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Laser Doppler velocimetry experiment with a water flow to measure the Fourier transform of the time-interval probability: Comparison between experimental and theoretical results

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In this paper a laser Doppler velocimetry (LDV) experiment with a water flow is performed to show that the experimental results for the Fourier transform of the two-photon time-interval probability $Q_F(\nu)$ agree with the theoretical results derived in previous papers. The conclusions of these papers can therefore be applied to a real LDV experiment. It is concluded that by using this technique for small scattered intensities the velocities can be obtained with much more accuracy than by measuring the autocorrelation function $g^{(2)}(\tau)$. The values of the intensity and of the Doppler frequencies that can be used are discussed.

INTRODUCTION

When carrying out a laser Doppler velocimetry (LDV) experiment in the low-scattered-intensity limit, one usually measures the intensity autocorrelation function[1] $g^{(2)}(\tau)$ from which the velocity distribution function is obtained. Recently, a technique consisting of measuring the Fourier transform $Q_F(\nu)$ of the two-photon time-interval probability was proposed[2] in order to apply it to LDV experiments where a small signal was obtained. Later,[3,4] more detailed studies of this technique were made. A theoretical model was obtained and verified by computer- and optical-simulation methods.[3] It was found that, for small signals, $Q_F(\nu)$ has a very simple mathematical expression[3] and it is able to improve greatly the errors involved in the velocity determination.[4]

The aim of this paper is to measure $Q_F(\nu)$ in a LDV experiment with a water flow to verify that the theoretical model studied previously works well in a real experiment, in order to generalize the results obtained from computer- and optical-simulation methods.

EXPERIMENTAL SETUP

In LDV experiments, lasers operating in the TEM_{00} mode are used whose intensity varies with the distance to the axis of the beam as a Gaussian function. Frequently, severe difficulties are encountered when LDV techniques are used to measure velocities in a turbulent flow because of the Gaussian character of the intensity profile of the laser beam. This laser beam can be transformed into another one whose intensity does not change with the distance to the axis, by using a holographic filter,[5] to avoid the difficulties explained above. The theoretical model for $Q_F(\nu)$ was obtained supposing that a differential LDV system was used with a device that changes the Gaussian intensity profile of the laser beam into another one with a constant intensity to simplify the results. In our experimental setup these conditions were taken into account as it is shown in Fig. 1.

A 10 mW He-Ne laser beam (TEM_{00} mode) was used to produce an interference-fringe system at the point P where the velocity of the fluid was measured. Mirrors M_1 and M_2 were used to reduce the length of the setup that was mounted on a steel-honeycomb optical table. The pieces placed between M_1 and M_2 were used to change the Gaussian intensity profile of the laser beam into another with an approximately constant intensity by using the holographic method of Quintanilla.[5] The microscope objective O_1 and collimator C_1 were used to expand the diameter of the laser beam in order to filter it with the holographic filter H. Another system (C_2, O_2) was then used to reduce the diameter of the laser beam that passed through a spatial filter (SF) to eliminate diffracted light. A beam splitter (BS) and an optical system (L_1) were used to produce the interference fringes, with a fringe spacing equal to 2.47 μm. Light scattered

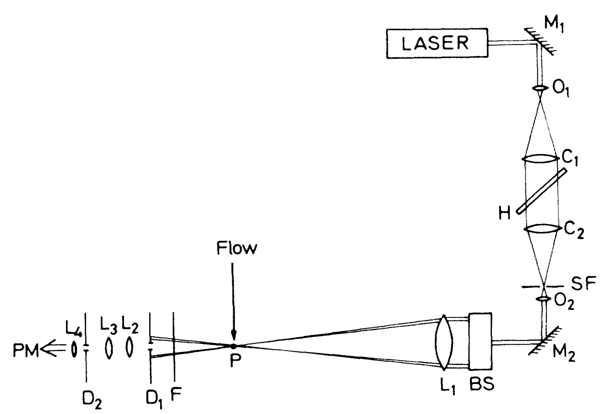

FIG. 1. Experimental setup: M_1, M_2, mirrors; O_1, O_2, microscope objectives; C_1, C_2, collimating lenses; H, holographic filter; SF, spatial filter; BS, beam splitter; L_1, L_2, L_3, L_4, lenses; F, filter; and D_1, D_2, diaphragms.

from seeded particles passing through the fringes was focused on a pinhole (D_2) by a zoom-lens system (L_2 and L_3 lenses). The light passing through D_2 was sent to the detection system (Fig. 2) by means of lens L_4. The filter F and diaphragm D_1 were used to adjust the value of the scattered intensity.

The detection system is schematized in Fig. 2 and consists of the photomultiplier (PM), amplifier-discriminator (AD), time-interval meter (TI, controlled by the computer, com), and a frequency meter (FM) to measure the signal photocount rate.

EXPERIMENT

To perform the experiment a water flow from a tap was introduced into a glass pipe with a 30-mm diameter, the distance from the tap to the interference fringes being larger than 4 m. The frequency of the LDV signal was always near 40 kHz. Therefore, to avoid signal perturbation by dark noise, we could not use photocount rates smaller than 0.2 photopulses in a period of the signal. Therefore we made measurements of $Q_F(\nu)$ for three values of the mean number $\bar{n}_p$ of photopulses in a period: 0.2, 0.5, and 0.75.

To compare these experimental results with the theoretical ones we must take into account that the theoretical model was derived for a flow with a constant velocity, whereas in a real experiment a velocity distribution is obtained because of velocity unstabilities and small turbuluences. On the other hand, light other than that scattered from seeded particles is also detected and contributes to a background in the signal that was not considered in the theoretical model.

Let us consider a LDV experiment where a laser beam with a constant intensity profile is used. Let us suppose that the fluid has a constant velocity and there is not background. When a seeded particle passes through the interference fringes the scattered intensity can be written as

$$I(t)=I_s\{1+M\cos[2\pi\nu_s(t-t_0)+\phi_i]\} \quad (t_0\le t\le t_0+\Delta) , \tag{1}$$

where M is a visibility factor, ν_s is the frequency of the signal, ϕ_i is the phase of the signal in the beginning of the fringes, Δ is the time the particle takes to cross the fringe system from the instant t_0 to the instant $t_0+\Delta$, and I_s is

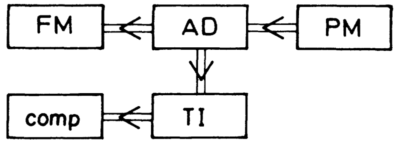

FIG. 2. Detection system: PM, photomultiplier; AD, amplifier discriminator; FM, frequency meter; TI, time-interval meter; and comp, computer.

the mean intensity of the signal in the interval $(t_0,t_0+\Delta)$. The frequency of the signal is related to the velocity V by

$$\nu_s=V/d , \tag{2}$$

d being the distance between two consecutive maxima in the interference fringes. In this particular case, $Q_F^{(s)}(\nu_s,\nu)$ denotes the function $Q_F(\nu)$, where the frequency (ν_s) corresponds to a signal (s) with no background. When $Q_F^{(s)}(\nu_s,\nu)$ is studied as a function of ν, it is found[3] that it passes through two narrow peaks. The higher one occurs for ν around zero and the smaller one for ν around ν_s. This later peak can then be used to determine ν_s and, therefore, V.

Let us now consider the case where there is a velocity distribution and no background. Since there corresponds a Doppler frequency to each velocity [Eq. (2)] the velocity distribution originates a frequency distributions $P(\nu_s)$. If $P(\nu_s)$ is a normalized function and we mean by $Q_F^{(s)}(\nu)$ the function $Q_F(\nu)$ in this case, it is obvious that

$$Q_F^{(s)}(\nu)=\int_0^{\infty}P(\nu_s)Q_F^{(s)}(\nu_s,\nu)d\nu_s . \tag{3}$$

In this case the smaller peak consists of a superposition of the peaks that correspond to the different values of ν_s.

If we now consider a case where there is no Doppler signal and only a background is detected, we have

$$I(t)=I_B , \tag{4}$$

and the values of $Q_F(\nu)$ can be calculated from the expression that corresponds to $Q_F^{(s)}(\nu_s,\nu)$ for $M=0$ and $I_S=I_B$ [see Eqs. (1) and (4)]. In this case where only a background is detected with an intensity I_B, $Q_F^{(B)}(\nu)$ will denote the function $Q_F(\nu)$ and only the peak around $\nu=0$ appears.

Finally, we shall consider the usual case where the detected photopulses are obtained from two different sources: the LDV signal due to a velocity distribution and the background due to the stray light. In this case $Q_F(\nu)$ must be a linear expansion of the Fourier transform of the time-interval probability corresponding to the signal $Q_F^{(s)}(\nu)$ and to the background $Q_F^{(B)}(\nu)$:

$$Q_F(\nu)=a_sQ_F^{(s)}(\nu)+a_BQ_F^{(B)}(\nu) . \tag{5}$$

Since in this paper we want to show that the experimental results agree with the theoretical ones in a LDV experiment, we measured $Q_F(\nu)$ for six different cases and we tried to fit Eq. (5) to these results. to do this we evaluated $Q_F^{(s)}(\nu)$ from Eq. (3) and the expression for $Q_F^{(s)}(\nu_s,\nu)$ obtained in Ref. 3. To evaluate $Q_F^{(B)}(\nu)$ we used the expression for $Q_F^{(s)}(\nu_s,\nu)$ with a modulation factor equal to zero. The numerical results obtained from these fittings are shown in Table I. Figures 3 and 4 show, as an example, the graphical results for $\bar{n}_p=0.2$.

It is worth mentioning that in the theoretical model it was supposed that the intensity was constant across the laser beam, whereas the laser beam obtained from the holographic filter (cases 1–3) has a central area where the intensity is approximately constant and decreases smoothly towards the edge of the beam. In spite of that

TABLE I. Results of the fitting between experiment and theory. In cases 1, 2, and 3, $Q_F(\nu)$ was measured with the holographic filter (rectangular intensity profile) and in cases 4, 5, and 6 $Q_F(\nu)$ was measured without it (Gaussian intensity profile). In this table $\bar{n}_p$ is the mean number of photopulses in the mean period of the signal $(1/\bar{\nu}_s)$; I_s and I_B are the signal and background intensities, M is the visibility factor of the signal [Eq. (1)], and a_s and a_B give us the proportion of signal and background in $Q_F(\nu)$ [Eq. (5)].

Case	$\bar{n}_p$	I_s (photopulses/s)	M	$\bar{\nu}_s$ (Hz)	I_B (photopulses/s)	a_s/a_B
1	0.2	7500	0.75	40 100	24 000	2
2	0.5	18 750	0.70	41 630	60 000	2
3	0.75	26 000	0.70	40 800	87 000	2
4	0.2	7500	0.78	41 750	24 000	2
5	0.5	18 750	0.78	41 550	60 000	2
6	0.75	20 250	0.70	41 350	87 000	2

it can be observed (Fig. 3) that there is a good agreement between experimental and theoretical results. It was found that the velocity of the water flow was not constant, so a frequency distribution $P(\nu_s)$ appeared because of velocity instabilities and possible turbulences. In all three cases $P(\nu_s)$ was found to be Lorentzian shaped having a width [full width at half maximum (FWHM)] approximately equal to 4 KHz. Therefore the results for $P(\nu_s)$ were independent of the photocount rate as expected. It can also be observed (cases 1–3 in Table I) that the variations of I_s/I_B, M, $\bar{\nu}_s$, and a_s/a_B with $\bar{n}_p$ are negligible if one takes into account that the water flow was not perfectly stable (see values of $\bar{\nu}_s$ in Table I). From the theoretical model,[3] noise in $Q_F(\nu)$ can be calculated. In the example of Fig. 3(b) [detail of the peak from which $P(\nu_s)$ can be obtained] 10^5 samples of the time interval were used to obtain $Q_F(\nu)$. This is equivalent to a data accumulation time of 12 sec. From the theoretical model[3] it is obtained that noise is of the order of 0.04 times the value of Q_F at the maximum of the peak, that is in

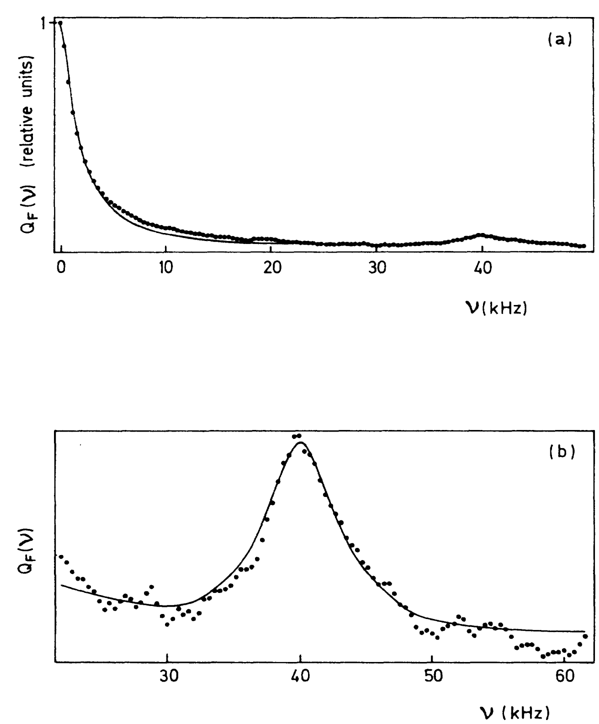

FIG. 3. Experimental (dotted line) and theoretical (solid line) results for case 1 in Table I: (a) curve with the two expected peaks and (b) a detail of the small peak, from which $P(\nu_s)$ can be obtained.

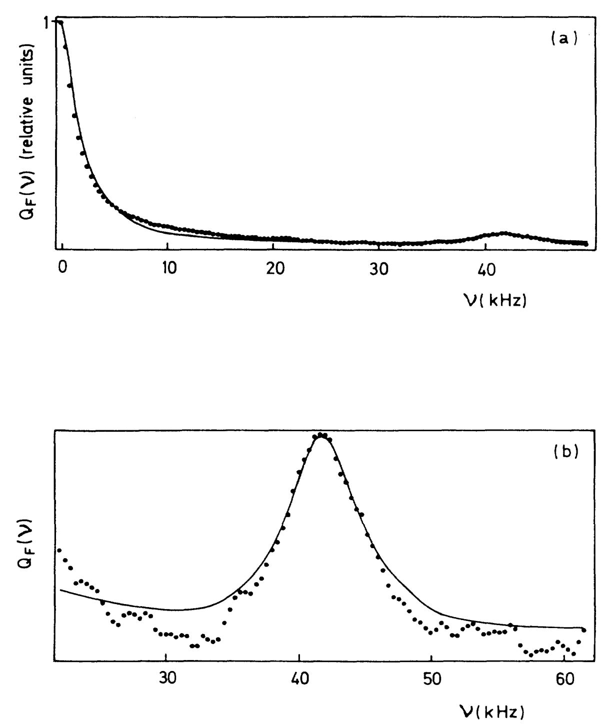

FIG. 4. Experimental (dotted line) and theoretical (solid line) results for case 4 in Table I: (a) curve with the two expected peaks and (b) a detail of the small peak, from which $P(\nu_s)$ can be obtained.

good agreement with Fig. 3(b). So it is obvious that the theoretical model for $Q_F(\nu)$ works well in a real experiment.

Measurements in cases 4–6 in Table I were carried out once the pieces placed between mirrors M_1 and M_2 (Fig. 1) were removed in order to use the laser beam with its own Gaussian intensity profile. In Fig. 4 it can be observed that in spite of the substantial difference between Gaussian and rectangular profiles, there is a good agreement between experimental and theoretical results. On the other hand (cases 4–6 in Table I) the values of I_s/I_B, M, $\bar{\nu}_s$, and a_s/a_B are approximately equal to the ones obtained when using the holographic filter. The same occurs with the distribution $P(\nu_s)$ and noise in Q_F. Therefore the theoretical model derived for rectangular intensity profiles seems to be applicable to LDV experiments where laser beams with an ordinary Gaussian profile are used.

CONCLUSIONS

From the above results we can conclude that the theoretical model for $Q_F(\nu)$ works well in a real experiment. Therefore, the conclusions previously derived from this model agree with the results of a real LDV experiment.

In particular, we note that for small intensities the velocities in a flow can be obtained from $Q_F(\nu)$ with a much smaller error[4] than the velocity distribution obtained from $g^{(2)}(\tau)$ (that error can be divided by a factor ranging from approximately 10 to 100, if $Q_F(\nu)$ is measured). Therefore in those experiments where high-power lasers are used to obtain enough signal to derive a velocity distribution from $g^{(2)}(\tau)$ with a small error, a lower laser power can be used if the velocity distribution is obtained from measuring $Q_F(\nu)$. On the other hand, the measurement of $Q_F(\nu)$ allows us to improve those LDV experiments where a very small signal is obtained when using high-power laser beams.

Furthermore, the signal for a constant velocity is very simple when the scattered intensity is small. In this case[3] $Q_F^{(s)}(\nu_s,\nu)$ consists of two peaks centered at $\nu=0$ and $\nu=\nu_s$. For values[3] of $I_s/\nu_s \leq 0.2$ and $I_s\Delta \geq 4$ [see Eq. (1)] these peaks are Lorentzian shaped. For values[3] of $I_s/\nu_s \leq 0.1$ and $I_s\Delta \geq 2$, the values of ν_s can be directly obtained from the maximum of the peak around ν_s with a systematic error smaller than 0.1%. Therefore, for small intensities, ν_s can be obtained from $Q_F^{(s)}(\nu_s,\nu)$ in a much simpler way than from $g^{(2)}(\tau)$.

As in all LDV processing techniques there is an upper limit for the velocities that can be measured from $Q_F(\nu)$. We have found[6] that when measuring $Q_F(\nu)$ with a photon-counting system having a cutoff frequency ν_c, the frequency ν_s of a periodic signal can be obtained from $Q_F(\nu)$ with an error smaller than 0.1% for $\nu_s \leq 0.1\nu_c$ and smaller than 1% for $\nu_s \leq 0.4\nu_c$. Since, at present, photon–counting systems with $\nu_s = 1$ GHz are available, frequencies up to 100 MHz (0.1% error) that corresponds to a velocity near of the sound velocity or up to 400 MHz (1% error) can be measured. Therefore, $Q_F(\nu)$ allows us to measure high velocities.

When the velocity of the fluid varies with time the values of the Fourier transform of the time-interval probability are related to the frequency distribution by Eq. (3). Since for small intensities $Q_F^{(s)}(\nu_s,\nu)$ is Lorentzian shaped[3] $Q_F^{(s)}(\nu)$ is a convolution integral from which $P(\nu_s)$ can be obtained. At present we are studying the problem of obtaining the velocity distribution from $P(\nu_s)$.

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