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Effects of optical gain and cavity-mode squeezing on the Mollow spectrum

G. S. Agarwal
School of Physics, University of Hyderabad, Central University Post Office, Hyderabad 500 134, Andhra Pradesh, India
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A theoretical model for studying the effects of optical gain and the cavity-mode squeezing on the spectrum of radiation produced by coherently driven two-level atoms is developed. The squeezing of the cavity mode dramatically affects the central component leading to a spike in the spectrum which becomes more pronounced with the increase in optical gain and squeezing. The squeezing can also affect the three photon Rabi sideband and can facilitate laser oscillation at such frequencies.

Mollow[1,2] considered how the radiative properties of a system are changed dramatically if the system is driven by an intense coherent field. These modifications can be studied either by examining the spectrum of the spontaneously emitted radiation[1] or by monitoring the absorption[2] from a weak probe field. The spectrum is also sensitive to the environment in which the atoms are radiating. For example, recently considerable work has been done on how the spectrum of the radiation emitted by a coherently driven system is modified due to the optical-gain processes.[3-6] Such processes may arise, say, from four-wave-mixing processes in free space or one might consider gain processes in a cavity. The latter case has also been studied experimentally. The gain processes modify the widths and heights of different resonances in Mollow spectrum[7] and also make the off-resonance spectrum asymmetric. A question that arises is—what happens if the nature of the vacuum in the cavity is changed; e.g., one can replace the vacuum by squeezed vacuum. Thus we investigate the combined effects of optical gain and the squeezed cavity mode on the Mollow spectrum. We specifically study the radiation emitted by coherently driven atoms passing through a squeezed cavity. We develop a theoretical model for studying the characteristics of the emitted radiation. We demonstrate the spectral narrowing of the central component of the Mollow spectrum. We show that in the limit of good cavity and good degree of squeezing the central component narrows from a width of the order of the free-space atomic width γ to a width of the order of κ (or even less), which is the loss rate from the cavity.

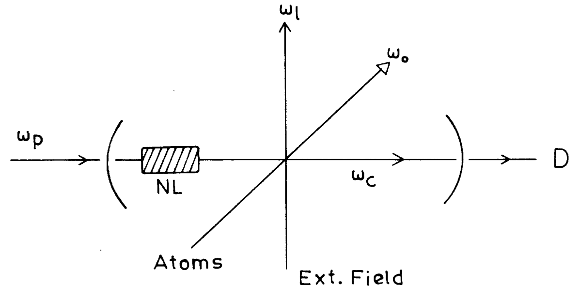

FIG. 1. Schematic illustration of the model with NL denoting the nonlinear medium which is pumped by the field ω_p. The atoms are pumped by ω_l.

Consider an atomic beam of two-level atoms, which are also pumped by a coherent field, passing through a squeezed cavity (Fig. 1). The mode of the cavity is squeezed—the squeezing might be achieved by degenerate parametric processes or by other methods.[8] It is assumed that the atoms spend sufficient time in the cavity so that steady state is reached. The output of the cavity, i.e., the number of photons in the cavity mode, can be studied as a function of the cavity detuning parameter. The Hamiltonian for the dynamics of the two-level atom in a squeezed cavity[9] can be written in the form

$$H = \omega_0 S^z + \omega_c a^\dagger a + [G(a^\dagger)^2 e^{-i\omega_p t} + G^* a^2 e^{i\omega_p t}] + (g_c S^+ a + g_c^* S^- a^\dagger) + (g S^+ e^{-i\omega_l t} + \text{H.c.}) + \cdots , \tag{1}$$

where the ellipsis denotes coupling to various heat baths which would simulate the effects of losses from the cavity and the effects of spontaneous emission by the atoms into all other modes. Various terms in (1) have the following obvious interpretation: (a) g terms describe the coherent pumping of the atoms; (b) g_c terms describe the interaction of the atoms with the cavity mode; (c) G terms describe the generation of squeezed vacuum in the cavity, ω_p is the carrier frequency of the pump used to produce squeezed light in the cavity; (d) terms involving ω_c and ω_0 are the unperturbed terms. The total Hamiltonian is obtained by summing over all the atoms. For simplicity, we have not shown the phase factors such as $\exp(i\mathbf{k}_l \cdot \mathbf{r}_j)$, etc. The density matrix ρ for the combined atom-field system obeys the master equation

$$\frac{\partial \rho}{\partial t} = -i[H,\rho] - \kappa(a^\dagger a\rho - 2a\rho a^\dagger + \rho a^\dagger a) - \gamma(S^+ S^- \rho - 2S^- \rho S^+ + \rho S^+ S^-) . \tag{2}$$

Here κ is related to the quality factor Q ($\kappa = \omega_c/2Q$) of the cavity and γ is half of the Einstein—A coefficient for emission in free space. The master equation (2) is rather complex; however, it can be simplified further by invoking the physical situation. In this paper we consider the case when γ *is large compared to* κ *and* $|G|$. In such a case we can derive the master equation[10] for the field density matrix ρ by *adiabatically eliminating the atomic degree of freedom.* Our calculations show that the field density matrix ρ (from now onwards ρ will represent the field density matrix) obeys

$$\frac{\partial\rho}{\partial t}=-i[\delta_0 a^\dagger a,\rho]-i[G(a^\dagger)^2+G^* a^2,\rho]-\kappa(a^\dagger a\rho-2a\rho a^\dagger+\rho a^\dagger a)$$
$$-[R_e(\delta)(aa^\dagger\rho-a^\dagger\rho a)+R_a(\delta)(\rho a^\dagger a-a\rho a^\dagger)+\text{H.c.}],\ \ \delta=\omega_c-\omega_l,\ \delta_0=\omega_c-\frac{\omega_p}{2}\,. \tag{3}$$

Here $R(\delta)$'s are defined by

$$R_e(\delta)=R_e'+iR_e''=g^2N\int_0^\infty d\tau e^{-i\delta\tau}\lim_{t\to\infty}[\langle S^+(t+\tau)S^-(t)\rangle-\langle S^+(t)\rangle\langle S^-(t)\rangle]\,, \tag{4}$$

$$R_a(\delta)=R_a'+iR_a''=g^2N\int_0^\infty d\tau e^{-i\delta\tau}\lim_{t\to\infty}[\langle S^-(t)S^+(t+\tau)\rangle-\langle S^-(t)\rangle\langle S^+(t+\tau)\rangle]\,, \tag{5}$$

where N is the total number of atoms. The derivation assumes that different atoms are uncorrelated. We have also dropped the so-called interference terms under the condition that γ is much bigger than κ and G. The two time-correlation functions in (4) and (5) are to be obtained from the solution of the master equation for the coherently driven two-level atom in free space, i.e., from

$$\frac{\partial\rho_A}{\partial t}=-i[\Delta S^z+(gS^++g^*S^-),\rho_A]$$
$$-\gamma(S^+S^-\rho_A-2S^-\rho_A S^++\rho_A S^+S^-)\,. \tag{6}$$

Note that optical Bloch equations follow from (6). The two-time correlations for a two-level system have been calculated by many authors.[1,2] Note that $\text{Re}[R_e(\delta)/g^2N]$ is just the Mollow spectrum; i.e., the incoherent part of the spectrum of the radiation emitted by a coherently driven two-level atom in free space. The interpretation of R's is also clear from our fundamental Eq. (3) if we examine the rate at which the number of photons in the cavity mode changes. We find from (3)

$$\frac{d}{dt}\langle a^\dagger a\rangle=-2\langle a^\dagger a\rangle[\kappa+R_a'(\delta)-R_e'(\delta)]$$
$$+2R_e'(\delta)-2iG\langle(a^\dagger)^2\rangle+2iG^*\langle a^2\rangle\,. \tag{7}$$

Clearly $2[R_a'(\delta)-R_e'(\delta)]$ gives the rate of absorption (by the atoms) of the cavity photons and $2R_e'(\delta)$ gives the rate of spontaneous emission into the cavity mode. The term involving κ gives the rate of loss from cavity mirrors. The number of photons in the cavity is influenced by the phase-sensitive coupling terms G. Note that if the atoms were pumped incoherently to the excited state, then $R_a(\delta)$ will be zero. We are considering a coherently driven system and thus we have a mixed state involving ground and excited states and hence both R_a and R_e enter our calculations.

The master equation (3) can be solved exactly and all the physical quantities can be calculated. In what follows we examine the modifications[11] in the Mollow spectrum due to gain processes and the squeezing of the cavity mode. Using (7) and the equations for $\langle a^2\rangle$ and $\langle(a^\dagger)^2\rangle$ that follow from (3), we have proved that the number of photons in the steady state is

$$\langle a^\dagger a\rangle=\frac{R_e'+f}{\kappa+R_a'-R_e'-2f}\,, \tag{8}$$

$$f=|G|^2\left\{\left[R_a-R_e+\kappa\left(1-\frac{i\delta_0}{\gamma}\frac{\gamma}{\kappa}\right)\right]^{-1}+\text{c.c.}\right\}. \tag{9}$$

It should be remembered that all R's are functions of $\delta=\omega_c-\omega_l$. These R's also depend on the strength of the coherent field pumping the atoms, the pump atom detuning Δ, and the density of atoms. The derivation of (8) assumes that the steady state exists. The condition for the existence of steady state can be obtained by examining the eigenvalues of the drift matrix formed from equations for $\langle a\rangle$ and $\langle a^\dagger\rangle$. Calculations show that the steady state exists if

$$\kappa+R_a'-R_e'>0,\ \ |\kappa+R_a-R_e-i\delta_0|^2>4|G|^2\,, \tag{10}$$

otherwise the cavity field grows and no steady state exists. In such a case one has the possibility of laser oscillation which can be discussed by generalizing (3) to include nonlinear terms in a and $a^\dagger$. The condition for the existence of the steady state and laser oscillation depends on the parameters κ, G, g, Δ, δ, and $g_c^2N/\kappa\gamma$.

The free-space result is recovered if we set $|G|=0$ and take the limit of large κ whence $\langle a^\dagger a\rangle=R_e'(\delta)/\kappa$. For $|G|=0$, (8) reduces to the result obtained by Holm *et al.* for the unsqueezed cavity. In this case gain processes are most significant if $\kappa\sim|R_a'-R_e'|$ and when $R_a'-R_e'<0$. This will be the case when the absorption spectrum exhibits regions of amplification. In the absence of the atom ($R_e=R_a=0$),

$$\langle a^\dagger a\rangle=2|G|^2/(\kappa^2+\delta_0^2-4|G|^2)\,. \tag{11}$$

Clearly, the effect of the squeezing parameter is important when $4|G|^2\lesssim(\kappa^2+\delta_0^2)$. The amount of squeezing in the cavity itself depends on the parameter $\delta_0=\omega_c-\omega_p/2$. We can make different choices of δ_0 depending on the frequency region of Mollow spectrum under consideration.

We evaluate (8) numerically for a range of parameters. Let α_0 be the optical gain parameter defined by

$$\alpha_0=Ng_c^2/\gamma\,. \tag{12}$$

Thus, from now on κ and $|G|$ would be taken in units of a_0. The external field parameters Δ, g, and δ will be expressed in units of γ. We also need to know the relative magnitudes of γ and κ. Note that we have assumed a good cavity and hence $\gamma \gg \kappa$. For computations we take $\gamma = 20\kappa$. We display the numerical results for a range of parameters in Figs. 2–4. We plot total output of the cavity as a function of cavity detuning parameter δ. For $|G| = 0$, we recover the known results for the effect of optical gain on Mollow spectrum. As the parameter a_0 increases (κ decreases) the sidebands grow. In fact, the sidebands become more intense than the central component. We also find that the height of the central peak scales as $1/\kappa$. Figures 2–4 give the effect of the cavity-mode squeezing. Here we have to make an appropriate choice of the detuning parameter $\delta_0 = \omega_c - \omega_p/2$. The choice depends on the region of Mollow spectrum under consideration. If we are examining the central component of the Mollow spectrum, then we choose δ_0 close to zero. This can be done by taking $\omega_p = 2\omega_l$ and thus $\delta_0 = \delta = \omega_c - \omega_l$ which is close to zero. For Fig. 2 we choose a value of κ for which the effects of optical gain are rather unimportant. We find that as the squeezing parameter G increases, the central component of the three-peak Mollow spectrum becomes more and more prominent leaving the sidebands practically unaffected by the squeezing of the cavity mode. As a matter of fact, as G approaches $\kappa/2$ (note that G is always smaller than $\kappa/2$ for $\delta \sim 0$) the central part of the spectrum consists of a narrow spike riding on top of a broad structure. For $\kappa = 0.02$ (Fig. 3), the case in which the optical-gain effects are very significant, the squeezing of the cavity mode has a very dominant effect on the central component of the Mollow spectrum—the spike is quite pronounced. The half-width of the spike is approximately equal to κ and can become less than κ. We have also calculated the output when the field driving the atoms is detuned from resonance. For $G = 0$, the spectra are asymmetric due to optical-gain processes. For $G \neq 0$, the central peak grows in magnitude. Again for G approaching $\kappa/2$ a spike appears on top of a broad resonant background. Note that for δ values close to the sideband frequencies $(\Delta^2 + 4|g|^2)^{1/2}$, the denominator $(\kappa + R_a' - R_e')^2 + (R_a'' - \delta)^2$ is large and hence Eq. (8) reduces to

$$\langle a^\dagger a \rangle \approx R_e'/(\kappa + R_a' - R_e') \, . \qquad (13)$$

Thus, the parameter G has no effect on sidebands unless $2G \sim \delta$. This is because for sidebands the cavity is too far detuned from resonance and the squeezing is insignificant.

In order to make the squeezing more effective at the sidebands, we consider an alternate situation. We imagine that the cavity mode is close to resonance with $\omega_p/2$. Note that ω_c itself is in the vicinity of sideband frequencies, i.e., near $\omega_l \pm (\Delta^2 + 4|g|^2)^{1/2}$. We thus choose

$$\delta_0 = \omega_c - \omega_l + (\Delta + 4|g|^2)^{1/2} \qquad (14)$$

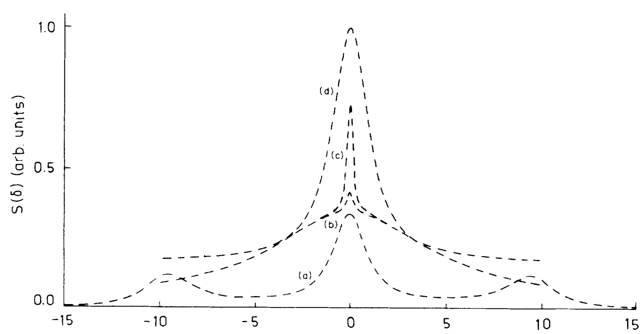

FIG. 2. Effect of squeezing the cavity mode on its output $S(\delta) = \langle a^\dagger a \rangle$ as a function of detuning $\delta = (\omega_c - \omega_l)/\gamma$ for $\kappa/a_0 = 0.1$ and for $|G|/a_0 = 0$ [curve (a)], 0.02 [curve (b)], 0.035 [curve (c)], 0.045 [curve (d)]. For $|G| \neq 0$ only the central resonance is shown. The actual values for curves (a)–(c) are half of those shown. The scale on x axis for curves (b) and (c) [curve (d)] is $\frac{1}{5}$ [$\frac{1}{50}$] of that shown. The other parameters are $g = 5\gamma$, $\omega_0 = \omega_l$.

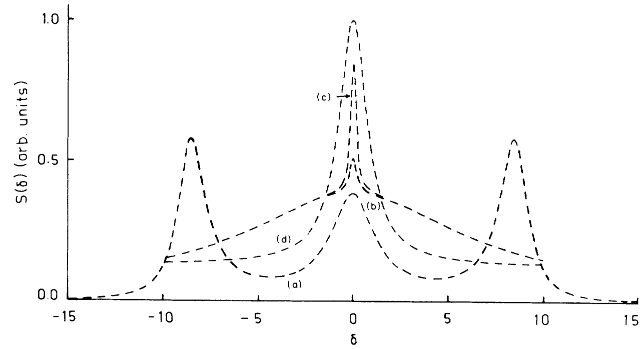

FIG. 3. Same as in Fig. 2 but for $\kappa/a_0 = 0.02$ and $|G|/a_0 = 0$ [curve (a)], 0.005 [curve (b)], 0.0075 [curve (c)], and 0.0095 [curve (d)]. The actual values for curves (a)–(c) are $\frac{1}{3}$ of those shown and the scale on x-axis is same as in Fig. 2.

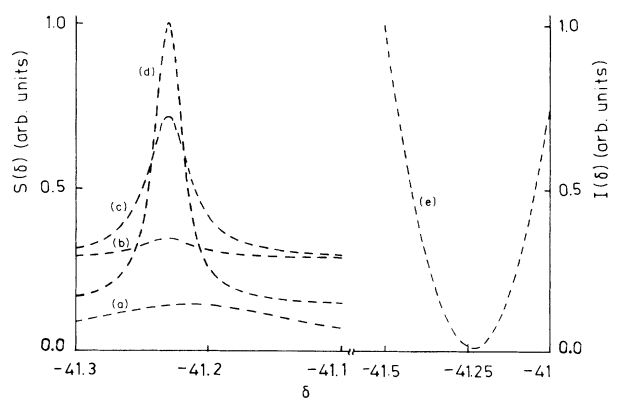

FIG. 4. Cavity output when the cavity is tuned close to the three-photon peak (14) for $g = 20\gamma$, $\omega_0 - \omega_l = 10\gamma$, $\kappa = 0.1a_0$, and $|G|/a_0 = 0$ [curve (a)], 0.01 [curve (b)], 0.02 [curve (c)], and 0.025 [curve (d)]. The actual values in cases (b) and (c) are half of those shown. The scale on x axis for $G = 0$ is from -40 to -42. The curve (e) gives the left-hand side of the inequality (15). The minimum (maximum) value for curve (e) in the shown range is 0.307×10^{-2} (0.281).

for studying the effects of optical gain and cavity-mode squeezing on the "three-photon" sideband. We show in Fig. 4 the typical behavior of the cavity output when ω_c is tuned in the neighborhood of the three-photon Rabi sideband. The behavior that we observe is similar to that of Figs. 2 and 3; i.e., as the radiation in the cavity becomes more and more squeezed, the spike at the sideband frequency becomes more and more pronounced. As a matter of fact, the three-photon sideband experiences a lot of gain and soon the system starts oscillating and the laser action occurs. When this happens then one has to go beyond the linearized theory on which the result (8) is based. Note that the threshold condition for laser operation depends on the parameter $|G|$. Thus, one can have a situation that optical-gain processes do not lead to an instability (i.e., $\kappa+R_a'-R_e'>0$), but the cavity-mode squeezing leads to laser oscillation, i.e.,

$$I(\delta)=|\kappa+R_a-R_e-i\delta_0|^2<4|G|^2. \quad (15)$$

The left-hand side of this inequality is also plotted in Fig. 4, from which one can read the maximum value of $|G|$ beyond which the laser oscillation would occur.

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[1]B. R. Mollow, Phys. Rev. **188**, 1969 (1969).

[2]B. R. Mollow, Phys. Rev. A **5**, 2217 (1972).

[3]G. S. Agarwal and R. W. Boyd, Phys. Rev. A **38**, 4019 (1988).

[4]D. Holm and M. Sargent III, Phys. Rev. A **33**, 4001 (1986).

[5]D. Holm, M. Sargent III, and S. Stenholm, J. Opt. Soc. Am. B **2**, 1456 (1985).

[6]Y. Zhu, A. Lezama, and T. W. Mossberg, Phys. Rev. A **39**, 2268 (1989).

[7]The Mollow spectrum also depends on the nature of relaxation processes in the atom. The relaxation processes arising from collisions and fluctuations in the driving field have been investigated [B. R. Mollow, Phys. Rev. A **15**, 1023 (1977); G. S. Agarwal, Phys. Rev. Lett. **37**, 1383 (1976); J. H. Eberly, *ibid.* **37**, 1387 (1976)]; C. W. Gardiner [Phys. Rev. Lett. **56**, 1917 (1986)] proposed that the atom immersed in a broadband squeezed bath has a very interesting type of phase-sensitive relaxation. The effects of this latter relaxation have been examined by H. J. Carmichael, A. S. Lane, and D. F. Walls, Phys. Rev. Lett. **58**, 2539 (1987).

[8]See, for example, D. F. Walls, P. D. Drummond, A. S. Lane, M. A. Marte, M. D. Reid, and H. Ritsch, in *Squeezed and Nonclassical Light,* edited by P. Tombesi and R. Pike (Plenum, New York, 1989), p. 1.

[9]A previous study [G. S. Agarwal and S. Dutta Gupta, Phys. Rev. A **39**, 2961 (1989)] deals with emission from *undriven atoms* in a squeezed cavity when the atomic dynamics is approximated by a oscillator. In contrast, we deal here with coherently driven two-level atoms.

[10]G. S. Agarwal, Phys. Rev. A **34**, 4055 (1986).

[11]Our work refers to a situation which is different from the one treated previously by Carmichael, Lane, and Walls (Ref. 7). These authors consider the emission from a *single* atom in *free space* and assume that the linewidth of the squeezed radiation is *much larger* than the atomic linewidth. In contrast, we study the behavior of a *collection* of atoms *inside* the squeezed cavity under the assumption that the *atomic linewidth is much bigger* than the characteristic parameters κ and G of the cavity.