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Feynman–Monte Carlo calculations of electron capture at relativistic collider energies

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Calculations are presented for electron capture associated with pair production in collisions of heavy ions at relativistic energies. We employ the recently introduced technique of evaluating Feynman perturbation integrals exactly by Monte Carlo methods. The results agree well with virtual-photon approximations if they are carefully formulated, but appear to disagree with some published results. The cross sections for symmetric collisions of uranium, gold, iodine, copper, and silicon are presented as a function of the beam energy. These results are discussed within the context of beam storage times of a relativistic heavy-ion collider.

The use of relativistic heavy ions to study strongly coupled electromagnetic phenomena is a field of growing interest.[1–3] At the present time, one experiment at the Brookhaven alternating gradient synchrotron (AGS) is undertaking the study of extreme peripheral heavy-ion collisions with ^{28}Si beams up to laboratory energies per nucleon of 14.5 GeV. This experiment, among other objectives, will measure Coulomb and diffractive dissociation processes at large impact parameters.[4] In the near future, the promise of a relativistic heavy-ion collider (RHIC) will allow the study of electromagnetic phenomena with heavy-ion reactions such as ^{197}Au+^{197}Au at equivalent laboratory energies per nucleon of 23 TeV.[5]

At increasing relativistic energies, the electromagnetic field associated with the fully stripped heavy ions becomes sharply peaked in a direction transverse to the heavy-ion motion. In the laboratory frame, at impact parameters of a few nuclear radii, transverse electric fields in excess of a few GeV/fm may be expected for heavy ions at the highest energies available to RHIC. Under these conditions, we may expect copious production of electron-position pairs, as well as heavy lepton production.[6–8]

In this paper, we focus on one important aspect of peripheral collisions of heavy ions at relativistic energies by calculating the cross section for electron capture in the two-photon limit using Monte Carlo integration over intermediate and final momenta. Our calculation is an extension of earlier published work on pair production via two-photon diagrams.[9] The present study spans an energy range that includes the present AGS experiments and the future RHIC program. In particular, we focus on the reliability of the equivalent photon or Weizsäcker-Williams method.[10–13] We introduce a more rigorous version of this method, which we call the *single-peak* approximation. This relatively simple expression approximates the full Monte Carlo calculations over a wide energy range.

The new physics associated with strong and pulsed electromagnetic fields from relativistic heavy ions has important practical implications for detector design and machine performance at RHIC. The largest nuclear depletion mechanism of the RHIC beam is estimated to be the electron-capture process following pair production during beam crossing. A previous study[14] has obtained a value of 80 b per beam for Au+Au using the Weizsäcker-Williams method. However, a range of values has been advocated by other authors.[15] The present study seeks to resolve this uncertainty. We stress that the Feynman–Monte Carlo method evaluates the appropriate Feynman diagrams exactly, for practical purposes. The various virtual-photon expressions represent attempts to approximate the Feynman diagrams.

The electron-capture mechanism is shown schematically in the two-photon diagrams of Fig. 1. We use time-dependent perturbation theory to expand the S matrix in orders of the electromagnetic interactions of the two colliding ions a and b. The leading contributions to pair production arise from the two Feynman diagrams, in which each ion interacts exactly once.[9] When the direct and crossed diagrams are correctly included, the resulting amplitude is Lorentz covariant and gauge invariant.[16] Capture is then described in a sudden (or impulse) approximation by convoluting the electron line in Fig. 1 into the momentum wave function of the final bound state. This ansatz can also be derived by summing the ladder diagrams for repeated interactions of the electron and the capturing nucleus.

We work in natural units ($\hbar=c=m=e=1$), and assume a geometry in the c.m. or collider frame that has the target [Fig. 1(b)] moving from right to left, parallel to the z axis with velocity β; its energy in mass units is $\gamma=(1-\beta^2)^{-1/2}$. The projectile [Fig. 1(a)] moves from left to right with velocity $-\beta$. All results will be presented in terms of the *collider* energy γ; the equivalent *fixed target* energy is given by $\gamma_{\rm FT}=2\gamma^2-1$. The pair production cross section is given by

$$\sigma_{\rm pair}=\frac{1}{(2\beta)^2}\sum_{\sigma_k,\sigma_q}\int d^3k\,d^3q\,d^2p_\perp\frac{1}{(2\pi)^8}|B(k,q;\mathbf{p}_\perp)|^2, \tag{1}$$

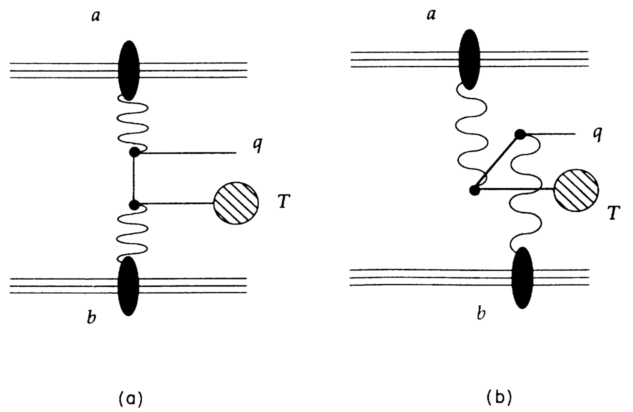

FIG. 1. (a) Direct and (b) crossed Feynman diagrams for pair production with electron capture into a target state T in a heavy-ion collision.

where

$$B(k,q;\mathbf{p}_\perp)=A^{(+)}(k,q;\mathbf{p}_\perp)+A^{(-)}(k,q;\mathbf{k}_\perp+\mathbf{q}_\perp-\mathbf{p}_\perp)\,. \tag{2}$$

The superscripts $(\pm)$ refer to the direct and cross diagrams, respectively. Closely following Ref. 9, the notation of (1) is as follows: $\mathbf{q},\mathbf{k}$ are the momenta of the positron and electron in the final state, $\mathbf{p}$ is the intermediate fermion momentum, and σ_q,σ_k run over the spins of the positron and electron. The label k is short for the quantum numbers $\mathbf{k}$, σ_k, and s_k, where $s_k=\pm 1$ for positive- and negative-energy states. The frequencies of the virtual photons determine the components p_0 and p_z so that only an integration over $\mathbf{p}_\perp$ remains in (1).

The amplitude $A^{(+)}(k,q;\mathbf{p}_\perp)$ is expressed as a product,

$$A^{(+)}(k,q;\mathbf{p}_\perp)=F(\mathbf{k}_\perp-\mathbf{p}_\perp:\omega_a)F(\mathbf{q}_\perp-\mathbf{p}_\perp:\omega_b)\mathcal{T}_{kq}(\mathbf{p}_\perp;\beta)\,, \tag{3}$$

where F is the scalar part of the field associated with each heavy ion,

$$F(\mathbf{u}:\omega)=\frac{4\pi Z}{\mathbf{u}^2+(\omega/\beta\gamma)^2}\,, \tag{4}$$

and ω_a,ω_b are the frequencies associated with the fields a,b, respectively. For electron pair production, as opposed to heavy-lepton pair production, the nuclei may be treated as point charges. The amplitude $\mathcal{T}_{kq}$ relates the intermediate photon lines to the outgoing fermion lines. To be precise, $\mathcal{T}_{kq}$ is a function of the fermion momenta k,p,q and the photon frequencies ω_a,ω_b, but in no way depends on the distribution of charge within the sources; it is not the physical $\gamma\gamma$ amplitude, since the arguments place the photon lines off shell, in general. An expression for $A^{(-)}(\mathbf{k},\mathbf{q},\mathbf{p}_\perp)$ is obtained by reversing the sign of β and thereby interchanging ω_a and ω_b.

The cross section for electron capture is obtained from (1)–(4) by convoluting the amplitude B into the momentum representation of the bound-state wave function $\Phi_T(s_k,\mathbf{k})$. In the target frame, we can expand the bound-state wave function in Dirac plane waves,

$$|T\rangle=\sum_{s_k}\int d^3k\frac{1}{(2\pi)^3}\Phi_T(s_k,\mathbf{k})\,|s_k,\sigma_k(=\sigma_T),\mathbf{k}\rangle\,. \tag{5}$$

The bound state $|T\rangle$ is defined by the usual Dirac hydrogenic quantum numbers $(n_T,\kappa_T,m_T,\sigma_T)$. Most of the capture is into the K shell ($1s_{1/2}$ state), for which Φ_T is independent of σ_T and m_T. Expressions for $\Phi_T(s_k,\mathbf{k})$ are given elsewhere.[17] In this way, our expression for the capture cross section derived from (1) becomes

$$\sigma_{\rm cap}=\frac{1}{(2\beta)^2}\sum_{\sigma_q}\int d^3q\,d^2p_\perp\frac{1}{(2\pi)^5}\left|\sum_{s_k}\int d^3k\frac{1}{(2\pi)^3}\tilde{\Phi}_T(s_k,\mathbf{k})B(k,q;\mathbf{p}_\perp)\right|^2, \tag{6}$$

where $s_q=-1$, $s_k=\pm 1$, $\sigma_q=\pm 1$, and $\sigma_k=\sigma_T$. The bound-state wave function $\tilde{\Phi}_T(s_k,\mathbf{k})$ is expressed here in the collider frame by the transformation

$$\tilde{\Phi}_T(s_k,\mathbf{k}_\perp,k_z)=\cosh(\omega/2)\Phi_T(s_k,\mathbf{k}_\perp,\tilde{k}_z)+\sinh(\omega/2)\Phi_T(-s_k,\mathbf{k}_\perp,\tilde{k}_z)\,, \tag{7}$$

$$\tilde{k}_z=\gamma[k_z+\beta(1+\mathbf{k}^2)^{1/2}]\,,$$

where $\cosh\omega=\gamma$. It remains to cast (6) in a form which can be evaluated by Monte Carlo methods.[9]

As it stands, (6) is not suitable for Monte Carlo evaluation: The amplitude B in the inner integration is complex with a rapidly varying phase. However, we can use the fact that $\tilde{\Phi}$ is sharply and symmetrically peaked around $\tilde{k}_z=0$ or $k_z=\beta\gamma(1+\mathbf{k}_\perp^2)^{1/2}$. Under these circumstances (6) can be replaced by

$$\sigma_{\rm cap}\simeq\frac{1}{(2\beta)^2}\sum_{\sigma_k,s_q}C(s_q)\int d^3k\,d^3q\,d^2p_\perp\frac{1}{(2\pi)^8}\times|\tilde{\Phi}_T(s_k,\mathbf{k})|^2\,|B(k,q;\mathbf{p}_\perp)|^2\,, \tag{8}$$

where the renormalization factor is given by

$$C(s_k)=\frac{\left|\int\tilde{\Phi}_T(s_k,\mathbf{k})d^3k\right|^2}{\int|\tilde{\Phi}_T(s_k,\mathbf{k})|^2d^3k}\,. \tag{9}$$

The integral (8) can be evaluated as it stands, the integrand being positive definite. The error is $O(1/\gamma^4)$ and hence negligible in the range we consider ($\gamma \gtrsim 3$).

Although it is not obvious, the formulation (6) can be related to effective photon methods. To this end, we recall the first-order perturbation ansatz for capture,

$$S_{\text{cap}}(s_q,\mathbf{q};\rho) = \int dt \exp(i\omega_q t)\langle s_q,\mathbf{q} \,|\, V_a(t) \,|\, T\rangle . \tag{10}$$

We are now working in the target frame, in which V_a is the interaction supplied by the projectile at an impact parameter ρ, and $\omega_q = E_q^{(-)} - E_T$. In contrast to (6), Eq. (10) is not gauge invariant. However, if we express V_a in the radiation gauge, the two methods should be equivalent to the leading orders in V_a and V_b. Thus, (6) becomes

$$S_{\text{cap}}(s_q,\mathbf{q};\rho) = \int d^2\lambda_\perp \frac{1}{4\pi^2\beta\omega_q} \exp(-\tfrac{1}{2} i\boldsymbol{\lambda}_\perp \cdot \boldsymbol{\rho}) F(\boldsymbol{\lambda}_\perp{:}\omega_q)\Phi_T(s_k,\mathbf{k})\langle s_q,\mathbf{q} \,|\, \boldsymbol{\lambda}_\perp \cdot \boldsymbol{\alpha} \,|\, s_k,\mathbf{k}\rangle , \tag{11}$$

where $\boldsymbol{\lambda}_\perp = \mathbf{q}_\perp - \mathbf{k}_\perp$ and $\lambda_z = q_z - k_z = \omega_q/\beta$. Finally, the capture cross section follows from integrating $|S|^2$ over all impact parameters,

$$\sigma_{\text{cap}} = \int d^3q \frac{1}{(2\pi)^3} \sum_{s_q} \int d^2\rho \,|\, S_{\text{cap}}(s_q,\mathbf{q};\rho) \,|^2$$
$$= \int d^3q \frac{1}{(2\pi)^3} \sum_{s_q} \int d^2\lambda_\perp \frac{1}{(2\pi\beta\omega_q)^2} \,|\, F(\boldsymbol{\lambda}_\perp{:}\omega_q)\Phi_T(s_k,\mathbf{k})\langle s_q,\mathbf{q} \,|\, \boldsymbol{\lambda}_\perp \cdot \boldsymbol{\alpha} \,|\, s_k,\mathbf{k}\rangle \,|^2 . \tag{12}$$

The *single-peak approximation* is based on the observation that the form factor F is sharply peaked around $\lambda_\perp = 0$, especially for $\gamma \gg 1$. Then we can separate

$$\mathcal{F}_a = Z_a^2 \int d^2\lambda_\perp (\lambda_\perp \lambda_\perp) \,|\, F(\lambda_\perp{:}\omega_q) \,|^2 , \tag{13}$$

and evaluate the rest of the integrand at $\lambda_\perp = 0$, or

$$\mathbf{k} = \mathbf{q} - \frac{\omega_q}{\beta}\mathbf{e}_z . \tag{14}$$

Within a constant factor, $\mathcal{F}_a$ is the equivalent photon flux tensor associated with the projectile.[9] The integral over $\lambda_\perp$ in (13) must be cut off at $\lambda_\perp \simeq m_\perp$, where

$$m_\perp = (1 + \mathbf{q}_\perp^2)^{1/2} .$$

If (12) is evaluated exactly, this cutoff arises naturally. Then (12) becomes

$$\sigma_{\text{cap}} = \left[\frac{Z_a \alpha}{2\pi\beta}\right]^2 \sum_{s_q} \int d^3q \frac{1}{\omega_q^2} \mathcal{P}\left[\frac{\beta\gamma m_\perp}{\omega_q}\right] \times \mathcal{M}(\mathbf{k},\mathbf{q}) \,|\, \Phi_T(s_k,\mathbf{k}) \,|^2 . \tag{15}$$

For most purposes $\mathcal{P}(x) \sim \ln x$; $\mathcal{M}$ comes from the spinors, and is of order unity or less.[9]

In Fig. 2, the capture cross section is plotted for Au+Au collision as a function of the collider energy. The scale of the cross section is given by $\sigma_0 = (\lambda Z_a Z_b \alpha^2)^2$, where λ is the Compton wavelength of the electron, and α is the fine-structure constant. The value of σ_0 for Au+Au is 164.7 b. Figure 2 shows three calculations: the full Monte Carlo from (6), the single-peak approximation from (12), and a Weizsäcker-Williams calculation from Baur and Bertulani.[15] The error bars reflect the remaining statistical errors in the Monte Carlo evaluation of the integral. The single-peak approximation agrees well with the Monte Carlo one over the entire energy range, and is algebraically simpler. The Weizsäcker-Williams curve is consistently lower than the Monte Carlo method by a factor of 3. We do not have an explanation for this discrepancy. However, we note: (i) Our numerical calculations for total pair production have been checked out against all other calculations available to us,[15,18] to an appropriate level of accuracy. (ii) An independent calculation by Lee and Weneser, based on the standard assumptions of the Weizsäcker-Williams method, agrees well with our results.[14]

From the perspective of RHIC accelerator performance, the top RHIC energy per nucleon for Au is 100 GeV. At this energy the capture cross section is 72 b per

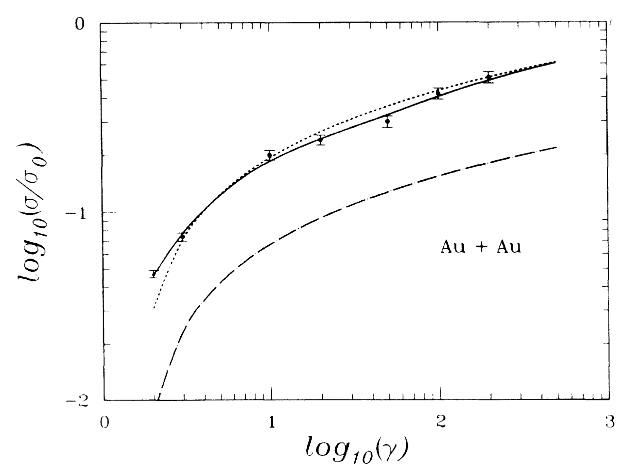

FIG. 2. Scaled capture cross sections for Au+Au. The scaling factor $\sigma_0 = 164.7$ b. The three curves are as follows: solid line, Feynman–Monte Carlo calculations, with statistical uncertainties indicated by error bars; dashed line, the single-peak approximation (15); dot-dashed line, Weizsäcker-Williams calculations from Ref. 15.

beam. This result is about 10% smaller than the value obtained by Lee and Weneser using a variant of the Weizsäcker-Williams method.[14] For comparison, the total pair production is about 32 kb, or 440 times larger.

In Fig. 3, the capture cross sections are shown for a sample of symmetric heavy-ion collisions used in the design of RHIC. These are full Monte Carlo calculations. To convey the variation with Z, all the results are scaled according to $\lambda^2 = 1.49$ kb. It appears that the cross sections are very small for ions lighter than ^{63}Cu, and indeed very few heavy ions would be lost to this mechanism for $A \lesssim 100$ The result for ^{197}Au is in harmony with a 10-h beam lifetime at RHIC. An equally encouraging aspect of Fig. 3 is the saturation at large values of Z: The electron-capture cross section for U+U is only twice that for Au+Au. It has recently been determined that ^{238}U may be injected into RHIC.[19]

We have addressed the problem of electron capture in relativistic heavy-ion collisions, and attempted to resolve apparent discrepancies among published Weizsäcker-Williams estimates for this mechanism. We evaluated the leading Feynman diagrams numerically by Monte Carlo methods. The results were compared with simpler single-peak approximation, and found to be in reasonable agreement. It is important to realize that the exact second-order perturbation amplitudes for pair production by point charges are free of any singularities. This was made clear in the work of Brodsky, Kinoshita, and Terazawa.[18] Equivalent photon methods do indeed reduce the complexity of the two-photon diagrams by using on-shell expressions for the amplitude $\mathcal{T}_{kq}$ in (3). In addition, the form factors are separately integrated over all transverse momenta. For point charges, these approximations introduce divergences which are removed by cutoffs in the transverse mass. The nonuniqueness of this procedure explains why a wide variety of estimates for the same cross section can all be labeled "Weizsäcker-Williams."

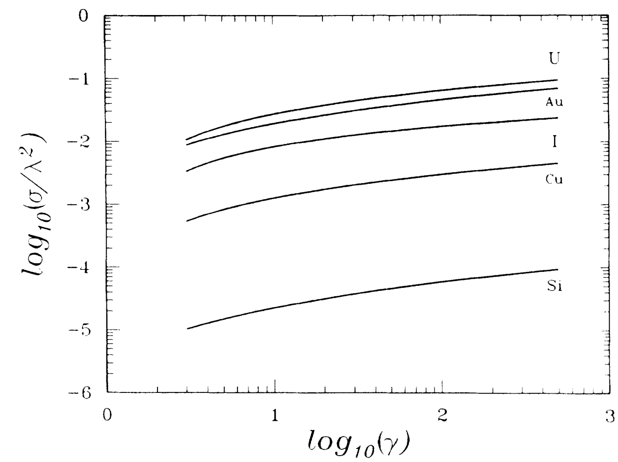

FIG. 3. Capture cross sections for symmetric $zA + zA$ collisions, scaled with respect to $\lambda^2 = 1.49$ kb. The curves correspond, as labeled, to the ions $A(Z)$ = Si(14), Cu(29), I(53), Au(79), and U(92).

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