PHYSICAL REVIEW A VOLUME 40, NUMBER 11 DECEMBER 1, 1989

Reduced-noise nonreciprocal transducer based upon vacuum tunneling

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(Received 5 July 1989)

Displacement sensors that work by taking advantage of the nonlinear displacement-current relationship of an electron vacuum-tunneling probe (VTP) are theoretically analyzed in this paper. We show, using the language of electromechanical two-port transducers, that the VTP is nonreciprocal and that its noise is intrinsically quantum limited. We present a semiquantitative analysis of a VTP used to monitor the displacement of a simple mechanical harmonic oscillator and show that the Heisenberg uncertainty relation for the position and momentum of the mechanical oscillator is enforced by the noise in the VTP. The results of an optimal filter calculation of the sensitivity of the VTP-mechanical oscillator system for impulsive force detection are presented. These results are contrasted with results for a conventional capacitive transducer, and we show that the VTP may offer vastly increased sensitivity as a consequence of its nonreciprocity. The maximum sensitivity of the VTP system is calculated as a function of the temperature, the dc tunneling current, and the mass, frequency, and quality factor of the mechanical oscillator. For typical operating conditions the maximum sensitivity is obtained for small-mass systems, which makes the VTP ideal for miniature accelerometers and related devices.

I. INTRODUCTION AND OUTLINE

The marvelous capability of the scanning tunneling microscope[1] and the atomic force microscope[2] to produce atomic-resolution images of surfaces is widely appreciated. However, the unique properties of a vacuum-tunneling probe used as an electromechanical transducer are not widely recognized. In this paper we theoretically examine electromechanical transduction which is based upon the vacuum-tunneling phenomenon, and present some startling conclusions. The vacuum-tunneling probe, which we shall refer to as the VTP, for fundamental reasons can be far superior to conventional means of electromechanical transduction in some applications. The VTP is active and nonreciprocal. We show that as a consequence of nonreciprocity the noise of a VTP may be thousands of times less than optimized conventional transducer schemes. The VTP is especially well suited to applications that involve monitoring very small test masses characteristic of micrometer-size micromechanical devices.

The essence of the VTP is that it is nonreciprocal. It is a much better sensor that actuator. Therefore the back-action force of the transducer on the test mass which is monitored by the VTP is very weak. Unavoidable electronic noise is converted by a transducer to a fluctuating back-action force on the mechanical test mass, which is a significant factor in the determination of the sensitivity of a transduction system. The near elimination of the back-action force by the VTP allows substantial noise reduction and increased sensitivity.

For comparison consider another example of a nonreciprocal electromechanical transducer, the so-called back-action-evasion (BAE) scheme.[3,4] The BAE strategy is nonreciprocal by virtue of the time-dependent coupling of the transducer to the mechanical test object, which is conveniently modeled as a mechanical harmonic oscillator. The characteristic of the BAE scheme is that it is intrinsically phase sensitive; there is information about one phase of the mechanical harmonic oscillator at the transducer output and the transducer back-action force is confined to the orthogonal phase of the harmonic oscillator. The vacuum-tunneling probe displays another type of back-action reduction, distinct from the phase-sensitive BAE strategy. The VTP is phase insensitive, and there is reduced back action on both phases as well.

One of the issues which we will explore in this paper is the fundamental quantum limit to the sensitivity of a VTP. The extraordinary feature of the VTP is that it should be relatively easy to reach this limit. To achieve quantum-limited performance with a conventional transducer requires that the amplifier following the transducer be quantum limited. With a VTP transducer it should be possible to make quantum-limited measurements using common room-temperature amplifiers.

There are two equivalent points of view one may take in discussing the quantum-mechanical limits of electromechanical transducer sensitivity. If we treat the mechanical test mass as a quantum oscillator, the standard quantum limit for the sensitivity of a continuous displacement measurement is given by, $\Delta x \geq (\hbar/2m\omega)^{1/2}$

(see Ref. 3). This result follows from the Heisenberg uncertainty relation between the momentum and the position of the test mass. An alternate point of view, which we prefer, is to ascribe quantum properties to the apparatus which is used to make the measurement and to treat the test mass classically. This is the point of view taken in the Heisenberg microscope paradigm.[5] One invokes the deBroglie relation between the momentum and wavelength of the photons used to locate a particle in the microscope. The momentum kick to the particle is in inverse proportion to the diffraction-limited resolution of the "microscope." The compromise between the momentum and position measurement accuracy is expressed by the uncertainty relation, $\Delta x \Delta p \geq \hbar/2$. To detect a force which acts on the test mass one must make a series of measurements. If, at one instant, one makes a precise position measurement at the expense of momentum precision, in accord with the Heisenberg uncertainty relation, the predictions of position at subsequent times will be uncertain because the momentum "feeds back" to affect the position. The optimum strategy, which gives the minimum position uncertainty for a continuous measurement, leads to the above standard quantum limit.

In the same spirit one can represent the quantum-measurement noise in a transducer by putting the quantum fluctuations in the amplifier which follows the transducer. One model has noise generators at the input and output of the amplifier, and the product of the spectral densities of the two noise generators is constrained by quantum mechanics to be greater than a minimum value. If one accounts for the amplifier output noise and the fluctuating back action caused by the amplifier input noise, one finds that for measurements of the mechanical harmonic oscillator position the standard quantum limit $\Delta x \geq (\hbar/2m\omega)^{1/2}$ is obeyed. If we adopt this point of view it is easier to understand why the VTP is superior to conventional transducers. The amplifier noise becomes unimportant because of the gain of the transducer.

The paper is divided into the following sections. In the next section we present a semiquantitative analysis of the VTP, and we show that it is intrinsically quantum limited, i.e., that the tunneling probe enforces the Heisenberg uncertainty principle for a position measurement of a mechanical harmonic oscillator. In Sec. III we present the two-port representation of the VTP so that we can make a comparison to a conventional capacitive transducer to emphasize the essential features of the VTP. In Sec. IV we present the results of a detailed calculation of the sensitivity of a VTP which accounts for the noise of the probe and the Brownian motion of the mechanical test object. This calculation uses a random-variable approach and results from optimal filter theory. In the following section (Sec. V) we present the results a similar calculation for a capacitive probe and a comparison of the two schemes. The major conclusion of Sec. V is that the tunneling probe offers the most advantage when used to monitor small test masses. Following a summary we include Appendixes A and B in which we discuss the limits of applicability of our results. We determine the level at which the amplifier noise may be disregarded and the requirements for the VTP capacitance to be negligible.

II. SIMPLIFIED THEORY OF THE VACUUM-TUNNELING PROBE

The vacuum-tunneling probe is a pointed conducting tip held a few Å from the conducting surface of a test mass, the motion of which we wish to monitor (see Fig. 1). Normally the scanning tunneling microscope is operated with feedback to hold the tip a fixed distance from a surface, but we assume that this distance is variable and is given by $d-x$, where d is the nominal separation of the tip and test mass and x represents the small deviation from the nominal gap. The test mass is assumed to be voltage biased $(+V_0)$ positive with respect to the tip, so electrons tunnel from the tip to the surface of the test mass. One may define an effective tunneling resistance R_T which is dependent upon the separation of the tip from the test-mass surface,

$$R_T = R_0 e^{-2\kappa x} \approx R_0(1-2\kappa x) \quad \text{for } 2\kappa x \ll 1 , \tag{1}$$

where R_0 is typically $10^7\Omega$ for a nominal separation of several Å, and $\kappa=(2m_e\phi)^{1/2}/\hbar$, where ϕ is the probe material work function, m_e is the electron mass, and $\hbar$ is Planck's constant.

There is also capacitance between the tunneling tip and the test-mass surface which is a relatively large, stray capacitance from the fringing fields as well as a capacitance which is dependent upon the separation of the tip from the test-mass surface in the following manner:

$$C = C_0/(1-x/d) \approx C_0(1+x/d) \quad \text{for } x \ll d . \tag{2}$$

In our analysis it will be assumed that this capacitance is small enough that its effects can be ignored. In an earlier paper[6] we give the conditions under which the capacitance can be ignored, and this issue is also addressed in Appendix A. We defer a full treatment of the capacitance effects to a planned forthcoming paper.[7]

With the tip voltage biased, the tunneling current will be a function of the tip–to–test-mass separation. We assume that one monitors the tunneling current to infer the motion of the test mass,

$$I(t) = I_0(1+2\kappa x) , \tag{3}$$

where $I_0 = V_0/R_0$ is the nominal tunneling current. The quantity $2\kappa I_0$ gives the "forward" transfer characteristic of a tunneling probe. For a value of $I_0 = 10^{-7}$ A and $\kappa = 10^{10}\ \text{m}^{-1}$, the sensitivity is 2×10^3 A/m.

The tunneling probe has an influence on the motion of the test mass, the back action, because the tunneling electrons transfer momentum. The rate of electron transfer from the probe to the test mass is $I(t)/e$. If we assume that each electron carries momentum p_e, then the back-action force of the probe on the test mass is $F_{\text{BA}} = p_e I(t)/e$.

To determine the minimum detectable motion of the test mass, we must consider the fluctuations of the tunneling current and the back-action force. It is a general feature that two sources of uncorrelated noise are required to properly describe the noise limits of a detection system.[8] In the VTP these are the current-shot noise and the momentum-transfer noise (back-action force). For now we neglect the noise of the amplifier which follows the tunneling probe; we will show in a later section that this can be safely ignored under quite lenient conditions. The test mass Brownian motion is also left out at this stage, but we show later that it can be a significant factor.

The fluctuation of the tunneling current ΔI is given by the square root of the current-shot-noise spectral density $\sqrt{2eI}$ multiplied by the square root of the measurement bandwidth, which is approximately 4 divided by the measurement time.[9] The measurement time is taken to be the time for N electrons to arrive with the mean time between arrivals being e/I_0. Then

$$\Delta I \approx I_0(8/N)^{1/2} . \quad (4)$$

Thus the rms apparent displacement fluctuation is

$$\Delta x = \Delta I / 2\kappa I_0 \approx (1/\kappa)(2/N)^{1/2} . \quad (5)$$

The fluctuations in the back-action force are the other source of noise. The back-action force can be written $F_{BA} = (1/e)(I_0 + \Delta I)(p_1 + \Delta p)$; there are two ways in which the back-action force may fluctuate–either by a fluctuating rate of electron transfer ΔI (the current-shot noise) or by the fluctuation about the mean of the momentum of the transferred electrons Δp. We have the following expression for the fluctuating force of the VTP on the test mass:

$$\Delta F_{BA} = (1/e)(I_0 \Delta p + p_1 \Delta I + \Delta p \Delta I) . \quad (6)$$

We will ignore the last term which is second order in the small fluctuating quantities. The second term in Eq. (6) arises from the shot noise and therefore is completely correlated with the apparent displacement noise. In the presence of correlations there is always a way to make such a term vanish, and the results of a detailed calculation (Sec. IV) demonstrate that this indeed may happen. Furthermore, this term depends upon the mean momentum of the tunneling electrons which could have nearly any value with engineered band-gap materials. Therefore, the first term remains as the irreducible part of the back-action noise.

We can estimate a lower bound for this by considering the quantum uncertainty of the tunneling electrons. We assert our knowledge of the electrons which tunnel is such that at some instant they are confined to the probe–test-mass gap d. Using the Heisenberg relation $\Delta x \Delta p \gtrsim \hbar/2$ and setting $\Delta x = d$, we have $\Delta p \gtrsim \hbar/2d$. The quantity κd is roughly of order unity, although usu-

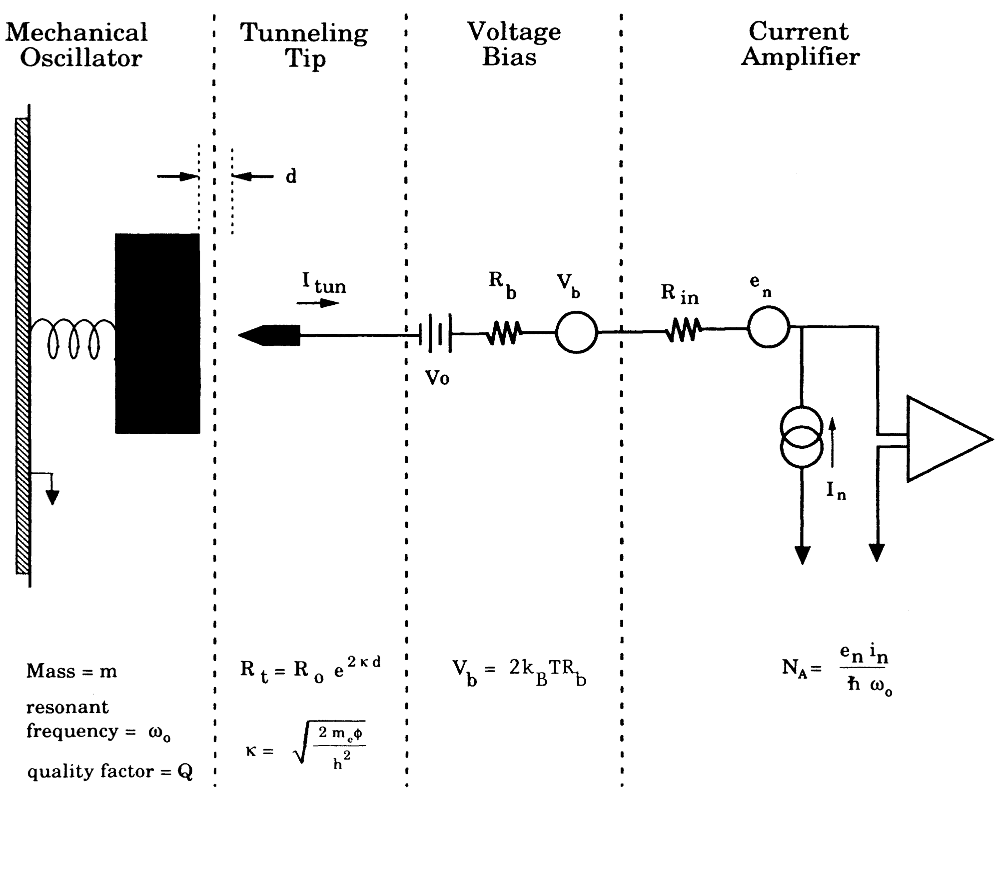

FIG. 1. Electromechanical schematic of a motion transducing system using a vacuum-tunneling probe.

ally closer to 10, so we have approximately $\Delta p \gtrsim \hbar\kappa$. This is the momentum uncertainty imparted to the test mass by a single-electron tunneling event. The test-mass momentum uncertainty grows as a random walk because the individual tunneling events are uncorrelated. Thus the momentum uncertainty of the test mass after N events is $\Delta p_{\text{test mass}} \gtrsim \hbar\kappa\sqrt{N}$. Taking the product of Δx and Δp, we see that the uncertainty relation for the test mass is obeyed $\Delta x\,\Delta p_{\text{test mass}} \gtrsim \hbar$.

The preceding argument demonstrates that the tunneling probe is intrinsically a quantum-limited sensor. In our argument we assert that the tunneling electrons are confined to the gap and that this leads to a momentum uncertainty of approximately $\hbar\kappa$. It seems that increasing the tip-mass separation d would decrease the momentum uncertainty and allow the uncertainty product $(\Delta x\,\Delta p)$ to be made arbitrarily small. However, if d is increased, the uncertainty product remains bounded. While the momentum uncertainty does become smaller, κd increases and the tunneling current is exponentially dependent upon this quantity. The current quickly becomes so small that the shot noise is no longer the dominant source of current fluctuation, and the amplifier noise begins to dominate. Thus the apparent displacement becomes inversely proportional to the square root of the tunneling current, and Δx increases faster than Δp decreases. A correct quantum-mechanical calculation of the momentum uncertainty is necessary, but we believe the result should not be significantly different from our heuristic estimate.

III. TWO-PORT REPRESENTATION OF THE VTP AND NONRECIPROCITY

A convenient representation of an electromechanical transducer is as a two-port network. There is a matrix which relates the mechanical variables (force F and velocity u) and the electrical variables (voltage V and current i). One representation is the impedance matrix in which the velocity and the current are treated as inputs:

$$\begin{bmatrix} F \\ V \end{bmatrix} = \begin{bmatrix} Z_m & M_e \\ M_m & Z_e \end{bmatrix} \begin{bmatrix} u \\ i \end{bmatrix} . \tag{7}$$

The constant M_e is a measure of the transducer efficiency at converting electrical current into mechanical force. The constant Z_e is the electrical input impedance of the transducer, M_m is a measure of the transducer capacity to convert mechanical velocity into electrical voltage, and Z_m is the mechanical input impedance of the transducer.

A passive two-port device is one in which the energy which flows into one port must equal the sum of the energy flowing out of the other port and the energy dissipated within the network. The energy flowing into the mechanical port is

$$P_m = \text{Re}(Fu^*/2) , \tag{8}$$

where Re( ) signifies the real part, and u^* is the complex conjugate of the velocity. The energy flowing into the electrical port is

$$P_e = \text{Re}(Vi^*/2) , \tag{9}$$

where i^* is the complex conjugate of the current. An ideal reciprocal two-port is described by $P_e = P_m$, and it follows that $M_m = -M_e^*$.

If $M_m \neq -M_e^*$, then the transducer is nonreciprocal. Therefore it is active in the sense that it either draws power from a source or loses power to a sink. If the difference of the magnitudes of M_m and M_e is large enough to overcome any power dissipation which may be present there can be an overall increase in signal power, and the device has gain.

It is straightforward to determine M_m and M_e for the tunneling probe and a simple comparison case, a capacitive probe. A capacitive probe normally operates with zero-average velocity and zero-average current. Consequently a small- and large-signal analysis of the device are equivalent. A tunneling probe normally operates with zero-average velocity but with some steady-state current I_0. A small-signal analysis will be done; the variables F, V, u, and i will represent fluctuations from the average values.

First we present the result for a typical capacitive probe. If we assume that there is an electric field E_0 in a capacitor (of nominal capacitance C_0) which is coupled to a test mass then the impedance matrix is

$$Z_c = \begin{bmatrix} \dfrac{C_0E_0^2}{j\omega} & \dfrac{E_0}{j\omega} \\ \dfrac{E_0}{j\omega} & \dfrac{1}{j\omega C_0} \end{bmatrix} . \tag{10}$$

We are working in the frequency domain; $j = \sqrt{-1}$ and ω is the angular frequency. The ratio of output voltage to velocity (V/u) is given by $M_m = E_0/j\omega$ and the ratio of force to current (F/i) is given by $M_e = E_0/j\omega$ as well. In this case $M_e = -M_m^*$, and the device satisfies the reciprocity condition.

A similar analysis for the tunneling probe leads to a very different conclusion. When the basic equations describing the vacuum-tunneling probe are put into impedance matrix form and capacitance is neglected, the small-signal impedance matrix is

$$Z_t = \begin{bmatrix} \dfrac{I_0p_e(1-2\kappa d)}{ed} & \dfrac{p_e}{e} \\ \dfrac{2\kappa V_0}{j\omega} & R_0 \end{bmatrix} . \tag{11}$$

The ratio of the force exerted by the probe to the current passing through the probe is $M_e = p_e/e$. The ratio of the output voltage to the velocity of the test mass is $M_m = 2\kappa V_0/j\omega$. This device is clearly nonreciprocal since $M_e \neq -M_m^*$. The ratio of the magnitudes of M_m to M_e is

$$\left| \frac{M_m}{M_e} \right| \approx \left[\frac{2\kappa e V_0}{p_e} \right] \frac{1}{\omega} . \tag{12}$$

If we assume that the tunneling electrons have the Fermi momentum and that the Fermi energy is equal to the work function of the tunneling probe material then we have $|M_m|/|M_e|=eV_0/\hbar\omega$. For a typical bias of 0.1 V and a test-mass resonant frequency of 1 kHz, this quantity is approximately 10^{10}. The transfer characteristic for mechanical-to-electrical conversion is vastly greater than the reverse transfer characteristic for electrical-to-mechanical conversion. The very weak coupling of the electrical fluctuations in the tunneling probe to the test mass via M_e is the key to the noise reduction of the VTP.

In conventional transducers the optimum performance is reached in a compromise between the apparent fluctuations, or additive noise, and the back-acting noise. A test mass which is sensed by a conventional probe is subjected to the full effect of the electrical fluctuations. In the example of a capacitive probe, a detailed analysis leads to the conclusion that the dominant source of electrical fluctuations is the amplifier which follows the transducer. The limiting sensitivity is called the amplifier limit and will be discussed in Sec. V.

At this point we come back to the problem of the tunneling probe capacitance. If the capacitance is too large, the probe behaves as a conventional reciprocal capacitive transducer which is subject to the amplifier limit. For monitoring small masses ($\lesssim 10^{-9}$ kg), capacitance of less than 10^{-17} F is required for all capacitive effects to be negligible. For larger masses the restrictions are less severe. (See Appendix A for a detailed discussion.)

IV. OPTIMAL SIGNAL-TO-NOISE RATIO AND VTP SENSITIVITY

The output of any transducer-amplifier system contains both signal information and unwanted noise. We can express this as $y(t)=f(t)+n(t)$, where $y(t)$ is the output of the transducer, $f(t)$ is the output signal, and $n(t)$ is the noise. The output can be filtered to optimize the signal-to-noise ratio. The maximum signal-to-noise ratio for a linear system with stationary noise is given by[10]

$$\left[\frac{S}{N}\right]^2_{\max}=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{|F(\omega)|^2}{S(\omega)}d\omega\ , \tag{13}$$

where $S(\omega)$ is the spectral density of the noise and $F(\omega)$ is the Fourier transform of the output signal. The filter of the output which maximizes the signal-to-noise ratio is called the optimal filter. It is implicit in the above and, in fact, does not need to be explicitly specified to calculate the maximum signal-to-noise ratio. It is straightforward to find the optimal filter but for the present purpose we omit this.

In the following we find the spectral density of the noise at the output of the tunneling probe which, combined with the expected signal, will allow us to evaluate Eq. (13) for the optimal signal-to-noise ratio. We assume that the object which is monitored by the tunneling probe is a mechanical harmonic oscillator, with displacement x, and which has the equation of motion

$$m\left[\ddot{x}+\frac{\omega_0}{Q_m}\dot{x}+\omega_0^2x\right]=\sum_i F_i\ , \tag{14}$$

where m is the mass of the oscillator, ω_0 is the resonant frequency, and Q_m is the mechanical quality factor which is the number of cycles of oscillation for the amplitude to decay by a factor of $(1/e)$. The forces which act on the mechanical oscillator include the back action force F_{BA}, the Langevin force which is responsible for the Brownian motion, and the signal force F_{sig}. Other forces such as the Casimir force, van der Waals force, and magnetic forces are assumed to be negligible.

The back action consists of two parts, one which is due to the image charge force associated with the unavoidable probe capacitance F_{cap}, and a part F_{tun} which is the rate of momentum transfer by the tunneling electrons. The capacitive back-action force is given by

$$F_{cap}=qE\ , \tag{15}$$

where q is the charge on the tunneling probe and E is the electric field in the gap. For now we exclude this term but it may become important if the mass of the mechanical oscillator is less than 10^{-9} kg and the probe capacitance exceeds 10^{-17} F (refer to Appendix A). The momentum transfer back action is given by

$$F_{tun}=I(t)p_e/e\ , \tag{16}$$

where $I(t)$ is the tunneling current, p_e is the electron momentum, and e is the electron charge. The tunneling current $I(t)$ includes shot noise with a spectral density of $S_{i_s}=2eI_0$. The momentum of the tunneling electrons p_e has an average value p_1 and fluctuations Δp which have a spectral density S_p.

The exact expression for the spectral density of the momentum fluctuations S_p is an open question which must be answered by a full quantum-mechanical calculation of the fluctuations in the momentum flux through the tunneling probe gap. In the present paper we do not attempt to solve this problem. Rather we present our results in a way that is independent of the value of S_p, and we adopt an expression for S_p which we derive heuristically.

One may assert that the tunneling electrons are localized to the gap during the tunneling process and that the electron's wave function in the gap is a decreasing exponential $\psi=Ae^{-\kappa x}$. The probability of finding the electron at a particular point is given by $\psi^*\psi$ and hence equals $A^2e^{-2\kappa x}$. If the position uncertainty is defined as the wavepacket width at half amplitude, then $\Delta x=(\ln 2)/2\kappa$. By the Heisenberg uncertainty relation the minimum momentum uncertainty of each electron is $\hbar/\Delta x$ or $2\hbar\kappa/\ln 2$. The square of the momentum uncertainty of one electron equals the momentum fluctuation spectral density times an effective bandwidth which we take to be four divided by the mean time between electron arrivals (e/I_0). Thus the momentum fluctuation spectral density is $(\hbar/\kappa)^2e/[I_0(\ln 2)^2]$ which we approximate by

$$S_p\sim 2(\hbar\kappa)^2e/I_0\ . \tag{17}$$

The Langevin force responsible for the Brownian motion of the mechanical oscillator has a single-sided spectral density given by

$$S_{fl}=2k_BT\omega_0m/Q_m\ , \tag{18}$$

where T is the absolute temperature and k_B is Boltzmann's constant.

There are a number of sources of electrical fluctuations in the system, see the equivalent circuit in Fig. 1. The Johnson-Nyquist noise from the bias resistance is depicted by a voltage source (labeled v_b) and has a single-sided spectral density S_{v_b} given by $S_{v_b}=2k_BTR_b$. The noise of the current amplifier is represented by a current noise source i_n (with spectral density S_{i_n}) which is purely additive and an input voltage noise source e_n (with spectral density S_{e_n}) which includes, in part, the fluctuations associated with the input resistance of the amplifier R_{in}.

Given the noise terms listed above, one can determine both the signal part and the noise part of the output current I_{out}. We assume that the signal which we are trying to detect is an impulse $p_0\delta(t)$, which arrives at $t=0$, that the capacitance is negligible and that the bias resistance and the amplifier input resistance are both negligible compared to the tunneling resistance. The Fourier transform of the signal is then

$$F(\omega)=\frac{p_0I_0\xi}{m\omega_0^2d}\frac{1}{G(y)}\ , \tag{19}$$

where

$$\xi=2kd\ ,$$

$$G(y)=-y^2+\frac{jy}{Q}+1+\xi\varepsilon\ ,$$

$$\varepsilon=\frac{I_0p_1}{em\omega_0^2d}\ ,$$

and

$$y=\frac{\omega}{\omega_0}\ .$$

We find the following expression for the spectral density of the noise in the output current:

$$S(\omega)=\frac{S_{i_n}}{|G(y)|^2}\left[y^4(\gamma_t^2+\sigma+1)+y^2\left[(\gamma_t^2+\sigma+1)\left[-2+\frac{1}{Q^2}\right]-2\varepsilon\xi\right]+\left[\gamma_t^2+\sigma+(1+\varepsilon\xi)^2+\frac{S_p}{S_{i_n}}\left[\frac{I_0^2\xi}{em\omega_0^2d}\right]^2+\frac{S_{fl}}{S_{i_n}}\left[\frac{I_0\xi}{em\omega_0^2d}\right]^2\right]\right]\ , \tag{20}$$

where S_{i_n} is the spectral density of the amplifier additive noise, S_p is the spectral density of the electron momentum fluctuations, S_{fl} is the spectral density of the Langevin force, $\gamma_t=e_n/(i_nR_0)$ is the noise impedance of the amplifier, and σ is the ratio of the shot noise to the amplifier additive noise. Under the assumption that the signal is an impulse we calculate the impulse strength p_0, which gives a signal-to-noise ratio of unity. The figure of merit we adopt is the noise number N_I, which is defined by

$$N_I=\frac{p_0^2}{2\hbar m\omega_0}\ , \tag{21}$$

The noise number of the transducer system is the number of energy quanta which would be deposited in the unexcited mechanical oscillator by the minimum detectable impulse, i.e., that which gives a signal-to-noise ratio of unity.

Under the following set of assumptions that the tunneling probe capacitance is negligible, the current flowing through the probe is large enough so that the shot noise dominates the amplifier noise, and the bias resistance and amplifier input resistance are small compared to the tunneling resistance, we find the following expressions for the transducer noise number:

$$N_{I_{tun}}=\frac{\mu\sqrt{2}}{\varepsilon\xi}\left[1+\frac{(\varepsilon\xi)^2}{\mu^2}\frac{S_pI_0}{\hbar^2\kappa^22e}+\frac{2E_t}{Q_m}\frac{\varepsilon\xi}{\mu}\right]^{1/2}\times\left[\left[1+\frac{(\varepsilon\xi)^2}{\mu^2}\frac{S_pI_0}{\hbar^2\kappa^22e}+\frac{2E_t}{Q_m}\frac{\varepsilon\xi}{\mu}\right]^{1/2}-1\right]^{1/2}, \tag{22}$$

where

$$\mu=\frac{p_1}{\hbar\kappa}\ \text{and}\ E_t=\frac{k_bT}{\hbar\omega_0}\ .$$

This expression is minimized when the term under the first radical approaches unity. Then the last factor can be simplified using the approximation $\sqrt{(1+z)}\approx 1+z/2$ for $z\ll 1$. The expression then reduces to

$$N_{I_{tun}}\approx\left[1+\frac{(\varepsilon\xi)^2}{\mu^2}\frac{S_pI_0}{\hbar^2\kappa^22e}+\frac{2E_t}{Q_m}\frac{\varepsilon\xi}{\mu}\right]^{1/2}\times\left[\frac{S_pI_0}{\hbar^2\kappa^22e}+\frac{2E_t}{Q_m}\frac{\mu}{\varepsilon\xi}\right]^{1/2}. \tag{23}$$

In principle the temperature can be lowered and the mechanical quality factor increased so that $E_t/Q_m\ll\varepsilon\xi/\mu$. In this case, and with all previous assumptions including $\varepsilon\xi/\mu\ll 1$, the quantum limit $N_I=1$

is approached if the spectral density of the momentum fluctuations has the form given in Eq. (17).

V. SENSITIVITY OF A CAPACITIVE (RECIPROCAL) TRANSDUCER

For comparison we include a noise number calculation for a conventional, capacitive transducer, see Fig. 2. The essential conclusions of this calculation are the same for any conventional, passive transducer. We emphasize that in this calculation we consider a purely capacitive probe, not the effect of the stray capacitance on the tunneling probe result.

For a conventional capacitive probe the amplifier noise and the Langevin force are the major limiting factors. When the bias resistance and amplifier input resistance are small compared to the impedance of the capacitor at the mechanical frequency and the Johnson noise of the bias resistance is negligible compared to the amplifier input noise, the noise number of the capacitive transducer is found to be

$$N_{I_{\text{cap}}}=\frac{N_A}{2\beta\gamma_c}2\pi\left[\int_{-\infty}^{+\infty}y^2\left\{y^6(\gamma_c^2)+y^4\left[\gamma_c^2\left[\frac{1}{Q^2}-2\right]+1\right]+y^2\left[\frac{4E_t\beta\gamma_c}{QN_A}-2(1-\beta)+\left[\frac{1}{Q}\right]^2+\gamma_c^2\right]+(1-\beta)^2\right\}^{-1}dy\right]^{-1}, \tag{24}$$

where

$$\gamma_c=\frac{e_n\omega_0C_0}{i_n},\quad N_A=\frac{e_ni_n}{\hbar\omega_0},$$

$$\beta=\frac{C_0V_0^2}{m\omega_0^2d^2},\quad E_t=\frac{k_bT}{\hbar\omega_0}.$$

It can be shown that the minimum noise number for this transducer occurs when $\gamma_c^2\gg(1-\beta)^2$. The noise number has a minimum of $N_{I_{\text{cap}}}=N_A$, where N_A is the noise number of the amplifier, which is defined by $N_A=(S_{e_n}S_{i_n})^{1/2}/\hbar\omega_0$. One says that $N_I=N_A$ is the amplifier limit. In summary conventional reciprocal

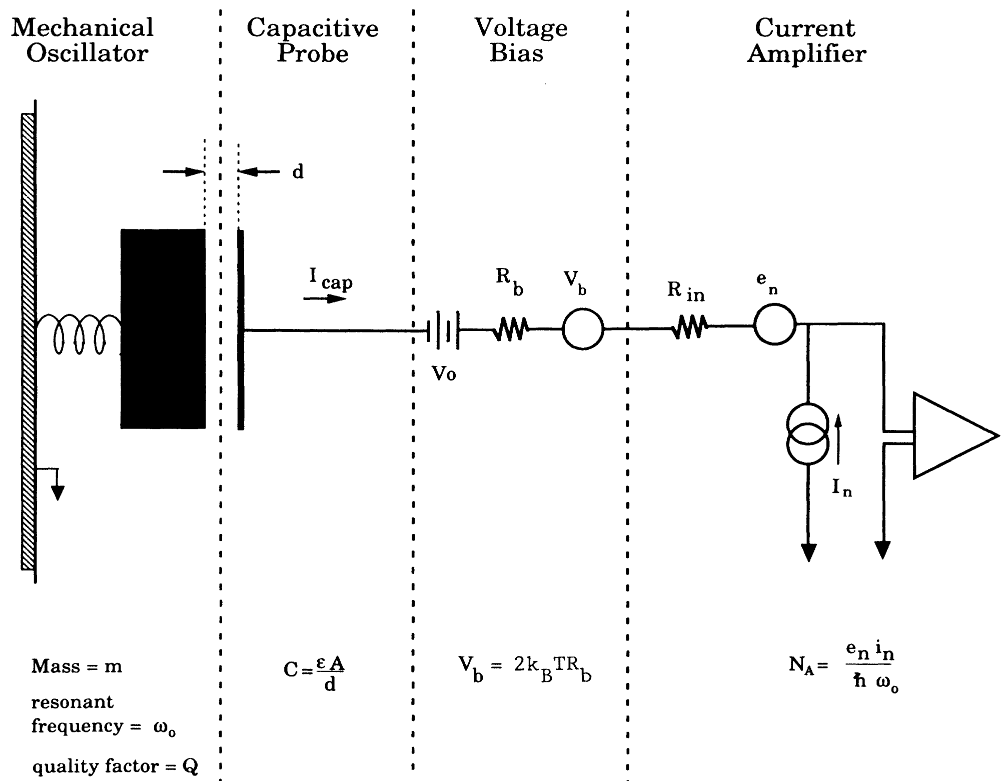

FIG. 2. Electromechanical schematic of a motion transducing system using a capacitive probe.

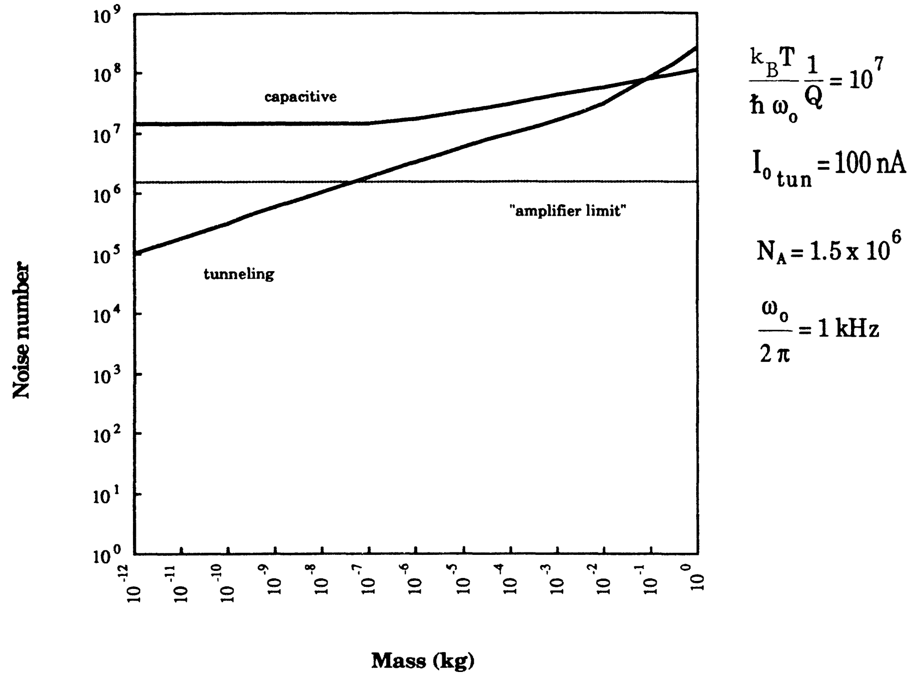

FIG. 3. Calculated noise number of a vacuum-tunneling probe transducer and of a capacitive transducer at room temperature. The mechanical quality factor Q is approximately 600.

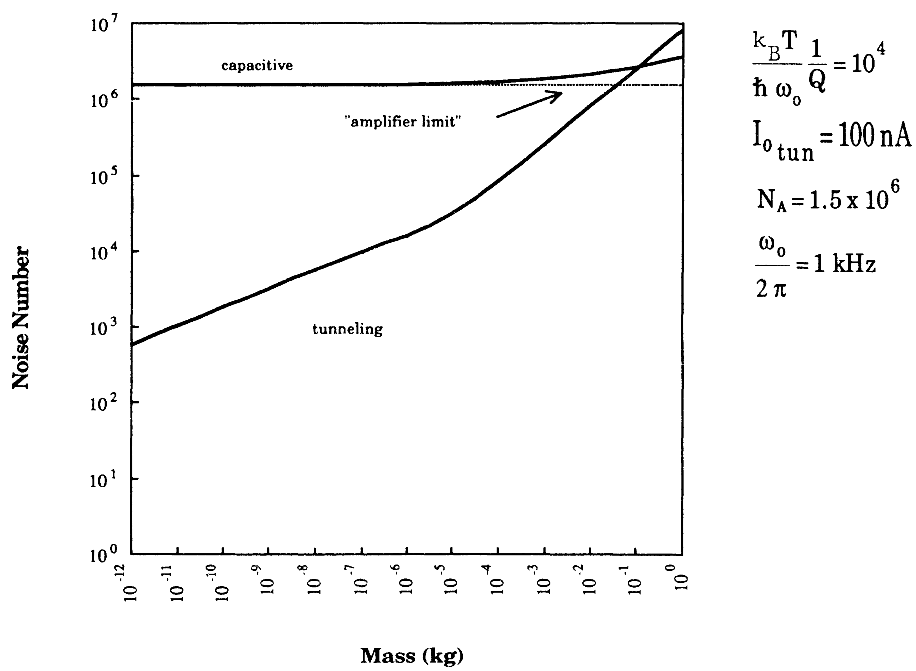

FIG. 4. Calculated noise number of a vacuum-tunneling probe transducer and of a capacitive transducer at 77 K. The mechanical quality factor Q is approximately 1.6×10^5.

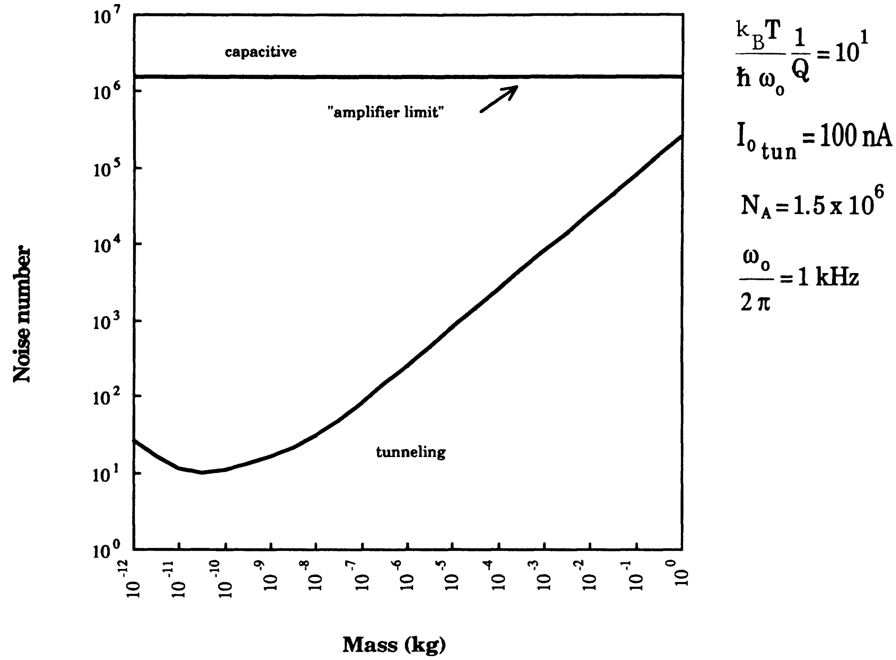

FIG. 5. Calculated noise number of a vacuum-tunneling probe transducer and of a capacitive transducer at 4.2 K. The mechanical quality factor Q is approximately 9×10^6.

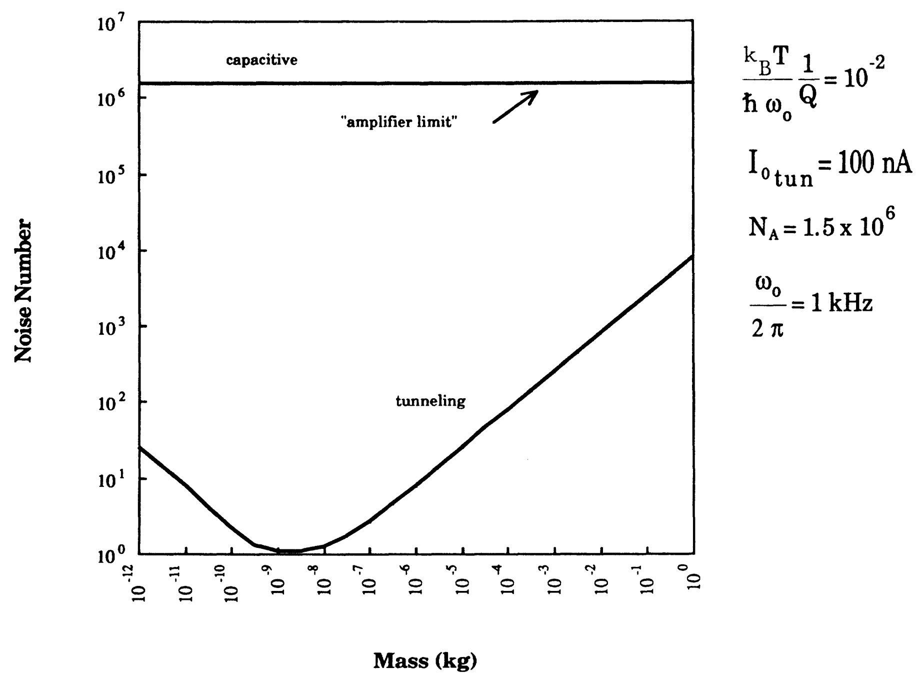

FIG. 6. Calculated noise number of a vacuum-tunneling probe transducer and of a capacitive transducer at 0.05 K. The mechanical quality factor Q is approximately 10^8.

transducers obey the constraint[11]

$$N_I \geq N_A \geq 1 \ . \quad (25)$$

In Figs. 3–6 we plot the noise number for the tunneling transducer and the purely capacitive transducer as a function of the mass of the mechanical test body. For the tunneling transducer the momentum fluctuations of the electrons are assumed to take the form which was previously postulated (i.e., $S_p = \hbar^2\kappa^2 2e/I_0$). Figure 3 corresponds to a system with a relatively low-quality oscillator ($Q \sim 600$) near room temperature ($T = 300$ K). While the capacitive transducer is more sensitive for kilogram-scale masses, the tunneling transducer is much more sensitive for small masses. One of the major noise sources is the Brownian motion. The capacitive probe noise number approaches the amplifier limit as the mass is reduced but the tunneling probe noise number continues to drop, only limited by the Brownian noise which is a less severe limitation than the amplifier noise.

Figure 4 corresponds to a system with a better mechanical oscillator ($Q = 1.6 \times 10^5$) at liquid-nitrogen temperatures ($T = 77$ K). Under these conditions the capacitive transducer is completely limited by the amplifier. However the tunneling transducer exceeds the amplifier limit over most of the mass range, again being limited by the Brownian motion.

Figure 5 and 6 continue the trend toward lower temperatures and higher mechanical Q's. Figure 5 is for a mechanical oscillator with $Q = 9 \times 10^6$ at liquid-helium temperatures ($T = 4.2$ K). Figure 4 is for an extremely high-Q mechanical oscillator ($Q = 10^8$) at mK temperatures ($T = 0.05$ K). In both cases the capacitive transducer is completely limited by the amplifier noise. The tunneling transducer performance continues to improve until the quantum limit is reached. These two figures also demonstrate that for the tunneling probe operated with a specific average current, there is an optimum mass which gives the best system sensitivity. This is due to a tradeoff between two effects. As the oscillator mass is decreased the Brownian noise, in terms of the noise number, decreases. Simultaneously, the influence of the fluctuations of the momentum of the tunneling electrons on the mass of the mechanical oscillator becomes more dominant. The optimum mass is that for which the fluctuations which are due to Brownian motion are the same magnitude as the noise which is due to the fluctuating momentum transfer of the electrons. The mass which represents the optimum is a function of the temperature, the mechanical oscillator quality factor, and the current flowing through the tunneling tip. For reasonable VTP operating conditions the optimum mass is typically a few micrograms or less.

Figure 7 shows the relation between the current and the optimum mechanical oscillator mass. This figure demonstrates that to optimize the VTP for masses in the gram to kilogram range (which is typical of transducers used in a gravitational wave detector), tunneling currents many orders of magnitude greater than are currently used would be required. This is presently a practical impossibility.

VI. SUMMARY

The vacuum tunneling probe (VTP) transducer is a new type of transducer which is nonreciprocal and inherently quantum limited. The nonreciprocal nature of the VTP

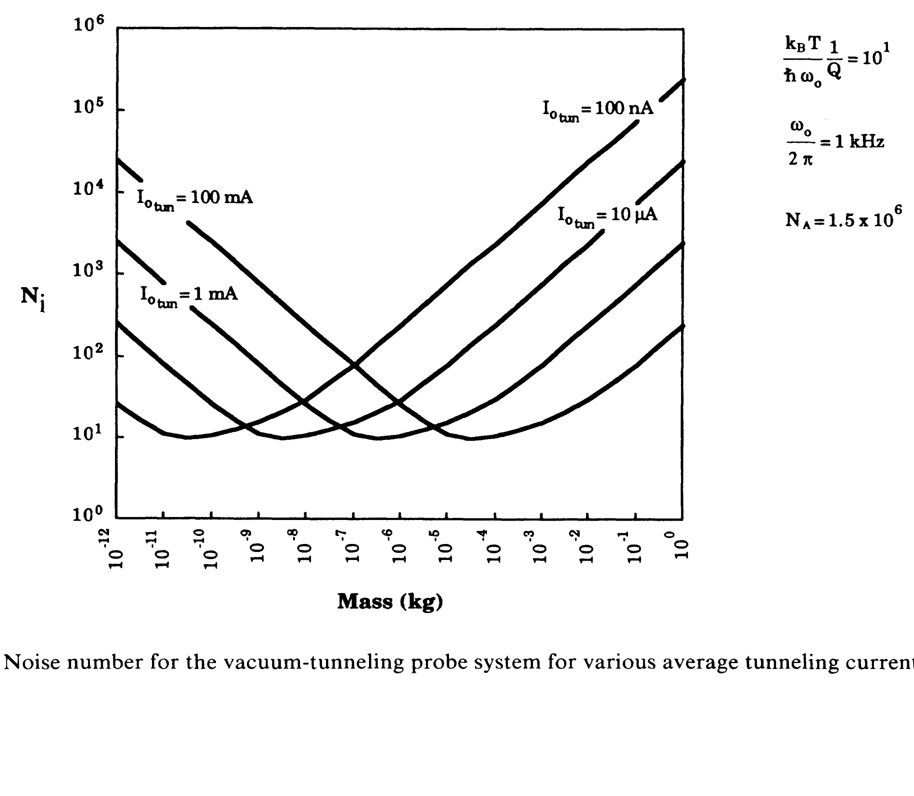

FIG. 7. Noise number for the vacuum-tunneling probe system for various average tunneling currents.

transducer makes it insensitive to the noise which is fed back from the following amplifier which is a major limiting factor in the performance of a conventional transducer. This has the result of a greatly increased sensitivity. A figure of merit for impulsive force detection using a mechanical oscillator was defined and calculated for both a VTP system and a capacitive transducer. While the performance of the capacitive transducer is limited by the noise of the following amplifier, the performance of the vacuum-tunneling probe is only limited by quantum fluctuations. If typical room-temperature amplifiers are used, this difference can be many orders of magnitude. The maximum sensitivity of the VTP system is a function of the temperature, the steady-state tunneling current, the mechanical quality factor, and the mass of the mechanical oscillator. For typical operating conditions the maximum sensitivity is obtained for small mechanical oscillator masses—less than a microgram—making the VTP ideal for miniature accelerometers and related devices.

ACKNOWLEDGMENTS

Partial funding for this work was provided by the National Science Foundation.

APPENDIX A

In the text of the paper we assumed that the current-shot noise was the predominate source of apparent noise (noise at the system output which is not attributable to noise present at the input). In order for this to be true, the following conditions must apply: $\sqrt{2eI_0} \gg i_n$ and $\sqrt{2eI_0} \gg e_n/R_0$. If the amplifier has $S_{i_n}=3.2\times10^{-28}$ A^2/Hz and $S_{e_n}=1.4\times10^{-21}$ V^2/Hz and a typical tunneling resistance is used ($R_0 > 10^6$ Ω), then the most stringent of these two requirements dictates that $I_0 \gg 10^{-9}$ A. These values correspond to an amplifier with a noise number of $N_A=10^6$ and correspond to a commercially available device. Bias currents of 1 nA to a few nanoamps are typical of VTP operation although larger currents have been used.

The most stringent requirements for the capacitance is a result of the capacitive back-action term. In order for capacitive back action to be completely negligible, the requirement is

$$C_0 \ll \frac{m\omega_0^2 d^2}{V_0^2} .$$

If the operating conditions are assumed to be $\omega_0=10^4$ rad/sec, $d=10^{-9}$ m, $V_0=0.1$ V, $R_0=10^7$ Ω, and $m=10^{-9}$ kg, then the capacitance would have to be less than 10^{-17} F. While capacitance in the range of 10^{-18} F have been reported for point contact junctions,[12] clearly the capacitive back action becomes a concern for very small mass systems. Note also that the above operating conditions correspond to a current of 10^{-8} A which only marginally satisfies the requirements for the amplifier noise being negligible. One way of alleviating this problem is to use an amplifier which has less additive noise, thereby easing the minimum current requirement. Then the current and voltage can be reduced, and the shot noise will continue to dominate. Since the capacitance requirement goes as V_0^{-2}, an order-of-magnitude decrease in the current and voltage would relax the capacitance requirement by two orders of magnitude. For systems utilizing larger mechanical oscillator masses the requirements of negligible capacitive back action are easily met for typical operating conditions.

APPENDIX B

The sampling function in the time domain has a magnitude of one for $|t| \le t_0/2$, where t_0 is the sampling time. The Fourier transform of this function is a sin*c* function with ωt_0 as the argument and an amplitude of t_0. The sin*c* function is at half-height when the argument is approximately ±1.90. This corresponds to $\omega=\pm1.9/t_0\approx\pm2/t_0$. If the bandwidth is taken to be the width at half-height then $\Delta_{BW}\approx4/t_0$.

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