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Muon spectrum and convoy effects after muon-catalyzed fusion

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We study final-state interactions of the muon after muon-catalyzed D-T fusion reaction with the α particle and with target matter. The yield of convoy muons, traveling with the α particle but remaining unbound is calculated. Energy loss in the dense target may lead to capture of a fraction of these muons into outer shells of the α particle. We show that the final capture probability can be strongly density dependent.

Understanding the final-state interaction of the negative muon with, and its capture by, the charged fusion products is of crucial importance because the loss probability ω_s due to capture ultimately limits the number of fusion cycles per muon. In previous work only capture into bound states has been studied, which amounts to less than 1% for the most relevant $(dt \rightarrow an)$ fusion reaction.

Here we would like to draw attention to capture into comoving continuum states, leading to enhancement of the muon phase-space distribution in the vicinity of the outgoing α particle. Some of these so-called "convoy" muons will recombine with the α particle due to interactions with the surrounding hydrogen-target matter: Muons escaping with velocity somewhat greater than that of the emerging helium nucleus are decelerated more rapidly while traversing the target matter than the α particle due to their smaller mass. Hence, the α particle eventually catches up with these muons which can then be captured into bound states by emission of radiation or by external Auger processes. Environmental conditions, e.g., target density or presence of deflecting magnetic fields, will influence the fraction of muons available to secondary capture.

We now begin to study the relevant processes in detail. We assume in this work that at the moment of the $d+t$ fusion the muon wave function $|\psi_i\rangle$ is not too different from the (Heμ) $1s$ state. Immediately after fusion, the muon finds itself in the traveling Coulomb field of the α particle which moves with velocity $n_\alpha = 5.84$ (a.u.). As shown in Ref. 1, the final-state Coulomb interactions can be taken into account *exactly* by expanding the initial wave function into traveling Coulomb states

$$\psi_n^{(v_\alpha)}(r) = e^{im_\mu \mathbf{v}_\alpha \cdot r}\phi_n(r) , \tag{1}$$

where $\phi_n(r)$ is the wave function of an eigenstate of muonic helium. The time-dependent amplitude for finding the muon in one of the traveling eigenstates is[1]

$$a_n = \langle \psi_n^{(v_\alpha)} | \psi_i(r)\rangle e^{-iE_i t} . \tag{2}$$

In terms of these amplitudes the full time-dependent wave function of the muon is given by

$$\psi_\mu(r,t) = \sum_n a_n \psi_n^{(v_\alpha)}(r)\exp(-iE_n t - \tfrac{1}{2} i m_\mu v_\alpha^2) , \tag{3}$$

where the sum runs over bound and continuum states.

In order to demonstrate the relative magnitude of the relevant quantities, i.e., sticking versus convoy muons, we formulated Eq. (2) here, ignoring the influence of nuclear interactions. In such an approach[2] the primary capture (usually called "sticking") probability is given only by a sum over bound states:

$$\omega_s^{(0)} = \sum_n^{\text{bound}} |a_n|^2 . \tag{4}$$

The other coefficients a_n determine the muon spectrum. In particular, here we address the observation that the asymptotic momentum distribution of muons in the continuum is influenced by the presence of the attractive Coulomb potential of the α particle. The occupation of continuum states in the vicinity of this moving charge, i.e., with $\mathbf{k} \approx m_\mu \mathbf{v}_\alpha$, is strongly enhanced. This "convoy" effect, often called capture to the continuum, has been extensively studied for electrons captured by a fast charged particle.[3,4] The asymptotic laboratory momentum spectrum of muons is obtained by using Coulomb scattering states $\phi_{\mathbf{k}'}^{(-)}(r)$ with outgoing asymptotic momentum $\mathbf{k}' = \mathbf{k} - m_\mu \mathbf{v}_\alpha$ with respect to the moving α particle:

$$f_\mu(\mathbf{k}) = |\langle \psi_{\mathbf{k}'}^{(v_\alpha)} | \psi_i \rangle|^2 . \tag{5}$$

Following Ref. 3, we find the expression

$$f_\mu(\mathbf{k}) = \frac{8(A^2+B^2)Z'/k' e^{2CZ'/k'}}{D(k^2+Z^2)^4 \sinh(\pi Z'/k')} , \tag{6}$$

where $Z' = 2$ denotes the charge of the α particle and $Z = 2$ characterizes the initial state $|\psi_i\rangle$. We used the abbreviations

$$\begin{aligned} A &= (k'^2 - Z^2)(Z - Z') - m_\mu^2 v_\alpha^2 (Z+Z') - 2Z' m_\mu \mathbf{v}_\alpha \cdot \mathbf{k}' , \\ B &= \frac{2Z}{k'}[k'^2(Z-Z') + Z' m_\mu \mathbf{v}_\alpha \cdot \mathbf{k}'] , \\ C &= \tan^{-1}\frac{m_\mu^2 v_\alpha^2 - k'^2 + Z^2}{2Zk'} , \\ D &= (m_\mu^2 v_\alpha^2 - k'^2 + Z^2)^2 + 4Z^2 k'^2 . \end{aligned} \tag{7}$$

The muon laboratory energy spectrum,

$$f_\mu(E) = \int d^3k\, f_\mu(\mathbf{k}) \delta[E - E(\mathbf{k})] , \tag{8}$$

after $d+t$ fusion is shown in Fig. 1(a) (solid line), in comparison with the energy spectrum obtained by neglecting final-state Coulomb effects (dotted line). For energies above 10 m.a.u. (muonic atomic units) the Coulomb distortion effects clearly lead to a strong enhancement in the muon distribution. The small bulge at $E \approx 17$ m.a.u. (about 90 keV) corresponds to muons moving with the speed of the α particle.

The effect is quite opposite in the case of $d+d$ fusion, shown in Fig. 1(b), where the escaping ^3He nucleus corresponds to a muon energy of only 5 m.a.u. Here, the Coulomb attraction leads to a depletion of high-energy muons, and the convoy effect is much less prominent, because the nuclear charge sits in a region of phase space which is strongly populated anyway. The normalization of the continuum spectrum determines the fraction of muons bound in discrete states. We find the expected results both for $dt\mu$ and $dd\mu$ fusions, which verifies the correctness of our approach.

It is obvious that for the purpose of convoy muon capture the fraction of muons traveling ahead of the α particle is of particular interest. We have therefore also shown in Fig. 1 the energy spectrum of such advanced convoy muons (dashed line), composed of those muons with momentum component $k_{\parallel} > m_\mu v_\alpha$ along the direction of motion of the α particle. As can be seen, all high-energy muons are traveling *ahead* of the α particle, a striking consequence of the attraction exerted by its Coulomb potential. Integrating over this part of the muon spectrum we find an advanced muon fraction of 0.83%, compared with less than 0.1% if we switch off the Coulomb attraction of the α particle. The magnitude of this enhancement is not negligible in comparison with the amount of primary bound-state capture of about 1%, when $|\psi_i\rangle$ is the muonic helium $1s$ state and no attention is paid to nuclear interactions. We have performed the same calculation also for the muon spectrum after $dd\mu$ fusion, where we find that a fraction of 4.2% of muons are advanced, amounting to less than one-third of those captured into bound states.

After their initial liberation, muons, as well as the α particles, suffer gradual energy loss while moving through the target. In the range of interest, the energy loss $dE/dt = -v\rho S$ is in good approximation only a function of the velocity v and charge Z of the moving charge $S = ZS_0(v)$, and the target density ρ. Generally, the rate of decrease in the velocity of a charged particle of mass M is given by

$$\frac{dv}{dt} = -\frac{Z}{M} S_0(v)\rho . \tag{9}$$

Initially, the muon is so close to the α particle that the unbound $(\alpha\mu)^+$ ion scatters coherently on target electrons. However, both particles gradually move apart since their velocities are unequal. Once they are separated by a distance of atomic scale, the muon is decelerated almost 20 times faster than the α particle. The energy-loss processes do not alter the direction of motion of the muon or the α particle in the laboratory frame, but a deflection may occur occasionally by a close collision with a hydrogen nucleus from the target. Neglecting those for the moment, the α particle will eventually catch up again with all muons traveling in the same direction, which initially were moving at greater speed. As the capture cross sections are extremely large at low relative μ-α velocity, the muon may be captured by the α particle during this secondary encounter, most likely by inelastic energy loss to target electrons. Our results discussed above indicate that the relative magnitude of this effect, as compared with primary capture, is much larger in $d+t$ fusion than in $d+d$ fusion.

Because muons at low energy are predominantly captured into higher orbitals, they can be easily lost again due to subsequent hard collisions with target molecules, unless they have sufficient time to cascade into the K or L shell before the next collision. The probability of final capture, i.e., "sticking," of these muons must therefore be a strong function of target density. To illustrate this effect, we have calculated the reactivation probability R_{nl} of a captured muon as a function of its initial state (nl) in helium and of target density.[5] As shown in Fig. 2, most captured muons are lost again in liquid hydrogen if $n \geq 4$, whereas

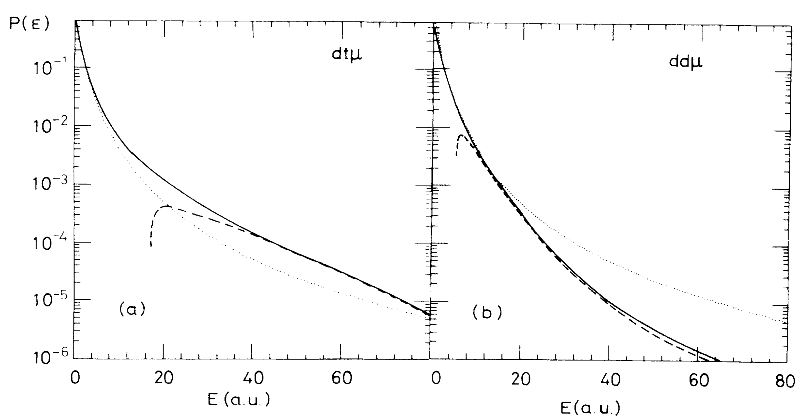

FIG. 1. Energy spectra of muons after muon-catalyzed fusion: full Coulomb-distorted spectrum (solid lines), spectrum without Coulomb attraction (dotted lines), spectrum of muons traveling ahead of the helium nucleus (dashed lines). (a) $d+t$ fusion; (b) $d+d$ fusion. All quantities are in muonic atomic units.

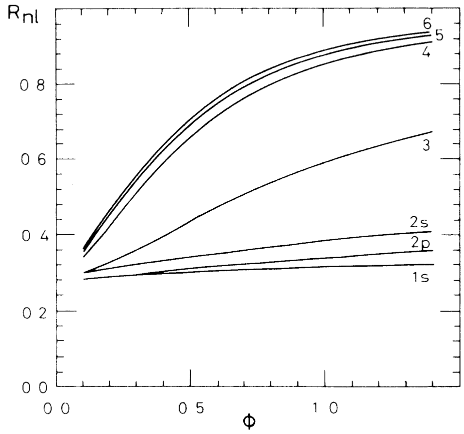

FIG. 2. Density dependence of the reactivation probability of muons initially captured into states with quantum numbers (nl) in the moving helium nucleus after $d+t$ fusion.

most muons "stick" under gas conditions.

This mechanism offers an opportunity to explain the density dependence of the effective sticking coefficient in d-t fusion. The observation[6] of (i) a sticking fraction decreasing with density by about a factor of 2 and (ii) a relatively small sticking fraction (about 0.4%) at densities above liquid hydrogen density ($\phi=1.2$ LHD) suggests that the intrinsic sticking probability to the $1s$ state is smaller than the conventional Coulomb value corrected for stripping (0.65%). It is believed that a significant Coulomb-nuclear interference effect[2] is responsible for the density-independent reduction of the sticking probability in the d-t system. The convoy muons will contribute an additional term to the sticking probability, which is density dependent due to the competition between the stripping of muons from outer shells and the radiative (hence, density independent) transitions to the tightly bound inner shells, as has been shown in Fig. 2. Thus, at low densities the sum of intrinsic sticking and convoy muon capture is observed, while at high densities the convoy contribution to sticking is negligible.

An additional factor to consider in the complex α-μ-matter interaction is the amount of initial transverse motion between the α particle and the muon. Obviously, our arguments do not apply if their trajectories were to diverge too fast. In order to study this effect quantitatively, we have calculated the distribution in $y=\cos\beta$ of muons moving ahead of the α particle, where β is the laboratory angle between the momentum vectors of α and μ. In Fig. 3, this probability distribution is shown for the d-t system. It seems that we can ignore the effect of transverse motion in the context of the present qualitative discussion, but we plan to return to this point in a more detailed investigation based on the motion of the muon in coordinate *and* momentum space [see Eq. (10)].

Since the whole deceleration process takes several picoseconds, the muon trajectory in the target may be influenced by external forces. In fact, a muon and an α particle originally traveling in the same direction are bent in opposite ways by a magnetic field, so that their trajectories separate. For a magnetic field of 1.5 T, the separation at the end of the muon trajectory amounts to 10^{-5} cm. It is therefore easily conceivable that the probability of secondary capture by the α particle can be influenced, and probably reduced, if a magnetic field of such strength is present. This effect may help to understand the result of an experiment performed under such conditions,[7] in which a significantly reduced sticking fraction was observed. However, in order to compute this effect we must obtain the full phase-space probability $f_\mu(\mathbf{k},\mathbf{r},t)$ of finding the muon with momentum $\mathbf{k}$ at position $\mathbf{r}$ in the laboratory frame.

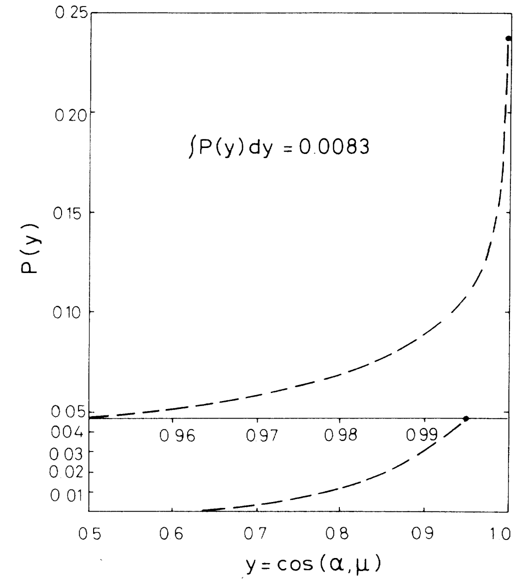

FIG. 3. $y=\cos(\alpha,\mu)$ spectra $P(y)$ of forward muons after muon-catalyzed d-t fusion. Note that two different scales for y are used in the upper and lower part of the figure.

Clearly the capture processes mentioned above will only operate if the muon is not much further away from the α particle than about one hydrogen atomic radius. The full phase space (momentum and position) distribution of the muon as a function of time, before any interactions with target atoms have occurred, is given by the Wigner transform of the wave function (3):

$$f_\mu(\mathbf{k},\mathbf{r},t)=(2\pi)^{-3}\int d^3x\, e^{-i\mathbf{k}\cdot\mathbf{x}}\psi_\mu^*(\mathbf{r}-\tfrac{1}{2}\mathbf{x},t)\times\psi_\mu(\mathbf{r}+\tfrac{1}{2}\mathbf{x},t)\,. \qquad (10)$$

At time $t=0$ it corresponds to the Wigner transform of the initial wave function ψ_i which resembles a muonic helium $1s$ state, i.e., the muons initially can be taken to move almost radially. A muon originally traveling in the vicinity of the emerging α particle will hence also preferentially have a velocity vector in that direction.

The equation governing the time evolution of the Wigner function (10) becomes the Boltzmann or Fokker-Planck equation in the classical limit,[8] which may be applied after coherence of the components of different muon energy is lost (about 10^{-17} s). The subsequent evolution of the muon distribution can then be calculated with the help of the known rate of energy loss and straggling in hydrogen. We plan to return to this complex problem in a future publication in the hope that this method will permit us to describe the influence of magnetic fields and target density on the reported very puzzling results concerning the effective sticking of the muon.[6,7]

In conclusion, we have shown that an appreciable fraction of muons are pushed in the Coulomb cusp of the fusion α particle and are likely to be captured during the slowdown process. This muon convoy effect potentially explains the observed density dependence of the sticking probability.

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