# Amplitude noise reduction in atomic and semiconductor lasers

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The quantum noise properties of atomic and semiconductor lasers are compared. The intensity fluctuation spectrum of the output field is calculated for the case of regularly, i.e., sub-Poissonian, pumped lasers. The effects of spontaneous emission on the output amplitude fluctuations may be suppressed in regularly pumped atomic lasers by requiring the lifetime of the lower lasing level to be short in comparison with that of the upper laser level. The output amplitude fluctuations then tend to zero at the laser frequency. In regularly pumped semiconductor lasers, net stimulated emission of photons dominates over spontaneous emission because of the combination of pumping and rapid intraband thermalization due to Coulomb scattering.

## I. INTRODUCTION

Recent experiments have demonstrated that the intensity noise level of a semiconductor laser may be reduced below the shot-noise limit by suppressing the pump noise.[1] Conventional laser theory predicts that the photon statistics of a laser well above threshold will tend to a Poissonian. The intensity fluctuation spectrum of the output field of such a laser has a flat noise level at the shot-noise limit. There are three intrinsic noise sources which contribute to the noise in the laser output. These are pump noise, spontaneous emission noise, and vacuum fluctuations entering the cavity through the mirror. In order to produce a quieter laser output, it is necessary to reduce one or more of these noise sources. A number of theoretical papers[2–6] have analyzed the effect of reduced pump fluctuations on both the photon statistics of the light inside the laser cavity and the intensity fluctuations of the light emerging from the cavity. Golubev and Sokolov,[2] Walls, Haake, and Collett,[5] and Benkert *et al.*[6] all consider modifications of the Scully-Lamb theory[7] of the laser appropriate to regular pump excitation. Yamamoto and co-workers[3] have applied the semiconductor laser theory of Haug[8] to treat a semiconductor laser with regular pump excitation. Marte and co-workers[4] have considered pumping lasers with squeezed light with reduced amplitude fluctuations. Haake, Tan, and Walls[5] have numerically simulated a sequence of regular pump cycles using a master equation derived from Haken's[9] treatment of the laser. All these analyses have shown that regular, i.e., sub-Poissonian, pump excitation will result in sub-Poissonian photon statistics and a reduction of the intensity fluctuations below the shot-noise limit. Similar results have been obtained for the internal field of a micromaser with regular pump excitation.[10] The aim of this paper is to elucidate the similarities and differences in the quantum noise properties of atomic and semiconductor lasers with regular pumping. We present calculations of the intracavity photon statistics and output intensity fluctuation spectrum well above threshold.

For atomic systems our treatment is based on the quantum theory of Lax and Louisell,[11] and modifications thereof, which describe the so-called *four-level* laser (only three levels appear explicitly in the model; the fourth level to which the pump is coupled decays so rapidly that effectively pumping takes place to the upper lasing level). Yamamoto and co-workers[3] have given a thorough discussion of the quantum noise properties of regularly pumped semiconductor lasers from threshold to well above. We concentrate on the photon statistics and output intensity fluctuation spectra well above threshold. By comparison of the results for the atomic and semiconductor lasers, it is possible to understand when and why the suppression of pump noise leads to reduced amplitude fluctuations in the output. For atomic systems, the relative decay rates of the lasing levels is of central importance here, while in semiconductors the quite different physical mechanism of intraband Coulomb scattering plays a similarly important role.

In Sec. II we discuss intensity fluctuations of an atomic laser inside and outside the laser cavity. The results are compared with those for a semiconductor laser in Sec. III. A summary of our results is presented in Sec. IV. The Appendix includes a discussion of the effects of spontaneous emission between the lasing levels.

## II. INTENSITY FLUCTUATIONS OF AN ATOMIC LASER

### A. The model of Lax and Louisell

We use the three-level laser model discussed by Lax and Louisell (LL).[11] The level scheme is shown in Fig. 1, and the lasing transition is between the upper levels 2 and 1. In atomic or solid-state lasers, elastic collisions may produce a large phase damping rate for this transition. LL showed that when the polarization decay is the largest rate, the polarization may be adiabatically eliminated. The resultant quantum Langevin equations are then, in the notation of LL

$$\dot{D}=R_2-\Gamma_2 D-2\Pi nD+(\Gamma_1-\Gamma_2)N_1+G_2-G_1\ , \quad (2.1)$$

 

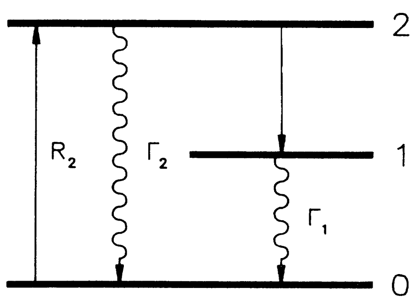


FIG. 1. The atomic laser level scheme. Lasing occurs between levels 2 and 1, which decay spontaneously at rates $\Gamma_2$ and $\Gamma_1$, respectively. The upper level is pumped at a rate $R_2$.

$$\dot{N}_1 = -\Gamma_1 N_1 + \Pi n D + G_1 , \tag{2.2}$$

$$\dot{n} = -\gamma n + \Pi n D + G_p , \tag{2.3}$$

where $N_i$ ($i=1,2$) is the population of level $i$ which decays at rate $\Gamma_i$, with $G_i$ the corresponding noise operator. The inversion $D \equiv N_2 - N_1$ increases at a rate $R_2$, by virtue of the pumping to the upper laser level 2. The photon number $n$ decays at rate $\gamma$ due to the cavity output coupling, but is fed at a rate $\Pi n D$ by the net stimulated emission. Here $\Pi \equiv 2\mu^2\Gamma$ is a coupling constant, where $\mu$ is the atomic dipole matrix element and $\Gamma = \frac{1}{2}(\Gamma_1+\Gamma_2)+\Gamma_{ph}$ is the polarization decay rate with $\Gamma_{ph}$ the contribution due to phase damping collisions. The model is slightly simplified with respect to that presented in LL. Only pumping of the upper level is considered and inelastic collisional excitation between the lasing levels is ignored. We also at this stage ignore any spontaneous emission between the lasing levels on the assumption that this is a small fraction of the total decay rate $\Gamma_2$, of the upper level. (The effects of spontaneous emission between the lasing levels is discussed in the Appendix.) Spontaneous emission into the laser mode is also ignored as it is small in the region of interest well above threshold.

The diffusion coefficients of the noise operators in (2.1) and (2.3), defined by the notation $\langle G_i(t)G_j(t')\rangle = 2D_{ij}\delta(t-t')$, are given by

$$2D_{pp} = \Pi n (N_1+N_2) + \gamma n + \Pi N_2 , \tag{2.4}$$

$$2D_{22} = R_2 + [\Gamma_2 + \Pi(n+1)]N_2 + \Pi n N_1 , \tag{2.5}$$

$$2D_{11} = \Pi(n+1)N_2 + (\Gamma_1 + \Pi n)N_1 , \tag{2.6}$$

$$2D_{12} = 2D_{21} = -\Pi n(N_1+N_2) - \Pi N_2 , \tag{2.7}$$

$$2D_{p2} = 2D_{2p} = -\Pi n(N_1+N_2) - \Pi N_2 , \tag{2.8}$$

$$2D_{p1} = 2D_{1p} = \Pi n(N_1+N_2) + \Pi N_2 . \tag{2.9}$$

**B. Steady-state operating conditions above threshold**

The operating states are found by dropping the noise terms in Eqs. (2.1) to (2.3) and setting the time derivatives equal to zero. These are denoted by a subscript zero. Above threshold, the atomic inversion is clamped at the value

$$D_0 = \frac{\gamma}{\Pi} . \tag{2.10}$$

The intracavity photon number is ($R_2 \geq R_{th}$)

$$n_0 = n_s(R_2/R_{th} - 1) , \tag{2.11}$$

where the characteristic photon number $n_s$ is

$$n_s = \frac{\Gamma\overline{\Gamma}}{2\mu^2} = \frac{\overline{\Gamma}}{\Pi}, \quad \frac{1}{\overline{\Gamma}} \equiv \frac{1}{\Gamma_1} + \frac{1}{\Gamma_2} , \tag{2.12}$$

and the threshold pumping rate

$$R_{th} = \frac{\gamma\Gamma_2}{\Pi} = \frac{\gamma\Gamma\Gamma_2}{2\mu^2} \tag{2.13}$$

is very weakly dependent on $\Gamma_1$ since phase damping $\Gamma_{ph}$ dominates the polarization decay rate, i.e., $\Gamma = \frac{1}{2}(\Gamma_1 + \Gamma_2) + \Gamma_{ph} \approx \Gamma_{ph}$. The atomic populations satisfy

$$N_{10} = \frac{\gamma}{\Gamma_1} n_0, N_{20} = \frac{\gamma}{\Gamma_1} n_0 + \frac{\gamma}{\overline{\Gamma}} n_s . \tag{2.14}$$

From Eq. (2.11) we note that well above threshold $R_2 \gg R_{th}$, $n_0 \gg n_s$, and Eq. (2.14) shows that in the same limit the inversion is small compared with the level populations.

**C. Linearized analysis of quantum fluctuations**

In Eqs. (2.1) to (2.3) we set $D = D_0 + \Delta D$, $N_1 = N_{10} + \Delta N_1$, and $n = n_0 + \Delta n$ and linearize about the steady-state operating point, to find

$$\Delta\dot{D} = -\Gamma_2 \Delta D - 2\Pi(D_0\Delta n + n_0 \Delta D) + (\Gamma_1 - \Gamma_2)\Delta N_1 + G_2 - G_1 , \tag{2.15}$$

$$\Delta\dot{N}_1 = \Pi(D_0 \Delta n + n_0 \Delta D) - \Gamma_1 \Delta N_1 + G_1 , \tag{2.16}$$

$$\Delta\dot{n} = -\gamma\Delta n + \Pi(D_0\Delta n + n_0\Delta D) + G_p . \tag{2.17}$$

We now calculate the steady-state variance in intracavity photon number, since as Golubev and Sokolov,[2] and Haake and co-workers[5] have shown, knowledge of the photon mean number ($n_0$) and variance are sufficient to determine the output intensity fluctuation spectrum of the laser well above threshold. In this regime, where the stimulated transition rate is large, we may adiabatically eliminate the atomic fluctuations by setting $\Delta\dot{D} = \Delta\dot{N}_1 = 0$ in Eqs. (2.15) and (2.16), respectively, giving

$$\Delta N_1 = \frac{1}{\Gamma_1}[\gamma\Delta n + \Pi n_0 \Delta D + G_1] , \tag{2.18}$$

$$\Delta D = \frac{1}{\Gamma_2 + \Pi n_0 \left[1 + \frac{\Gamma_2}{\Gamma_1}\right]} \times \left[-\gamma\left[1+\frac{\Gamma_2}{\Gamma_1}\right]\Delta n + G_2 - \frac{\Gamma_2}{\Gamma_1}G_1\right] . \tag{2.19}$$

Substituting these results into Eq. (2.17) one finds, for $n_0 \gg n_s$,

$$\Delta\dot{n} = -\gamma\Delta n + G(t) , \quad (2.20)$$

where

$$G(t) \equiv G_p(t) + \frac{1}{1+\frac{\Gamma_2}{\Gamma_1}}\left[G_2(t) - \frac{\Gamma_2}{\Gamma_1}G_1(t)\right] . \quad (2.21)$$

We define $\langle G(t)G(t')\rangle = \langle GG\rangle\delta(t-t')$, then using Eqs. (2.4) to (2.9) we find the steady-state variance in photon number is given

$$\sigma^2 = \frac{\langle GG\rangle}{2\gamma} = \tfrac{1}{2}n_0 + \frac{1}{2\gamma\left[1+\frac{\Gamma_2}{\Gamma_1}\right]^2}\left[R_2 + \Gamma_2\left[N_{20} + \frac{\Gamma_2}{\Gamma_1}N_{10}\right]\right] . \quad (2.22)$$

The first term on the right-hand side is due to vacuum fluctuations transmitted by the output coupling mirror. The other contributions are from pumping ($R_2$) and spontaneous emission. We expect that well above threshold the intracavity photon statistics are Poissonian so that $\sigma^2 \to n_0$. Using the operating conditions (2.11) and (2.14) we find

$$R_2 = \gamma\left[1+\frac{\Gamma_2}{\Gamma_1}\right](n_0+n_s) , \quad (2.23)$$

$$\Gamma_2\left[N_{20} + \frac{\Gamma_2}{\Gamma_1}N_{10}\right] = \frac{\Gamma_2}{\Gamma_1}R_2 , \quad (2.24)$$

giving well above threshold $n_0 \gg n_s$:

$$\sigma^2 = \tfrac{1}{2}n_0 + \tfrac{1}{2}n_0 + O(n_s) = n_0 + O(n_s) . \quad (2.25)$$

The photon statistics do indeed become Poissonian. Pumping and spontaneous emission together contributing $\frac{1}{2}n_0$ to the variance $\sigma^2$. When $\Gamma_1 = \Gamma_2$ the pumping term [proportional to $R_2$ in (2.22)] and the spontaneous emission each produce $\frac{1}{4}n_0$. However, when $\Gamma_1 \gg \Gamma_2$ the lower lasing level decays rapidly and stimulated absorption is minimized, pumping noise dominates the spontaneous emission, contributing the same amount as vacuum fluctuations ($\frac{1}{2}n_0$) to the photon noise.

It has been shown that well above threshold the intensity fluctuation spectrum $\langle\delta I^2\rangle_\omega$ is given by[2,5]

$$\langle\delta I^2\rangle_\omega = \int_{-\infty}^{\infty} d\tau\, e^{i\omega\tau}\langle I_{\rm out}(t+\tau), I_{\rm out}(t)\rangle = \gamma n_0\left[1+\frac{2Q\delta^2}{\delta^2+\omega^2}\right] , \quad (2.26)$$

where $\langle a,b\rangle \equiv \langle ab\rangle - \langle a\rangle\langle b\rangle$, $I_{\rm out}$ is the output intensity operator, and

$$\delta = \frac{\gamma n_0/n_s}{1+n_0/n_s} \approx \gamma \quad (n_0 \gg n_s) , \quad (2.27)$$

$$Q = \frac{\sigma^2 - n_0}{n_0} . \quad (2.28)$$

Well above threshold $Q \to 0$ and the spectrum becomes flat (shot noise) $\langle\delta I^2\rangle_\omega \to \gamma n_0$. This is the expected result. We have thus checked that our simple analysis yields the correct results for the noise well above threshold.

### D. Reduced pump fluctuations

We now show how the output intensity fluctuations may be reduced by reducing the pump fluctuations, and how spontaneous emission affects the reduction possible. One might naively hope that setting the pump-noise term $R_2$ to zero in the diffusion coefficients would model a perfectly quiet pump. However, such a procedure must be consistent with the laws of quantum mechanics, in particular equal time commutation relations between atom and field operators must be preserved under time evolution. LL have shown this to be true for the full model including pump noise. Remarkably, their argument is independent of the pumping noise since it only appears in diagonal diffusion coefficients which do not effect the evolution of commutators. Hence we may indeed set $R_2 = 0$ in these coefficients and maintain quantum-mechanical consistency. Of course, it is necessary to keep $R_2$ in the drift term since this determines the laser operating conditions. That the resulting theory does indeed correspond to a laser with an antibunched pump is confirmed by the agreement of our results with other analytic and numerical results.[5]

The calculation of the variance in photon number goes through as before except that we set $R_2 = 0$ in Eq. (2.22), as this arises from the diffusion coefficient $D_{22}$ of (2.5). We find

$$\sigma^2 = \tfrac{1}{2}n_0 + \frac{\Gamma_2\left[N_{20} + \frac{\Gamma_2}{\Gamma_1}N_{10}\right]}{2\gamma\left[1+\frac{\Gamma_2}{\Gamma_1}\right]^2} , \quad (2.29)$$

where the two terms arise from vacuum fluctuations and spontaneous emission, respectively. Using the operating conditions (2.23) and (2.24) we find, well above threshold $n_0 \gg n_s$,

$$\sigma^2 = n_0\left[1 - \frac{1}{2\left[1+\frac{\Gamma_2}{\Gamma_1}\right]}\right] + O(n_s) . \quad (2.30)$$

This result is in agreement with that of Golubev and Sokolov[2] on the basis of an extension of the Scully-Lamb laser model[7] to treat regular excitation, and numerical simulations of a regularly pumped laser.[5]

Note that for $\Gamma_1 = \Gamma_2$, equal spontaneous decay rates,

$$\sigma^2 \to \tfrac{3}{4}n_0 , \quad (2.31)$$

while for $\Gamma_1 \gg \Gamma_2$

$$\sigma^2 \to \tfrac{1}{2} n_0 \ . \tag{2.32}$$

The limiting value given by (2.32) for the intracavity photon number is due to the vacuum fluctuations, transmitted by the output coupling mirror. However, even for a perfectly quiet pump this may only be approached if the lower lasing level decays much more rapidly than the upper level, since then the effects of spontaneous emission are reduced. In this way spontaneous emission limits the intracavity photon noise reduction.

The output intensity fluctuations are given by Eq. (2.26) with

$$2Q \to \begin{cases} -\frac{1}{2}, & \Gamma_1 = \Gamma_2 \\ -1, & \Gamma_1 \gg \Gamma_2 \end{cases} \tag{2.33}$$

so the minimum intensity noise at zero frequency is

$$\langle \delta I^2 \rangle_{\omega=0} \to \gamma n_0 (1+2Q) \to \begin{cases} \frac{1}{2}\gamma n_0, & \Gamma_1 = \Gamma_2 \\ 0, & \Gamma_1 \gg \Gamma_2 \ . \end{cases} \tag{2.34}$$

The output intensity fluctuations are reduced by 50% below shot-noise level (3-dB squeezing) for equal decay rates, but may be reduced to almost zero (perfect squeezing) in the limit $\Gamma_1 \gg \Gamma_2$. Almost perfect noise reduction is possible in principle extracavity as the output field consists of a superposition of the vacuum field reflected from the cavity output port, and the transmitted laser field. In the limit $\Gamma_1 \gg \Gamma_2$ the noise amplitudes of these components at $\omega = 0$ are equal (corresponding to the laser frequency) but oppositely phased so that they cancel. In the case $\Gamma_1 = \Gamma_2$ such complete cancellation is not possible. The spectra are illustrated in Fig. 2.

## III. INTENSITY FLUCTUATIONS: ATOMIC VERSUS SEMICONDUCTOR LASER

### A. Semiconductor laser

The standard quantum theory of semiconductor lasers is based on the work of Haug and Haug and Haken.[8] Recently this has been extended to include the details of many-body Coulomb effects by Haug and Koch.[12] Yamamoto and co-workers[3] have discussed the effects of reduced pump noise both theoretically and experimentally in some detail. For comparison with our discussion of atomic lasers, we give a parallel analysis of the semiconductor laser.

Since intraband Coulomb effects occur on a very fast timescale, the interband polarization may be adiabatically eliminated. Then assuming that Coulomb scattering does not change the number of carriers in a band, the total number of electrons in the conduction band $N_c = \sum_{\mathbf{k}} n_{\mathbf{k}c}$ is locally equal to the number of holes in the valence band. Laser action occurs due to electronic transitions between the conduction and valence band when the medium gain equals the cavity losses. The laser is described by two equations for the macroscopic variables $N_c$ and the photon number $n$, where[8]

$$\dot{N}_c = P - n(E_{cv} - E_{vc}) - R_{sp} + F_c(t) \ , \tag{3.1}$$

$$\dot{n} = -\gamma n + n(E_{cv} - E_{vc}) + F(t) \ , \tag{3.2}$$

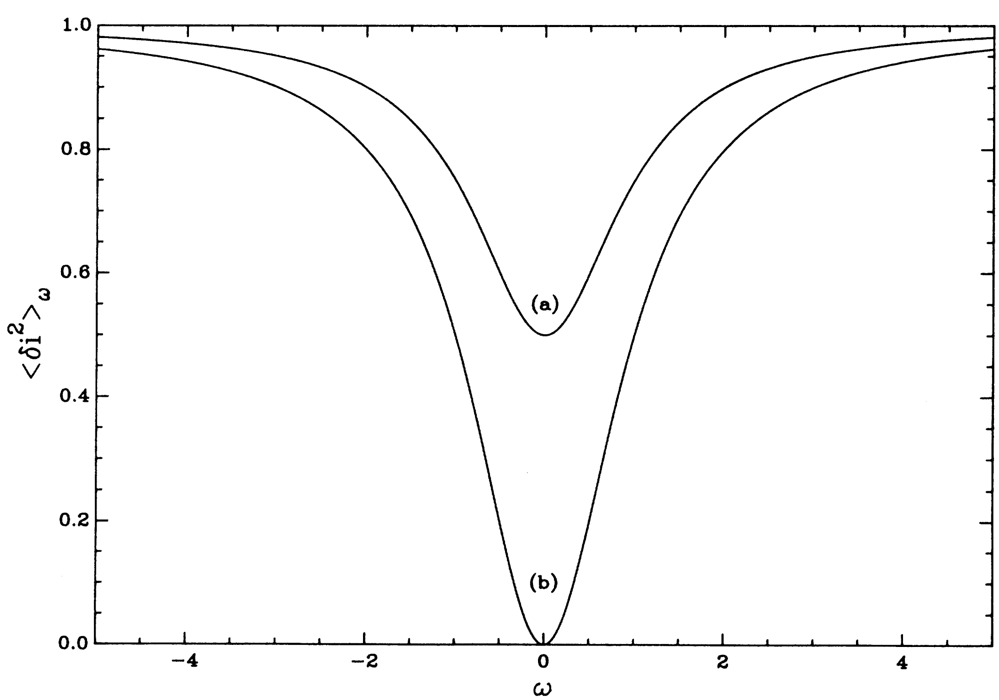


FIG. 2. Intensity fluctation spectrum for (a) $\Gamma_1 = \Gamma_2$ (b) $\Gamma_1 \gg \Gamma_2$. The shot-noise level is unity and frequency is measured in units of the cavity bandwidth.

where following conventional notation $P$ is the pump rate, $R_{sp}$ is the spontaneous emission rate, and $E_{cv}$ ($E_{vc}$) is the stimulated emission (absorption) rate between the conduction and valence bands. Photons are lost from the laser at rate $\gamma$ and $F_c(t)$, $F(t)$ are noise forces which have the nonzero correlations:

$$\langle F_c(t)F_c(t')\rangle=\langle P+R_{sp}+E_{cv}(n+1)+E_{vc}n\rangle\delta(t-t') , \tag{3.3}$$

$$\langle F(t)F(t')\rangle=\langle \gamma n+E_{cv}(n+1)+E_{vc}n\rangle\delta(t-t') , \tag{3.4}$$

$$\langle F(t)F_c(t')\rangle=\langle F_c(t)F(t')\rangle = -\langle E_{cv}(n+1)+E_{vc}n\rangle\delta(t-t') , \tag{3.5}$$

with

$$E_{cv}=\sum_{\mathbf{k}_1\mathbf{k}_2}|g_{\mathbf{k}_1\mathbf{k}_2}|^2\frac{2\gamma_{\mathbf{k}_1\mathbf{k}_2}}{[(\varepsilon_{\mathbf{k}_1\mathbf{k}_2}-\Omega)^2+\gamma^2_{\mathbf{k}_1\mathbf{k}_2}]}n_{\mathbf{k}_1c}(1-n_{\mathbf{k}_2v}) , \tag{3.6}$$

and $E_{vc}$ given by a similar expression with $C$ and $V$ interchanged. Here $g_{\mathbf{k}_1\mathbf{k}_2}$ is the optical matrix element between states $\mathbf{k}_1c$ with wave vector $\mathbf{k}_1$ in the conduction band $c$, and $\mathbf{k}_2v$ in the valence band $v$, $\gamma_{\mathbf{k}_1\mathbf{k}_2}$ is the damping rate between $\mathbf{k}_1c$ and $\mathbf{k}_2v$ and $\varepsilon_{\mathbf{k}_1\mathbf{k}_2}=\varepsilon_{\mathbf{k}_1c}-\varepsilon_{\mathbf{k}_2v}$ is the frequency between $\mathbf{k}_1c$ and $\mathbf{k}_2v$. $\Omega$ is the laser frequency. The characteristic Fermi factor $n_{\mathbf{k}_1c}(1-n_{\mathbf{k}_2v})$ indicates that an electronic transition only occurs between $\mathbf{k}_1c$ and $\mathbf{k}_2v$ if the former is occupied and the latter vacant. In the atomic laser where only one electron is active, there is no such restriction on transitions since we know any other state (apart from the core) is unoccupied and the Pauli exclusion principle is not manifest.

The spontaneous emission rate also contains the Fermi transition factor

$$R_{sp}=2\pi\hbar\sum_{\mathbf{k}_1,\mathbf{k}_2}|g_{\mathbf{k}_1\mathbf{k}_2}|^2\rho_L n_{\mathbf{k}_1c}(1-n_{\mathbf{k}_2v}) , \tag{3.7}$$

where $\rho_L$ is the density of states for the light field. In common with our analysis of the atomic laser our interest is in operation well above threshold, so in (3.1) and (3.2) we have ignored the term $E_{cv}$ representing spontaneous emission into the laser mode relative to the stimulated term $nE_{cv}$, and the spontaneous emission into modes other than the lasing one $R_{sp}$.

The operation conditions for the laser are

$$P=n_0(E^0_{cv}-E^0_{vc})+R^0_{sp} , \tag{3.8}$$

$$E^0_{cv}-E^0_{vc}=\gamma , \tag{3.9}$$

where subscript or superscript zero indicates steady-state mean value. Combining (3.8) and (3.9) one finds

$$P=\gamma n_0+R^0_{sp} . \tag{3.10}$$

We now linearize Eqs. (3.1) and (3.2) about the steady states, and following Yamamoto and co-workers[3] we adabatically eliminate the electron number fluctuations well above threshold by setting $\Delta\dot{N}_c=0$. This is valid if the stimulated emission rate is large enough. In general since the photon lifetime is so short for semiconductor lasers such an adiabatic elimination is not valid above threshold in the usual operation regimes, and precludes for example relaxation oscillations. Here we proceed with the elimination to compare with our results for the atomic laser. Quantitative numerical results for the more general case are given in Ref. 3, and these tend towards our results well above threshold. Thus

$$0=\Delta\dot{N}_c=-\Delta n(E^0_{cv}-E^0_{vc}) -n_0(\Delta E_{cv}-\Delta E_{vc})-\Delta R_{sp}+F_c , \tag{3.11}$$

$$\Delta\dot{n}=-\gamma\Delta n+\Delta n(E^0_{cv}-E^0_{vc}) +n_0(\Delta E_{cv}-\Delta E_{vc})+F . \tag{3.12}$$

Now the stimulated emission rate is much larger than the spontaneous decay rate so one may ignore $\Delta R_{sp}$ with respect to $n_0(\Delta E_{cv}-\Delta E_{vc})$ well above threshold.[3] Note that for an atomic laser the effect of upper-level spontaneous emission is only negligible when $\Gamma_1 \gg \Gamma_2$, i.e., the lower level decays rapidly. This is discussed in detail in Sec. III B.

We find

$$\Delta\dot{n}=-\gamma\Delta n+F_c(t)+F(t) . \tag{3.13}$$

The steady-state variance in photon number is

$$\sigma^2=\frac{\langle\bar{F}\bar{F}\rangle}{2\gamma}=\tfrac{1}{2}n_0+\frac{1}{2\gamma}(P+R^0_{sp}) , \tag{3.14}$$

where we define $\bar{F}(t)=F_c(t)+F(t)$, $\langle\bar{F}(t)\bar{F}(t')\rangle=\langle\bar{F}\bar{F}\rangle\delta(t-t')$, and use Eqs. (3.3) to (3.5). Using the operating condition (3.10) for $P$ in (3.14), we find

$$\sigma^2=\tfrac{1}{2}n_0+\tfrac{1}{2}\left[n_0+\frac{2R^0_{sp}}{\gamma}\right]=n_0+\frac{R^0_{sp}}{\gamma} . \tag{3.15}$$

It is evident from Eq. (3.10) written in the form

$$n_0=\frac{R^0_{sp}}{\gamma}\left[\frac{P}{R^0_{sp}}-1\right] \quad (P>R^0_{sp}) , \tag{3.16}$$

that $R^0_{sp}/\gamma$ plays the role of a characteristic photon number, and well above threshold $P\gg R^0_{sp}$ is it negligible compared with $n_0$. Thus in (3.15)

$$\sigma^2=n_0+O\left[\frac{R^0_{sp}}{\gamma}\right]\approx n_0 , \tag{3.17}$$

i.e., Poissonian number fluctuations. Correspondingly, from (2.26), the intensity spectrum is flat (shot noise). These results are in agreement with the detailed numerical calculations of Yamamoto and co-workers.[3]

As was also established by Yamamoto and co-workers,[3] setting $P=0$ in the noise correlations (3.3) pertains to a perfectly quiet pump. That this is quantum mechanically consistent follows from an analysis similar to that discussed in II D. One then finds from (3.14) with $P=0$

$$\sigma^2=\tfrac{1}{2}n_0+\frac{R_{sp}^0}{2\gamma}\approx\tfrac{1}{2}n_0\ . \tag{3.18}$$

The intensity fluctuation spectrum is given by (2.26) with $2Q=-1$. Thus at zero frequency $\langle \delta I^2\rangle_{\omega=0}\approx 0$ corresponding to perfect squeezing of output amplitude fluctuations. These results are similar to the atomic laser when the lower lasing level decays rapidly compared to the upper level $\Gamma_1 \gg \Gamma_2$.

### B. Discussion

Why are the intensity fluctuation properties of a semiconductor laser similar to an atomic system whose spontaneous decay rates satisfy $\Gamma_1 \gg \Gamma_2$, rather than $\Gamma_1=\Gamma_2$, say, when pump fluctuations are suppressed? From the operating conditions for the atomic laser, one sees that in general, the net rate of stimulated transitions is equal to the total spontaneous decay from the lower level, i.e.,

$$\Gamma_1 N_{10}=\Pi n_0 D_0\ . \tag{3.19}$$

Well above threshold the atoms are saturated in the sense that $N_{10}\approx N_{20}$. The relative amounts of spontaneous emission from the upper level $\Gamma_2 N_{20}$ and lower level $\Gamma_1 N_{10}$, then depend on the ratio $\Gamma_2/\Gamma_1$ only. Using (3.19) we observe that the net rate of stimulated transitions from the upper level is only large compared to the spontaneous emission $\Gamma_2 N_{20}$, if $\Gamma_1 \gg \Gamma_2$. That is to say, in this limit, there is a high probability that pump excitation of the upper level produces a laser photon which exits the cavity, rather than a spontaneous photon.

A complementary view of this is also useful. The ratio of stimulated absorption to spontaneous decay from the lower level is

$$\frac{\Pi n_0 N_{10}}{\Gamma_1 N_{10}}=\frac{\dfrac{R_2}{R_{th}}}{1+\dfrac{\Gamma_1}{\Gamma_2}}\ , \tag{3.20}$$

which for a given pumping rate, is *minimized* in the limit $\Gamma_1 \gg \Gamma_2$. Thus conditional on a stimulated emission event occurring rapid spontaneous emission from the lower level reduces the probability of absorption, so that the quantum may exit the cavity. Also in the limit, one may argue heuristically that if the pumping is regular (or antibunched) one may expect the efficient conversion into laser photons to be regular, and that this be reflected in the output noise properties after the boundary condition at the output mirror has been accounted for.

For a semiconductor, the lasing transition occurs over some range of **k** values determined by the medium gain versus cavity loss. Since the photon momentum is negligible, selection rules indicate that the electronic momentum should be conserved in an optical transition. In practice, carrier concentration and doping modify this selection rule so that a range of **k** values are involved. Of course, a transition may only occur if the Pauli exclusion principle allows it.

A stimulated transition occurs when an electron in the conduction band combines with a hole in the valence band, leaving a hole in the conduction band, and producing a laser photon. Now suppose that such an event has occurred. If the hole and electron remained in the states they occupied immediately after the event, the Pauli principle would not affect the probability for stimulated absorption (the possibility of laser action would be strongly affected by this). However, strong Coulomb interactions scatter electrons and holes to rapidly produce quasi-Fermi distributions with the bands. The probabilities that an electron and hole occupy the states that were occupied immediately after the event are thus *reduced* from unity to the appropriate Fermi factors.[13] The probability of stimulated absorption which depends upon the product of such Fermi factors is thus *reduced* by the Pauli exclusion principle which comes into play because of the Coulomb scattering.

## IV. SUMMARY

In this paper we have considered the basic physical mechanisms behind intensity noise reduction in the output from atomic and semiconductor lasers pumped by a regular or antibunched source. We summarize our results as follows. In general, for atomic lasers spontaneous emission from the lasing levels limits the noise reduction achievable. When $\Gamma_1=\Gamma_2$ the noise is 50% (3 dB) below the shot noise level at the laser frequency, for a perfectly quiet pump. In the limit $\Gamma_1 \gg \Gamma_2$ 100% noise reduction, or perfect amplitude squeezing is possible in the output at the laser frequency since spontaneous emission and pump fluctuations give negligible contributions, and complete destructive interference between the transmitted and reflected vacuum fluctuations removes their contribution (at the laser frequency). These results are consistent with other analytical and numerical models of regularly pumped atomic lasers.[2,5,6,14,15] For the semiconductor laser, we obtained agreement with the results of Yamamoto and co-workers[3] by a simple analytical treatment which paralleled out theory for the atomic laser. This showed almost perfect amplitude noise reduction to be possible with a noiseless pump.

To understand the physics behind these noise reductions we consider first the operation of an ideal source of antibunched light. We require that for each of the regularly spaced pump excitations, photons are produced into the cavity mode with unit efficiency. The field emerging from the output mirror having finite mean amplitude but zero noise amplitude at the cavity oscillation frequency (assuming the vacuum-noise contributions cancel due to destructive interference).

To approach unit efficiency in an atomic laser, one needs to circumvent spontaneous emission from the upper level and reduce stimulated absorption. These conditions are both achieved if the lower level decays much faster than the upper level $\Gamma_1 \gg \Gamma_2$. This is just the usual design criterion for an efficient laser. Intuitively one may argue that once a stimulated event occurs, the rapid lower-level decay prevents absorption and allows the photon to exit the cavity. In semiconductors the absorption is suppressed by the strong intraband Coulomb scattering which brings the Pauli exclusion principle into play.

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## APPENDIX: ATOMIC LASER WITH SPONTANEOUS DECAY BETWEEN THE LASING LEVELS

In our discussion of the atomic laser, we have ignored spontaneous decay between the lasing levels. The assumption being that it is a small fraction of the total decay rate of the upper lasing level. Here we consider explicitly the role of spontaneous decay between lasing levels on the output intensity fluctuations.

Following the notation of LL, let $W_{12}$ represent the spontaneous decay rate from level 2 to 1. Thus since $\Gamma_2$ is the total decay rate of the upper level $\Gamma_2 - W_{12}$ represents the emission rate to ground. The theoretical analysis proceeds as in Sec. II. We find that the mean photon number above threshold is given by

$$n_0 = \bar{n}_s \left[ \frac{R_2}{\bar{R}_{\text{th}}} - 1 \right] , \tag{A1}$$

where

$$\bar{n}_s = \frac{\bar{\Gamma}\Gamma}{2\mu^2 \left[ 1 - \dfrac{W_{12}}{\Gamma_1 + \Gamma_2} \right]} , \tag{A2}$$

and

$$\bar{R}_{\text{th}} = \frac{\gamma \Gamma_1 \Gamma_2}{\pi(\Gamma_1 - W_{12})} \tag{A3}$$

are the characteristic photon number and threshold pumping rate, respectively. Proceeding with a linearized analysis of the quantum fluctuations we drive, analogous to Eq. (2.20), for $n_0 \gg \bar{n}_s$,

$$\Delta \dot{n} = -\gamma \Delta n + \bar{G}(t) , \tag{A4}$$

where

$$\bar{G}(t) \equiv G_p(t) + \frac{(\Gamma_1 - W_{12}) G_2(t) - \Gamma_2 G_1(t)}{\Gamma_1 + \Gamma_2 - W_{12}} \tag{A5}$$

may be compared with Eq. (2.21).

Note that since we have spontaneous emission $W_{12}$ present, the noise forces $G_1(t)$ and $G_2(t)$ are not precisely the same as in Sec. II. The inclusion modifies their diffusion coefficients of (2.6) and (2.7) by

$$\begin{aligned} 2D_{11} &\to 2D_{11} + W_{12} N_2 , \\ 2D_{12} &\to 2D_{12} - W_{12} N_2 , \\ 2D_{21} &\to 2D_{21} - W_{12} N_2 . \end{aligned} \tag{A6}$$

To investigate how $W_{12} \neq 0$ changes our earlier results, we now concentrate on the extreme (worst) case $W_{12} = \Gamma_2$, where level 2 decays only to level 1. For lasing to occur under such conditions requires $\Gamma_2 < \Gamma_1$ [see (A3)]. The noise $\bar{G}(t)$ reduces to

$$\bar{G}(t) = G_p(t) - \frac{\Gamma_2}{\Gamma_1} G_1(t) + \left[ 1 - \frac{\Gamma_2}{\Gamma_1} \right] G_2(t) , \tag{A7}$$

and the variance in photon number

$$\sigma^2 = \frac{\langle \bar{G}\bar{G} \rangle}{2\gamma} = \tfrac{1}{2} n_0 + \left[ 1 - \frac{\Gamma_2}{\Gamma_1} \right]^2 \frac{R_2}{2\gamma} + \frac{\Gamma_2}{2\gamma} \left[ N_{20} + \frac{\Gamma_2}{\Gamma_1} N_{10} \right] , \tag{A8}$$

where $\langle \bar{G}(t) \bar{G}(t') \rangle = \langle \bar{G}\bar{G} \rangle \delta(t-t')$. Then using the operating conditions

$$\Gamma_2 \left[ N_{20} + \frac{\Gamma_2}{\Gamma_1} N_{10} \right] = \left[ 1 - \frac{\Gamma_2}{\Gamma_1} \right] \bar{R}_{\text{th}} + \frac{\Gamma_2}{\Gamma_1} \left[ 1 + \frac{\Gamma_2}{\Gamma_1} \right] R_2 , \tag{A9}$$

$$R_2 = \bar{R}_{\text{th}} + \frac{\gamma n_0}{1 - \dfrac{\Gamma_2}{\Gamma_1}}$$

gives for $n_0 \gg \bar{n}_s$

$$\sigma^2 = \tfrac{1}{2} n_0 + \tfrac{1}{2} n_0 \left[ 1 + \frac{2 \left[ \dfrac{\Gamma_2}{\Gamma_1} \right]^2}{1 - \dfrac{\Gamma_2}{\Gamma_1}} \right] + O(\bar{n}_s) , \tag{A10}$$

where the first factor is due to vacuum fluctuations and the remainder is due to pumping and spontaneous emission. Equation (A10) should be compared with (2.25). Poissonian photon statistics $\sigma^2 = n_0$ are not produced well above threshold as spontaneous emission between lasing levels produces an excess component. However, when $\Gamma_2 \ll \Gamma_1$ the excess contribution is minimized. Note that this inequality also reduces the laser threshold pumping rate.

Now we consider the photon statistics when the pump is perfectly antibunched. Following the discussion of Sec. II we set $R_2 = 0$ in Eq. (A8) and using the operating conditions (A9) we find ($n_0 \gg \bar{n}_s$):

$$\sigma^2 = \tfrac{1}{2} n_0 \frac{1 + \left[ \dfrac{\Gamma_2}{\Gamma_1} \right]^2}{1 - \dfrac{\Gamma_2}{\Gamma_1}} + O(\bar{n}_s) . \tag{A11}$$

From (A11) it is clear that provided $\Gamma_2/\Gamma_1 \lesssim \sqrt{2}-1 = 0.4$, subpoissonian photon statistics are still achievable, and correspondingly, from (2.26) output intensity fluctuations below shot-noise level are possible. Indeed provided $\Gamma_2/\Gamma_1 \ll 0.4$, we may still approach perfect squeezing of the output intensity fluctuation component resonant with the laser frequency. This is the same conclusion we found when spontaneous emission between laser levels was ignored and all decay went to the ground level. Note also that when $\Gamma_2 \ll \Gamma_1$, the operating conditions, threshold pump rate, characteristic photon number, etc. are relatively independent of the proportion of upper-level spontaneous emission into the lower lasing level.

We conclude that provided $\Gamma_2 \ll \Gamma_1$, significant reduction of output intensity fluctuations are achievable irrespective of spontaneous emission between the lasing levels. These results have been confirmed by a numerical model.[15]


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