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Potential scattering transitions in a strong chaotic non-Markovian radiation field

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The theory of the nonrelativistic potential scattering in the presence of a strong laser field is extended to consider a fluctuating radiation field, having a chaotic non-Markovian statistics. The statistical properties are assumed on the electric field and, carrying on a suitable approximation on the characteristic time scales entering the process, the stochastic properties of the vector potential are derived. As a consequence of the assumed non-Markovian statistics, the field spectrum is non-Lorentzian, and its role on the temporal coherence of the field is discussed. Scattering linewidths and line shapes, as well as coherence factors as functions of intensity, are calculated and discussed in the context of recent experimental results obtained with multimode fields.

I. INTRODUCTION

Free-free transitions (also known as potential scattering) in the presence of a strong radiation field are among the new processes widely studied in laser atomic physics,[1] besides maintaining their fundamental importance within nonrelativistic quantum electrodynamics. The presence of the external radiation field when an electron is scattered by a potential enlarges considerably the interest in the process, in that several multiphoton channels are open while the collision takes place. A fortunate circumstance is that the theoretical description of the process may be accomplished very accurately, because under the conditions of experimental interest the field may be accounted for almost exactly, beyond any perturbative scheme. These features have made the multiphoton free-free transitions of particular interest not only from the standpoint of atomic physics but also from that of quantum optics and electronics, both experimentally[2] and theoretically.[3]

It is by now well known that very strong fields are never purely coherent and the fluctuations of the electromagnetic field, space, and/or time inhomogeneities may affect significantly the atomic processes, while the information attainable by means of perturbative treatments of the field are likely to be approximate and restricted to particular ranges of the parameters.[4–6] An understanding of these circumstances has recently provided a renewed interest in the processes in which the radiation field is accounted for accurately, in order to study multiphoton processes induced by strong fluctuating radiation fields and to isolate where possible the specific role played by the radiation field properties. Among these processes, the multiphoton free-free transitions have a particular place, because several models for the field fluctuations may be studied with great accuracy, without approximations as far as the statistical properties of the stochastic field parameters are concerned. Thus two aspects have been explored along the years: on the one hand, the modification of the scattering process depending on the properties of the field;[7] on the other hand, the role played by the field models in characterizing the multiphoton processes.[8–12]

The consideration of the field fluctuation properties has been carried on, introducing different models, mainly that of the chaotic field[8,9] (in which both amplitude and phase of the field fluctuate), of the Gaussian-amplitude field[4] (in which only the amplitude is fluctuating, with a Gaussian distribution) and of the phase diffusion field[10] (only the phase fluctuates); the latter has also been investigated, removing the assumption of the Markovian character of the phase fluctuations,[11] and it has been also shown that the Markov property breaks down for strong fields, a feature confirmed later in other contexts as well.[13]

In this paper, we consider the effects of a chaotic non-Markovian field on the multiphoton free-free transitions, namely, we suppose that the field assisting the process has both the amplitude and phase fluctuating, and the stochastic process describing the amplitude is not a Markov process. Below we shall describe in detail the physical situations in which this feature actually applies. The emphasis is on the role played by the Markov property; as this property affects the spectrum of the field making its wings fall more rapidly, we shall first study the scattering linewidths and line shapes as functions of the field statistics; we discuss then the role of the Markov property in modifying the probabilities of the various multiphoton channels. To this aim, we present an approach which, assuming the stochastic properties on the electric field, guarantees the stationarity also of the process which directly enters the average of interest, i.e., the vector potential of the field. The scattering process is treated within the first Born approximation (FBA) in the scattering static potential, while the field and its fluctuations are treated exactly. A representation set of numerical calculations is presented, clearly showing the range of parameters where the Markov property has to be removed in the atomic process considered in this paper.

The paper is organized as follows. In Sec. II we briefly review the field-assisted potential scattering theory, showing how fluctuating fields, in particular, Gaussian and non-Markovian, are included in the treatment; in Sec. III we discuss the field statistical models, with particular emphasis on the Gaussian, Markovian, and stationary properties of the fluctuating quantities and on the general (statistics-independent) non-Lorentzian features arising by the non-Markovian character; further, we derive the transition probabilities within the assumed non-Markovian model for the field. In Sec. IV we present some numerical results, displaying the role of the Markov property in affecting the coherence properties of the field and in modifying the scattering process. Section V contains the final remarks.

II. FREE-FREE TRANSITIONS IN STRONG FIELDS

Let us consider an electron of mass m and charge $-e$ moving in the presence of a static, local and central potential $V(r)$ and of a radiation field, taken in dipole approximation and described by its vector potential $\mathbf{A}(t)$. Its Schrödinger equation is

$$\frac{1}{2m}\left[\hat{\mathbf{P}}+\frac{e}{c}\mathbf{A}(t)\right]^2\psi+V(r)\psi=i\hbar\dot{\psi}\ . \tag{2.1}$$

Assuming the potential V as the perturbation causing the transition, as in conventional scattering theory, the unperturbed initial and final states are, then, solutions of

$$\frac{1}{2m}\left[\hat{\mathbf{P}}+\frac{e}{c}\mathbf{A}(t)\right]^2\chi=i\hbar\dot{\chi}\ , \tag{2.2}$$

given, for any $\mathbf{A}(t)$, by the nonrelativistic Volkov waves

$$\chi_k(\mathbf{r},t)=\exp(i\mathbf{k}\cdot\mathbf{r})\times\exp\left[-\frac{i}{2m\hbar}\int^t d\tau\left[\hbar\mathbf{k}+\frac{e}{c}\mathbf{A}(\tau)\right]^2\right] \tag{2.3}$$

labeled by $\hbar\mathbf{k}$, the particle momentum averaged over the field period or statistics, as required.

The exact S matrix for the transition from the initial momentum $\hbar\mathbf{k}_i$ to the final momentum $\hbar\mathbf{k}_f$ is

$$S_{fi}=-\frac{i}{\hbar}(\chi_{k_f},V\psi_i^+)\ , \tag{2.4}$$

where χ_{k_f} is a Volkov wave and ψ_i^+ is the exact solution of (2.1) for the incident channel, with standard causal boundary conditions. Parentheses for the scalar product indicate both space and time integrations. As

$$\psi_i^+=\chi_{k_i}+G^+V\chi_{k_i}\ , \tag{2.5}$$

G^+ being the retarded Green function in the presence of both V and $\mathbf{A}(t)$, (2.4) yields

$$S_{fi}=-\frac{i}{\hbar}(\chi_{k_f},V\chi_{k_i})-\frac{i}{\hbar}(\chi_{k_f},VG^+V\chi_{k_i})\ . \tag{2.6}$$

The first term in (2.6) gives the Born approximation to the exact S matrix for the field-assisted potential scattering; the second term accounts for higher-order terms in the scattering potential. In fact, expanding G^+ in powers of V as

$$G^+=G_0^++G_0^+VG_0^++\cdots\ , \tag{2.7}$$

G_0^+ being the retarded Green function in the absence of V, one obtains the expansion

$$S_{fi}=\sum_{\nu=1}^{\infty}S_{fi}^{(\nu)}\ , \tag{2.8}$$

where the νth term contains ν times V and $(\nu-1)$ times the field-assisted propagator G_0^+. In the coordinate representation, this latter one is given by

$$G_0^+(\mathbf{r},t;\mathbf{r}',t')=-\frac{i}{\hbar}V(t-t')\int d\mathbf{k}\,\chi_k^*(r',t')\chi_k(r,t)\ , \tag{2.9}$$

where U is the step function and χ_k are Volkov waves. Explicitly, the first two orders of the expansion (2.8) are given by

$$S_{fi}^{(1)}=-\frac{i}{\hbar}\lim_{T\to\infty}\int_{-T}^{T}dt\int d\mathbf{r}\,\chi_{k_f}^*(\mathbf{r},t)V(r)\chi_{k_i}(\mathbf{r},t)\ , \tag{2.10}$$

$$S_{fi}^{(2)}=\left[-\frac{i}{\hbar}\right]^2\lim_{T\to\infty}\int_{-T}^{T}dt\int_{-T}^{t}dt'\int d\mathbf{k}\int d\mathbf{r}\,\chi_{k_f}^*(\mathbf{r},t)V(r)\chi_k(\mathbf{r},t)\int d\mathbf{r}'\chi_k^*(\mathbf{r}',t')V(r')\chi_{k_i}(r',t')\ . \tag{2.11}$$

By (2.8) and (2.9), for every ν (i.e., at any order in the scattering potential) the field enters exactly, through the initial, intermediate, and final Volkov waves. If the field is fluctuating, $A(t)$ is a stochastic process and what is physically meaningful is the average over all the possible realizations of the transition probability

$$\langle|S_{fi}|^2\rangle=\left\langle\left|\sum_{\nu=1}^{\infty}S_{fi}^{(\nu)}\right|^2\right\rangle\ . \tag{2.12}$$

Due to (2.8), (2.9), and (2.3), (2.12) requires the average of exponentials of the form

$$\left\langle \exp\left[i\sum_m \alpha_m \int_{t_m}^{t'_m} d\tau_m A(\tau_m) \right] \right\rangle . \tag{2.13}$$

If one assumes a chaotic model for the field, the electric field amplitude is Gaussian and (2.13) depends on the generating functional of the Gaussian process $\int A(t)$. Thus it is given by

$$\exp\left[-\tfrac{1}{2} \sum_{m,m'} \alpha_{m'} \alpha_{m'} \int_{t_m}^{t'_m} d\tau_m \int_{t_m}^{t'_m} d\tau'_m \langle A(\tau_m) A(\tau'_m) \rangle \right] , \tag{2.14}$$

which depends only on the $A(t)$ first-order correlation function related to the field spectrum. We remark that only the Gaussian character of the electric field has been exploited, and not the Markov property of the fluctuating quantities. The removal of this latter assumption is usually described in terms of a departure of the field spectrum from the Lorentzian shape resulting by the simplest Markovian models. To study this feature, in Sec. III we will work out a non-Markovian Gaussian model for the field fluctuations, that can be included in the above treatment for the laser-assisted scattering.

III. A NON-MARKOVIAN FIELD MODEL AND THE SCATTERING TRANSITION PROBABILITY

The first-order transition probability, averaged over the field fluctuations, using the first-order S-matrix element (2.10) and the Volkov waves (2.3), is found as

$$\langle |S_{fi}|^2 \rangle = \lim_{T\to\infty} \int_{-T}^{T} dt \int_{-T}^{T} dt' \exp[i\omega_{fi}(t-t')] \times F(t,t') |\tilde{\mathbf{V}}(\Delta)|^2 , \tag{3.1}$$

where

$$\omega_{fi} = (\varepsilon_f - \varepsilon_i)/\hbar, \quad \varepsilon_\gamma = \hbar^2 k_\gamma^2 / 2m, \quad \gamma = i,f , \tag{3.2}$$

and

$$F(t,t') = \left\langle \exp\left[i\alpha_{fi} \int_{t'}^{t} d\tau A(\tau) \right] \right\rangle , \tag{3.3}$$

$$\tilde{\mathbf{V}}(\Delta) = (1/\hbar) \int d\mathbf{r} V(r) \exp(i\boldsymbol{\Delta}\cdot\mathbf{r}) , \tag{3.4}$$

$$\alpha_{fi} = \frac{e}{mc} \hat{\mathbf{e}}_L \cdot \boldsymbol{\Delta}, \quad \hat{\mathbf{e}}_L = \mathbf{A}(t)/A(t), \quad \boldsymbol{\Delta} = \mathbf{k}_i - \mathbf{k}_f . \tag{3.5}$$

For simplicity, we assumed a linear polarization for the field. In this first-order treatment of the scattering potential, the fluctuating field enters only the exponential (3.3), and so far has been treated exactly, whatever its statistics may be. For a chaotic field, using Eqs. (2.13) and (2.14), we have

$$F(t,t') = \exp\left[-\tfrac{1}{2}\alpha_{fi}^2 \int_t^{t'} d\tau \int_t^{t'} d\tau' \langle A(\tau) A(\tau') \rangle \right] . \tag{3.6}$$

An expansion of the exponential and use of the property for a Gaussian process,

$$\langle A(\tau_1) A(\tau_2) \cdots A(\tau_k) \rangle = \sum_P \langle A(\tau_1) A(\tau_2) \rangle \cdots \langle A(\tau_{k-1}) A(\tau_k) \rangle , \tag{3.7}$$

P denoting the index permutation operator, shows that instead of (3.6) we may, for a stationary process $A(t)$, compute the following average:

$$F(t,t') = \exp\left[-\tfrac{1}{2}\alpha_{fi}^2 \int_0^{|t-t'|} d\tau \int_0^{|t-t'|} d\tau' \langle A(\tau) A(\tau') \rangle \right] \equiv F(|t-t'|) . \tag{3.8}$$

This feature permits one to define a transition probability per unit time independent of time and then a cross section, and physically corresponds to the fact that the statistical properties of the assisting field do not change during the atomic process. Thus, in the present treatment, the stationarity of $A(t)$ is a crucial requirement in order to proceed further.

Writing the electric field and the vector potential as

$$E(t) = [\mathcal{E}(t)\exp(-i\omega t) + \text{c.c.}]/2 , \tag{3.9a}$$

$$A(t) = [\mathcal{A}(t)\exp(-i\omega t) + \text{c.c.}]/2 , \tag{3.9b}$$

$\mathcal{E}(t)$ and $\mathcal{A}(t)$ being the respective complex amplitudes and ω the central frequency, by the relation $E(t) = (-1/c)\partial A/\partial t$ it follows that the amplitudes are related by

$$\dot{\mathcal{A}} = i\omega\mathcal{A}(t) - c\mathcal{E}(t) . \tag{3.10}$$

The $A(t)$-correlation function is then found as

$$\langle A(t) A(t') \rangle = \text{Re}\{ \langle \mathcal{A}(t) \mathcal{A}^*(t') \rangle \exp[-i\omega(t-t')] \} . \tag{3.11}$$

Generally speaking, the observable fluctuating quantity is the field intensity, so that the statistical properties should be assumed on the electric field amplitude. Nevertheless, the statistical properties are often assumed directly on $A(t)$ too, because the quantities to be averaged contain $A(t)$ instead of $E(t)$, as in our treatment. Between the complex amplitudes of the vector potential and of the electric field, defined in Eqs. (3.9), the following relationship holds:

$$\langle \mathcal{E}(t)\mathcal{E}^*(t') \rangle = \left[\frac{\omega}{c} \right]^2 \langle \mathcal{A}(t)\mathcal{A}^*(t') \rangle + \frac{1}{c^2} \frac{\partial^2}{\partial t\, \partial t'} \langle \mathcal{A}(t)\mathcal{A}^*(t') \rangle + i\frac{\omega}{c^2} \left[\frac{\partial}{\partial t} - \frac{\partial}{\partial t'} \right] \langle \mathcal{A}(t)\mathcal{A}^*(t') \rangle . \tag{3.12}$$

We observe that the well-known relation $|\mathcal{E}|^2 = \omega^2 |\mathcal{A}|^2 / c^2$ between $\mathcal{E}(t)$ and $\mathcal{A}(t)$ is strictly valid only when they are time-independent; this is true, in particular, for nonfluctuating fields. Besides, from the statistical standpoint, only the Gaussian property of $\mathcal{E}(t)$ is shared by $\mathcal{A}(t)$, and not, in general, its stationarity or the Markovian character.

To gain more insight into the statistical relationship between the electric field and the vector potential for fluctuating fields, let us regard Eq. (3.10) as a complex Langevin equation for $\mathcal{A}(t)$, driven by the "noise" $\mathcal{E}(t)$ and let us assume the statistical properties on the "physical" field $E(t)$; then, we derive the statistical properties of $A(t)$ through Eqs. (3.10) and (3.11). Within the simplest Markovian model for a chaotic field, the complex amplitude $\mathcal{E}(t)$ is assumed to be an Ornstein-Uhlenbeck process, i.e., it fulfills

$$\dot{\mathcal{E}} = -b\mathcal{E}(t) + \mathcal{F}(t) , \tag{3.13}$$

$\mathcal{F}(t)$ being a complex, Gaussian, zero-mean and white-noise

$$\langle \mathcal{F}(t)\mathcal{F}^*(t')\rangle = 2b\langle |\mathcal{E}|^2\rangle \delta(t-t') , \tag{3.14}$$

where the meanings of b and $\langle |\mathcal{E}|^2\rangle$ will be clear in the following. By solving (3.13), (3.14), and (3.10), the $\mathcal{A}(t)$-correlation function is found as

$$\begin{aligned}\langle \mathcal{A}(t)\mathcal{A}^*(t')\rangle = {} & [\langle |\mathcal{E}|^2\rangle c^2/(\omega^2+b^2)] \\ & \times\{2bt_0 \exp[i\omega(t-t')] \\ & +[(\omega-ib)^2/(\omega^2+b^2)] \\ & \times \exp(-b|t-t'|)\} ,\end{aligned} \tag{3.15}$$

where $t_0 = \min(t,t')$. Thus it turns out that, even for a simple Markovian chaotic field, $A(t)$ is a nonstationary process.

The problem of the propagation of the statistical properties from $E(t)$ to $A(t)$ has been in the past either ignored [assuming the statistical properties directly on $A(t)$] or circumvented by dropping the nonstationary term in (3.15), invoking, for $b \ll \omega$, its smallness;[9] or even assuming *ab initio* a time-independent correlation function for $A(t)$.[14] The task of getting a stationary $A(t)$ may be trivially accomplished by assuming that $\mathcal{E}(t)$ is δ correlated; then $\mathcal{A}(t)$ would be an Ornstein-Uhlenbeck process, i.e., Markovian and stationary. Here, our interest is in the inclusion of non-Markovian features of the electric field but retaining a stationary $A(t)$; we show now that, by assuming a slow variation of $\mathcal{A}(t)$, it is possible to account for both these features.

Let us assume that the driving force $\mathcal{F}(t)$ in Eq. (3.13) is not δ correlated, but that it itself fulfills the Langevin equation[15]

$$\dot{\mathcal{F}} = -\beta\mathcal{F}(t) + f(t) , \tag{3.16}$$

where

$$\langle f(t)f^*(t)\rangle = 2b\beta\langle |\mathcal{E}|^2\rangle (b+\beta)\delta(t-t') . \tag{3.17}$$

Equations (3.16) and (3.17) permit to account for the nonvanishing correlation time β^{-1} of $\dot{\mathcal{F}}$, and this is crucial when the times occurring in the atomic process become eventually so small as to be comparable with β^{-1}.[11,13,15] If $\beta\to\infty$, the process (3.16) and (3.17) becomes the white noise described by (3.14) and the field spectrum is Lorentzian; instead, (3.16) and (3.17) describe Lorentzian-like field spectra, having a cutoff at β. By Eqs. (3.10), (3.13), and (3.16), the following third-order differential equation for $\mathcal{A}(t)$ is obtained

$$\begin{aligned}\dddot{\mathcal{A}} + (b+\beta-i\omega)\ddot{\mathcal{A}} + [b\beta - i\omega(b+\beta)]\dot{\mathcal{A}} \\ - i\omega b\beta\mathcal{A}(t) + cf(t) = 0 .\end{aligned} \tag{3.18}$$

Roughly, the scale of the $\mathcal{A}(t)$ variation is b^{-1} (see below); thus for $\beta\approx\omega$ and $b\ll\omega$ one may see that the third and the second derivative terms are of the order b^3 and $b^2\beta$, respectively, while the first derivative is of order $\beta^2 b$. Then, in the regime at hand, we may simplify (3.18) as

$$\dot{\mathcal{A}} = -\Gamma\mathcal{A}(t) + G(t) , \tag{3.19}$$

where

$$\Gamma = ib\beta\omega/[i\omega(b+\beta)-b\beta] \tag{3.20}$$

and $G(t)$ is a complex, Gaussian, zero-mean process, whose correlation function is found as

$$\langle G(t)G(t')\rangle = \frac{2c^2 b\beta(b+\beta)\langle |\mathcal{E}|^2\rangle}{(b\beta)^2+\omega^2(b+\beta)^2}\delta(t-t') . \tag{3.21}$$

Thus, $\mathcal{A}(t)$ is now an Ornstein-Uhlenbeck process, with a complex damping term Γ. The $\mathcal{A}(t)$-correlation function is then found as

$$\langle \mathcal{A}(t)\mathcal{A}^*(t')\rangle = 2\langle |\mathcal{A}|^2\rangle \exp(-\Gamma|t-t'|) , \tag{3.22}$$

$$\langle |\mathcal{A}|^2\rangle \equiv c^2\langle |\mathcal{E}|^2\rangle /2\omega^2 . \tag{3.23}$$

Letting in Eq. (3.20) ω and β much larger than b, the stationary part of (3.15) is recovered.

From Eq. (3.11), using (3.22), one finds

$$\begin{aligned}\langle A(t)A(t')\rangle \\ = \langle |\mathcal{A}^2|\rangle \exp(-\gamma|t-t'|)\cos[\Omega(t-t')] ,\end{aligned} \tag{3.24}$$

where

$$\gamma = \mathrm{Re}\Gamma = b[\beta(b+\beta)\omega^2]/[(b\beta)^2+\omega^2(b+\beta)^2] , \tag{3.25}$$

$$\Omega = \omega[(b+\beta)^2\omega^2]/[(b\beta)^2+\omega(b+\beta)^2] . \tag{3.26}$$

Thus, in this non-Markovian chaotic model for the field, the finite correlation time β^{-1} of the force $\mathcal{F}(t)$ driving the electric field affects both the correlation time γ^{-1} of $\mathcal{A}(t)$ and the central frequency Ω. When ω and β are much larger than b, one has $\gamma\approx b$ and $\Omega\approx\omega$; the non-Markovian features are instead expected to emerge for $b\approx\beta$, when, for instance, $\gamma\approx b/2$. In the calculations we shall also consider the region $\beta\sim b$, just to get some qualitative information on the behavior of our approximation in the full non-Markovian regime.

Using now the stationary correlations function (3.24) in the average (3.6), we find (t denoting now $|t-t'|$)

$$\begin{aligned}F(t) = \exp\{-(\lambda^2/2)[\cos\varphi_0 + \gamma t \\ - \exp(-\gamma t)\cos(\Omega t - \varphi_0)]\} ,\end{aligned} \tag{3.27}$$

where

$$\lambda=\frac{e}{m\omega}\left[\frac{\langle|\mathcal{E}|^2\rangle}{(\gamma^2+\Omega^2)}\right]^{1/2}\hat{\mathbf{e}}_i\cdot\boldsymbol{\Delta}\ ,$$
$$\varphi_0=\arctan\left[\frac{2\gamma\Omega}{\gamma^2+\Omega^2}\right]. \tag{3.28}$$

Performing now the time integration in (3.1), after the required amount of algebraic manipulations, the following doubly differential transition probability is arrived at:

$$\frac{d^2W}{d\Omega\, d\varepsilon_f}=\sum_n\left[\frac{d^2W}{d\Omega\, d\varepsilon_f}\right]_n , \tag{3.29}$$

where

$$\left[\frac{d^2W}{d\Omega\, d\varepsilon_f}\right]_n=2\hbar\exp(-\frac{\lambda^2}{2}\cos\varphi_0)|\tilde{V}(\Delta)|^2\sum_{k=0}^{\infty}f_{\nu k}(\lambda^2/2)\frac{\delta_{\nu k}\cos(n\varphi_0)-\varepsilon_n\sin(n\varphi_0)}{\delta_{\nu k}^2+\varepsilon_n^2}\ , \tag{3.30}$$

where $\nu=|n|$ and

$$f_{\nu k}(x)=\frac{(x/2)^{\nu+2k}}{k!(\nu+k)!}\ ,$$
$$\delta_{\nu k}=\hbar\gamma(\nu+2k+\lambda^2/2)\ , \tag{3.31}$$
$$\varepsilon_n=\varepsilon_f-\varepsilon_i-n\hbar\Omega\ .$$

The scattering line in the nth channel, given in Eq. (3.30), is a superposition of line shapes, each one centered at the final energy $\varepsilon_f=\varepsilon_i+n\hbar\Omega$ and having a bandwidth roughly equal to $\delta_{\nu k}$. Each line shape is weighted by $f_{\nu k}(\lambda^2/2)$; for weak fields $f_{\nu 0}$ will give the main contribution, and for Markovian narrow bandwidth fields the scattering line becomes a Lorentzian of bandwidth $\nu\hbar b$, as expected on the basis of the lowest-order perturbation theory; generally, the bandwidth also depends on the field intensity through λ. It is also easy to see that Eq. (3.30) describes asymmetric sub-Lorentzians, due to the presence of ε_n also in the numerator of the line shape.

The Markovian limit of (3.29) is readily obtained by the replacements $\gamma\rightarrow b$ and $\Omega\rightarrow\omega$, and yields the result already found in Ref. 9; the vanishing bandwidth limit, $b\rightarrow 0$, (in which, of course, any non-Markovian feature disappears) is also easily obtained and yields

$$\left[\frac{d^2W}{d\Omega\, d\varepsilon_f}\right]_n^{b=0}=2\hbar\exp(-\lambda_0^2/2)I_n(\lambda_0^2/2)|\tilde{V}(\Delta)|^2 \times\delta(\varepsilon_f-\varepsilon_i-n\hbar\omega)\ , \tag{3.32}$$

where I_n is the Bessel function of imaginary argument and λ_0 is obtained by λ in Eq. (3.28) setting $\gamma=0$ and $\Omega=\omega$. Equation (3.32) reproduces a well-known result.[8]

The expressions (3.29) and (3.30) have been numerically evaluated for several values of the parameters entering it and in Sec. IV we present a representative set of results. The simple equation (3.32) has been used frequently in the recent past to get information on the effects of a chaotic field on multiphoton free-free transitions. With due modifications, the same simple zero-bandwidth field model has been widely used in other multiphoton elementary processes as well. Now, the exact equations (3.29) and (3.30) offer the possibility to assess the limit of validity of Eq. (3.32) when the very strong fields come into play, and such information may be of use for other contexts as well. We hope to report on this aspect of the problem in the near future.

IV. NUMERICAL CALCULATIONS AND DISCUSSIONS

The numerical calculations of the transition rates reported below are aimed at showing how the cutoff in the laser spectrum modifies the scattering line shapes and widths, and the probability of the various multiphoton channels with respect to the coherent field case. The calculations are performed assuming as scattering potential a screened Coulomb potential of unitary charge and screening radius $r_0=50a_0$, a_0 being the Bohr radius. The electron initial energy is $\varepsilon_i=100$ eV.

The first set of calculations concerns the ratio R of the scattering linewidth ($\hbar\Gamma$) to the field spectrum bandwith ($\hbar b$, taken equal to 10^{-4} eV) as a function of the field intensity I, for several values of the cutoff β and of the number n of exchanged photons (Fig. 1). For $\beta\gg b$ (Lorentzian limit) and weak field the perturbative result of $n!$ is recovered; increasing the field strength, $\hbar\Gamma$ increases and, for strong fields, it becomes roughly independent of n. In this range, it is proportional to I, as already found in Ref. 10 also for a phase diffusion model for the field; the same conclusion is arrived at by looking at the expression $\delta_{\nu k}$ in (3.31) for the linewidth of each sub-Lorentzian. For $\beta\approx b$ (non-Lorentzian limit) and weak fields, Γ is considerably smaller than in the Lorentzian case; increasing the field intensity again yields an independence of n, leaving only a linear dependence on the intensity; further, the strong field linear behavior produces decreasing Γ when β is decreased. Thus, as far as the scattering linewidths are concerned, the main role of β is to decrease them with respect to the Lorentzian case, at relatively low intensity. At higher intensities, the linewidths in the two models become almost the same. However, further increasing the intensity shows a trend towards a prevailing of the non-Lorentzian linewidths

[Fig. 1(b)].

A second set of calculations concerns the so-called coherence factor; this is defined by

$$\chi_n = \frac{(dW/d\Omega)_n^{\text{ch}}}{(dW/d\Omega)_n^{\text{coh}}} , \tag{4.1}$$

where

$$\left[\frac{dW}{d\Omega}\right]_n^{\text{ch}} = \int_\alpha^\beta d\varepsilon_f \left[\frac{d^2W}{d\Omega\, d\varepsilon_f}\right]_n , \tag{4.2}$$

with $\alpha=\varepsilon_i+n\hbar\Omega-\hbar\Omega/2$ and $\beta=\varepsilon_i+n\hbar\Omega+\hbar\Omega/2$, the integrand given by (3.30). The denominator in Eq. (4.1) is the by now familiar result of the differential probability per unit time for the same process, within the first Born approximation in the presence of a purely coherent field[16]

$$\left[\frac{dW}{d\Omega}\right]_n^{\text{coh}} = 2\hbar J_n^2(\lambda_0(n))|\tilde{V}(\Delta(n))|^2 , \tag{4.3}$$

where J_n denotes the Bessel function of the first kind and the electron final wave vector is derived from the energy $\varepsilon_n=\varepsilon_i+n\hbar\Omega$.

The ratio (4.1) has been analyzed for vanishing field spectrum bandwidths,[8] showing that for weak fields $\chi_n \approx n!$, while for intense fields it strongly oscillates, averaging to $\sqrt{\pi}$. In Refs. 4, 5, and 17, an accurate analysis of the intermediate domains has yielded that when $\lambda_0(n)\approx n$, one has instead $\chi_n \ll 1$, i.e., the chaotic field may strongly damp the n-photon process. This important new result may be understood considering that at $n\approx\lambda_0$ the n-photon process is particularly (say, resonantly) favored within the coherent field model. The consideration of a fluctuating field, implying operations of averaging, introduces into the physical process mechanisms which give a smoother redistribution of the probabilities among different multiphoton channels; and it takes place at the expense of the channels which in the

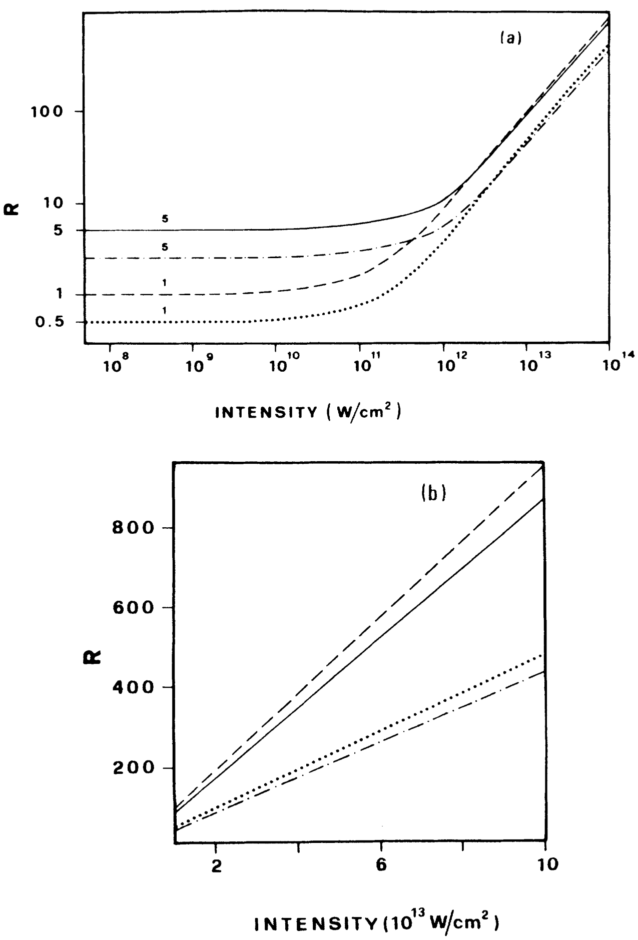

FIG. 1. (a) Ratio R of the scattering linewidth to the field spectrum bandwidth as a function of the field intensity (in W/cm^2) for $n=1$ and 5 exchanged photons (numbers on the curves) and different values of the cutoff parameter $\hbar\beta=1$ eV and $\hbar\beta=10^{-4}$ eV. Electron energy $\varepsilon_i=100$ eV. Field photon energy $\hbar\omega=1$ eV, bandwidth $\hbar b=10^{-4}$ eV. Sold curve, $n=5$ and $\hbar\beta=1$ eV; dashed curve, $n=1$ and $\hbar\beta=1$ eV; dotted curve, $n=1$ and $\hbar\beta=10^{-4}$ eV; dot-dashed curve, $n=5$ and $\hbar\beta=10^{-4}$ eV. (b) Details of (a). Notation of the curves the same as in (a).

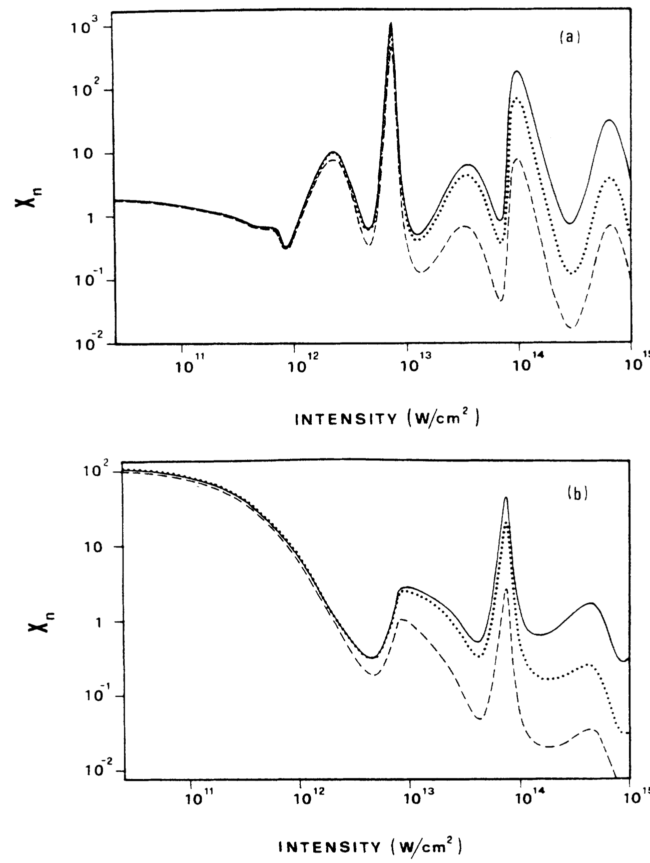

FIG. 2. Coherence factor χ_n as a function of the field intensity (in W/cm^2) for $n=2$ photons exchanged and different values of the field bandwidth b. The scattering angle is $\theta=45°$ and the cutoff parameter $\hbar\beta=1$ eV. Solid curve, $\hbar b=10^{-4}$ eV; dotted curve, $\hbar b=10^{-3}$ eV; dashed curve, $\hbar b=10^{-2}$ eV. Other parameters as in Fig. 1. (b) As in (a) but with $n=5$ photons exchanged.

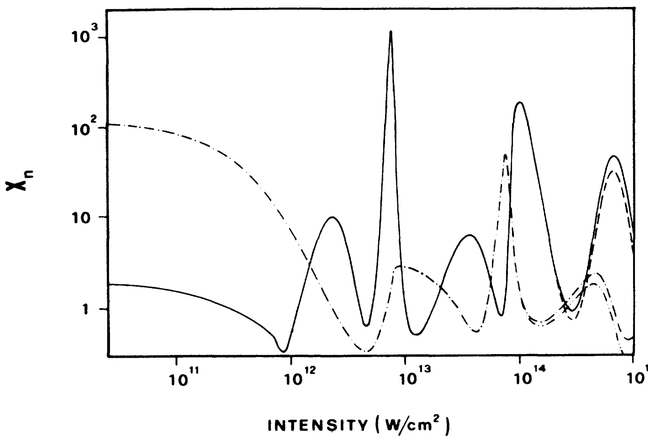

FIG. 3. Coherence factor χ_n as a function of the field intensity (in W/cm²) as $n=2$ and 5 photons exchanged and different values of the cutoff parameter $\hbar\beta=1$ eV and $\hbar\beta=10^{-4}$ eV. The scattering angle is $\theta=45°$ and the field bandwidth $\hbar b=10^{-4}$ eV. Other parameters as in Fig. 1. Solid curve, $n=2$ and $\hbar\beta=10^{-4}$; dashed curve, $n=2$ and $\hbar\beta=1$ eV; dot-dashed curve, $n=5$ and $\hbar\beta=10^{-4}$; double dot-dashed curve, $n=5$ and $\hbar\beta=1$ eV. Other parameters as in Fig. 1.

coherent field model have the greatest probabilities, yielding then $\chi_n \ll 1$. Very recently,[2] this result has received an experimental confirmation in a two-photon free-free transition and appears to qualify itself as a feature independent of the particular elementary process considered. Here, we generalize the above result to include (i) a finite spectrum bandwidth, and (ii) a non-Lorentzian spectrum shape.

The results for the coherence factor in the Lorentzian limit ($\beta \gg b$) and for $n=2$ and 5 are presented in Fig. 2, each one for two values of b. For weak fields, the $n!$ behavior is recovered and it is weakly affected by b; increasing the field strength, a minimum appears ($\chi_n \ll 1$) and it becomes deeper the larger b is; namely, the $b=0$ result underestimates the smoothing action produced by the chaotic field in the $\lambda_0 \approx n$ regime. Increasing further the field strength, an oscillation regime begins, again with the curves corresponding to larger b showing lower values; the mean value of the oscillations, equal to $\sqrt{\pi}$ for vanishing b, is now seen to be inversely proportional to b. Of course, for large b the field is more incoherent compared to the $b=0$ case and this enhances the averaging role of the fluctuating field.

The role of the cutoff β in the coherence factor is analyzed in Fig. 3; the $n!$ weak-field result is recovered, as well as the minimum occurring for $\lambda_0 \approx n$ and the strong-field oscillatory behavior; only for strong fields may a difference between the curves with $\beta=1$ and 10^{-4} be seen. This suggests that the coherence of the field is affected by memory effects of the stochastic parameters only for high intensities.

The last set of calculations concerns complete (i.e., summed over all the multiphoton channels) doubly differential (DDCS) and total (TCS) cross sections, respectively defined by

$$\frac{d\sigma}{d\Omega}=\left[\frac{m}{2\pi\hbar^2}\right]^2\left[\frac{k_f}{k_i}\right]\frac{dW}{d\Omega}\,, \tag{4.4}$$

$$\sigma=\int_{4\pi} d\Omega\left[\frac{d\sigma}{d\Omega}\right]\,. \tag{4.5}$$

In Fig. 4 we present DDCS as functions of the electron final energy. The curves are peaked at the conservation of energy corresponding to different numbers of exchanged (emitted or absorbed) photons; at a fixed field bandwidth ($\hbar b=10^{-4}$ eV), Fig. 4 displays the role of $\hbar\beta$ ($=1$ and 10^{-4} eV) and of the field intensity ($I=10^{12}$ W/cm² and 10^{14} W/cm²). The main role of decreasing β

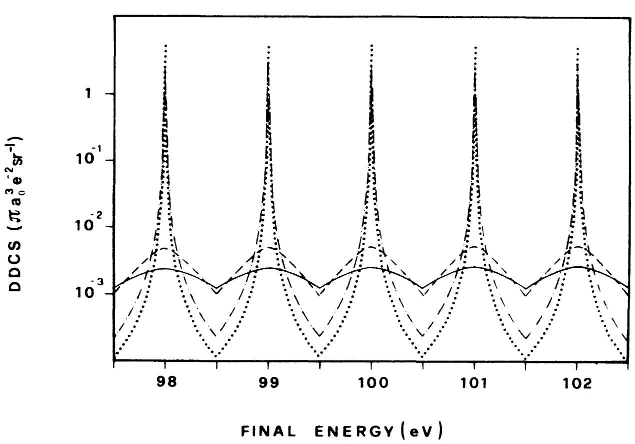

FIG. 4. Double differential cross section DDCS (in units of $\pi a_0^3 e^{-2}$ sr^{-1}) as a function of the electron final energy (in eV) and different values of the field intensity I and cutoff parameter $\hbar\beta$. Solid curve, $I=5\times10^{14}$ W/cm² and $\hbar\beta=1$ eV; dashed curve, $I=5\times10^{14}$ W/cm² and $\hbar\beta=10^{-4}$ eV; dot-dashed curve, $I=5\times10^{12}$ W/cm² and $\hbar\beta=1$ eV; dotted curve, $I=5\times10^{12}$ W/cm² and $\hbar\beta=10^{-4}$ eV. Other parameters as in Fig. 1.

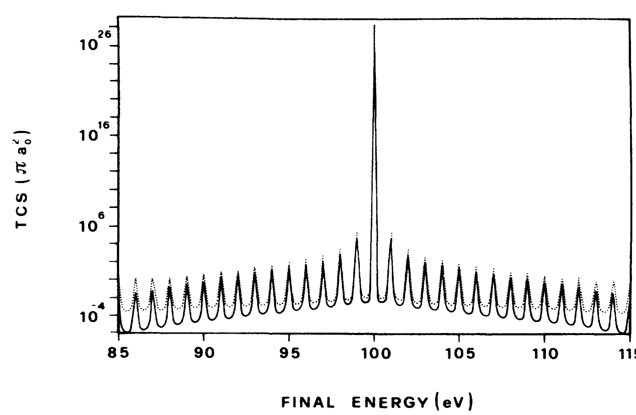

FIG. 5. Total cross sections TCS (in πa_0^2) as a function of the electron final energy (in eV), for the cutoff parameter $\hbar\beta=1$ eV and different values of the field intensity I. Solid curve, $I=10^{11}$ W/cm²; dashed curve, $I=10^{12}$ W/cm². Other parameters as in Fig. 1.

(and thus of increasing the non-Markovian features) is to lower the wings, as expected; the role of increasing the intensity is instead to decrease and widely broaden the lines as many multiphoton channels open and, as seen above, the linewidths rapidly increase with intensity. For high field strengths, the multiphoton peaks become less and less distinguishable.

Finally, in Fig. 5 we present the TCS as functions of the electron final energy, for $\hbar\beta=1$ and two field intensities. The elastic channel ($n=0$) is largely the most probable, even for an intensity of 5×10^{12} W/cm^2; increasing the photon multiplicity, a smooth, uniform decreasing of the cross sections is seen, with the absorption and emission channels roughly of the same orders. The cutoff in the spectrum only slightly affects the cross sections, mostly their shapes.

V. CONCLUDING REMARKS

We have extended the theory for potential scattering in the presence of a strong laser, considering the case of a chaotic non-Markovian field. In potential scattering, the field can be included nonperturbatively, whatever its time dependence (deterministic or stochastic), so that this process is able to provide general information on the role of the field statistical properties in highly nonlinear domains of the radiation-matter interaction.

Developing a slowly varying approximation for the amplitude of the field vector potential, we have been able to include the non-Markovian character of the field fluctuations, maintaining the stationarity of the stochastic process involved; as a consequence, the spectrum of the assisting field results in being non-Lorentzian, close to many experimental laser outputs.

Several numerical results have been presented, generalizing what is already known from the Markovian treatments and making also a contact with recent experimental results.[2] Scattering line shapes and widths have been presented for several field intensities and a number of exchanged photons, showing that, particularly for strong fields, the non-Markovian features of the field may play an important role; this result confirms a similar finding obtained within a phase diffusion model of the field[11] and qualifies as a general feature of strong field situations, largely independent of the particular physical process considered.

A study has also been presented of the coherence factors (ratios of the chaotic field probabilities to the coherent ones); it has been shown that the damping produced by a chaotic field with respect to the coherent one for particular intermediate intensity domains, recently confirmed experimentally,[2] is further enhanced by a non-Markovian field, due to the increased loss of time coherence. In this sense, it is believed that an understanding of field fluctuation effects in the laser-assisted potential scattering requires an accurate statistical description of the field properties.

The cross sections of the process are weakly affected by the non-Lorentzian spectrum of the assisting field, as expected for a process in which transitions near the spectrum wings are not particularly important. For strong enough fields, the main role is played by the intensity, which tends to widely broaden and to smear out the multiphoton peaks; the cutoff in the spectrum lowers the wings, generally making the peaks more distinguishable (the single multiphoton scattering events).

A new set of experiments in multiphoton free-free transitions has been recently started, and it poses a challenge to the theory, pointing towards new, more refined theoretical approaches. It is now clear that a description of the dynamics of the elementary process as accurate as that of the field statistical properties is required, this being at moment beyond the reach of lowest-order perturbation-theory methods. Even for not very strong fields, the parameters of the process may be such as to enhance nonperturbatively features arising from the field stochasticity, so that the forthcoming theoretical treatments, in order to compare themselves with the experiments, should take into account *ab initio* as much as possible of the actual stochastic properties of the field. However, our results show also that if one is neither interested in the line shape and widths of the scattering line nor in witnessing the statistical properties of the field, a zero-bandwidth treatment of the chaotic field is probably sufficient to extract the main information on the role the intensity fluctuations have on scattering parameters like total and (to a lesser extent) differential cross sections. This last statement needs, however, to be checked with detailed comparisons in the various domains of the process parameters.

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