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Generalized Navier-Stokes equations and light-induced gas-kinetic effects

Gerard Nienhuis
Fysisch Laboratorium, Rijksuniversiteit Utrecht, Postbus 80 000, 3508 TA Utrecht, The Netherlands
and Huygens Laboratorium, Rijksuniversiteit Leiden, Postbus 9504, 2300 RA Leiden, The Netherlands
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We present a general formalism for the description of gas-kinetic effects of light in a single-component gas, by elimination of rapid variables in the evolution equations. These equations contain both Boltzmann terms describing velocity-changing collisions and radiative transitions. The general result is a set of generalized Navier-Stokes equations for the density, the temperature, and the hydrodynamic velocity. The effects of the incident radiation on the transport properties of the gas are illustrated by deriving explicit transport equations in a simple model case. This shows that both the intensity and the intensity gradient serve as a thermodynamic force.

I. INTRODUCTION

Nearly resonant light scattering can modify the velocity distribution of gas particles. This modification arises provided that the excitation rate depends on the particle velocity, and that the gas-kinetic cross sections are different for the two internal states involved in the resonance transition. The modified velocity distribution may cause a substantial change in the macroscopic state of the gas and its transport properties.

The most studied manifestation of light-induced modification of the velocity distribution is light-induced drift. This effect may occur when a vapor, immersed in a buffer gas, is radiatively excited in a velocity-selective way.[1,2] Since usually excited particles have larger cross sections, leading to a lower mobility in the buffer gas, the net result of velocity-selective excitation is a lower mobility of the selected velocity group. The optically active particles may then obtain a nonvanishing average velocity. For appreciable optical thickness, this effect of light-induced drift may cause a very steep variation of the particle density in the region where the intensity decreases along the propagation direction of the light. For a drift velocity in the propagation direction, the light can effectively sweep the particles towards the dark end of the cell. This effect of the optical piston[2] has been observed and analyzed in detail.[3-6]

When the density of the buffer gas is much higher than the density of active particles, the equations of motion for the velocity distributions of the states of the active particles are linear, and the description of light-induced drift is relatively simple.[7,8] The buffer gas is then a heat bath, which may be considered in thermal equilibrium at all times. During the rapid local evolution of the velocity distribution due to collisions and radiative transitions, the active-particle density is the only conserved quantity. Momentum and kinetic energy of these particles is rapidly exchanged with the buffer gas.

The situation is considerably more complex for a pure (single-component) gas irradiated by nearly resonant light. The combined action of velocity-dependent radiative excitation and state-dependent collision rates can again lead to strong deviation of the velocity distribution from a Maxwellian. However, due to momentum conservation, the average velocity cannot be modified. In the absence of inelastic collisions, also the kinetic energy is conserved, and the effective translational temperature remains unchanged. Nevertheless, the modified velocity distribution can yield a nonvanishing heat flow and pressure anisotropy, so that the radiation field can induce changes in the thermodynamic and hydrodynamic state of the system.

These types of changes in the macroscopic state of a pure gas have been discussed in a few papers.[9-11] The derivations were based upon a special model description for the collisions. It was demonstrated that the incident light can induce stationary temperature inhomogeneities and particle fluxes, even though no temperature or density gradients were externally imposed.

The radiation field not only modifies the steady state of the gas, but also the decay of fluctuations around this steady state. Therefore we may expect a change in the transport properties of the gas. For a given propagation direction of the light, the usual spherical symmetry of the microscopic evolution is reduced to a cylindrical symmetry. Hence we may expect a difference in transverse and longitudinal components of the transport coefficients. Furthermore the general rule that the response to a thermodynamic force must have the same symmetry character as the force is now much less restrictive than in standard gas-kinetic theory. For instance, the response to an imposed temperature gradient can be a heat flow, a particle flow, and a contribution to the pressure anisotropy.

The first goal of the present paper is to present a general derivation of macroscopic equations of motion for a pure gas in a nearly resonant radiation field. We start from the microscopic equations for the velocity-dependent density matrix for the internal state of the particles. These equations add radiative transitions to the Boltzmann equation. No restrictive assumptions are made for the Boltzmann operator at this point. We distinguish the rapid evolution, due to collisions and radiative transitions and the slow evolution resulting from macroscopic gradients. By eliminating the rapid variables, we arrive at equations for the quantities that do not vary on the rapid time scale. These quantities, the local density, the hydrodynamic velocity, and the gas-kinetic temperature correspond to the conservation of the particle number, the momentum, and the kinetic energy. The method is a generalization of gas-kinetic methods for deriving the Navier-Stokes equations from the Boltzmann equation. The original gas-kinetic treatment, which is generally known as the Chapman-Enskog method,[12] is in fact a prototype of a general scheme for elimination of fast variables.[13,14] In the present case of an irradiated system the steady state with respect to the rapid time scale is not a local Maxwell distribution. In fact, the steady-state distribution cannot be described exactly, except in special model cases. Nevertheless, the general solution scheme gives new insight in the structure of the macroscopic equations, and in the classification of possible light-induced gas-kinetic effects. For illustrative purposes we shall discuss several cases explicitly.

II. STRUCTURE OF EVOLUTION EQUATIONS

We consider a pure gas irradiated by nearly resonant light. The formalism we are presenting does not require any specific assumption as to the number of degenerate substates or the coherence of the radiation. However, for notational simplicity we shall give explicit equations for the case that the radiation field with a bandwidth that is larger than the homogeneous linewidth couples the nondegenerate ground state $|g\rangle$ to a nondegenerate excited state $|e\rangle$. Then the stimulated radiative transitions may be described by a velocity-dependent transition rate $B(\mathbf{c},\mathbf{r})$, which is proportional to the local field intensity $I(\mathbf{r})$ at position $\mathbf{r}$. Coherence between the two internal states may then be ignored. The microscopic state of the gas is described by the two distribution function $f_e(\mathbf{c},\mathbf{r},t)$ and $f_g(\mathbf{c},\mathbf{r},t)$, defined by requiring that $f_g(\mathbf{c},\mathbf{r},t)d\mathbf{c}\,d\mathbf{r}$ is the number of particles at time t in the ground state with position in $d\mathbf{r}$ and velocity within $d\mathbf{c}$. The evolution equations for f_e and f_g are Boltzmann-type equations with radiative transitions added, and we write

$$\begin{aligned}\frac{\partial}{\partial t}f_e(\mathbf{c},\mathbf{r},t)=&-\mathbf{c}\cdot\nabla f_e(\mathbf{c},\mathbf{r},t)+B(\mathbf{c},\mathbf{r})f_g(\mathbf{c},\mathbf{r},t)\\&-[A+B(\mathbf{c},\mathbf{r})]f_e(\mathbf{c},\mathbf{r},t)\\&+J_{ee}[f_e,f_e]+J_{eg}[f_e,f_g]\ ,\\ \frac{\partial}{\partial t}f_g(\mathbf{c},\mathbf{r},t)=&-\mathbf{c}\cdot\nabla f_g(\mathbf{c},\mathbf{r},t)-B(\mathbf{c},\mathbf{r})f_g(\mathbf{c},\mathbf{r},t)\\&+[A+B(\mathbf{c},\mathbf{r})]f_e(\mathbf{c},\mathbf{r},t)\\&+J_{ge}[f_g,f_e]+J_{gg}[f_g,f_g]\ ,\end{aligned}\tag{2.1}$$

with A the spontaneous-decay rate. The Boltzmann operators J_{ij} $(i,j=e,g)$ are bilinear functionals of the standard form[15]

$$J_{ij}[a,b]=\int d\mathbf{c}_1 d\hat{\mathbf{u}}'[a(\mathbf{c}')b(\mathbf{c}_1')-a(\mathbf{c})b(\mathbf{c}_1)]\times u\,\sigma_{ij}(\mathbf{u},\mathbf{u}')\tag{2.2}$$

where a and b are functions of velocity. [The dependence on position $\mathbf{r}$ and time t has been suppressed in (2.2) for simplicity.] An elastic collision is fully specified by the velocities $\mathbf{c}$ and $\mathbf{c}_1$ of the two colliding particles before the collision, and the direction $\hat{\mathbf{u}}'=\mathbf{u}'/u'$ of the relative velocity

$$\mathbf{u}'=\mathbf{c}'-\mathbf{c}_1'\tag{2.3}$$

after the collision. Hence the velocities $\mathbf{c}'$, $\mathbf{c}_1'$, and the relative velocity $\mathbf{u}$ in (2.2) should be considered as functions of the independent variables $\mathbf{c},\mathbf{c}_1$, and $\hat{\mathbf{u}}'$. The quantity $\sigma_{ij}(\mathbf{u},\mathbf{u}')$ is the differential cross section for an elastic collision between a particle in state i and a particle in state j, with relative velocity $\mathbf{u}$ and $\mathbf{u}'$ before and after the collision. The two Boltzmann terms in each of the two equations (2.1) indicate that a particle in any one of the two internal states may suffer a velocity-changing collision with particles in either state. Since we allow the cross sections σ_{ee}, $\sigma_{eg}=\sigma_{ge}$, and σ_{gg} to be different, the collisional evolution of the distribution functions f_e and f_g is the same as for a mixture of two species. The radiative transitions may be regarded as inducing a monomolecular reaction with a velocity-dependent rate.

We can conveniently summarize the structure of the evolution equations (2.1) by introducing the two-vector function

$$F(\mathbf{c},\mathbf{r},t)=\begin{bmatrix}f_e(\mathbf{c},\mathbf{r},t)\\ f_g(\mathbf{c},\mathbf{r},t)\end{bmatrix}\ .\tag{2.4}$$

Obviously, the radiative terms in (2.1) are linear in F, and the collisional terms are bilinear. We formally express (2.1) in the form

$$\frac{\partial}{\partial t}F=-\mathbf{c}\cdot\nabla F+\frac{1}{\epsilon}(R[F]+C[F,F])\ ,\tag{2.5}$$

where the radiation operator R and the collisional operator C are defined by the radiative and the collisional terms in (2.1). The small expansion parameter ϵ indicates that the radiative and collisional processes occur at a time scale that is rapid compared with the effects of the macroscopic gradients, expressed by the free-flow term.[13] We also introduce the total distribution function

$$f(\mathbf{c},\mathbf{r},t)=f_e(\mathbf{c},\mathbf{r},t)+f_g(\mathbf{c},\mathbf{r},t)\tag{2.6}$$

for the particles, irrespective of their state.

The separation of the evolution equation in a free-flow term, a linear radiative term, and a bilinear collision term applies, irrespective of the number of the levels coupled by radiative transitions, and for arbitrary spectral and coherence properties of the radiation field.

III. CONSERVED QUANTITIES

On the rapid time scale, the combined action of radiation and collisions drives F to a steady state. This state is fully determined by specifying the values of those quantities that are conserved under the action of $R+C$. Then on the slow macroscopic time scale, the free-flow term modifies the value of these conserved quantities, while the operator $R+C$ continues to drive F to its steady state.

The result is that F follows these slow changes while remaining close to the steady state pertaining to the values of these locally conserved quantities. This is the picture of the Chapman-Enskog method of kinetic theory.

Radiative transitions and collisions have as conserved quantities the number of particles, and the momentum and kinetic energy of the particles. Here we use explicitly that inelastic collisions may be ignored. These conservation laws imply that the number density n, the hydrodynamic velocity $\mathbf{v}$ and the temperature T can change only due to macroscopic flows, resulting from the free-flow term in (2.5). These quantities are expressed in the total distribution function f in the usual way,[15] and we write

$$\begin{aligned} n(\mathbf{r},t) &= \int d\mathbf{c}\, f(\mathbf{c},\mathbf{r},t)\ , \\ \mathbf{v}(\mathbf{r},t) &= \int d\mathbf{c}\, \mathbf{c} f(\mathbf{c},\mathbf{r},t)/n(\mathbf{r},t)\ , \\ T(\mathbf{r},t) &= \int d\mathbf{c} \tfrac{1}{2} m[\mathbf{c}-\mathbf{v}(\mathbf{r},t)]^2 f(\mathbf{c},\mathbf{r},t)/[\tfrac{3}{2}n(\mathbf{r},t)k]\ , \end{aligned} \tag{3.1}$$

where k is Boltzmann's constant. Since only the free-flow term in (2.5) can contribute to the rate of change of these quantities, the conservation laws take the same form as in standard gas-kinetic theory,[15] and one directly derives

$$\begin{aligned} \frac{\partial}{\partial t} n &= -\mathbf{v}\cdot\nabla n - n\nabla\cdot\mathbf{v}\ , \\ n\frac{\partial}{\partial t}\mathbf{v} &= -n(\mathbf{v}\cdot\nabla)\mathbf{v} - \frac{1}{m}\nabla\cdot\overleftrightarrow{\mathbf{P}}\ , \\ \tfrac{3}{2}nk\frac{\partial}{\partial t}T &= -\tfrac{3}{2}nk\mathbf{v}\cdot\nabla T - \overleftrightarrow{\mathbf{P}}:\nabla\mathbf{v} - \nabla\cdot\mathbf{q}\ . \end{aligned} \tag{3.2}$$

Here $\mathbf{q}$ is the heat flow, defined as

$$\mathbf{q}(\mathbf{r},t) = \int d\mathbf{c}[\mathbf{c}-\mathbf{v}(\mathbf{r},t)]\tfrac{1}{2}m[\mathbf{c}-\mathbf{v}(\mathbf{r},t)]^2 f(\mathbf{c},\mathbf{r},t) \tag{3.3}$$

and

$$\overleftrightarrow{\mathbf{P}}(\mathbf{r},t) = \int d\mathbf{c}\, m[\mathbf{c}-\mathbf{v}(\mathbf{r},t)][\mathbf{c}-\mathbf{v}(\mathbf{r},\mathbf{t})]f(\mathbf{c},\mathbf{r},t) \tag{3.4}$$

is the pressure tensor.

Equations (3.2) are an exact result from the evolution equations (2.1), or their generalization for general spectral properties of the radiation and level degeneracies. They demonstrate that our evolution of n, $\mathbf{v}$, and T is determined by macroscopic gradients only, so that it takes place exclusively on the slow time scale. The first equation of Eqs. (3.2) is just the continuity equation for the particle density, but the equation for $\mathbf{v}$ and T involve the pressure tensor $\overleftrightarrow{\mathbf{P}}$ and the heat flow $\mathbf{q}$. The pressure tensor $\overleftrightarrow{\mathbf{P}}$ can be separated as usual in its isotropic and its anisotropic part, according to

$$\overleftrightarrow{\mathbf{P}}(\mathbf{r},t) = p(\mathbf{r},t)\overleftrightarrow{\mathbf{I}} + \overleftrightarrow{\mathbf{\Pi}}(t,t)\ , \tag{3.5}$$

where $\overleftrightarrow{\mathbf{I}}$ is the unit tensor and

$$p(\mathbf{r},t) = n(\mathbf{r},t)kT(\mathbf{r},t) = \tfrac{1}{3}\mathrm{Tr}\overleftrightarrow{\mathbf{P}}(\mathbf{r},t) \tag{3.6}$$

is the isotropic pressure. The remaining part $\overleftrightarrow{\mathbf{\Pi}}$ is the pressure anisotropy, which is a symmetric tensor with trace zero.

In order to turn Eqs. (3.2) into a closed set of equations for n, $\mathbf{v}$, and T, we have to express the quantities $\mathbf{q}$ and $\overleftrightarrow{\mathbf{\Pi}}$ in terms of n, $\mathbf{v}$, and T. This is what the Chapman-Enskog method achieves in standard gas-kinetic theory. In Sec. IV we generalize this method for the situation of an irradiated gas.

IV. GENERALIZED NAVIER-STOKES EQUATIONS

In this section we derive formal expressions for the heat flow $\mathbf{q}$ and the pressure tensor $\overleftrightarrow{\mathbf{P}}$, to first order in the expansion parameter ϵ. We shall focus our attention on the formal structure of these expressions, rather than on their closed analytical form. If we substitute these results in the conservation equations (3.2), we obtain equations of motion for n, $\mathbf{v}$, and T. The driving forces in these equations are not only the gradients of these quantities, but also the strength and the gradient of the intensity of the radiation field.

Our starting point is the evolution equation (2.5) for the pair of distribution functions f_g and f_e. If we expand

$$F = F_0 + \epsilon F_1 + \cdots \tag{4.1}$$

we notice that the lowest-order term in (2.5) has the order -1. This gives an equation for F_0, in the form

$$0 = R[F_0] + C[F_0, F_0]\ . \tag{4.2}$$

Even this lowest-order equation can only be solved explicitly for special model systems. Here we notice that for given local values of $n(\mathbf{r})$, $\mathbf{v}(\mathbf{r})$, and $T(\mathbf{r})$, there is a unique solution $F_0(\mathbf{c},\mathbf{r})$. This follows from the assumption that the particle number, the momentum, and the energy are the only conserved quantities for the combined action of radiation and collisions. Obviously, this solution will depend in addition on the local intensity $I(\mathbf{r})$ of the radiation field.

There are two situations where the solution of (4.2) is simple. First, when the collisional cross sections do not depend on the internal state of the collision partners, we may substitute one single Boltzmann term J for each J_{ij} in (2.1). Adding the two equations that are contained in the two-vector equation (4.2) then leads to the simple equation

$$J[f_0, f_0] = 0 \tag{4.3}$$

for the total distribution function to zeroth order in ϵ. This equation has the unique solution

$$\begin{aligned} f_0(\mathbf{c},\mathbf{r}) &= n(\mathbf{r})\left[\frac{m}{2\pi kT(\mathbf{r})}\right]^{3/2} \\ &\quad\times\exp\{-\tfrac{1}{2}m[\mathbf{c}-\mathbf{v}(\mathbf{r})]^2/kT(\mathbf{r})\} \\ &\equiv nW(\mathbf{c})\ , \end{aligned} \tag{4.4}$$

which is the Maxwell-Boltzmann distribution at the specified density, average velocity, and temperature.

The second situation where the solution for f_0 following from (4.2) is trivial occurs when the stimulated transition rate B in (2.1) does not depend on the velocity $\mathbf{c}$.

Then it is obvious that in the solution to (4.2), both f_{e0} and f_{go} are proportional to the Maxwell distribution f_0 with the given average velocity $\mathbf{v}$ and temperature T. The Boltzmann operators vanish in this case, and the ratio of the partial densities n_e and n_g is simply equal to $B/(A+B)$. In these two simple cases, the zeroth-order heat flow $\mathbf{q}_0$ and the pressure anisotropy $\overleftrightarrow{\Pi}_0$ vanish. Substituting these zero-order values in the conservation laws (3.2) gives the standard Euler equations.[15]

The zeroth-order distribution f_0 differs from the Maxwell distribution nW only when the Boltzmann terms J_{ij} in (2.1) are not identical, and when the stimulated transition rate B depends on the velocity. In this general case of state-dependent collision cross sections and velocity-dependent excitation, the unique solution of (4.2) for given n, $\mathbf{v}$, and T gives the total velocity distribution $f_0(\mathbf{c},\mathbf{r})$ to zeroth order. This in turn determines the zeroth-order heat flow $\mathbf{q}_0$, and pressure anisotropy $\overleftrightarrow{\Pi}_0$. When the light has a propagation direction in the z direction, the stimulated transition rate B can only depend on c_z. For symmetry reasons $\mathbf{q}_0$ and $\overleftrightarrow{\Pi}_0$ must then be invariant for rotations about the z axis, so that we may write

$$\mathbf{q}_0(\mathbf{r})=q_0(\mathbf{r})\hat{z} \; , \qquad \overleftrightarrow{\Pi}_0(\mathbf{r})=G_0(\mathbf{r})(\hat{x}\hat{x}+\hat{y}\hat{y}-2\hat{z}\hat{z}) \; , \tag{4.5}$$

where $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$ denote unit vectors in the three Cartesian directions. The two quantities q_0 and G_0 depend on $\mathbf{r}$ only through the density n, the hydronamic velocity $\mathbf{v}$, the temperature T, and the intensity I. Explicit expressions for $q_0(n,\mathbf{v},T,I)$ and $G_0(n,\mathbf{v},T,I)$ can only be obtained in special model cases.

Now we turn to the next order in ϵ of (2.5). The zeroth-order contribution is

$$\frac{\partial F_0}{\partial t}=-\mathbf{c}\cdot\nabla F_0+R[F_1]+C[F_1,F_0]+C[F_0,F_1] \; . \tag{4.6}$$

The gradient may be expressed as

$$\nabla F_0=\nabla n\frac{\partial F_0}{\partial n}+\nabla\mathbf{v}\cdot\frac{\partial F_0}{\partial \mathbf{v}}+\nabla T\frac{\partial F_0}{\partial T}+\nabla I\frac{\partial F_0}{\partial I} \; . \tag{4.7}$$

Likewise, for a given stationary intensity distribution the time derivative in (4.6) is

$$\frac{\partial F_0}{\partial t}=\frac{\partial n}{\partial t}\frac{\partial F_0}{\partial n}+\frac{\partial \mathbf{v}}{\partial t}\cdot\frac{\partial F_0}{\partial \mathbf{v}}+\frac{\partial T}{\partial t}\frac{\partial F_0}{\partial T} \; . \tag{4.8}$$

To zeroth order, we may substitute for the time derivatives of n, $\mathbf{v}$ and T in (4.8) the conservation laws (3.2) to this same order. This means that we have to substitute for $\mathbf{q}$ and $\mathbf{P}$ in (3.2) their zeroth order in ϵ, so that we may use the zeroth-order terms (4.5). After substituting (4.7) and (4.8) in (4.6), and eliminating the time derivatives by using the zeroth-order form of the conservation laws (3.2), we obtain the equation

$$\frac{\partial F_0}{\partial n}[(\mathbf{c}-\mathbf{v})\cdot\nabla n-n\nabla\cdot\mathbf{v}]+\frac{\partial F_0}{\partial \mathbf{v}}\left[[(\mathbf{c}-\mathbf{v})\cdot\nabla]\mathbf{v}-\frac{1}{nm}\nabla\cdot\overleftrightarrow{\mathbf{P}}_0\right]$$
$$+\frac{\partial F_0}{\partial T}\left[(\mathbf{c}-\mathbf{v})\cdot\nabla T-\frac{2}{3nk}(\overleftrightarrow{\mathbf{P}}_0:\nabla\mathbf{v}+\nabla\cdot\mathbf{q}_0)\right]+\frac{\partial F_0}{\partial I}\mathbf{c}\cdot\nabla I=R[F_1]+C[F_1,F_0]+C[F_0,F_1] \; . \tag{4.9}$$

The zeroth-order pressure tensor $\overleftrightarrow{\mathbf{P}}_0$ is obtained by substituting $\overleftrightarrow{\Pi}_0$ in (3.5). The right-hand side of Eq. (4.9) is a linear expression in the first-order correction F_1. The left-hand side of (4.9) is just a rewrite of $\partial F_0/\partial t+\mathbf{c}\cdot\nabla F_0$. This implies that this left-hand side gives zero when it is multiplied with 1, $\mathbf{c}$, or $\frac{1}{2}mc^2$, and subsequently integrated over the velocity and summed over the two components. This can be checked explicitly. The solution of (4.9) for F_1 is unique with the additional requirement that it gives a vanishing contribution to n, $\mathbf{v}$, and T.

We may conclude that Eq. (4.9) has a single unique solution for F_1, which is determined by n, $\mathbf{v}$, T, I, and their gradients. From F_1 we can in turn determine the first-order contributions $\mathbf{q}_1$ and $\overleftrightarrow{\Pi}_1$ to the heat flow and the pressure anisotropy. Substituting $\mathbf{q}_0+\mathbf{q}_1$ and $\overleftrightarrow{\Pi}_0+\overleftrightarrow{\Pi}_1$ for $\mathbf{q}$ and $\overleftrightarrow{\Pi}$ in the conservation laws (3.2) gives a closed set of evolution equations for n, $\mathbf{v}$, and T. This set generalizes the Navier-Stokes equations.

In order to illustrate the structure of these generalized hydrodynamic equations we discuss several simple limiting cases in Sec. VI.

V. SYMMETRY CONSIDERATIONS

We have seen that Eq. (4.9) determines in principle the first-order contributions $\mathbf{q}_1$ and $\overleftrightarrow{\Pi}_1$ to the heat flux and the pressure anisotropy in terms of the gradients of n, $\mathbf{v}$, T, and I. This equation (4.9) generalizes a basic result of standard gas-kinetic theory, where the radiative term R is omitted, and F_0 may be replaced by the general Maxwellian f_0, as given in (4.4). Equation (4.9) then takes the form[12,15]

$$f_0\frac{m}{kT}\left[\left[\frac{1}{2}(\mathbf{c}-\mathbf{v})^2-\frac{5}{2}\frac{kT}{m}\right](\mathbf{c}-\mathbf{v})\cdot\nabla T/T+[(\mathbf{c}-\mathbf{v})(\mathbf{c}-\mathbf{v})-\tfrac{1}{3}(\mathbf{c}-\mathbf{v})^2\overleftrightarrow{\mathbf{I}}]:\nabla\mathbf{v}\right]=J[f_1,f_0]+J[f_0,f_1] \; . \tag{5.1}$$

This equation determines f_1 completely, since we have the additional requirement that f_1 gives a negligible contribution to n, $\mathbf{v}$, and T.

The Boltzmann collision operator J is isotropic in the space of functions of $\mathbf{c}-\mathbf{v}$, for the given Maxwellian $f_0=nW$. Therefore this operator cannot change the rank of an irreducible tensor with respect to the group $O(3)$. It is this feature which causes the heat flow $\mathbf{q}$ to be proportional to the temperature gradient, so that[12]

$$\mathbf{q}_1=-\lambda\nabla T \ , \tag{5.2}$$

with λ the heat conductivity. Likewise, the tensor $\overleftrightarrow{\mathbf{S}}$ with components

$$S_{ij}=\tfrac{1}{2}(\nabla_i v_j+\nabla_j v_i)-\tfrac{1}{3}\delta_{ij}\nabla\cdot\mathbf{v} \tag{5.3}$$

is the only thermodynamic force in (5.1) which is an irreducible (symmetric traceless) tensor of rank 2, so that the pressure anisotropy $\overleftrightarrow{\Pi}_1$ must be proportional to $\overleftrightarrow{\mathbf{S}}$, and we write[12]

$$\overleftrightarrow{\Pi}_1=-2\zeta\overleftrightarrow{\mathbf{S}} \ , \tag{5.4}$$

with ζ the viscosity. Substituting these two relations (5.2) and (5.3) in (3.2) gives the common Navier-Stokes equations in terms of the two transport coefficients λ and ζ.

In the present case of an irradiated gas, the full spherical symmetry is reduced to a cylindrical symmetry with the propagation direction $\hat{z}$ of the light as axis of symmetry. The right-hand side of (4.9) may be viewed as a linear operator acting on F_1. Now this operator can couple only irreducible tensors with respect to the group $O(2)$.[16] Generalizing the considerations leading to the proportionalities (5.2) and (5.4) now reveals that the scalar part of $\mathbf{q}_1$ and $\overleftrightarrow{\Pi}_1$ with respect to the group $O(2)$ is a linear combination of the scalar parts of the gradients of n, $\mathbf{v}$, T, and I. The scalar parts of $\mathbf{q}_1$ and $\overleftrightarrow{\Pi}_1$ are

$$q_{1z},\Pi_{1,zz},\Pi_{1,xx}+\Pi_{1,yy} \ . \tag{5.5}$$

The scalar parts of the gradients are given in Table I. It is easily seen that there are five scalars in these thermodynamic forces. The total number of transport coefficients for the scalar parts is thus equal to 15.

TABLE I. List of scalars, vectors, and second-rank tensors that can be composed from the gradients of density, velocity, temperature, and intensity.

Scalars	Vectors	Tensor
$\frac{\partial n}{\partial z}$	$\left[\frac{\partial n}{\partial x},\frac{\partial n}{\partial y}\right]$	
$\frac{\partial T}{\partial z}$	$\left[\frac{\partial T}{\partial x},\frac{\partial T}{\partial y}\right]$	
$\frac{\partial I}{\partial z}$	$\left[\frac{\partial I}{\partial x},\frac{\partial I}{\partial y}\right]$	$\left[\frac{\partial v_x}{\partial x}-\frac{\partial v_y}{\partial y},\frac{\partial v_x}{\partial y}+\frac{\partial v_y}{\partial x}\right]$
$\frac{\partial v_z}{\partial z}$	$\left[\frac{\partial v_z}{\partial x},\frac{\partial v_z}{\partial y}\right]$	
$\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}$	$\left[\frac{\partial v_x}{\partial z},\frac{\partial v_y}{\partial z}\right]$	

Furthermore, the heat flux $\mathbf{q}_1$ and the pressure anisotropy $\overleftrightarrow{\Pi}_1$ have each a vector part, transforming as a two-dimensional vector in the xy plane under rotation about the z axis, and reflection about any plane through the z axis. These vector parts with respect to the group $O(2)$ are

$$(q_{1x},q_{1y}), \quad (\Pi_{zx},\Pi_{zy}) \ . \tag{5.6}$$

The gradients of n, $\mathbf{v}$, T, and I have five vector parts, as indicated in Table I. Therefore the number of transport coefficients relating the vector fluxes (5.6) to these vector forces is equal to 10.

Finally, the pressure anisotropy has a part transforming under the group $O(2)$ as an irreducible tensor of rank 2, which is basically a symmetric traceless tensor in the xy plane.[16,17] This tensor is determined by the two components

$$(\Pi_{1,xx}-\Pi_{1,yy},\Pi_{1,xy}+\Pi_{1,yx}) \ . \tag{5.7}$$

The gradient forces contain a single second-rank tensor, as given in Table I. Hence, this tensor part (5.7) is determined by a single additional transport coefficient.

We conclude that in the general case, the Navier-Stokes equation of an irradiated gas may require a total number of 26 transport coefficients. Of course, in practical cases many of these will be negligible, and in special model cases most may be zero. Not only does the light intensity I create a zeroth-order contribution $\mathbf{q}_0$ and $\overleftrightarrow{\Pi}_0$ to the heat flux and the pressure anisotropy, but also the intensity gradient ∇I acts as a thermodynamic force in a similar fashion as the gradients of n, $\mathbf{v}$, and T.

VI. STRONG VELOCITY SELECTION

In this section we derive explicit expressions for the heat flux $\mathbf{q}$ and the pressure anisotropy $\overleftrightarrow{\Pi}$ along the lines of Sec. IV, in the special case that only a narrow velocity group is excited by the radiation. This implies that the fraction of excited particles is always small, even in the presence of saturation. Thus we assume that the excitation rate $B(\mathbf{c})$ differs from zero only when c_z is near the Doppler-selected velocity $c_0=(\omega_L-\omega_0)/k_L$, with ω_L the central frequency of the radiation, ω_0 the transition frequency of the particles, and k_L the wave number. Since particles can only be excited within this narrow group, it is natural to expect that the distribution functions f_e and f_g contain a narrow structure around $c_z=c_0$. Since the fraction of excited atoms is small, an additional expansion in this fraction is justified. Furthermore, we approximate the collision operator by assuming that an excited particle with a selected velocity recovers the Maxwell-Boltzmann distribution after a single collision, and that these thermalizing collisions occur at a rate κ_e. The corresponding rate for a ground-state particle with a light-selected velocity is κ_g. Note that these collision rates κ_e and κ_g refer to collisions of an excited or ground-state particle with collision partners that are predominantly in the ground state. We write the two velocity distributions in the form

$$f_e(\mathbf{c})=nW(\mathbf{c})[\varphi_e(\mathbf{c})+\chi_e(\mathbf{c})]$$
$$f_g(\mathbf{c})=nW(\mathbf{c})[1+\varphi_g(\mathbf{c})+\chi_g(\mathbf{c})] , \tag{6.1}$$

where nW is defined in (4.4). The factors χ_e and χ_g are assumed to differ from zero exclusively in the narrow velocity region where the excitation rate $B(\mathbf{c})$ is nonzero, and the factors φ_e and φ_g denote additional smooth corrections. The sharp factors χ_e and χ_g describe basically the Bennett peak and hole, corresponding to the particles that have not yet suffered a collision since their last radiative transition. The sharp factors χ have an appreciable strength within the selected velocity group only, whereas the smooth factors φ are nonzero but small over the full width of the Maxwell distribution. The contribution to Maxwellian averages, such as macroscopic fluxes, remains small for both χ and φ.

A. Zeroth order

The equation (4.2) for the zeroth-order terms gives equations for the factors $\varphi_{e0},\varphi_{g0}$ and χ_{e0},χ_{g0}. These equations can only hold if the terms containing sharp factors compensate each other. This gives the two equations

$$0=-(A+B+\kappa_e)\chi_{e0}+B(1+\chi_{g0}) ,$$
$$0=(A+B)\chi_{e0}-(\kappa_g+B)\chi_{g0}-B . \tag{6.2}$$

Here we used the fact that the only contribution from the Boltzmann operators yielding a sharp contribution is the loss terms, which describes the removal of particles from the selected velocity group. According to the strong-collision assumption, particles entering a collision with a velocity in the narrow selected group will leave the collision with a smooth distribution. Note that we have neglected the smooth contribution within the selected group. The solution of (6.2) is

$$\chi_{e0}=\kappa_g B/N, \quad \chi_{g0}=-\kappa_e B/N , \tag{6.3}$$

with

$$N=(A+2B+\kappa_e)\kappa_g+B(\kappa_e-\kappa_g) . \tag{6.4}$$

Furthermore, the zeroth-order terms f_{e0} and f_{g0} also contain smooth correction factors φ_{e0} and φ_{g0}. These smooth factors are needed to compensate for modifications in n, $\mathbf{v}$, and T that the sharp factors χ alone would create. In line with the assumption of strong collisions, we assume that the smooth factor $\varphi_0=\varphi_{e0}+\varphi_{g0}$ has the form

$$\varphi_0=\alpha+\boldsymbol{\beta}\cdot(\mathbf{c}-\mathbf{v})+\gamma\tfrac{1}{2}m(\mathbf{c}-\mathbf{v})^2 , \tag{6.5}$$

where the parameters α, $\boldsymbol{\beta}$, and γ are determined by the requirement that $nW(\chi_0+\varphi_0)$ yields a vanishing contribution to n, $\mathbf{v}$, and T. The heat flux to zeroth order may then be evaluated from the identity

$$q_0=\int d\mathbf{c}\tfrac{1}{2}m(\mathbf{c}-\mathbf{v})^2(\mathbf{c}-\mathbf{v})nW(\mathbf{c})(1+\chi_0+\varphi_0)$$
$$=\int d\mathbf{c}[\tfrac{1}{2}m(\mathbf{c}-\mathbf{v})^2-\tfrac{5}{2}kT](\mathbf{c}-\mathbf{v})nW\chi_0 , \tag{6.6}$$

for φ_0 of the form (6.5), and when $\varphi_0+\chi_0$ do not modify n, $\mathbf{v}$, and T.

For later use it is convenient to express the strength of the sharp structure

$$\chi_0=-(\kappa_e-\kappa_g)B/N \tag{6.7}$$

in the absorption rate. If we call U the number of photon absorptions per unit time and per unit volume, then the balance of loss and gain of excited particles in the sharp distribution χ_{e0} gives

$$(A+\kappa_e)\int d\mathbf{c}\, nW(\mathbf{c})\chi_{e0}(\mathbf{c})=U . \tag{6.8}$$

With (6.7), this equality gives

$$\int d\mathbf{c}\, nW(\mathbf{c})\chi_0(\mathbf{c})=-\frac{\alpha}{A+\kappa_e}U , \tag{6.9}$$

with

$$\alpha=\frac{\kappa_e-\kappa_g}{\kappa_g} \tag{6.10}$$

the relative difference of collision rates.

With (6.6) and (6.9) we easily obtain an expression for the zeroth-order heat flux $\mathbf{q}_0$. The narrow structure χ_0 may be treated as a δ peak at $c_z=c_0$, with strength determined by (6.9). The hydrodynamic velocity $\mathbf{v}$ is assumed small compared to typical thermal velocities, so that $\mathbf{v}$ may be ignored in (6.6). We find that the zeroth-order heat flux $\mathbf{q}_0$ takes the form given in (4.5) with

$$q_0=-\frac{\alpha}{A+\kappa_e}(\tfrac{1}{2}mc_0^2-\tfrac{3}{2}kT)c_0U . \tag{6.11}$$

The zeroth-order pressure anisotropy $\overleftrightarrow{\Pi}_0$ is evaluated in the same fashion. The result takes the form given in (4.5), with

$$G_0=\frac{\alpha}{A+\kappa_e}\tfrac{1}{3}(mc_0^2-kT)U . \tag{6.12}$$

In order to appreciate these results, one should realize that their variation with the light frequency ω_L is determined by the selected velocity component c_0. The absorption rate U as a function of ω_L is basically proportional to the Doppler absorption profile.

Results of the same structure as (6.11) and (6.12) were obtained by Folin *et al.*[9] on the basis of a rather special scaling assumption of the potentials. Measurements of the light-induced pressure difference $\Pi_{zz}-\Pi_{xx}=-3G_0$ have been reported for the case of CH_3F.[18]

The qualitative behavior of q_0 and G_0 as a function of the light frequency ω_L is illustrated in Fig. 1. Assuming that $\alpha>0$ (or $\kappa_\epsilon>\kappa_g$), the heat flux is in the propagation direction for a light frequency ω_L just above the resonance frequency. When ω_L is more than a few Doppler widths above resonance, the heat flux changes sign. The pressure in the x direction is changed by an amount G_0.

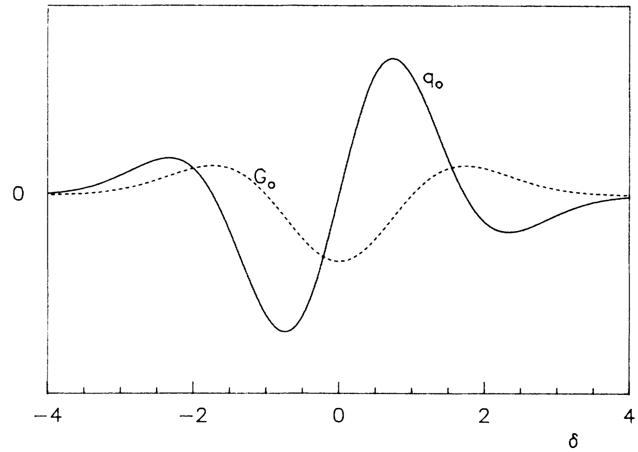

FIG. 1. Zeroth-order heat flux q_0 and pressure anisotropy G_0 in arbitrary units, as a function of the reduced frequency detuning in units of the Doppler width $\delta=(\omega_L-\omega_0)/k_L\sqrt{(kT/m)}$. These plots represent Eqs. (6.11) and (6.12).

This pressure change is negative for ω_L near resonance, and it becomes positive for excitation in the Doppler wings.

B. First order

Now we turn to the first-order results, which are formally determined by (4.9). In the light of the approximations mentioned above, we treat as small parameters the gradient of n, $\mathbf{v}$, T, and I, the smooth factors φ_0, and the average of the sharp factors χ_0, and we consider the first order in these parameters. First we consider the sharp contribution to (4.9). For simplicity we assume that the gradients of n, $\mathbf{v}$, and T give contributions which are small compared with the effect of the intensity gradient. This is a realistic assumption in practice, since the hydrodynamic quantities usually vary little over the dimensions of the system, whereas the intensity can show drastic variations over small distances at the edge of the beam. Therefore, the only first-order contribution to the sharp structure from the left-hand side is due to the term proportional to $\partial F_0/\partial I$. On the right-hand side of (4.9), the sharp terms may be treated in the same way as in (6.2). The sharp structures χ_1 in F_1 are damped due to collisions with particles in the F_0 distribution, which gives rise to damping rates κ_e and κ_g. The damping due to partners in the F_1 distribution may be ignored in the present model, since F_1 has a vanishing contribution to the density. With these arguments, the equations determining χ_{e1} and χ_{g1} are obtained as

$$\begin{aligned}\frac{\partial\chi_{e0}}{\partial I}\mathbf{c}\cdot\nabla I&=-(A+B+\kappa_e)\chi_{e1}+B\chi_{g1}\ ,\\ \frac{\partial\chi_{g0}}{\partial I}\mathbf{c}\cdot\nabla I&=(A+B)\chi_{e1}-(B+\kappa_g)\chi_{g1}\ .\end{aligned}\tag{6.13}$$

These equations are easily solved for χ_{e1} and χ_{g1}. Since we are only interested in the total velocity distribution, we consider $\chi_1=\chi_{e1}+\chi_{g1}$. For simplicity we consider only the common case that κ_e and κ_g differ slightly, so that $\alpha\ll 1$. Then we find to a good approximation

$$\chi_1=-\frac{1}{\kappa_g}\mathbf{c}\cdot\nabla I\frac{d\chi_0}{dI}\ .\tag{6.14}$$

This sharp structure must be compensated by a corresponding smooth factor, of the form (6.5), in order to compensate for changes in n, $\mathbf{v}$, and T. These factors are accounted for by evaluating the heat flux according to Eq. (6.6).

Finally we consider the smooth factor arising from the gradients in n, $\mathbf{v}$, and T. Since each term on the left-hand side of (4.9) contains a gradient, we must ignore the contribution from φ_{e0} and φ_{g0}. Hence, we take a Maxwellian for F_0. Furthermore, we may neglect $\overleftrightarrow{\Pi}_0$ and $\mathbf{q}_0$ in (4.9), since these quantities are also small. On the right-hand side of (4.9), we ignore the difference in cross sections for particles in different states, since the fraction of excited particles is small. Hence, by adding the two equations implicit in the two-vector equation (4.9), we obtain exactly the form (5.1). The first-order smooth correction is therefore equal to the first-order correction to the Maxwellian in standard gas kinetics for a simple gas without a radiation field. The additional contribution to the heat flux and the pressure anisotropy due to the gradient of n, $\mathbf{v}$, and T are therefore given by (5.2) and (5.4), with λ and ζ the heat conductivity and the viscosity of the gas in the absence of light.

With these ingredients we can directly evaluate $\mathbf{q}_1$ and $\overleftrightarrow{\Pi}_1$. We obtain for the heat flux in the propagation direction

$$q_{1z}=\frac{\alpha}{\kappa_g(A+\kappa_e)}\frac{1}{2}c_0^2(mc_0^2-3kT)\frac{\partial U}{\partial z}-\lambda\frac{\partial T}{\partial z}\ .\tag{6.15}$$

For the components normal to the propagation direction we find

$$q_{1x}=\frac{\alpha}{\kappa_g(A+\kappa_e)}\frac{kT}{2m}(mc_0^2-kT)\frac{\partial U}{\partial x}-\lambda\frac{\partial T}{\partial x}\ ,\tag{6.16}$$

and a similar equation for q_{1y}. This shows that gradients both of the temperature and of the intensity (or the absorption rate) serve as a thermodynamic force creating a heat flux. For a light frequency ω_L near resonance, the gradients of U normal to the propagation direction are most effective in creating a heat flux, and for a frequency ω_L in the Doppler wings, the parallel gradient is more important. The intensity-gradient contributions to the heat flux are sketched in Fig. 2.

In the same way we may evaluate the first-order pressure anisotropy. We can separate $\overleftrightarrow{\Pi}$ in the form

$$\overleftrightarrow{\Pi}_1=\overleftrightarrow{\Pi}_{1r}-2\zeta\overleftrightarrow{S}\ ,\tag{6.17}$$

where $\overleftrightarrow{\Pi}_{1r}$ is the radiative contribution due to the sharp structure (6.14) and the last term is the usual contribution due to the velocity gradients, as specified in (5.4). The radiative contribution has diagonal elements, which are given by

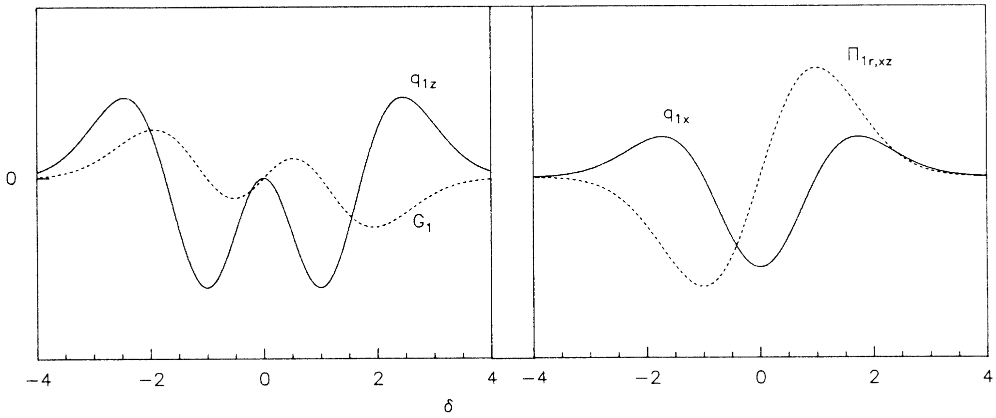

FIG. 2. Components of the first-order heat flux and pressure anisotropy in arbitrary units. The plots correspond to Eqs. (6.15), (6.19), (6.16), and (6.20).

$$\Pi_{1r,xx}=\Pi_{1r,yy}=-\tfrac{1}{2}\Pi_{1r,zz}\equiv G_1 \,, \tag{6.18}$$

with

$$G_1=\frac{\alpha}{\kappa_g(A+\kappa_e)}\frac{1}{3}(kT-mc_0^2)c_0\frac{\partial U}{\partial z} \,. \tag{6.19}$$

Furthermore, $\overleftrightarrow{\Pi}_{1r}$ has nonvanishing off-diagonal components

$$\Pi_{1r,xz}=\Pi_{1r,zx}=\frac{\alpha}{\kappa_g(A+\kappa_e)}kTc_0\frac{\partial U}{\partial x} \tag{6.20}$$

and a similar expression holds for $\Pi_{1r,yz}$. The remaining component $\Pi_{1r,xy}$ vanishes. These radiative contributions to the pressure anisotropy are sketched in Fig. 2.

We find that intensity gradients cause pressure anisotropies as well as a heat flux. A gradient in the propagation direction gives a contribution to the heat flux in this same direction, and an axially symmetric pressure anisotropy. An intensity gradient normal to the propagation direction creates a heat flow (6.16) in this same direction, and an off-diagonal component in the pressure tensor, as given in (6.20). Note that these intensity-gradient contributions to the heat flux are even in the detuning $\omega_L-\omega_0$, whereas contributions to the pressure tensor are odd in the detuning.

Effects of the intensity gradients on thermodynamic fluxes were discussed before in Refs. 10 and 11. Our derivation is based, however, on a systematic expansion of the evolution equation for the velocity distributions F. Therefore, our method allows generalization to other situations than the case of strong velocity selection and strong collisions.

Now that we have obtained explicit expressions for $\mathbf{q}$ and $\overleftrightarrow{\Pi}$ up to first order, we can substitute

$$\mathbf{q}=\mathbf{q}_0+\mathbf{q}_1 \tag{6.21}$$

and

$$\overleftrightarrow{\Pi}=\overleftrightarrow{\Pi}_0+\overleftrightarrow{\Pi}_1 \tag{6.22}$$

into the conservation laws (3.2). The result is an explicit form of generalized Navier-Stokes equations. The evolution of n, $\mathbf{v}$, and T is determined by these equations, where the intensity and its gradients now serve as additional thermodynamic forces.

C. Simple solutions

A number of solutions of the equations of motion can be easily characterized. First we consider the heat flux and the steady-state temperature. When a closed cell is surrounded by adiabatic walls, no heat can leak out of the cell. When a light beam passes through the cell, a light-induced heat flow will arise, that builds up temperature differences. In the steady state, the total heat flux will usually disappear. Hence the sum of (6.11) and (6.15) will be zero and also (6.16) must vanish. For a light beam with a given spatial intensity profile, and a negligible intensity decrease in the propagation direction, the temperature will vary linearly in the z direction, and the temperature variation in the normal direction is proportional to the intensity profile. We find

$$T(x,y,z)=\frac{\alpha}{2\lambda(A+\kappa_e)}\times U(x,y)\left[\frac{c_0^2}{\kappa_g}(mc_0^2-kT_0)(z-z_0)-c_0(mc_0^2-3kT)\right]+T_0 \,, \tag{6.23}$$

with T_0 and z_0 integration constants. We assumed that the temperature variations remain small. When the cell is surrounded by walls that are kept at a constant temperature T, a steady-state heat flux will arise, as described by (6.11), (6.15), and (6.16).

Analogous effects arise due to the light-induced contributions to the pressure anisotropy. When light passes through an open tube embedded in a larger vessel, the pressure at the open ends is constant. The diagonal part of the pressure tensor as described by G_0 and G_1, Eqs. (6.12) and (6.19) will not affect the hydrodynamic velocity in the propagation direction, at least when the intensity does not vary in this direction. Of course, the intensity variations over the cross sections of the cell give rise to pressure gradients in this direction, which will lead to a density profile in the steady state. The pressure on the tube wall is therefore modified by the light. A more interesting effect arises from the off-diagonal components in $\overleftrightarrow{\Pi}$, created by the intensity profile. These components are given in (6.20). In the steady state and for zero pressure difference at the ends of the tube the total off-diagonal components of $\overleftrightarrow{\Pi}_1$ must vanish, since the net force in the z direction on a volume element must be zero. Hence, the transverse gradient of the component v_z of the hydrodynamic velocity is proportional to the transverse gradient of U. We obtain from (6.17) and (6.20)

$$v_z(x,y)=\frac{\alpha}{\zeta\kappa_g(A+\kappa_e)}kTc_0U(x,y)+v_0 \ . \qquad (6.24)$$

The integration constant v_0 is determined by the wall properties. This effect of viscous flow, already predicted in Refs. 10 and 11, has recently been observed experimentally in CH_3F.[19] The net flow of particles induced by light in a simple gas may be understood by noting that in each point in the gas, the ground-state particles have suffered their last collision on a further distance than the excited particles, since the ground-state particles have a larger mean free path. In the presence of an intensity gradient, this leads to a noncompensating influx of momentum, and thereby to a force. In a way, the effect is analogous to light-induced drift, where now the momentum-absorbing role of the buffer gas is taken over by neighboring layers in the gas with a different intensity.

In general, also the diagonal terms in the pressure anisotropy $\overleftrightarrow{\Pi}$ can give rise to steady-state drift velocities in the absence of external pressure differences. When a tube is oriented in a direction that is not parallel to the propagation direction of the light, these diagonal elements give stresses along the tube axis, which will lead to viscous flow.

VII. CONCLUSIONS

We present a general formalism leading to evolution equations for the density n, the temperature T, and the hydrodynamic velocity $\mathbf{v}$ of a single-component gas in a nearly resonant radiation field. The method consists of elimination of the rapid variables, which are affected by collisions and radiative transitions. It may be viewed as a generalization of the Chapman-Enskog technique.[12,13] The velocity distributions of the two states involved in the radiative transition are expanded in a parameter of the order of the mean free path divided by a typical macroscopic distance. The zeroth and first order are determined, respectively, by (4.2) and (4.9). These distributions determine the heat flux $\mathbf{q}$ and the pressure anisotropy $\overleftrightarrow{\Pi}$ in terms of n, $\mathbf{v}$, T, and their gradients, and of the light intensity and its gradient. Substitution of these results for $\mathbf{q}$ and $\overleftrightarrow{\Pi}$ in the conservation laws (3.2) of particle number, momentum, and energy leads to macroscopic evolution equations for n, $\mathbf{v}$, and T. These equations generalize the Navier-Stokes equations, and contain in principle the full class of gas-kinetic effects of light.

A striking difference with standard gas kinetics is that now even the zeroth-order velocity distributions cannot be found exactly, and additional approximations are needed to obtain explicit results. As an example, we treat the case of excitation of a narrow velocity group, so that the fraction of excited atoms remains small. Furthermore, the thermalization of this narrow group is assumed to occur by a single collision, in analogy to the BGK model.[12] In this case, explicit expressions are obtained for the heat flux and the pressure anisotropy. The results are contained in (6.11), (6.12), and(6.15)–(6.20). Similar expressions were obtained in Refs. 9–11 on the basis of a rather special collision model. We feel that the merit of our method is that the effects arise naturally within a unified and systematic description. Furthermore, the general results of Sec. IV may serve as a basis for obtaining explicit results with quite different model assumptions.

The physical origin of the zeroth-order heat flux and pressure anisotropy is simply the modification of the total velocity distribution by the combined action of velocity-selective excitation and state-dependent collision rates. This purely local mechanism cannot modify n, $\mathbf{v}$, or T, due to the conservation laws. But higher moments of the velocity distribution, such as $\mathbf{q}$ and $\overleftrightarrow{\Pi}$, are affected in a way determined by the local intensity. First-order contributions to $\mathbf{q}$ and $\overleftrightarrow{\Pi}$ arise from the difference in mobility between ground-state and excited particles, due to their different mean free paths. Therefore the contribution from ground-state particles to the exchange of momentum and energy occurs on a slightly larger length scale than the contribution from excited particles. The net result is a nonzero exchange of energy or momentum between neighboring regions with different intensity. This explains that the first-order contributions to heat flux and pressure anisotropy are proportional to the intensity gradient.

The general formalism indicates the existence of many more gas-kinetic effects of light than obtained explicitly so far. When an appreciable fraction of the particles is excited, the transport coefficients must be modified, since the gas is basically a binary mixture. Furthermore, the transport coefficients will attain a tensorial character, due to the symmetry-breaking effect of the velocity-selective excitation. Finally, the pressure anisotropy may attain a contribution proportional to the gradient of the temperature, and likewise the heat flux may be directly affected by velocity gradients. These effects, which are implicit in our results of Sec. IV, need further investigation.

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