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Screening effect on the inverse bremsstrahlung in a plasma in the presence of two laser fields

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The effect of Coulomb screening on the inverse bremsstrahlung heating process in a plasma illuminated by two laser fields is discussed. It is shown that, although the screening effect actually lowers the Coulomb interaction, one might accomplish a reduction of the weakening effect and consequently an enhancement of the collisional plasma heating, by illuminating the plasma with the two electromagnetic waves having a difference in frequency close to the plasma frequency.

I. INTRODUCTION

There has been increasing interest in the study of the interaction of intense radiation fields (e.g., lasers) with plasmas,[1–6] regarding, for instance, laser fusion experiments. In particular, the investigation of the rate of absorption, by inverse bremsstrahlung, of two laser fields (namely, weak and strong fields) has been done by Fonseca *et al.*,[6] who have found the important result that the rate of energy absorption is nonvanishing but rather large in the regime of ultrahigh intensity of the strong laser, contrary to the case in which only one laser is present.[2] They have shown that plasma heating by two laser fields via inverse bremsstrahlung may be one of the most efficient mechanisms for heating a plasma to the desired thermonuclear temperature. In their calculations,[6] however, the effect of the screening of the Coulomb interaction between the electron and the nuclei has been neglected. As is well known, the interaction of a charged particle with electrons is substantially lowered by Coulomb screening, thereby affecting the energy absorption rate.

Therefore, the purpose of this paper is to consider the effect of Coulomb screening on the collisional absorption of two laser fields in a plasma, and to investigate the conditions under which the weakening effect of screening could be reduced.

The plasma is assumed to be infinite and homogeneous, and we neglect the effects of external magnetic fields. The laser fields are treated as classical plane electromagnetic waves in the dipole approximation. This is justifiable if the distance over which the amplitude of the electromagnetic waves changes is large in comparison with the size of the charges placed in the plasma, the initial Debye screening radius, and the amplitude of the electron oscillations in the wave fields. The electron states are described by the solution to the Schrödinger equation for an electron in the fields of classical electromagnetic waves. The inverse bremsstrahlung process is treated using a first-order perturbation theory, as in Ref. 6.

In Sec. II we shall give the derivation of the screened potential of a static charge Ze placed in a plasma subjected to two electromagnetic waves. The transition probabilities for the electron collision with a nucleus are calculated in Sec. III, and are used to write a kinetic equation for the electrons. In Sec. IV we calculate the rate of change in the kinetic energy of the electrons, and compare it with the one of Ref. 6. Finally, in Sec. V we give our conclusions.

II. SCATTERING POTENTIAL

The modification of Coulomb screening due to the presence of two electromagnetic waves has been discussed in Ref. 7. Here we shall briefly outline the main results. We begin by writing the Hamiltonian of our system as

$$H(t)=\frac{1}{2m}\sum_{\mathbf{p}}\left[\hbar\mathbf{p}+\frac{e}{c}\mathbf{A}(t)\right]^2 C_{\mathbf{p}}^{\dagger}C_{\mathbf{p}}-e\sum_{\mathbf{p},\mathbf{k}}\phi(\mathbf{k},t)C_{\mathbf{p}+\mathbf{k}}^{\dagger}C_{\mathbf{p}}, \tag{1}$$

where

$$\mathbf{A}(t)=(c/\omega_1)\mathbf{E}_1\cos(\omega_1 t)+(c/\omega_2)\mathbf{E}_2\cos(\omega_2 t)$$

describes the two laser fields, and the scalar potential ϕ describes the field of the static charge and the self-consistent fields. The Fourier components of this scalar potential are given by the Poisson equation

$$k^2\phi(\mathbf{k},t)=4\hbar\rho(\mathbf{k})-4\pi e\sum_{\mathbf{p}}\langle C_{\mathbf{p}-\mathbf{k}}^{\dagger}C_{\mathbf{p}}\rangle_t\ , \tag{2}$$

where $\rho(\mathbf{k})$ is the Fourier component of the static charge, and $\langle\ \rangle_t$ denotes averaging with the complete Hamiltonian. Constructing the equation of motion for $\langle C_{\mathbf{p}-\mathbf{k}}^{\dagger}C_{\mathbf{p}}\rangle_t$ within the usual random-phase approximation[8] (RPA), solving it with the initial condition $\langle C_{\mathbf{p}-\mathbf{k}}^{\dagger}C_{\mathbf{p}}\rangle_{t=-\infty}=0$, and substituting into Eq. (2), one gets[7]

$$\begin{aligned}\phi(\mathbf{k},t)=\frac{4\pi\rho(\mathbf{k})}{k^2}-\frac{4\pi ie^2}{k^2}\int_{-\infty}^{t}dt'\phi(\mathbf{k},t')\sum_{\mathbf{p}}(f_{\mathbf{p}-\mathbf{k}}-f_{\mathbf{p}})\exp\left[-\frac{(\varepsilon_{\mathbf{p}}-\varepsilon_{\mathbf{p}-\mathbf{k}})(t-t')}{\hbar}\right]\\ \times\exp\{-i\mathbf{k}\cdot\mathbf{a}_1[\sin(\omega_1 t)-\sin(\omega_1 t')]\}\\ \times\exp\{-i\mathbf{k}\cdot\mathbf{a}_2[\sin(\omega_2 t)-\sin(\omega_2 t')]\}\ .\end{aligned} \tag{3}$$

Here $\mathbf{a}_i = e\mathbf{E}_i/m\omega_i^2$ is the electron oscillation amplitude in the field of wave i, $\varepsilon_{\mathbf{p}} = \hbar^2\mathbf{p}^2/2m$ and $f_{\mathbf{p}}$ is the electron occupation number. If we now define

$$\tilde{\phi}(\mathbf{k},t)=\phi(\mathbf{k},t)\exp[i\mathbf{k}\cdot\mathbf{a}_1\sin(\omega_1 t)] \times\exp[i\mathbf{k}\cdot\mathbf{a}_2\sin(\omega_2 t)]\ ,$$
$$\tilde{\rho}(\mathbf{k},t)=\rho(\mathbf{k})\exp[i\mathbf{k}\cdot\mathbf{a}_1\sin(\omega_1 t)] \times\exp[i\mathbf{k}\cdot\mathbf{a}_2\sin(\omega_2 t)] \tag{4}$$

and substitute them into Eq. (3), we get

$$\tilde{\phi}(\mathbf{k},\omega)=4\pi\tilde{\rho}(\mathbf{k},\omega)/k^2\epsilon(\mathbf{k},\omega)\ , \tag{5}$$

where $\epsilon(\mathbf{k},\omega)$ is the usual dielectric constant in RPA.[8] It then follows from Eqs. (4) and (5) that $\phi(\mathbf{k},t)$ can be written[7]

$$\phi(\mathbf{k},t)=\sum_{n,s,l,m=-\infty}^{+\infty} J_{n+l}(Z_1)J_n(Z_1)J_{s+m}(Z_2)J_s(Z_2) \times\frac{4\pi\rho(\mathbf{k})\exp[i(l\omega_1+m\omega_2)t]}{k^2\epsilon(\mathbf{k},n\omega_1+s\omega_2)}\ , \tag{6}$$

where $Z_i=\mathbf{k}\cdot\mathbf{a}_i$ $(i=1,2)$ and $J_n(Z_1)$ and $J_s(Z_2)$ are Bessel functions of order n and s, respectively. Equations (6) tell us that in the presence of high-frequency fields, the potential, besides becoming anisotropic, has components at the fields' frequencies and their harmonics. In particular, the static component $\phi_0(\mathbf{r})$ (i.e., where $l=m=0$) will be[7]

$$\phi_0(\mathbf{r})=\frac{1}{(2\pi)^3}\int d^3k\frac{4\pi Ze}{k^2\epsilon_{\text{eff}}}e^{-i\mathbf{k}\cdot\mathbf{r}}\ , \tag{7}$$

where

$$\frac{1}{\epsilon_{\text{eff}}}=\sum_{n,s=-\infty}^{+\infty}\frac{J_n^2(\mathbf{k}\cdot\mathbf{a}_1)J_s^2(\mathbf{k}\cdot\mathbf{a}_2)}{\epsilon(\mathbf{k},(n\omega_1+s\omega_2))}\ . \tag{8}$$

That is, the effect of two laser fields on the static potential of a point charge can be taken into account by introducing an effective dielectric constant dependent on both the frequency and the polarization of these fields. In the zero-field limit ($a_i=0$) only the $n=s=0$ term in Eq. (8) survives, so that ϵ_{eff} reduces to the usual static dielectric constant $\epsilon(\mathbf{k},0)$.

III. TRANSITION PROBABILITIES AND KINETIC EQUATION

In Sec. II we have obtained the modification of the Coulomb potential of a fixed-point charge in the plasma due to the presence of two electromagnetic fields. Let us now consider the electron scattering by these static charges. In doing so, however, we shall consider only the static component $\phi_0(\mathbf{r})$ of the potential (i.e., $l=m=0$). This is justifiable, first, because the alternating components of $\phi(\mathbf{k},t)$ are weaker than the static one, and second, in order to make a comparison with the usual inverse bremsstrahlung problem in one or two laser fields in which only static potentials are considered.

Accordingly, treating the electron scattering with a nucleus as a perturbation and using first-order perturbation theory, the transition-probability amplitude for a transition from state 1 ($\mathbf{p}_1$) to state 2 ($\mathbf{p}_2$), due to the collision with a nucleus, is found to be

$$a(1\to 2)=-\frac{i}{\hbar}\int\int d^3r\,dt\,\psi_2^*(\mathbf{r},t)[-e\psi_0(\mathbf{r})]\psi_1(\mathbf{r},t)\ , \tag{9}$$

where $\psi_0(\mathbf{r})$ is the static component of Eq. (6) and $\psi_i(\mathbf{r},t)$ are the solutions to the Schrödinger equation for an electron in the field of classical electromagnetic waves[6]

$$\psi(\mathbf{r},t)=\frac{1}{\sqrt{V}}e^{i\mathbf{p}\cdot\mathbf{r}}e^{i\mathbf{p}\cdot\alpha(t)}e^{-i\varepsilon_{\mathbf{p}}t/\hbar}e^{i\beta(t)}\ , \tag{10}$$

where

$$\alpha(t)=\frac{e}{mc}\int^t dt'A(t')=\frac{e\mathbf{E}_1}{m\omega_1^2}\sin(\omega_1 t)+\frac{e\mathbf{E}_2}{m\omega_2^2}\sin(\omega_2 t)\ ,$$
$$\beta(t)=\frac{e^2A^2(t)}{4mc^2h}t,\quad \varepsilon_{\mathbf{p}}=\frac{\hbar^2\mathbf{p}^2}{2m}\ . \tag{11}$$

Substituting Eqs. (7) and (10) into Eq. (9), and performing the integrations, as in Ref. 6, we can write

$$a(1\to 2)=2\pi i\sum_{\mu,\nu=-\infty}^{+\infty}\frac{J_\mu^2(Z_1)J_\nu^2(Z_2)4\pi Ze^2}{V|\mathbf{p}_2-\mathbf{p}_1|^2\epsilon_{\text{eff}}(\mathbf{p}_2-\mathbf{p}_1,n\omega_1+s\omega_2)} \times\delta(\varepsilon_2-\varepsilon_1-\mu\hbar\omega_1-\nu\hbar\omega_2)\ , \tag{12}$$

with $Z_i=(\mathbf{p}_2-\mathbf{p}_1)\cdot\mathbf{a}_i$. From the well-known relation between the scattering amplitude and the T matrix,[9] we can then use Eq. (12) to obtain the transition probability per unit time $T(1\to 2;\mu,\nu)$, for the transition from state 1 to state 2 with absorption ($\mu,\nu>0$) or emission ($\mu,\nu<0$) of $|\mu|$ photons ω_1 and $|\nu|$ photons ω_2. One obtains

$$T(1\to 2;\mu,\nu)=\frac{2\pi}{\hbar}J_\mu^2(Z_1)J_\nu^2(Z_2)\left|\frac{4\pi Z_e^2}{V(\mathbf{p}_2-\mathbf{p}_1)^2\epsilon_{\text{eff}}}\right|^2 \times\delta(\varepsilon_2-\varepsilon_1-\mu\hbar\omega_1-\nu\hbar\omega_2)\ . \tag{13}$$

The kinetic equation for the number of electrons of momentum $\hbar\mathbf{p}_2, N_e(\mathbf{p}_2)$ is given in terms of the transition probability as[6]

$$\frac{\partial N_e(\mathbf{p}_2)}{\partial t}=\sum_{\mu,\nu=-\infty}^{+\infty}\sum_{\mathbf{p}_1}T(\mathbf{p}_2-\mathbf{p}_1;\mu,\nu)[N_e(\mathbf{p}_1)-N_e(\mathbf{p}_2)], \tag{14}$$

where we have assumed that the electrons are far from degeneracy [i.e., $N_e(\mathbf{p})\ll 1$]. Letting the sum over $\mathbf{p}_1$ become an integral, assuming a maxwellian distribution for the electrons, and using Eq. (13), Eq. (14) becomes

$$\frac{\partial f_e(\mathbf{p}_2)}{\partial t}=f(\mathbf{p}_2)\sum_{\mu,\nu=-\infty}^{\infty}\frac{V}{(2\pi)^3}\int d^3k\frac{2\pi}{\hbar}\frac{J_\mu^2(\mathbf{k}\cdot\mathbf{a}_1)J_\nu^2(\mathbf{k}\cdot\mathbf{a}_2)}{V^2|\mathbf{k}^2\epsilon_{\text{eff}}(\mathbf{k})|^2}(4\pi Ze^2)^2[\exp(\mu\hbar\omega_1+\nu\hbar\omega_2)/k_BT-1]$$
$$\times\delta\left[\frac{\hbar^2\mathbf{k}\cdot\mathbf{p}_2}{m}-\mu\hbar\omega_1-\nu\hbar\omega_2-\frac{\hbar^2k^2}{2m}\right], \tag{15}$$

where we have written $\mathbf{p}_1$ as $\mathbf{p}_1=\mathbf{p}_2-\mathbf{k}$. In Eq. (15) $f_e(\mathbf{p})$ is the electron distribution function.

IV. RATE OF ENERGY ABSORPTION

The rate of change of the average kinetic energy of the electrons should now be evaluated. This is done using Eq. (15) for the kinetic equation of the electrons. The result for the rate of change $d\langle\varepsilon\rangle/dt$ is given by

$$\frac{d\langle\varepsilon\rangle}{dt}=\sum_{\mathbf{p}_2}\varepsilon(\mathbf{p}_2)\frac{\partial f(\mathbf{p}_2)}{\partial t}=\sum_{\mathbf{p}_2}\varepsilon(\mathbf{p}_2)f(\mathbf{p}_2)\sum_{\mu,\nu=-\infty}^{+\infty}\frac{V}{(2\pi)^3}\int d^3k\frac{2\pi}{\hbar}\frac{J_\mu^2(\mathbf{k}\cdot\mathbf{a}_1)J_\nu^2(\mathbf{k}\cdot\mathbf{a}_2)(4\pi Ze^2)^2}{V^2|\mathbf{k}^2\epsilon_{\text{eff}}(\mathbf{k})|^2}$$
$$\times[\exp(\mu\hbar\omega_1+\nu\hbar\omega_2)/k_BT-1]$$
$$\times\delta\left[\hbar^2\frac{\mathbf{k}\cdot\mathbf{p}_2}{m}-\mu\hbar\omega_1-\nu\hbar\omega_2-\frac{\hbar^2k^2}{2m}\right]. \tag{16}$$

Upon taking the classical limit of Eq. (16) by letting $\hbar\to0$, such that[10]

$$\frac{\hbar\mathbf{p}}{m}\to\mathbf{v},\quad \sum_{\mathbf{p}}(\cdots)f(\mathbf{p})\to V\int d^3v(\cdots)f(\mathbf{v}), \tag{17}$$

Eq. (16) becomes

$$\frac{d\langle\varepsilon\rangle}{dt}=\sum_{\mu,\nu=-\infty}^{+\infty}\int d^3v_2\int d^3k\frac{mv_2^2f(\mathbf{v}_2)}{2(2\pi)^2}\frac{(\mu\omega_1+\nu\omega_2)^2}{(k_BT)^2}$$
$$\times(4\pi Ze^2)^2\frac{J_\mu^2(\mathbf{k}\cdot\mathbf{a}_1)J_\nu^2(\mathbf{k}\cdot\mathbf{a}_2)}{|\mathbf{k}^2\epsilon_{\text{eff}}(\mathbf{k})|^2}$$
$$\times\delta(\mathbf{k}\cdot\mathbf{v}_2-\mu\omega_1-\nu\omega_2). \tag{18}$$

Expressions for the effective rate of energy absorption, Eq. (19), can now be evaluated for the different regimes of the two laser fields, namely, laser frequency ω_1 weak and laser frequency ω_2 weak, laser frequency ω_1 weak and laser frequency ω_2 strong, etc. Since we are mainly interested in looking at the effects of screening on the collision absorption rate, we shall restrict ourselves here to the simple case of two weak-laser fields for which $\mathbf{k}\cdot\mathbf{a}_i\ll1$. In this case, the Bessel functions can be approximated by

$$J_x^2(\mathbf{k}\cdot\mathbf{a})=\frac{1}{(x!)^2}(\tfrac{1}{2}\mathbf{k}\cdot\mathbf{a})^{2|x|},\quad x=\mu,\nu, \tag{19}$$

and, consequently, only the $\mu,\nu=\pm1$ terms should be retained; i.e., in the weak-field regimes only single-photon processes are significant. Using Eq. (19) and retaining only the $\mu,\nu=1$ terms (we neglect photoemission terms), we get the following expression for $d\langle\varepsilon\rangle/dt$:

$$\frac{d\langle\varepsilon\rangle}{dt}=\int d^3k\int d^3v_2\frac{mv_2^2f(\mathbf{v}_2)}{2(2\pi)^2}\frac{(\omega_+)^2}{(k_BT)^2}$$
$$\times\frac{(4\pi Ze^2)^2}{|k^2\epsilon_{\text{eff}}(\mathbf{k})|^2}\left[\frac{\mathbf{k}\cdot\mathbf{a}_1}{2}\right]^2\left[\frac{\mathbf{k}\cdot\mathbf{a}_2}{2}\right]^2$$
$$\times\delta(\mathbf{k}\cdot\mathbf{v}_2-\omega_1-\omega_2)\quad(\omega_+\equiv\omega_1+\omega_2). \tag{20}$$

In order to perform the integration in Eq. (20), we should first specify $\epsilon_{\text{eff}}(\mathbf{k})$. Going back to Eq. (8) we see that ϵ_{eff} simplifies noticeably when the two laser beams have a difference of frequency nearly equal to the plasma frequency, i.e., $\omega_-=\omega_1-\omega_2\simeq\omega_p$, where ω_p is the plasma frequency. In this case we recall that the high-frequency dielectric constant is $\epsilon=1-\omega_p^2/\omega_-^2$, and since the dominant term in Eq. (8) is the resonant one, we can approximate

$$\frac{1}{\epsilon_{\text{eff}}}\simeq\frac{2J_1^2(\mathbf{k}\cdot\mathbf{a}_1)J_2^2(\mathbf{k}\cdot\mathbf{a}_2)}{1-\omega_p^2/\omega_-^2}$$
$$=\frac{1}{4}\frac{(\mathbf{k}\cdot\mathbf{a}_1)^2(\mathbf{k}\cdot\mathbf{a}_2)^2}{1-\omega_p^2/\omega_-^2}. \tag{21}$$

Replacing $f(\mathbf{v}_2)$ by

$$n_0(\pi v_T^2)^{-3/2}\exp(-v_2^2/v_T^2),$$

where $v_T^2=2k_BT/m$, and performing the integrations indicated in Eq. (20), one gets

$$\frac{d\langle\varepsilon\rangle}{dt}=\frac{\eta}{18}\frac{\pi Z^2e^4n_0(a_1k_D)^4(a_2k_D)^4}{mv_T(1-\omega_p^2/\omega_-^2)}, \tag{22}$$

where

$$\eta=\int_0^1 x^3(1-x^2)e^{-1/x^2}dx \ ,$$

and $k_D=\omega_p/v_T$ is the Debye wave number.

V. CONCLUSIONS

Equation (22) is the expression for the effective rate of energy absorption of the two weak-laser fields, which we want to discuss. In order to compare Eq. (22) with the results obtained previously without screening effects,[6] we have to find the expression for $d\varepsilon/dt$ corresponding to the limit of weak fields for the two lasers (here assumed to be both weak fields). The expression for $d\varepsilon/dt$ in Ref. 6 is for strong and weak fields, respectively. In the case of two weak fields, $d\varepsilon/dt$ is easily obtained, considering Eq. (11) in Ref. 6 and the expansions of Bessel functions in which the arguments are small, namely, $\lambda_i \ll \hbar\omega_i$ (i.e., considering the terms for which $\mu,\nu=\pm 1$). Solving the indicated integrals we find that $d\varepsilon/dt \propto I_1 I_2$, I_1, I_2 being the field intensities. Comparing Eq. (22) of the present paper with this new result ($d\langle\varepsilon\rangle/dt$ is proportional to the intensities of the two laser beams), we notice that they differ essentially by the extra factor of Eq. (22), namely,

$$(k_D a_1)^2(k_D a_2)^2/[1-\omega_p^2/(\omega_1-\omega_2)^2] \ .$$

In the first place, when screening effects in the electron-nucleus interaction are taken into account, the effective collisional-energy-absorption rate varies with the intensities of the two laser fields squared, rather than being linearly proportional.[6] Second, as $\omega_-=\omega_1-\omega_2$ gets closer to the plasma frequency, $d\langle\varepsilon\rangle/dt$ becomes increasingly large, whereas, as one gets away from resonance, Eq. (22) indicates that collisional absorption becomes a negligible heating mechanism. Physically, all these features may be understood as follows. The introduction of screening effects should, actually, weaken collisional absorption. This is because the first consequence of screening is a reduction in the strength of the Coulomb interaction, and thereby a reduction in the effective number of electron-nuclei collisions. However, if the plasma is illuminated by two radiation fields having a difference of frequency nearly equal to the natural frequency of oscillation of the screening cloud ω_p, a resonant condition is reached, with the result that the screening cloud is destroyed. This screening breakdown in turn ensures that the electron-nuclei interaction will then regain strength, and, therefore, an enhancement of the plasma heating by the two radiation fields should be expected.[6]

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