# Plasma-wave instability in the presence of two laser fields

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Plasmon scattering by electrons in the simultaneous presence of two laser fields is considered. A kinetic equation for the plasmon population is derived, and the rate of change of the plasmon population is calculated. We found that plasma waves propagating parallel to the direction of polarization of the radiation fields may be amplified over a relatively narrow range of plasmon wave numbers.

## I. INTRODUCTION

There has been a renewed interest in the study of the interaction of intense laser fields with plasma.[1-6] In particular, the heating of a plasma by two laser fields has recently been discussed[5,6] in connection with the problem of nuclear hot fusion. It has been shown that the correct and efficient way to achieve rapid energy absorption and a large heating rate, in contrast to the mechanisms considered previously,[1-4] is to illuminate the plasma with two laser fields, namely, a strong field (pumping) and weak field (probing), respectively.

An even more interesting aspect of the interacting laser-plasma problem is the one in which one considers the effects of laser field on the several wave-particle processes occurring in the plasma. This problem has been investigated by some authors[7] where the electron-plasmon scattering was studied in the presence of external fields (laser plus dc magnetic field). In particular, the changes induced by a laser field on the damping of plasma waves due to the electron-plasmon scattering has been calculated.[7]

It has been found[7] that the plasmon damping decreases and may revert its signal (amplification) whenever the drift velocity of the electrons, as imposed by the laser field, exceeds the phase velocity of the plasma wave. This is in complete analogy to the phonon amplification in semiconductors.[8]

Here in this paper we consider the influence of the simultaneous action of two laser beams as mentioned above on the damping of plasma waves in a plasma. The reason for this study is that, contrary to the case discussed in Ref. 7 in which only one laser (strong) is present, it will be shown that the threshold condition for plasma wave instability is now dependent upon the plasmon wave number $k$ instead of being the same for all values of $k$, i.e., there is a selective mechanism for plasmon amplification.

## II. FORMALISM

In this section we set up the theory for plasma-wave instability in the presence of two radiation fields. We have, therefore, considered the scattering of plasmon by electrons under the action of two laser fields. Our approach follows closely that of Ref. 7. The laser beams are treated as classical plane electromagnetic waves in the dipole approximation; the electron states are described by the solution to the Schrödinger equation for an electron in the laser fields. The electron-plasmon scattering is treated by first-order perturbation theory, but with retention of the laser fields to all orders. The transition probabilities are then used to write a kinetic equation for the plasmon population from which the damping rate is obtained.[9]

We begin with the solution to the time-dependent Schrödinger equation for an electron in the electromagnetic fields of the laser beams, namely,[1]

$$\Psi(\mathbf{x},t)=L^{-3/2}\exp\left[i\mathbf{p}\cdot\mathbf{x}-(i/2m\hbar)\int^{t}dt\,[\hbar\mathbf{p}-e\,\mathbf{A}(t)/c]^2\right]. \tag{1}$$

Here $\mathbf{p}$ is the electron wave vector, such that in the absence of the laser fields its energy $\varepsilon_p$ is $\hbar^2\mathbf{p}^2/2m$ and

$$\mathbf{A}(t)=(c/\omega_1)E_1\cos\omega_1 t+(c/\omega_2)E_2\cos\omega_2 t$$

is the vector potential of laser 1 and 2 within the dipole approximation.

The probability amplitude for a transition from state 1 ($\mathbf{p}_1=\mathbf{p}$) to state 2 ($\mathbf{p}_2=\mathbf{p}+\mathbf{k}$) due to a collision with a plasmon of momentum $\hbar\mathbf{k}$ is given by

$$a(1\rightarrow 2;\mathbf{k}\rangle=-(i/\hbar)\int\int d\mathbf{x}\,dt\,\Psi_2^*(\mathbf{x},t)V(\mathbf{k})\times\exp(i\mathbf{k}\cdot\mathbf{x}-i\omega_k t)\Psi_1(\mathbf{x},t), \tag{2}$$

where $|V(\mathbf{k})|^2=2\pi e^2\hbar\omega_k/\mathcal{V}\mathbf{k}^2$ is the electron-plasmon vertex,[10] $\omega_k$ is the plasmon dispersion relation, and $\mathcal{V}$ is the normalization volume. By substituting Eq. (1) into Eq. (2), performing the indicated integrations, and using the well-known relation between the scattering amplitude and the $T$ matrix,[11] we obtain the transition probability per unit time $T_{n,m}(1\rightarrow 2;\mathbf{k})$ for the transition from state 1 ($\mathbf{p}_1=\mathbf{p}$) to state 2 ($\mathbf{p}_2=\mathbf{p}+\mathbf{k}$) due to a collision with a plasmon $\mathbf{k}$ with absorption $(n,m>0)$ or emission $(n,m<0)$ of $|n|$ and $|m|$ photons:

$$T_{n,m}(1\to 2;\mathbf{k})=(2\pi/\hbar)|V(\mathbf{k})|^2 J_n^2(\lambda_1/\hbar\omega_1)J_m^2(\lambda_2/\hbar\omega_2)\times\delta(\varepsilon_{\mathbf{p}+\mathbf{k}}-\varepsilon_{\mathbf{p}}-\hbar\omega_{\mathbf{k}}-n\hbar\omega_1-m\hbar\omega_2)\ , \quad (3)$$

where $J_i$ is the Bessel function of order $i$ ($i=n,m$) and argument

$$\lambda_i/\hbar\omega_i=e\mathbf{k}\cdot\mathbf{E}_i/m\omega_i^2 \quad (i=1,2)\ .$$

The rate of change of the plasmon number of wave number $\mathbf{k}$ is then given in terms of the transition probability $T_{n,m}$ as[7]

$$\frac{dN_k}{dt}=\gamma_k N_k\ ,$$
$$\gamma_k=(2\pi/\hbar)|V(k)|^2 \times\sum_{m,n=-\infty}^{+\infty}\sum_p J_n^2(\lambda_1/\hbar\omega_1)J_m^2(\lambda_2/\hbar\omega_2)(f_{\mathbf{p}+\mathbf{k}}-f_{\mathbf{p}})\times\delta(\varepsilon_{\mathbf{p}+\mathbf{k}}-\varepsilon_{\mathbf{p}}-\hbar\omega_{\mathbf{k}}-\hbar n\omega_1-\hbar m\omega_2)\ . \quad (4)$$

Here $f_{\mathbf{k}}$ is the electron distribution function. Equation (4) tells us that if $\gamma_k$ is positive, the plasmon population grows with time, whereas if $\gamma_k$ is negative, it is damped.

In the following we assume that laser 1 is the weak field and laser 2 is the strong one. In the strong-field limit, $\lambda_2 \gg \hbar\omega_2$ and the argument of the Bessel function $J_m$ is large. The condition $\lambda_2\gg\hbar\omega_2$ is essentially $E_2$ large. The sum over $m$ in Eq. (4) may be written approximately[1]

$$\sum_{\substack{m=-\infty\\ m\neq 0}}^{+\infty} J_m^2(\lambda_2/\hbar\omega_2)\delta(\Sigma-\hbar m\omega_2)\simeq\tfrac{1}{2}[\delta(\Sigma-\lambda_2)+\delta(\Sigma+\lambda_2)]\ ,$$

where $\Sigma\equiv\varepsilon_{\mathbf{p}+\mathbf{k}}-\varepsilon_{\mathbf{p}}-\hbar\omega_k-\hbar n\omega_1$. In the weak-field limit we shall confine ourselves only to one-photon transitions ($n=\pm1$). Under the foregoing assumptions Eq. (4) may then be written as

$$\gamma_k=(V_k^2\pi/\hbar)J_1^2(\lambda_1/\hbar\omega_1)\sum_{\mathbf{p}}(f_{\mathbf{p}+\mathbf{k}}-f_{\mathbf{p}})\delta_\varepsilon\ , \quad (5)$$

where

$$\delta_\varepsilon=\delta(\varepsilon_{\mathbf{p}+\mathbf{k}}-\varepsilon_{\mathbf{p}}-\hbar\omega_k+\hbar\omega_1-\lambda_2)+\delta(\varepsilon_{\mathbf{p}+\mathbf{k}}-\varepsilon_{\mathbf{p}}-\hbar\omega_k+\hbar\omega_1+\lambda_2)+\delta(\varepsilon_{\mathbf{p}+\mathbf{k}}-\varepsilon_{\mathbf{p}}-\hbar\omega_k-\hbar\omega_1-\lambda_2)+\delta(\varepsilon_{\mathbf{p}+\mathbf{k}}-\varepsilon_{\mathbf{p}}-\hbar\omega_k-\hbar\omega_1+\lambda_2)\ .$$

Equation (5) gives the plasmon damping for multiphoton absorption, or emission of $|m|\gg1$ photons of intense laser field with the simultaneous absorption or emission of $n=\pm1$ photons of weak laser field.

We now assume a Maxwellian distribution function for the electrons, namely,

$$f_{\mathbf{p}}=N_0(\pi v_T^2)^{-3/2}\exp(-\hbar^2\mathbf{p}^2/2mk_bT)\ , \quad (6)$$

where $v_T^2=2k_bT/m$ and $N_0$ is the density of plasma electrons. This assumption is valid provided the electron heating in the laser fields may be neglected ($e^2E_i^2/2m\omega_i^2<k_bT$). By inserting Eq. (6) into Eq. (5) and performing the integrations assuming $\mathbf{k}$ parallel to $\mathbf{E}_1$ and $\mathbf{E}_2$, we obtain the expression for the damping of plasma waves in the presence of the two laser fields

$$\gamma_k=\{[\mathcal{V}\pi^{1/2}N_0\omega_k|V(k)|^2J_1^2(\lambda_1/\hbar\omega_1)]/2\hbar kmv_T^3\}F(\alpha,\beta,b)\ , \quad (7)$$

where

$$\alpha=(\lambda_2-\hbar\omega_1)/\hbar\omega_k,\quad \beta=(\lambda_2+\hbar\omega_1)/\hbar\omega_k\ ,$$
$$b=\omega_k/k_bT,\quad J_1^2(\lambda_1/\hbar\omega_1)=(\lambda_1/2\hbar\omega_1)^2\ ,$$
$$F(\alpha,\beta,b)=\exp[-b^2(1+\beta^2)]\times\{\beta\tanh(2b^2\beta-1)\cos(2b^2\beta)+\exp[-b^2(\alpha^2-\beta^2)]\times[\alpha\tanh(2b^2\alpha)-1]\cos(2b^2\alpha)\}\ . \quad (8)$$

The expression for $F$ is, in general, quite involved. A detailed analysis of it, however, indicates that it is more favorable for $F$ to be positive when $\hbar\omega_2\gg\lambda_1$ and $2b^2\beta\ll1$. Then $\beta=-\alpha=\omega_1/\omega_k$ and Eq. (8) reduces to

$$F\simeq4\exp(-b^2)\exp(-x^2)(2x^2-1)\ ,\quad x=\omega_1/k_bv_T\ , \quad (9)$$

provided $\omega_2\ll kv_2\ll\omega_1$ and $v_i<v_T$.

## III. DISCUSSION AND CONCLUSIONS

It follows from (7) and (9) that, in contrast to the case treated in Ref. 7, i.e., the case in which only one laser field is present, the threshold condition for plasma-wave instability is now dependent upon the value of $k$, instead of being the same for all values of $k$ ($v_1>v_{\text{phase}}$). This is seen from Eq. (9), which becomes positive for $x>1/\sqrt{2}$ (or $k<\omega_1\sqrt{2}/v_T$), has a maximum at $x=\sqrt{(3/2)}$, and then decreases quite rapidly with increasing $x$. In other words, in the simultaneous presence of a weak laser and a strong laser the plasmon population in a relatively narrow range of $k$ values may become unstable, i.e., there is a selective mechanism for plasma-wave instability.

In short, it should be emphasized that our calculation contains a number of simplifying assumptions. Nevertheless, some essential conclusions can be drawn. Among them the present mechanism has the ability of exciting plasma waves propagating essentially in the direction of polarization of the laser fields ($E_1$ and $E_2$ are assumed to be parallel). For **k** not parallel to $\mathbf{E}_i$ the Bessel function in Eq. (7) becomes very small, which leads to damping rather than a growth of the plasmon population. Secondly, the excited plasmons are restricted to a relatively narrow band of $k$ values in the vicinity of $k \sim \omega_1 / v_T$.

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