, and a multimode expansion introduced due to the small radius of the beam. Radiation diffraction is presented by including the transverse Laplacian and finding the eigenmode solutions at threshold gain condition. The channeling lasant medium is assumed to be surrounded by a nonlasant medium for a transverse square beam profile. The surrounding medium can be the same channeling crystal where Bragg reflections are avoided by impurities or dislocations, or a medium with a higher atomic number. It is found that for a beam radius in the range 100–1000 Å, reflection guiding and absorption guiding[8] can maintain the amplification of modes guided in the lasant medium. This avoids the diffraction of the radiation out of the amplification volume independent of the system length or gain factor. We also look for a gain guiding mechanism. We find that small radius gain guiding further increases the requirements on the beam current density.

It is noted that radiation guiding by reflections or absorptions can be maintained by a planar superlattice structure transverse to the electron-beam propagation, where lasant regions of 100–1000 Å wide are surrounded by nonlasant guiding regions. The radiation guiding in the other transverse dimension can be Fresnel guiding ($F_0 \gg 1$) in a range of 1 μm. Thus radiation guiding in a beam of dimension 100 Å$\times$1 μm can be maintained with a total beam current of 10 μA and a current density of 10^5 A/cm^2.

Some of the system parameters considered here are optimistic and a theoretical and an experimental effort should be performed to identify the physical mechanisms that can improve their values. The occupation length of the bound states is taken in the range 100–1000 μm. At room temperature this value is of the order of 50 μm and limits the cavity length. Cooling the crystal or increasing the beam particle energy may increase, to some extent, its value. It is also possible to increase the occupational length by applying a set of successive foils in the longitudinal direction to recapture the bound-state population. It is assumed that the population inversion in the lasing states is of the order of unity. An exact treatment of the population of the bound states may reduce this value and increase the current density requirements. Another optimistic parameter is the coherence length c/Γ, where $1/\Gamma$ is the decay time of the state polarization, and it is taken to be of the order of the occupation length. In general, the coherence length is smaller or equal to the occupation length and an effort should be made to increase their values. In this paper, we indicate the mechanisms which can be of interest for channeling x-ray lasers in high beam current densities and do not represent a complete solution of the problem.

The plan of the paper is as follows. In Sec. II we consider the DFB equations of motion for the lasant channeling medium. Eigenmodes solutions and the guiding conditions for the DFB system, including the nonlasant medium, are derived in Sec. III. The guiding in a one-pass system is considered in Sec. IV. Summary and conclusions are given in Sec. V.

II. DIFFRACTION MODEL IN A DFB X-RAY LASER

In this section we consider the radiation propagation in the lasant channeling medium. The inclusion of the surrounding nonlasant medium is presented in the following sections. We begin by characterizing the set of channeling transverse eigenstates as a two-level system with states $|1\rangle$ and $|2\rangle$, W and $\hbar\omega_0=\epsilon_2-\epsilon_1$ are the population and energy differences, respectively.[4] The directions of beam channeling and Bragg reflections are taken in the z direction. The Doppler up-shifted radiation frequency $\omega=\omega_0/[1-(v/c)]\simeq 2\gamma^2\omega_0$ in the forward direction is chosen to closely match the nth-order Bragg frequency $\omega\sim n\omega_B$, where v is the channeling electron speed, $\omega_B=\pi c/b$, and b is the periodic reflection plane spacing. Consequently, the channeling radiation can be tuned to satisfy the Bragg reflection condition and induce distributed feedback in the channeling crystal.[6]

The electric E and polarization P fields are taken in transverse x direction and are defined in terms of forward and backward traveling waves

$$\begin{aligned} E&=\epsilon_+ e^{-i\omega(t-z/c)}+\epsilon_- e^{-i\omega(t+z/c)}+\text{c.c.}\ , \\ P&=p_+ e^{-i\omega(t-z/c)}+p_- e^{-i\omega(t+z/c)}+\text{c.c.}\ , \end{aligned} \tag{1}$$

where ω is the electromagnetic wave frequency, $\epsilon_\pm$ and $p_\pm$ are slowly varying complex amplitudes depending on $\rho=(x,y)$, z, and t.

The radiation propagation is considered in the framework of the Maxwell-Bloch set of equations. The Maxwell wave equation is

$$\nabla^2\mathbf{E}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{E}=\frac{4\pi}{c^2}\frac{\partial}{\partial t}\left[\frac{\partial}{\partial t}\mathbf{P}+c\nabla\times\mathbf{M}+\mathbf{J}\right] , \tag{2}$$

where the magnetization $\mathbf{M}=\mathbf{P}\times\mathbf{v}/c$.[4] The crystal-induced current $\mathbf{J}$ can be represented as[6,9]

$$\frac{\partial}{\partial t}\mathbf{J}=\frac{c}{2\pi}\left[\omega K\mathbf{E}+\mu\frac{\partial\mathbf{E}}{\partial t}\right] , \tag{3}$$

where $K=2\pi e^2 n_e/cm_e\omega$ is the reflection function with n_e the spatially modulated atomic electron density, and μ is the modulated absorption function. An average is carried out on K and μ over the transverse dependence $\rho=(x,y)$ so that $K(z)=K(z+a)$ and $\mu(z)=\mu(z+a)$ are periodic functions in the z direction. Inserting Eq. (3) in Eq. (2) and using Eq. (1) we obtain

$$\sum_{\eta=\pm 1} e^{i\eta kz}\left[\left[-\frac{i}{2k}\nabla_\perp^2+\eta\frac{\partial}{\partial z}+\frac{1}{c}\frac{\partial}{\partial t}+\mu+iK\right]\epsilon_\eta -2\pi i\frac{\omega}{c}(1-\eta v/c)P_\eta\right]=0 , \tag{4}$$

where ϵ_η and P_η for $\eta=\pm 1$ are $\epsilon_\pm$ and $P_\pm$, respectively. Here $k=\omega/c$ and the second-order derivatives with respect to z and t are ignored because $\partial^2\epsilon/\partial t^2 \ll \omega\partial\epsilon/\partial t$ and $\partial^2\epsilon/\partial z^2 \ll k\partial\epsilon/\partial z$. The diffraction effect in Eq. (4) is represented by the transverse Laplacian $\nabla_\perp^2$.

Equation (4) is supplemented by the Bloch equation for $P_\pm$,[10]

$$\frac{\partial}{\partial t}P_\pm+v\frac{\partial}{\partial z}P_\pm=i\Delta_\pm P_\pm -i(1\mp v/c)d^2 n_b W(\epsilon_\pm/\hbar)-\Gamma P_\pm , \tag{5}$$

where $d=e\langle 1|x|2\rangle$ is the electric dipole moment, n_b is the beam number density depending on the transverse coordinate ρ, Γ is a phenomenological damping constant related to the channeling coherence length v/Γ, $\Delta_\pm=\omega(1\mp v/c)-\omega_0$ is a detuning frequency, and v/c represents a magnetic dipole interaction correction. In the limit of short coherence length V/Γ the left-hand side of Eq. (5) is small and near resonance ($\Delta_+=0$ and $\omega\simeq 2\gamma^2\omega_0$):

$$P_+=-id^2 n_b W(1-v/c)\epsilon_+/\hbar\Gamma . \tag{6}$$

In this limit $\Delta_- \sim \omega$, $\Delta_- \gg \Gamma$, and in the case of low gain P_- can be ignored in Eq. (4). Substituting Eq. (6) in Eq. (4) we obtain

$$\sum_{\eta=\pm 1} e^{i\eta k z}\left[-\frac{i}{2k}\nabla_\perp^2+\eta\frac{\partial}{\partial z}+\frac{1}{c}\frac{\partial}{\partial t}+\mu+iK-g_\eta\right]\epsilon_\eta=0 \ , \tag{7}$$

where g_η for $\eta=+1$ is $g_+=g=2\pi\omega d_1^2 n_b W/\hbar c\Gamma$ and is the forward gain factor with $d_1=d(1-v/c)$, and g_η for $\eta=-1$ is $g_-=0$ and is the backward gain factor.[6,11] The gain g_+ includes the beam transverse profile and depends on $\rho=(x,y)$.

In the following we obtain the equation of motion for a DFB x-ray laser by using the resonance parts of Eq. (7). Notice that K and μ are periodic functions in z and for $f(z)=f(z+a)$ we can use the Fourier series expansion

$$f(z)=\sum_{l=-\infty}^{\infty} f_l e^{2ilk_B z} \ , \tag{8}$$

where $k_B=\omega_B/c=\pi/b$ and

$$f_l=\frac{1}{b}\int_{-b/2}^{b/2} dz\, f(z) e^{-2ilk_B z} \ . \tag{9}$$

We insert the Fourier expansion Eq. (8) for $K(z)$ and $\mu(z)$ in Eq. (7). For the case that the radiation frequency is close to the lth-order Bragg reflection condition $k\sim lk_B$, and ignoring highly oscillatory terms, we obtain from Eq. (7)

$$-\frac{i}{2k}\nabla_\perp^2\epsilon_+ +\frac{\partial}{\partial z}\epsilon_+ +\frac{1}{c}\frac{\partial \epsilon_+}{\partial t} -(g_+-i\delta-\nu_0)\epsilon_+ +\nu_-\epsilon_-=0 \ , \tag{10}$$

$$-\frac{i}{2k}\nabla_\perp^2\epsilon_- -\frac{\partial}{\partial z}\epsilon_- +\frac{1}{c}\frac{\partial \epsilon_-}{\partial t} -(g_--i\delta-\nu_0)\epsilon_- +\nu_+\epsilon_+=0 \ , \tag{11}$$

where Eq. (10) and Eq. (11) are obtained from Eq. (7) for $\eta=+1$ and $\eta=-1$, respectively. In Eqs. (10) and (11) we redefined $\epsilon_\pm$ as $\epsilon_\pm \exp[\mp i(k-lk_B)z]$, and $\delta=lk_B-k$ is the small detuning from the lth-order Bragg reflection.[6] Here $\nu_0=\mu_0+iK_0$, $\nu_-=\mu_l+iK_l$, and $\nu_+=\mu_l^*+iK_l^*$, where we used the fact that K and μ are real functions, i.e., $K_{-l}=K_l^*$ and $\mu_{-l}=\mu_l^*$.

To include the atomic displacement effects and thermal motion $K_l\to K_l\exp(-W_l)$ and $\mu_l\to\mu_l\exp(-W_l)$, where W_l is the Debye-Waller factor of order l.[9,12] Here $W_l=2(lk_B)^2\langle(U^{(z)})^2\rangle$ and $\langle(U^{(z)})^2\rangle$ is the ensemble average of the square of the atomic displacement in the z direction.[13]

The coupled set Eqs. (10) and (11) are the DFB equations of motion, where diffraction effects are presented by the transverse Laplacian and by the transverse beam profile in g_+. In the following section eigenmode solutions of Eqs. (10) and (11) at threshold are considered and the conditions for radiation guiding are derived.

III. EIGENMODES AND RADIATION GUIDING IN DFB SYSTEMS

The system at threshold is presented by the set of equations (10) and (11) at steady state,[6,11]

$$-\frac{i}{2k}\nabla_\perp^2\epsilon_\pm \pm\frac{\partial}{\partial z}\epsilon_\pm-(g_\pm-i\delta-\nu_0)\epsilon_\pm+\nu\epsilon_\mp=0 \ , \tag{12}$$

where for the lasant channeling medium $\nu_0=\mu_0+iK_0$, $\nu=\mu_l+iK_l$, and for simplicity we take K_l and μ_l to be real.

To extend Eq. (12) for the surrounding nonlasant medium we define for that region $\nu_0=\tilde{\nu}_0=\tilde{\mu}_0+i\tilde{K}_0$, where $\tilde{\mu}_0$ and $\tilde{K}_0$ are the average absorption and reflection coefficient of the nonlasant medium, respectively. We further ignore the gain ($g_\pm=0$) and Bragg reflections ($\nu=0$) in the nonlasant medium. Equation (12) is a coupled set of Schrödinger-type equations with a complex potential.

A. Eigenmode solutions

For a transverse beam profile in the lasant medium of the form $\frac{1}{2}[f_+(x)+h_+(y)]$ the gain factors are

$$g_\pm(x,y)=\frac{g}{2}[f_\pm(x)+h_\pm(y)] \ , \tag{13}$$

where $g_-(x,y)=0$ or $f_-(x)=h_-(y)=0$. The eigenmode solution (n,m) for Eq. (12) can be written as

$$\epsilon_\pm^{(n,m)}(\rho,z)=e^{q_{n,m}z}U_\pm^{(n)}(x)V_\pm^{(m)}(y) \ , \tag{14}$$

where $q_{n,m}=\frac{1}{2}(q_n+q_m)$ and the eigenstates equations of motion are

$$\left[-\frac{i}{k}\frac{d^2}{dx^2}\pm q_n-[g\cdot f_\pm(x)-i\delta-\nu_0]\right]U_\pm^{(n)}(x) +\nu U_\mp^{(n)}(x)=0 \ , \tag{15}$$

$$\left[-\frac{i}{k}\frac{d^2}{dy^2}\pm q_m-[g\cdot h_\pm(y)-i\delta-\nu_0]\right]V_\pm^{(m)}(y) +\nu V_\mp^{(m)}(y)=0 \ . \tag{16}$$

For explicit solutions we consider in the following the case of a transverse square well gain profile: $f_+(x)=h_+(y)=1$ for the lasant medium $|x|<a$ and $|y|<a$, and zero gain outside this well. For this case the eigenstate solutions in the x and y direction are similar, i.e., $V_\pm^{(n)}(y)=U_\pm^{(n)}(y)$. For the x direction the symmetric solutions of Eq. (15) are

$$U_\pm^{(n)}(x)=\begin{cases} A_\pm(e^{i\alpha_n x}+e^{-i\alpha_n x}), & |x|\le a \\ B_\pm e^{i\beta_n x}, & x\ge a \end{cases} \tag{17}$$

where $\mathrm{Im}(\beta_n)>0$ in order that $\exp(i\beta_n x)\to 0$ for $x\to\infty$. Antisymmetric solutions of Eq. (15) can be obtained in a similar way to the symmetric solutions, but are not considered in the following.

To obtain a closed set of equations for α_n, β_n, and q_n

we insert Eq. (17) for the lasant region $|x|<a$ in Eq. (15):

$$\left[\frac{i\alpha_n^2}{k}\pm q_n-g_\pm+i\delta+\nu_0\right]A_\pm+\nu A_\mp=0\ , \tag{18}$$

where $g_+=g$ and $g_-=0$ for $|x|<a$. From the condition on the determinant of the coefficients $A_\pm$ in Eq. (18) to be zero we obtain

$$\left[q_n-g_++i\delta+\nu_0+\frac{i\alpha_n^2}{k}\right]\times\left[-q_n-g_-+i\delta+\nu_0+\frac{i\alpha_n^2}{k}\right]-\nu^2=0\ . \tag{19}$$

In a similar way we can get for the nonlasant region $x>a$, where $g_\pm=0$ and $\nu=0$ the condition

$$\left[q_n+i\delta+\tilde{\nu}_0+\frac{i\beta_n^2}{k}\right]\left[-q_n+i\delta+\tilde{\nu}_0+\frac{i\beta_n^2}{k}\right]=0\ . \tag{20}$$

The dispersion relation is obtained from the continuity conditions on $U_\pm(x)$ and $dU_\pm(x)/dx$ at $x=a$

$$\tan(\alpha_n a)=-\frac{i\beta_n}{\alpha_n}\ . \tag{21}$$

The closed set of equations (19)–(21) can be solved for α_n, β_n, and q_n.

Assuming that α_n is known then from Eq. (19), we can solve for q_n in terms of α_n

$$q_n^{(1)}=\frac{g}{2}+\lambda_n,\quad q_n^{(2)}=\frac{g}{2}-\lambda_n\ , \tag{22}$$

where

$$\lambda_n=\left[\left[\frac{g}{2}-i\delta-\nu_0-\frac{i\alpha_n^2}{k}\right]^2-\nu^2\right]^{1/2}\ . \tag{23}$$

For a slab of length L centered at $z=0$, the accompanying boundary conditions read $\epsilon_\pm(z=\mp L/2)=0$ and no external radiation sources are assumed. The eigenmode solutions in the range $|x|<a$ and $|y|<a$ consistent with the boundary condition are[6,11]

$$\begin{aligned}\epsilon_+^{(n,m)}(\rho,z)&=e^{gz/2}\sinh[\lambda_{n,m}(z+L/2)]\\&\quad\times\cos(\alpha_n x)\cos(\alpha_m y)\ ,\\ \epsilon_-^{(n,m)}(\rho,z)&=\pm e^{gz/2}\sinh[\lambda_{n,m}(z-L/2)]\\&\quad\times\cos(\alpha_n x)\cos(\alpha_m y)\ ,\end{aligned} \tag{24}$$

where $\lambda_{n,m}=(\lambda_n+\lambda_m)/2$. A formal solution for the allowed resonance frequencies δ and threshold values g for the eigenstate (n,m) can be obtained by inserting Eq. (24) in the equation for $\epsilon_\pm$ in Eq. (12), and we obtain that

$$\lambda_{n,m}=\pm\nu\sinh(\lambda_{n,m}L)\ , \tag{25}$$

$$\frac{g}{2}-i\delta=\nu_0+\frac{i}{2k}(\alpha_n^2+\alpha_m^2)\pm\nu\cosh(\lambda_{n,m}L)\ . \tag{26}$$

The coupled equations (25) and (26) yield the allowed δ and g for the mode (n,m).

Approximate solutions of Eqs. (25) and (26) can be obtained in the limit of strong reflections $|K_lL|^2\gg1+(gL)^2$ and $|\lambda_{n,m}L|\ll1$. Upon expanding Eq. (25) in this limit for $n=m$ and using that μ_0^2, $\mu_l^2\ll K_l^2$, we obtain the lowest threshold values for δ and g for the states (n,n)

$$\delta_t^{(n,n)}=-K_0+K_l-\frac{1}{k}\mathrm{Re}(\alpha_n^2)\ , \tag{27}$$

$$g_t^{(n,n)}=\frac{6}{K_l^2L^3}+2(\mu_0-\mu_l)-\frac{2}{k}\mathrm{Im}(\alpha_n^2)\ . \tag{28}$$

The first term in the threshold gain Eq.(28) is due to reflections. The second term in Eq. (28) is due to reduced absorption of the standing radiation waves generated with nodes on the atomic sites and is similar to the Borrmann anomalous transmission effect.[9] From numerical calculation of the anomalous absorption in single crystal the value of $1-\mu_l/\mu_0$ can be of the order of 10^{-3}.[12] Thus, for $\mu_0\sim10\ \mathrm{cm}^{-1}$, the threshold gain due to absorption can be reduced to the order of $10^{-2}\ \mathrm{cm}^{-1}$. The real and imaginary parts of α_n^2 in Eqs. (27) and (28) are the mode contributions to the selectivity and threshold gain, respectively. In the following, limiting values of α_n are considered and the conditions for radiation guiding in DFB x-ray lasers are derived.

B. Guiding conditions

For the square well gain profile guiding solutions are obtained for $|\alpha_n/\beta_n|\ll1$. For this case we obtain from the dispersion relation Eq. (21) that $\alpha_n a=(n+\frac{1}{2})\pi$ and the threshold values in the strong reflection limit Eqs. (27) and (28) are

$$\delta_t^{(n,n)}=-K_0+K_l-\frac{\pi^2}{ka^2}(n+\tfrac{1}{2})^2\ , \tag{29}$$

$$g_t^{(n,n)}=\frac{6}{K_l^2L^3}+2(\mu_0-\mu_l)\ . \tag{30}$$

The threshold gain in Eq. (30) is independent of n. But for a transverse gain profile which is maximized at the center $x=y=0$, then the threshold gain would increase with n.

The condition for radiation guiding is that $|e^{i\beta_n a}|\ll1$ or $\mathrm{Im}(\beta_n a)\gg1$. For low values of n, $\alpha_n a\sim1$ and the condition for guiding is consistent with the condition $|\alpha_n/\beta_n|\ll1$. To calculate β_n we use Eq. (20) in the strong reflection limit $[(K_lL)^2\gg1+(g/2)^2,\ |\lambda_nL|\ll1]$, where q_n, Eq. (22), can be ignored. The equation for β_n is

$$i\delta+\tilde{\mu}_0+i\tilde{K}_0+\frac{i\beta_n^2}{k}=0\ . \tag{31}$$

From Eq. (29) $\delta\approx-K_0+K_l$ and the solution for β in Eq. (31) is

$$\beta_n=[ik\tilde{\mu}_0-k(\tilde{K}_0-K_0+K_l)]^{1/2}\ . \tag{32}$$

From Eq. (32) there are two mechanisms for radiation guiding independent of the gain or length of the system: the absorption guiding and the reflection guiding.[8] The absorption guiding condition is obtained for $\tilde{\mu}_0 > |\tilde{K}_0 - K_0 + K_l|$ and $\mathrm{Im}(\beta_n a) \simeq (ka^2\tilde{\mu}_0/2)^{1/2} \gg 1$, where $\tilde{\mu}_0$ is the average absorption coefficient of the nonlasant medium. For this case the effective Fresnel number $F_\mu = \pi a^2 \tilde{\mu}_0/\lambda > 1$, with an effective length $L_{\text{eff}} = 1/\tilde{\mu}_0$. The reflection guiding is obtained for $\tilde{K}_0 > K_0 - K_l, \tilde{\mu}_0$ and $\mathrm{Im}(\beta_n a) \simeq (ka^2\tilde{K}_0)^{1/2} \gg 1$, where $\tilde{K}_0$ is the average reflection coefficient of the nonlasant medium. For this case the effective Fresnel number is $F_K = 2\pi a^2 \tilde{K}_0/\lambda > 1$ with $L_{\text{eff}} = \frac{1}{2}\tilde{K}_0$.

For the first-order Bragg reflection ($l=1$) typically $K_1 \simeq K_0/2 = 5\times 10^3$ cm^{-1} for channeling crystals, e.g., silicon and diamond.[6] Reflection guiding can be maintained by a surrounding medium of the same crystal, where Bragg reflections are avoided by impurities or dislocations, and $\tilde{K}_0 = K_0 > K_0 - K_1, \mu_0$. Another possibility for guiding is to use a surrounding medium with a higher atomic number with $\tilde{K}_0$ or $\tilde{\mu}_0$ of the order of 10^4 cm^{-1} and $L_{\text{eff}} \simeq 1\ \mu$. The beam radius for guiding should be $a > (\lambda L_{\text{eff}}/\pi)^{1/2}$ and for $\lambda = 3$ Å, $a > 100$ Å. Thus radiation guiding and amplification can be obtained for high current density of 10^5 A/cm^2 and for very low total current in the range of 10 μA.

For the case that the surrounding nonlasant medium is the same as the lasant medium (includes Bragg reflections but with no gain), then it is possible to write for this region a similar equation to Eq. (19), where $g_\pm = 0$ and α_n is replaced by β_n. Substituting it in Eq. (19), using the continuity conditions of the fields at the boundary and the strong reflection conditions ($|q_n L| \ll 1$, $K_0 L \gg 1$), we find that the only guiding mechanism is the gain guiding with $\mathrm{Im}(\beta_n a) \simeq (ka^2 g/2)^{1/2} \gg 1$. Here the effective Fresnel number $F_g = \pi a^2 g/\lambda > 1$ with $L_{\text{eff}} = 1/g$. The beam radius for gain guiding is $a > (\lambda/\pi g)^{1/2}$ and for $\lambda = 3$ Å, $g = 10^{-2}$ cm^{-1} then $a > 10\ \mu$m, and gain guiding increases the requirements on the total beam current.

IV. GUIDING IN A ONE-PASS CHANNELING X-RAY LASER

The equation of motion for a one-passage amplifier with no resonance Bragg reflections is obtained from Eq. (12) $\epsilon \equiv \epsilon_+$, $\epsilon_- = 0$, $\nu_\pm = 0$,

$$-\frac{i}{2k}\nabla_\perp^2 \epsilon + \frac{\partial}{\partial z}\epsilon - (g_+ - i\delta - \nu_0)\epsilon = 0 \ , \tag{33}$$

where for a square well profile $g_+ = g$, $\nu_0 = \mu_0 + iK_0$ for the lasant region $|x| < a$ and $|y| < a$, and $g_+ = 0$, $\nu_0 = \tilde{\nu}_0 = \tilde{\mu}_0 + i\tilde{K}_0$ out of the amplification region. Following the treatment of Sec. III, the eigenmodes solutions can be written as

$$\epsilon^{(n,m)}(\rho, z) = e^{q_{n,m} z} U^{(n)}(x) U^{(m)}(y) \ , \tag{34}$$

where $q_{n,m} = (q_n + q_m)/2$. The mode equation for $U^{(n)}(x)$ is

$$\left[-\frac{i}{k}\frac{d^2}{dx^2} + q_n - g + i\delta + \nu_0 \right] U^{(n)}(x) = 0 \ . \tag{35}$$

The symmetric solutions of Eq. (35) are

$$U^{(n)}(x) = \begin{cases} A(e^{i\alpha_n x} + e^{-i\alpha_n x}), & |x| \le a \\ Be^{i\beta_n x}, & x \ge a \ . \end{cases} \tag{36}$$

The relations between q_n, α_n, and β_n are similar to Eqs. (19)–(21) and are given by the relations

$$q_n - g + i\delta + \nu_0 + i\frac{\alpha_n^2}{k} = 0 \ , \tag{36a}$$

$$q_n + i\delta + \tilde{\nu}_0 + i\frac{\beta_n^2}{k} = 0 \ , \tag{36b}$$

$$\tan(\alpha_n a) = -i\frac{\beta_n}{\alpha_n} \ , \tag{36c}$$

with the guiding condition $\mathrm{Im}(\beta_n a) \gg 1$.

The guiding states are obtained for $|\alpha_n/\beta_n| \ll 1$ and from Eq. (36c) $\alpha_n a = (n + \frac{1}{2})\pi$. From Eq. (36a) the eigenvalues q_n are

$$q_n = g - \mu_0 - i\delta - iK_0 - i\frac{\pi^2}{a^2}(n + \tfrac{1}{2})^2 \ . \tag{37}$$

The gain factor of state (n,m) is $g_{n,m} = q_{n,m} + q_{n,m}^*$. For the guiding states $g_{n,m} = 2(g - \mu_0)$ and the amplification factor g should be larger than the average absorption μ_0.

From Eqs. (36a) and (36b) we have the relation

$$\nu_0 - \tilde{\nu}_0 - g + i\frac{\alpha_n^2}{k} - i\frac{\beta_n^2}{k} = 0 \ , \tag{38}$$

and for $|\alpha_n/\beta_n| \ll 1$ the solution for β_n is

$$\beta_n = [ik(g + \tilde{\mu}_0 - \mu_0) - k(\tilde{K}_0 - K_0)]^{1/2} \ . \tag{39}$$

In Eq. (39) there are three mechanisms for radiation guiding, where $\mathrm{Im}(\beta a) \gg 1$ independent of the system length. The absorption guiding is obtained for $\tilde{\mu}_0 > |g - \mu_0|$, $|\tilde{K}_0 - K_0|$, and $\mathrm{Im}(\beta a) \simeq (ka^2\tilde{\mu}_0/2) \gg 1$ with an effective Fresnel number $F_\mu = \pi a^2 \tilde{\mu}_0/\lambda > 1$. We obtain reflection guiding for $\tilde{K}_0 > K_0$, $|g + \tilde{\mu}_0 - \mu_0|$, and $\mathrm{Im}(\beta a) \simeq (ka^2\tilde{K}_0) \gg 1$, with an effective Fresnel number $F_K = 2\pi a^2 \tilde{K}_0/\lambda > 1$. The absorption or reflection guiding conditions in a one-pass channeling x-ray laser can only be maintained by a surrounding medium of larger atomic number, where $\tilde{K}_0$ or $\tilde{\mu}_0$ are larger than the lasant medium. Radiation guiding and amplification can be maintained for $\lambda = 3$ Å, $a > 100$ Å with $\tilde{\mu}_0$ and $\tilde{K}_0$ of the order of 10^4 cm^{-1}.

Gain guiding is obtained in Eq. (39) for $g > |\tilde{\mu}_0 - \mu_0|$, $|\tilde{K}_0 - K_0|$, and $\mathrm{Im}(\beta a) \simeq (ka^2 g/2)^{1/2} \gg 1$, with an effective Fresnel number $F_g = \pi a^2 g/\lambda > 1$. The beam radius for gain guiding should be $a > (\lambda/\pi g)^{1/2}$. For $\lambda = 3$ Å and $a \sim 100-1000$ Å, $g \ggg 1$ cm^{-1}. Thus gain guiding in very small beam radius can be obtained only by a further large increase in the current density.

V. DISCUSSION

In order to obtain significant gain from induced emission the current density in a DFB scheme should be high (10^4-10^5 A/cm^2). One of the concerns is the survival of the crystal. We consider heating of a diamond crystal by a beam of 20 MeV, a current density of $J=10^4$ A/cm^2, and a pulse duration of $\Delta t=1$ nsec. The temperature change is $\Delta T=n_b c \Delta t \Delta\epsilon / c_V$, where $\rho=2.55$ g/cm^3 (crystal density), $n_b=J/ec=2\times10^{12}$ cm^{-3} (beam density), $c_V=0.12$ cal/g K (specific heat), and $\Delta\epsilon=2.2$ MeV cm^2/g (electron energy loss[14]). The result is $\Delta T=40$ K and the heating is relatively low. The rms vibrational amplitude of the atoms scales with temperature as $1+T/T_D$, where T_D is the Debye temperature. Diamond has a very high Debye temperature ($\sim$2000 K), so even increasing the crystal temperature by 400 K increases the one-dimensional thermal vibration amplitude from 0.042 to 0.049 Å. Thus in diamond it is possible to increase the beam density or the pulse duration by an order of magnitude without an important effect on the crystal periodicity. The heating considered here is an upper bound because not all energy loss is coupled to the narrow lasing medium and part of it diffuses out. This may be the reason that an electron microscopy dc beam with a current density of 10^6 A/cm^2 does not destroy the emitter. Heating may be a problem for constant current accelerators that are usually employed for channel radiation measurement, but is not a problem for a short beam pulse of several nanoseconds and a cooled crystal with relatively high Debye temperature.

In such beams the total current can be very small (10 μA) and the transverse dimension $a\sim100-1000$ Å. Here radiation guiding in the amplification medium is important. In a system with $L=0.1$ cm, $\lambda=3$ Å the Fresnel spatial guiding is for $F_0=\pi a^2/\lambda L \gg 1$ and $a>1$ μm. Thus Fresnel guiding exists for relatively higher currents of an order of 1 mA. The gain guiding can be obtained for $F_g=\pi a^2 g/\lambda>1$. This guiding is not practical because it impose a further increase in the gain or current density. In a DFB scheme it is possible to consider radiation guiding by surrounding the lasant channeling medium by a nonlasant medium. The surrounding medium can be of the same channeling crystal, where Bragg reflections are avoided by impurities or dislocations, or by a surrounding medium with a higher atomic number. In such a system reflection guiding and absorption guiding by the nonlasant regions can maintain the amplification of radiation modes in the lasant regions independent of the system length or gain factor. The guiding can be maintained for $F=\pi a^2/\lambda L_{\mathrm{eff}}>1$, where the effective length $L_{\mathrm{eff}}\sim 1/\tilde{K}_0$ or $1/\tilde{\mu}_0$, with $\tilde{K}_0$ and $\tilde{\mu}_0$ the average reflection or absorption coefficient of the nonlasant medium, respectively.

It is possible to show by using the methods of Sec. III that the radiation guiding mechanisms in the x and y directions can be of different types. Thus it can be useful to consider a planar superlattice crystal with lasant channeling regions surrounded by nonlasant regions transverse to the beam propagation, where the lasant region dimension is of an order of 100 Å. Radiation guiding by reflection or absorption can be maintained in that transverse direction by the nonlasant regions. The guiding in the other transverse dimension can be Fresnel guiding ($F_0\gg1$) in a range of 1 μm. Thus radiation guiding in a beam of dimension (100 Å)$\times$(1 μm) can be maintained with a low beam current of 10 μA. The interaction of the beam with a lasant domain can be obtained by small scanning of the beam on the crystal surface.

The number of photons emitted in the spontaneous stage can be approximate from an emission of 10^{-3} photons/electron cm.[2] For a 10-nsec beam pulse, current density of 10^5 A/cm^2, beam radius 1 μm, and cavity length of 0.1 cm, the number of emitted photons is 2×10^4. In the stimulated emission stage the number of photons can be increased by several orders of magnitude if saturation can be attained. In this case, the number of photons is of the order of the number of electrons passing the cavity, which is 10^8. For x-ray laser application, as holographic imaging of biological systems, the sufficient number of coherent photons is 10^3.

High-current-density field emission beam sources (10^6 A/cm^2) with very low emittance ($G10^{-8}$ rad cm) and total current (10 nA) are dc sources used in scanning electron microscopy. In order to achieve an x-ray laser based on DFB scheme, it is interesting to consider an accelerator with such an electron source. If the source is pulsed with a pulse length of about 10 nsec, it is plausible that the current can be increased by a factor of 10^3 without increasing the emittance.

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