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From coherent to incoherent tunneling of squeezed states in double-well potentials

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Tunneling of initially squeezed wave packets in moderately asymmetric double-well potentials is studied in the presence of dissipation. The time dependence of the mean position is studied in order to investigate the features of the trajectory, which strongly depends on the involved parameters. Some questions related to the evaluation of the dissipative parameters are discussed.

I. INTRODUCTION

The study of the dynamics of a localized double-well oscillator coupled to a dissipative environment has been the object of considerable interest in recent years, and much of the recent work has been reviewed by Legget *et al.*[1] In all cases the hypothesis was made that only the two lowest levels resulting from tunneling splitting of the ground state needed to be taken into consideration: the conditions under which such two-level models work are discussed in detail in Ref. 1 and, basically, require that kT, the difference in energy between the potential minima (if any), and the tunneling splitting itself all be much smaller than the vibrational quantum in one well (or, more generally, the energy separation among the levels of the uncoupled wells). Even if one uses the two-level model the theory is very complicated whichever may be the formalism used, i.e., the Hamiltonian or the functional-integral one.[1]

On the other hand, there exists at least one important case in which the two-level model is intrinsically inadequate, irrespective of the parameters' values and of temperature, i.e., when the system is initially prepared as a squeezed wave packet localized in one of the potential wells. In fact, it is well known that wave functions relative to several excited levels must be superimposed to produce a squeezed wave packet. Thus, unless one is sure that the system is initially prepared in the exact ground state of the uncoupled well, higher levels should be taken into account. This kind of situation was considered recently by Dekker for a symmetric potential,[2,3] and by the present authors for a slightly asymmetric potential,[4] but in any case neglecting dissipation.

In the present paper we attempt an analysis of a squeezed problem in the presence of dissipation. A motivation for this study is that squeezing may affect the possibility of experimental observation of macroscopic quantum effect: tunneling through a barrier and coherent oscillation. For example, in order to observe quantum tunneling, it is necessary to reduce as far as possible classical, thermally activated tunneling without making the barrier too high. In general this condition as applied to superconducting junctions and superconducting quantum interference devices (SQUID's) requires that the barrier height be very small and, at the same time, that temperature be well below 100 mK.[5] It is expected that squeezing, by introducing excited-state components in the wave packet, will change the tunneling rates thereby imposing different experimental constraints. In a similar fashion, we can expect coherent oscillations to be affected as well.

Our approach is mainly phenomenological, because the treatment is extremely complicated even in the absence of squeezing, and we shall consider the case of moderate Ohmic dissipation and asymmetry, which (in the absence of squeezing) gives rise to damped oscillations with an incoherent background.

In Sec. II we extend the analysis of Ref. 4 in order to take dissipation into account and to obtain an expression for the mean trajectory which, as shown in Sec. III, in the absence of squeezing reproduces the results of previous treatments.[6] In this section we also explore the different behaviors of the time evolution of the tunneling system in the parameters space. In Sec. IV an estimate of the dissipative parameters is made, while in Sec. V we discuss some questions connected with the considered problem.

II. DISSIPATIVE TUNNELING DYNAMICS

To account for the time evolution of a tunneling system coupled to a thermal bath is a rather complicated problem for the coexistence of two different behaviors: quantum coherence and statistical incoherent thermalization. The presence of the squeezing further increases the complexity because of the contribution of excited states. A possible approach to the problem can be that of considering the temporal wave packet as the superposition of two packets each of which corresponds to a different regime. In such a way the total wave packet can be written as

$$\Phi(x,t,T)=\Phi_c(x,t,T)+\Phi_i(x,t,T)\ , \qquad (1)$$

where x is the spatial coordinate, t is the time, and T is the temperature. The first term Φ_c represents the coherent part of the wave packet and Φ_i the incoherent one. The coherent part will be given by

$$\Phi_c(x,t,T)=\int K(x,t;x_i,0;T)\Phi(x_i,0)dx_i \ , \tag{2}$$

here $\Phi(x_i,0)$ is the initial wave packet and the kernel is given by

$$K(x,t;x_i,0;T)=\sum_N \psi_N(x)\psi_N^*(x_i)\times\exp\left[-\frac{it}{\hbar}[E_N-i\varepsilon_N(T)]\right], \quad N=0,1,2,\ldots \tag{3}$$

where ψ_N and E_N are the eigenfunctions and the eigenvalues of the complete system, respectively; the imaginary part of the energy $\varepsilon_N(T)$ is related to the dissipative effects.[7] According to Ref. 4 we take the initial wave packet of a squeezed state as

$$\Phi(x_i,0)=\left[\frac{\omega e^{2R}}{\pi\hbar}\right]^{1/4}\exp\left[-\frac{y^2e^{2R}}{2}\right], \tag{4}$$

where R is the squeezing parameter and y a local coordinate centered at the minima of the potential

$$V(x)=\tfrac{1}{2}\omega^2(x\pm a)^2+V(\mp a) \tag{5}$$

being $V(-a)=0$ and $V(a)=-\sigma$. The eigenfunctions in Eq. (3) are given by

$$\psi_{4n}=(\sin\varphi_{2n})\psi_{-a}+(\cos\varphi_{2n})\psi_a \ ,$$
$$\psi_{4n+1}=(\cos\varphi_{2n})\psi_{-a}-(\sin\varphi_{2n})\psi_a \ , \tag{6}$$

where $\psi_{\pm a}$ are wave functions of harmonic oscillators centered at $x=\pm a$, respectively, and φ_{2n} are defined by the relation $\cos(2\varphi_{2n})=\sigma/(2\delta_{2n}+\sigma)$. For $n=0,1,2,\ldots$ the indexes choice of Eq. (6) allows us to take into account only the states which, for symmetry reasons, contribute to the wave packet. The energy shift δ_{2n} can be evaluated according to the relation[4]

$$\delta_{2n}=\frac{\exp(-2S_0/\hbar)}{(2\pi n')^2(\sigma+\delta_{2n})} \ , \tag{7}$$

where S_0 is the classical action in going between the two potential minima in absence of bias, and the density of states n' is given by

$$n'=\frac{1}{\hbar\bar{\omega}_{4n}} \ .$$

The frequencies $\bar{\omega}_{4n}$, for small asymmetry, can be expressed as[2]

$$\bar{\omega}_{4n}=2\frac{a^{2n}}{n!}\left[\frac{2\omega_0}{\hbar}\right]^{n+1/2}(\pi V_0)^{1/2} \ ,$$

where V_0 is the barrier height. By substituting into Eq. (7), we easily verify that the ground-state tunneling splitting is

$$\Delta E_0\equiv 2\delta_0=2\hbar\omega_0\left[\frac{S_0}{\pi\hbar}\right]^{1/2}e^{-S_0/\hbar} \ .$$

By resolving Eq. (7) with respect to δ_{2n} we obtain

$$\frac{2\delta_{2n}}{\hbar}=\left[\left[\frac{\sigma}{\hbar}\right]^2+\Delta\omega_{4n}^2\right]^{1/2}-\frac{\sigma}{\hbar} \ ,$$

where $\Delta\omega_{4n}=(\bar{\omega}_{4n}/\pi)\exp(-S_0/\hbar)$ is put equal to $\vartheta^{2n}/(2n)!$ being $\vartheta=4V_0/\hbar\omega$. Following the same procedure of Ref. 4, and taking into account Eq. (2), we find that

$$\Phi_c(x,t,T)=\sum_n \tilde{A}_{2n}\left[(\sin\varphi_{2n})\psi_{4n}\exp\left[-\frac{it}{\hbar}[E_{4n}-i\varepsilon_{4n}(T)]\right]+(\cos\varphi_{2n})\psi_{4n+1}\exp\left[-\frac{it}{\hbar}[E_{4n+1}-i\varepsilon_{4n+1}(T)]\right]\right], \tag{8}$$

where, by detailed balance, the parameters ε_{4n} and ε_{4n+1} are simply related by

$$\varepsilon_{4n}=\varepsilon_{4n+1}\exp[-(\sigma+2\delta_{2n})/kT]$$

being $\sigma+\delta_{2n}$ the energy separation [see also Eq. (21)], and

$$\tilde{A}_{2n}=\frac{\sqrt{(2n)!}}{n!}\left[\frac{1}{\cosh R}\right]^{1/2}\left[-\frac{\tanh R}{2}\right]^n . \tag{9}$$

As for the incoherent part of the wave packet, let us consider for the moment the case $R=0$, which requires consideration of only the ground tunnel-split doublet, $n=0$ of Eq. (6). In this case the incoherent wave packet will be modeled as a linear combination of statistical states of the type

$$\Phi_i(x,t,T)=[C_0(t,T)]^{1/2}\psi_0\exp\left[-\frac{iE_0t}{\hbar}+i\chi_r\right]+[C_1(t,T)]^{1/2}\psi_1\exp\left[-\frac{iE_1t}{\hbar}+i\chi_{r'}\right], \tag{10}$$

where χ_r and $\chi_{r'}$ are random phases,

$$C_0(t,T)=G(t)\left\{\tfrac{1}{2}\exp\left[\frac{\sigma+2\delta_0}{2kT}\right]\left[\cosh\left[\frac{\sigma+2\delta_0}{2kT}\right]\right]^{-1}\right\}, \tag{11}$$

and

$$C_1(t,T)=G(t)\left\{\tfrac{1}{2}\exp\left[-\frac{\sigma+2\delta_0}{2kT}\right]\left[\cosh\left[\frac{\sigma+2\delta_0}{2kT}\right]\right]^{-1}\right\}, \tag{12}$$

$G(t)$ being the temporal evolution given by

$$G(t)=1-\cos^2(\varphi_0)\exp\left[-\frac{2\varepsilon_1 t}{\hbar}\right]-\sin^2(\varphi_0)\exp\left[-\frac{2\varepsilon_0 t}{\hbar}\right]. \tag{13}$$

In this way the right thermal balance between the two states ψ_0 and ψ_1 is obtained in the asymptotic limit of $t\to\infty$. More exactly, we have assumed that this thermal balance holds at any time. The total wave-packet Φ, Eq. (1), maintains the right normalization as can be easily verified taking into account that the time-averaged overlap between Φ_c and Φ_i is zero owing to the random phases χ_r and $\chi_{r'}$ in Eq. (10).

The presence of a squeezed initial state, $R>0$ in Eq. (4), modifies the relative weights of the different states, i.e., the coefficients $\tilde{A}_{2n}$ in the coherent part of the wave packet [Eq. (8)]; we will assume that the same weights can be used for the incoherent part as well. Therefore we have that the total wave packet is given by

$$\Phi(x,t,T)=\sum_n \tilde{A}_{2n}[\Phi_{c,n}(x,t,T)+\Phi_{i,n}(x,t,T)], \tag{14}$$

where $\Phi_{c,n}$ is given in Eq. (8) and $\Phi_{i,n}$ in Eq. (10) is suitably indicized. By straightforward calculations we have that the expectation value of the coordinate

$$\langle x(t)\rangle=\int|\Phi(x,t,T)|^2 x\,dx$$

turns out to be

$$\begin{aligned}\frac{\langle x(t)\rangle}{\langle x(0)\rangle}=\sum_n(\tilde{A}_{2n})^2&\left\{\cos(2\varphi_{2n})\left[\cos^2\varphi_{2n}\exp\left[-\frac{2\varepsilon_{4n+1}}{\hbar}t\right]-\sin^2\varphi_{2n}\exp\left[-\frac{2\varepsilon_{4n}}{\hbar}t\right]\right]\right.\\&-\cos(2\varphi_{2n})\left[1-\cos^2\varphi_{2n}\exp\left[-\frac{2\varepsilon_{4n+1}}{\hbar}t\right]-\sin^2\varphi_{2n}\exp\left[-\frac{2\varepsilon_{4n}}{\hbar}t\right]\right]\tanh\left[\frac{\sigma+2\delta_{2n}}{2kT}\right]\\&\left.+\sin^2(2\varphi_{2n})\exp\left[-\frac{\varepsilon_{4n}+\varepsilon_{4n+1}}{\hbar}t\right]\cos\left[\frac{\sigma+2\delta_{2n}}{\hbar}t\right]\right\}.\end{aligned} \tag{15}$$

This expression, which in absence of dissipation ($\varepsilon_{4n}=\varepsilon_{4n+1}=0$) reduces to that of Ref. 4 describing a purely coherent regime, for moderate dissipation (small values of ε's) gives a description of the dynamical behavior which starts at $t=0$ with a quasicoherent motion and with increasing time becomes more and more incoherent. For large values of ε's the tunneling process becomes irreversible in a very short time. The third term in Eq. (15) represents the damped oscillatory part of the motion whose mean value is given by the first term: both tend to zero with increasing time. The second term, which is the only one which survives at large times, gives the asymptotic value of the coordinate.

III. COMPUTATIONAL ANALYSIS

Before considering the detailed different behaviors which can be obtained by Eq. (15) in different parameter regions, let us test our result in the limiting case of $R=0$.

At low temperature the asymptotic value of the coordinate tends to

$$\frac{\langle x(t\to\infty,T\to 0)\rangle}{\langle x(0)\rangle}=\sum_n \tilde{A}_{2n}\left[\frac{-\sigma}{[\sigma^2+(\hbar\Delta\omega_{4n})^2]^{1/2}}\right], \tag{16}$$

which in the absence of squeezing reduces to $-\sigma/[\sigma^2+(\hbar\Delta\omega_0)^2]^{1/2}$. This result is coincident with that of Ref. 6. Moreover, still for $R=0$, our result of Eq. (15) is very similar to Eq. (107) in Ref. 6 under the assumption that the total decay rate $\Gamma\approx\varepsilon_0+\varepsilon_1$. More properly, this assumption turns out to be exact for sufficiently large values of σ, since in this case ε_1 represents the decay rate from the upper to the lower minimum, while ε_0 is the rate of back tunneling.[8] A detailed evaluation of the decay rate Γ has been performed in Refs. 1, 6, and 8 for the Ohmic case. Here we will treat the ε's as parameters and we will discuss their evaluation in Sec. IV.

In Fig. 1 we show two examples of trajectories, computed by Eq. (15), with (a) or without (b) squeezing, under the assumption that ε_{4n+1} (for $R>0$) is independent of n. We note that, in addition to the typical damped oscillatory motion, the squeezing causes the presence of a chaotic behavior analogous to that obtained in the absence of dissipation.[4] In Fig. 2 we report the log-log plot of the trajectory length as a function of the inverse sampling interval for the case of Fig. 1(a). The fractal dimension of the trajectory is still comparable with the results previously obtained without damping[4] even for relatively

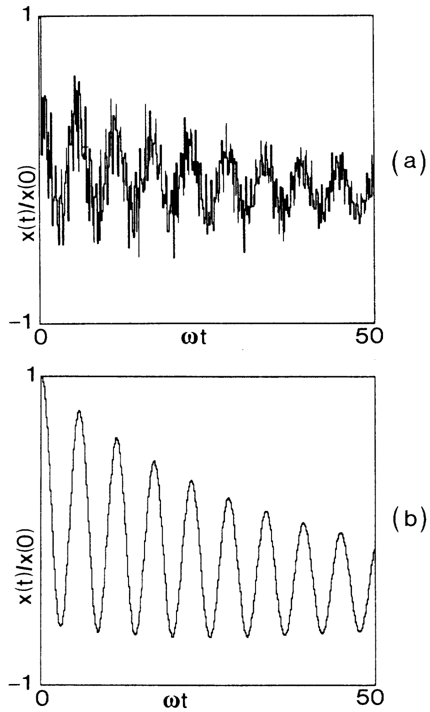

FIG. 1. Expectation value of the normalized position as a function of time with (a) or without (b) squeezing. Case (a): $R=2$, $\vartheta=40$, $\gamma=0$, $kT=0$, $\sigma=0.5$, $\varepsilon_{4n+1}=0.02$. R and ϑ are dimensionless quantities; kT, σ, ε are in units of the ground-state tunneling splitting. Case (b): same parameter values, but $R=0$.

high values of the ε's parameters. For $\varepsilon=0.5$ the fractal dimension is only slightly lowered to 1.906 even if the trajectory shape (see Fig. 3) looks completely different offering a case of quasi-incoherent tunneling.

In Fig. 4 we show three other examples of trajectories computed for moderate values of ε. We note that in addition to the damped oscillatory motion, a residual of chaotic behavior due to the squeezing is still present. By

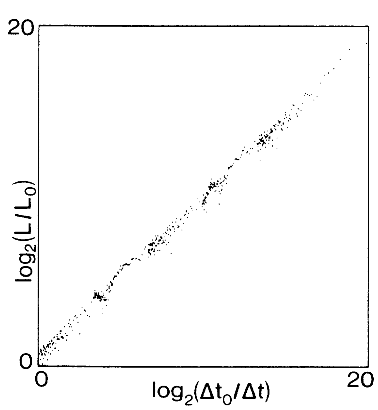

FIG. 2. Normalized length of the trajectory with the parameter values as in Fig. 1(a). The quantity L_0 is the length corresponding to $\Delta t_0=\pi/25$. The slope is fitted to 0.941, giving a fractal dimension $d=1.941$.

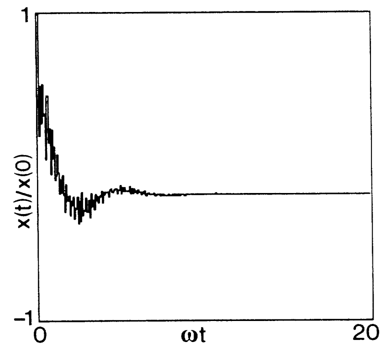

FIG. 3. Same as Fig. 1(a) for $\varepsilon_{4n+1}=0.5$.

comparing Figs. 4(a) and 4(b) we see that by increasing temperature the coherent oscillations die off sooner, and the asymptotic value tends to zero as expected. On the other hand, from Figs. 4(a) and 4(c) it appears that the effect of increasing asymmetry is to localize the system on

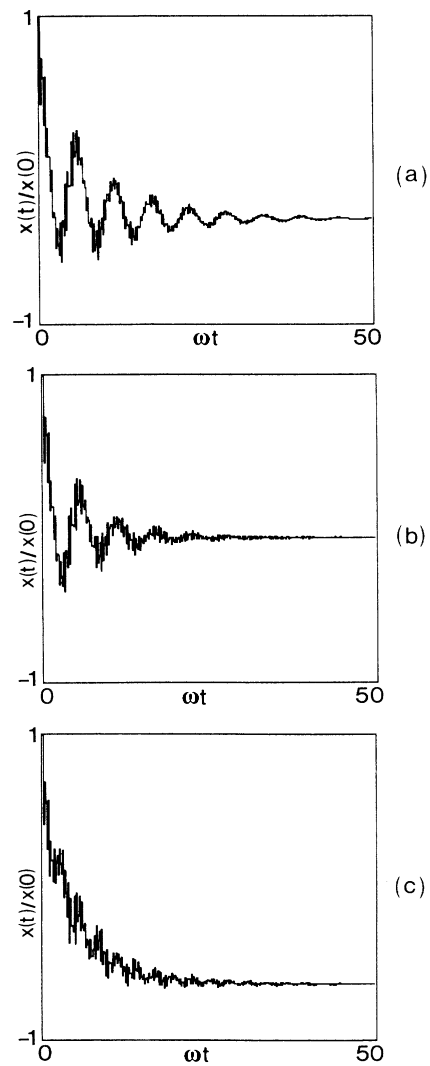

FIG. 4. Expectation value of the position as a function of time for $R=1$, $\vartheta=16$, $\varepsilon_{4n+1}=0.1$, $\gamma=0$. (a): $\sigma=0.5$, $kT=0$; (b): $\sigma=0.5$, $kT=3$; (c): $\sigma=2$, $kT=0$.

the lower well, and to make the motion more and more incoherent. These results are in qualitative agreement with those obtained in the absence of squeezing as reported in Ref. 6.

Until now the dissipation has been considered as operating only within the several pairs of levels, whose separation is $\sigma+2\delta_{2n}(\ll\hbar\omega)$, neglecting decay processes among the several pairs (even inside the same well). This is an incomplete picture which can be improved by considering that the initial squeezing is bound to disappear. We will take this fact into account by assuming that R evolves with the time according to the simple relation

$$R(t)=R_0 e^{-\gamma t} , \quad (17)$$

where γ is a suitable parameter.

In Fig. 5 we show a trajectory computed for $\gamma=0.1$. We see that the effect of a finite value of γ is that of reducing the extension of chaotic behavior and the amplitude of the oscillations depending on the value of R. The analysis of the log-log plot shows that the fractal dimension of the trajectory ($d=1.917$) is still comparable with the results previously obtained provided that the parameter γ is sufficiently small. When γ increases the fractal dimension tends to decrease, but for $\gamma\geq 0.3$ the results of the fractal analysis lose significance.

As said, the ε_{4n+1} are treated as parameters, and so far we have assumed that the dissipative effect is the same in each pair of levels; moreover, we have assumed that γ can be treated as an independent parameter. It is very difficult to predict the dependence of ε_{4n+1} on n and their relationship with γ without detailed knowledge of the microscopic mechanism that causes dissipation. We will consider this point again in the next section; for the moment, we will proceed with our phenomenological approach and consider also the case where ε_{4n+1} varies strongly with n, i.e., it is proportional to the square of the tunneling-splitting relative to the considered pair. In our notation this means to take

$$\varepsilon_{4n+1}\propto(\Delta\omega_{4n})^2=\frac{\vartheta^{4n}}{[(2n)!]^2} . \quad (18)$$

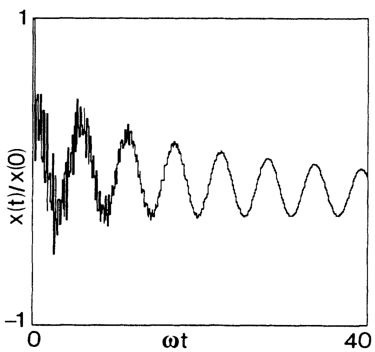

FIG. 5. Expectation value of the position vs time with a time-dependent squeezing parameter, Eq. (17), for $\gamma=0.1$ in units of the ground-state tunneling frequency. Other parameter values as in Fig. 1(a).

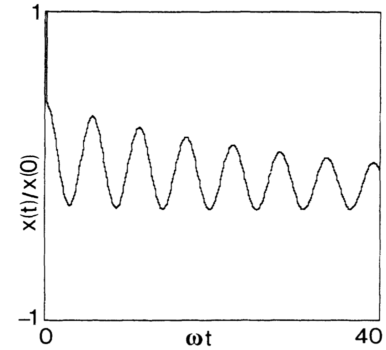

FIG. 6. Expectation value of the position vs time with dissipative parameters $\varepsilon_{4n+1}=(\Delta\omega_{4n})^2\varepsilon_1$, $\varepsilon_1=0.02$, $\gamma=0$. Other parameter values are $\sigma=0.5$, $kT=0$, $\vartheta=40$, $R=2$ [as in Fig. 1(a)].

In this framework we have that the oscillatory contributions from the upper levels tend to disappear very fast and the fractal nature is lost.

In Fig. 6 we show a trajectory computed under the last hypothesis. We see that even if the chaotic behavior is practically absent, the effect of the initial squeezing is still evident producing a sensible reduction of the amplitude of the oscillations and a particular shape in the initial region of the tunneling path depending on the value of the squeezing parameter R. A residual presence of the chaotic behavior can be evidenced even in this last scheme provided that the parameters ϑ and ε_1 are sufficiently small. A detailed description of this fact is given in Fig. 7.

From the above analysis it seems safe to conclude that our model, while providing a right description of the dynamical behavior already known in absence of squeezing, is able to give information also when the squeezing is present in different physical situations.

IV. DISSIPATIVE PARAMETERS

The model we have worked out in the previous sections treats the dissipative terms (namely the ε's and γ) essentially as parameters, considering two different schemes.

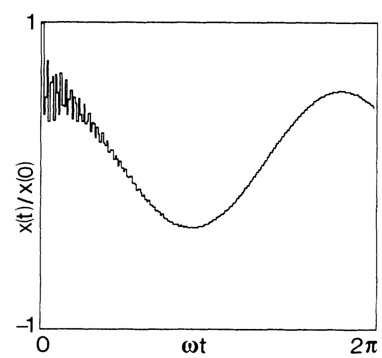

FIG. 7. Effect of the squeezing in the initial portion of the trajectory with dissipative parameters as in the case of Fig. 6 but $\varepsilon_1=0.0001$, $\vartheta=16$, $\sigma=0.5$, $kT=0$, $\gamma=0$, $R=2$.

As mentioned above, a consistent evaluation of the dissipative terms would require that the microscopic mechanisms which govern dissipation be known in sufficient detail. First of all, the treatment in terms of time-varying γ should be replaced by a proper consideration of the interpair relaxation, i.e., the new quantities $\varepsilon_{N-N'}$ ought to be introduced, let us label then generically K. The relationships that connect the ε's to each other, the K's to each other, and the first to the latter, will involve the density of states of the bath at energies of the order of the vibrational quantum in a separate well, which is much larger than $\Delta\omega_0$, and it is not evident that the derivation based on the Ohmic assumption would still be valid.[1] If it is, the choice $\varepsilon_{4n+1} \propto (\Delta\omega_{4n})^2$ can to some extent be justified by the considerations which follow concerning the damping for $R=0$, because even for $R>0$ intrapair relaxation is produced by tunneling. Interpair relaxation is a different process and will produce different relationships among the K's, not to mention the $\varepsilon - K$ connection.

In the remainder of this section, we will first see that our first choice does not contradict thermodynamics, and then we will examine in more detail our second choice.

The case of ε_{4n+1} values independent of n can be justified by considering the equation of the free energy, written as a complex quantity,

$$\exp\left[-\frac{F_r+iF_i}{kT}\right]=\sum_l \exp\left[-\frac{E_l+i\varepsilon_l}{kT}\right], \tag{19}$$

where $l=0,1,2,\ldots$ is the quantum number of the uncoupled initial well. This is satisfied by the trivial solution

$$F_i=\varepsilon_1=\varepsilon_2=\varepsilon_3\ldots, \tag{20}$$

which, as said in the previous section, it was assumed by us as a computational model although we were aware that other solutions exist. We note, however, that the plausibility of the solution (20) rests on the fact that for an uncoupled harmonic well the several levels are equally spaced (the spacing being $\hbar\omega$). The existence of a second well coupled to the first one produces several pairs of levels whose separation is $2\delta_{2n}+\sigma$. This suggests that other computational models could be more appropriate as discussed in the following.

The case of $\varepsilon_{4n+1} \propto (\Delta\omega_{4n})^2$ can be reasonably supported by extending to the excited states a result which was demonstrated to hold for the ground state. More exactly, as anticipated in Sec. III, this result for the ground state concerns the total decay (or relaxation) rate Γ which, in turn, is written as the sum of forward (Γ_1) and backward (Γ_0) tunneling rates of incoherent processes from the upper toward the lower minimum; that is,[8]

$$\Gamma=\Gamma_1+\Gamma_0=\Gamma_1\left[1+\exp\left[-\frac{\sigma}{kT}\right]\right]. \tag{21}$$

So, if σ is sufficiently high, we can establish a direct connection between the decay rates and our ε's, that is $\varepsilon_1 \propto \Gamma_1$ and $\varepsilon_0 \propto \Gamma_0$. Therefore, an estimate of ε's can be performed by evaluating, for instance by a path-integral method, the decay rate Γ as done in Ref. 8 for the Ohmic case. There are however, in our opinion, some aspects which need to be clarified.

Here we report the salient features of a similar treatment, based on an instanton analysis, in order to better understand the limits of application of the method. According to the Langer-Coleman method[9,10] the decay rate Γ can be related to the imaginary part of the free energy of a metastable state by the relation

$$\Gamma=\frac{2|\mathrm{Im}F|}{\hbar}=\frac{2}{\hbar}\mathrm{Im}\left|\frac{-\ln Z}{\beta}\right|, \tag{22}$$

where $\beta=1/kT$ and Z is the partition function which can be written as a functional integral

$$Z=\int \exp\left[-\frac{S[x(\tau)]}{\hbar}\right]\mathcal{D}[x(\tau)], \tag{23}$$

where $S[x(\tau)]$ is the Euclidean action of the system. On the other hand, the partition function can be expressed as

$$Z=Z_0+Z_1+\cdots=Z_0\left[1+\frac{Z_1}{Z_0}+\cdots\right], \tag{24}$$

where Z_0 represents the contributions from the paths of permanence in the initial minimum, $x(\tau)\equiv 0$; $Z_1 \ll Z_0$ is the contribution from one-bounce paths, etc. So, retaining only the leading terms we have

$$F\simeq -\frac{1}{\beta}\ln Z_0-\frac{Z_1}{\beta Z_0}, \tag{25}$$

where for $\beta\to\infty$ the first term gives the ground-state energy $\hbar\omega/2$. The second term has to be seen as a complex quantity whose real part is the energy shift (Lamb-shift), while the imaginary one is directly related to the decay rate [Eq. (22)].

In order to understand the origin of this imaginary part we have to consider in the partition function the contributions due to the quantum fluctuations, around the bounce trajectory, which are determined by the integral

$$\int \exp\left[-\frac{S(\sigma,\alpha,\tau_B)}{\hbar}\right]d\tau_B, \tag{26}$$

where S is the action integral for a bounce path of duration τ_B. For the potential model adopted here S can be expressed as $S=S_B-\Delta S$ where $S_B=2S_0=2\omega a^2$ is the bounce action in the absence of bias and dissipation[2] and

$$-\frac{\Delta S}{\hbar}=\sigma\tau_B-2\alpha\ln\omega\tau_B-\omega a^2\exp(-\lambda\omega\tau_B). \tag{27}$$

In Eq. (27) the first term is due to the bias, the second accounts for dissipation, $\alpha=2\eta a^2/\pi\hbar$ is a dimensionless coefficient directly related to the friction parameter η,[8] and the third one represents the interaction between kink and antikink.[11] This latter term can be made more or less important by varying the factor λ in front of the frequency ω. The bounce duration $\tau_B^{(0)}$ is given by the stationary condition of the action, that is by the solution of the transcendent equation

$$\omega\tau_B^{(0)}=\lambda^{-1}\ln\left[\frac{\lambda\omega a^2}{\hbar\mu}\right];\quad \mu=\frac{\sigma}{\omega}-\frac{2\alpha}{\omega\tau_B^{(0)}} . \tag{28}$$

This fact prevents μ to go to zero even for $\alpha\geq\sigma/\omega$; the bounce duration decreases with lowering α but remains finite for $\alpha\to 0$, as it must be, since the bounce duration can *never* be zero.

At zero temperature ($\beta\to\infty$) the integral (26) diverges: this is due to the first term in Eq. (27). The divergence can be eliminated by deforming the integration path into the complex plane, at the stationary point, Eq. (28), thereby obtaining the imaginary part of the energy.

By working out in the functional integral (23) the fluctuations due to all modes, apart from the breathing one which is factorized into the integral (26), by standard methods the following result is obtained:[8]

$$\Gamma=\left[\frac{\Delta\omega_0}{2}\right]^2\left|\frac{\mathrm{Im}\mathcal{J}}{\omega}\right| , \tag{29}$$

where

$$\mathcal{J}(\sigma,\alpha)=\int\exp\left[-\frac{\Delta S}{\hbar}\right]d\omega\tau_B , \tag{30}$$

and $\Delta\omega_0$ is the tunneling frequency relative to the ground state. Equation (29) at least qualitatively supports the second dissipative scheme adopted in Sec. III. At this point, however, some problems arise in the evaluation of the integral in Eq. (30). From the results reported in the literature, it can be seen that for any finite value of the bias ($\sigma\neq 0$) Γ is nearly proportional to the dissipative parameter α [see Eq. (7.20) in Ref. 1] so that $\Gamma\to 0$ for $\alpha\to 0$ as it must be, since finite decay is physically justified only in the presence of dissipation.

On the contrary, for the unbiased case $\sigma=0$, Γ tends to α^{-1} for $\alpha\to 0$ [see Eq. (5.31) in Ref. 1] thus, apparently, the two limits do not commute. Really, this incongruence does not exist. In fact, by substituting Eq. (27) into Eq. (30) and by evaluating the integral by the saddle-point method, we obtain the approximate result

$$\mathcal{J}=\pm i\left[\frac{\pi}{2}\right]^{1/2}\exp\left[\frac{\mu}{\lambda}\right](2\alpha)^{-1/2}\left[\frac{1}{\lambda}\ln\left[\frac{\lambda\omega a^2}{\hbar\mu}\right]\right]^{1-2\alpha}\times\left[\frac{\lambda\omega a^2}{\hbar\mu}\right]^{\sigma/\lambda\omega} , \tag{31}$$

which, for $\alpha\to 0$, shows a divergent unphysical behavior similar to that of the unbiased case.

We note, however, that for $\lambda\to\infty$ and $\mu\sim 0$ Eq. (31) becomes

$$\mathcal{J}\approx\pm i\sqrt{2\pi}\left[\frac{\sigma}{\omega}\right]^{2\alpha-1}(2\alpha)^{1/2-\alpha}e^{2\alpha} , \tag{32}$$

which is comparable with the result, quoted in Refs. 1 and 8,

$$\mathcal{J}\simeq 2\pi i\left[\frac{\sigma}{\omega}\right]^{2\alpha-1}\frac{1}{\Gamma(2\alpha)} , \tag{33}$$

simply obtained by neglecting in Eq. (27) the last term. By substituting this result into Eq. (29) we have in the limit of small α

$$\Gamma\simeq(\Delta\omega_0)^2\frac{\pi\alpha}{\sigma} , \tag{34}$$

which is practically coincident with the expression, relative to the relaxation rate of the incoherent process, reported in Eq. (106) of Ref. 6, for $\sigma\gg\Delta\omega_0$. This last result can be used in order to perform an estimate, at very low temperature, of the decay rate in the overdamped limit. We see that, in units of tunneling splitting ($\hbar\Delta\omega_0\equiv 1$), $\varepsilon\simeq\pi\alpha/2\sigma$ turns out to be of the same order as α when σ is of the order of unity. Thus, the values of ε considered in this work ($\varepsilon\leq 0.5$) all represent cases of moderate damping.[1]

V. CONCLUSION

In this paper we have examined the dynamical evolution of a localized double-well oscillator interacting with a thermal bath; in particular, we have studied the effects of initial squeezing of the wave packet on the coherent oscillations of the system.

The computation of the mean position versus time is essentially based on a traditional quantum-mechanical treatment while the estimate of the dissipative constants is based on a path-integral analysis, namely the Langer-Coleman approach to the metastability of the states. This is a powerful method, but its limits of applicability must be taken into account in order to avoid incorrect predictions, as discussed in Sec. IV and as anticipated by Sethna.[12] In fact, if we neglect the exponential term in the action integral [Eq. (27)], or underestimate its importance, we are just choosing a different approach to the problem, i.e., the so-called truncated matrix formulation.

Our approach has been a phenomenological one since, on the one hand, we treated the dissipation constants as parameters, and on the other hand we simulated interpair relaxation by a time-decaying R. The dissipation causes a quenching on the chaotic nature of the trajectory which in a finite time tends to disappear depending both on the relative importance of ε's and on the squeezing time evolution governed by the parameter γ. In particular, the fractal nature of the undamped trajectory is practically washed out in the second of our schemes, $\varepsilon_{4n+1}\propto(\Delta\omega_{4n})^2$, and the effect of squeezing in this case is primarily to quench coherent oscillations, a result that can be easily understood on intuitive grounds.

It seems that this has been a rather constant historical characteristic of the formal treatment of this problem so far: very complicated calculations produce perfectly "reasonable" results, especially as concerns the time evolution of the oscillations. Of course, this is satisfying, but it makes one think that there might have been a simpler way of getting to the same qualitative behavior, and we think that the present treatment helps towards this end.

A completely different alternative to the approach considered in this work can be offered by the thermo field dynamics (TFD).[13] An extension of this theory to treat nonequilibrium phenomena in the presence of dissipation can be employed in order to obtain further results to be compared with those here reported. In the case of the TFD the framework is the following: in a perturbative scheme the temporal evolution of the energy and the distribution of the "particles" are evaluated self-consistently both in the case of open (i.e., coupled to a thermal bath) systems and in the case of closed systems.

Recently,[14] it was suggested that this method could be applied to a ϕ^4 model in field theory in the case of a broken-symmetry situation, which simulates the behavior of the Higgs field. For a case which closely resembles our problem, the Lagrangian to be considered is of the type

$$\mathcal{L}=\frac{1}{2}[(\partial_\mu \phi)^2+\mu^2\phi^2]-\frac{\lambda}{4}\phi^4+c\phi \ , \qquad (35)$$

where $\phi=v+w$, v is the classical component of the field, w its quantum fluctuation, and $c\phi$ the bias.[15]

It is a shared opinion that the application of the framework of the TFD to a specific problem like that depicted here could be a convenient and useful expedient to clarify some aspects which are still uncertain in the topic of the dissipative tunneling processes. At the same time, this may represent an important test to demonstrate the usefulness of such theory to treat problems of practical interest.[16]

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