PHYSICAL REVIEW A VOLUME 40, NUMBER 1 JULY 1, 1989

Effects of a squeezed vacuum on a laser exhibiting phase locking: An application to a laser with injected atomic coherence

Ning Lu and János A. Bergou*
Center for Advanced Studies and Department of Physics and Astronomy, University of New Mexico, Albuquerque, New Mexico 87131
(Received 24 February 1989)

We study the steady-state operation and noise properties of a laser (or maser) exhibiting phase locking when an ordinary vacuum is replaced by a squeezed vacuum. Such a phase-sensitive laser can be realized by preparing the active medium initially in a coherent superposition of levels involved in the laser transition. We develop a general formalism to treat number-phase fluctuations and quadrature fluctuations. When applied to a one-photon laser with injected atomic coherence, we find that not only is a complete quenching of the (spontaneous-emission) quantum noise possible, but the field can actually be in a squeezed state. The variance in either amplitude or phase quadrature can be below the vacuum noise level by a factor of 2 (50%) at the expense of enhanced fluctuations in the other quadrature.

I. INTRODUCTION

The quantum theory of lasers was developed about two decades ago.[1–3] It proved to be an important step in understanding stochastic processes, in general, and statistical properties and noise performance of systems far from thermal equilibrium, in particular. Since then it has had numerous far-reaching generalizations,[4] many of them outside the area of quantum optics. Originally the theory was worked out to deal with incoherent atomic pumping. This means that active atoms are pumped to the upper levels of the lasing transition with no coherence between the different levels involved in the lasing operation. In density-operator language it corresponds to the vanishing of all off-diagonal elements of the initial atomic density operator. This approach is natural, if no special care is taken in the pumping process and the active atoms do not carry any phase information into the interaction region. In subsequent extensions of the theory, the assumption of incoherent pumping has always been tacitly assumed. This was the case in going from the traditional (one-mode, single-photon transition) laser to multimode lasers[5] or two-photon lasers.[6] It is not very surprising that the inherent quantum noise, which is due to spontaneous emission from the upper lasing level, is the same for all such systems. It is the phase-diffusion noise and its rate that was first derived theoretically by Schawlow and Townes.[7]

In recent works, however, the importance of atomic coherence has received much attention in connection with reaching beyond the traditionally accepted noise limits. In a series of recent papers[8] it has been demonstrated that complete quenching of the (spontaneous-emission) quantum noise from the relative phase of a two-mode laser is possible, if the two-mode lasing action takes place from two upper levels to a common lower level in a V-type three-level active medium and active atoms are prepared in a coherent superposition of two upper levels. This correlated-spontaneous-emission laser (CEL) effect shows the significance of atomic coherence in active systems. Likewise, in passive systems, it has been demonstrated[9] that the field emitted in spontaneous emission can be in a squeezed state,[10] i.e., the noise in one quadrature can be below the vacuum fluctuation limit. Very recently, we have shown[11] the intimate relationship between quantum noise quenching and squeezing. This led us to the prediction of an active system, the two-photon correlated-emission laser, which is capable of generating squeezed light of macroscopic intensity building up from ordinary vacuum.

Moreover, we have studied[12] a one-photon laser building up from an ordinary vacuum when two-level active atoms are prepared initially in a coherent superposition of the upper and lower levels. We have shown[12] that initial atomic coherence plays an important role in phase locking and noise reduction (via nonlinear process) in such a two-level one-photon laser. The photon-number distribution of the laser field can be exactly Poissonian and the laser-phase-diffusion coefficient is greatly reduced at the same time, leading to a laser field which is very close to that in a coherent state.

In another interesting series of recent papers[13,14] a different line of thought has been pursued. Namely, the effect of a broadband squeezed vacuum on quantum optical systems has been studied. In the present context, Ref. 14 is of particular interest where the effect of an injected squeezed vacuum on laser linewidth has been investigated. This work directed attention to the possibility of reducing the noise level in active systems (lasers) via the injection of a broadband squeezed vacuum.

These two different ways of reducing the inherent quantum noise in active optical systems, via correlated spontaneous emission and injected squeezed vacuum, naturally lead to the investigation of their combined effects (that is, how the inherent quantum noise is modified in a laser if the active medium is prepared in a coherent superposition of levels involved in the laser transition and the field builds up from the squeezed vacuum). The present work is devoted to the study of this problem.

The clarification of the noise performance of lasers is crucial to optical interferometry. At present, in the realm of high-precision measurements, optical interferometry provides one of the tools with the highest resolution. The resolution is often limited by the inherent quantum noise of the detection system. Any significant improvement on those traditionally accepted limits would bear an immediate impact on applications demanding a higher resolution than the currently accepted one. These applications include the laser gravity-wave detectors and laser gyroscopes. The main motivation of our research is the understanding of the ultimate quantum limits of resolution in optical interferometry.

The quantum theory of a traditional laser, as developed originally, deals with incoherent pumping and buildup of the field from the ordinary vacuum.[1–3] Recently, we have generalized[12] this theory for the case when initial atomic coherence is involved. These coherently pumped lasers exhibit phase locking of the laser fields and, besides photon-number variance, one can speak of the phase variance of the laser field in such a laser system. In Sec. II, we study in general the photon-number, phase, and quadrature variances of the laser fields exhibiting phase locking and discuss their relation. In so doing we follow the works by Drummond *et al.*[15] and find that the complex P representation is an appropriate tool to deal with the newly arising situation. Also, if appropriate care is taken, the Glauber-Sudarshan P representation[16,17] is an adequate tool to describe the noise properties. In Sec. III we include the possibility of shining a squeezed vacuum into the cavity and discuss the effects of a squeezed vacuum on laser operation and the quadrature variances of the laser field. In Sec. IV we apply this general formalism to the one-photon laser with two-level active medium (one-photon CEL). Our main finding is that not only is quenching of the spontaneous-emission noise of the active systems possible, but also the light field can actually be in a squeezed state. Finally, in Sec. V, we briefly summarize our main results and discuss their physical implications.

II. PHOTON-NUMBER, PHASE, AND QUADRATURE VARIANCES

We present in this section a general formalism for the variances in terms of the amplitude and phase of the laser field and in terms of two quadratures of the laser field. For a single-mode laser (or maser) exhibiting phase locking, such as a one-photon laser with injected atomic coherence[12] addressed in this paper, the master equation for the reduced field-density operator ρ in the interaction picture is obtained. Expanding ρ in diagonal coherent-state projection operators[16,17]

$$\rho=\int d^2v P(v,v^*,t)|v\rangle\langle v| , \tag{2.1}$$

where the coherent states $|v\rangle$ are eigenstates of the field annihilation operator a

$$a|v\rangle=v|v\rangle , \tag{2.2}$$

we obtain a Fokker-Planck equation in the Glauber-Sudarshan P representation

$$\frac{\partial}{\partial t}P(v,v^*,t)=\left[-\frac{\partial}{\partial v}d_v-\frac{\partial}{\partial v^*}d_{v^*}+2\frac{\partial^2}{\partial v\,\partial v^*}D_{v^*v}+\frac{\partial^2}{\partial v^2}D_{vv}+\frac{\partial^2}{\partial (v^*)^2}D_{v^*v^*}\right]P(v,v^*,t) , \tag{2.3}$$

where v and v^* are complex conjugate and

$$d_{v^*}=(d_v)^* , \tag{2.4a}$$

$$D_{v^*v^*}=(D_{vv})^* . \tag{2.4b}$$

The explicit expressions for the drift coefficient d_v and diffusion coefficients D_{v^*v} and D_{vv} do not concern us here. In order to study the properties of the laser amplitude (or photon number) and phase (for example, photon-number and phase variances), it is convenient to rewrite the above Fokker-Planck equation (2.3) in terms of laser intensity and phase variables I and φ via the relation[3,18] $v=\sqrt{I}e^{i\varphi}$,

$$\frac{\partial}{\partial t}P_1(I,\varphi,t)=\left[-\frac{\partial}{\partial I}d_I-\frac{\partial}{\partial \varphi}d_\varphi+\frac{\partial^2}{\partial I^2}D_{II}+\frac{\partial^2}{\partial \varphi^2}D_{\varphi\varphi}+2\frac{\partial^2}{\partial I\partial\varphi}D_{I\varphi}\right]P_1(I,\varphi,t) . \tag{2.5}$$

Here

$$P(v,v^*,t)=2P_1(I,\varphi,t) , \tag{2.6}$$

so that

$$\int P(v,v^*,t)d^2v=\int P_1(I,\varphi,t)dI\,d\varphi=1 . \tag{2.7}$$

In Eq. (2.5), d_I and d_φ are laser intensity- and phase-drift coefficients, D_{II} and $D_{\varphi\varphi}$ are photon-number- and phase-diffusion coefficients, respectively, and $D_{I\varphi}$ is a cross-diffusion coefficient representing the correlation between the intensity and phase. In the absence of initial atomic coherence, $D_{I\varphi}$ vanishes as in an ordinary laser.[18] In the presence of initial atomic coherence, all of these drift and diffusion coefficients are, in general, the functions of intensity I and phase φ. Again, their explicit forms are not important for our general discussion in this section.

The equations of motion for the intensity I and phase φ are found from Eq. (2.5) as

$$\frac{d}{dt}\langle I\rangle=\langle d_I\rangle , \tag{2.8a}$$

$$\frac{d}{dt}\langle \varphi\rangle=\langle d_\varphi\rangle . \tag{2.8b}$$

The photon-number and phase[19] variances can be obtained by calculating the following mean values with the distribution $P_1(I,\varphi,t)$:

$$\langle(\Delta\hat{n})^2\rangle = \langle:(\Delta\hat{n})^2:\rangle + \langle\hat{n}\rangle = \langle(\delta I)^2\rangle + \langle I\rangle \ , \tag{2.9a}$$

$$\langle(\Delta\varphi)^2\rangle = \langle(\delta\varphi)^2\rangle + \frac{1}{4\langle\hat{n}\rangle} \ , \tag{2.9b}$$

where :: denotes normally ordered variances, $\delta I = I - \langle I\rangle$, $\delta\varphi = \varphi - \langle\varphi\rangle$, and $\langle\hat{n}\rangle = \langle a^\dagger a\rangle = \langle I\rangle$. Note that $\langle(\delta I)^2\rangle$ and $\langle(\delta\varphi)^2\rangle$ are just normally ordered photon-number and phase variances. Along with $\langle\delta I\delta\varphi\rangle$, they obey the following equations:

$$\frac{d}{dt}\langle(\delta I)^2\rangle = 2\langle d_I\delta I\rangle + 2\langle D_{II}\rangle \ , \tag{2.10a}$$

$$\frac{d}{dt}\langle(\delta\varphi)^2\rangle = 2\langle d_\varphi\delta\varphi\rangle + 2\langle D_{\varphi\varphi}\rangle \ , \tag{2.10b}$$

$$\frac{d}{dt}\langle\delta I\delta\varphi\rangle = \langle d_I\delta\varphi\rangle + \langle d_\varphi\delta I\rangle + 2\langle D_{I\varphi}\rangle \ , \tag{2.10c}$$

which are obtained by using Eqs. (2.5) and (2.8). In the steady state, the laser intensity and phase are locked to the mean values $\langle I\rangle = n_0$ and $\langle\varphi\rangle = \varphi_0$ satisfying $d_I(n_0,\varphi_0)=0$ and $d_\varphi(n_0,\varphi_0)=0$, as indicated by Eqs. (2.8). The steady-state values of $\langle(\delta I)^2\rangle$ and $\langle(\delta\varphi)^2\rangle$ can be found from Eqs. (2.10) by setting $d/dt=0$ and expanding d_I, d_φ, D_{II}, $D_{\varphi\varphi}$, and $D_{I\varphi}$ around $I=n_0$, $\varphi=\varphi_0$ up to first order in δI and $\delta\varphi$. Generally one needs to solve three coupled first-order algebraic equations given by the three equations in Eqs. (2.10) to obtain $\langle(\delta I)^2\rangle_{SS}$ and $\langle(\delta\varphi)^2\rangle_{SS}$ as well as $\langle\delta I\delta\varphi\rangle_{SS}$. When $\partial d_I(n_0,\varphi_0)/\partial\varphi = \partial d_\varphi(n_0,\varphi_0)/\partial I = 0$, however, the stable laser-operation conditions are

$$\frac{\partial d_I(n_0,\varphi_0)}{\partial I} < 0, \quad \frac{\partial d_\varphi(n_0,\varphi_0)}{\partial\varphi} < 0 \ , \tag{2.11}$$

and the photon-number and phase variances are simply

$$\langle(\Delta\hat{n})^2\rangle_{SS} = n_0 + \frac{D_{II}(n_0,\varphi_0)}{|\partial d_I(n_0,\varphi_0)/\partial I|} \ , \tag{2.12a}$$

$$\langle(\Delta\varphi)^2\rangle_{SS} = \frac{1}{4n_0} + \frac{D_{\varphi\varphi}(n_0,\varphi_0)}{|\partial d_\varphi(n_0,\varphi_0)/\partial\varphi|} \ . \tag{2.12b}$$

Knowing $\langle(\Delta\hat{n})^2\rangle$, one also knows the normalized second-order correlation function

$$g^{(2)}(0) = 1 + \frac{\langle(\Delta\hat{n})^2\rangle - \langle\hat{n}\rangle}{\langle\hat{n}\rangle^2} \ . \tag{2.13}$$

A sub-Poissonian distribution $\langle(\Delta\hat{n})^2\rangle < \langle\hat{n}\rangle$ thus implies photon antibunching.

For light fields exhibiting nonclassical behavior such as sub-Poissonian distribution (i.e., photon antibunching) and squeezing, however, a well-behaved positive function for the Glauber-Sudarshan P representation does not exist.[10] In the following we choose the generalized (complex) P representation[15] to rediscuss the amplitude and phase noise of the laser field. In particular, for the sake of rigor, we use quadrature operators to study quadrature variances in the laser field.

We expand the reduced field operator ρ in nondiagonal coherent-state projection operators[15]

$$\rho = \int_{c,c'} dv\, dw P(v,w)\frac{|v\rangle\langle w^*|}{\langle w^*|v\rangle} \ , \tag{2.14a}$$

or in terms of the density-matrix elements

$$\rho_{nm} = \int_{c,c'} dv\, dw P(v,w) e^{-vw}\frac{v^n w^m}{\sqrt{n!m!}} \ , \tag{2.14b}$$

where v and w are not complex conjugates and c and c' are integration contours in the complex phase space of (v,w). The field master equation is then transformed into a Fokker-Planck equation in the complex P representation

$$\frac{\partial}{\partial t}P(v,w,t) = \left[-\frac{\partial}{\partial v}d_v - \frac{\partial}{\partial w}d_w + 2\frac{\partial^2}{\partial v\,\partial w}D_{wv} + \frac{\partial^2}{\partial v^2}D_{vv} + \frac{\partial^2}{\partial w^2}D_{ww}\right]P(v,w,t) \ . \tag{2.15}$$

Here the drift and diffusion coefficients have the same expressions as those in Eq. (2.3) with v^* replaced by w. This can be seen by comparing Eq. (2.14b) with the density-matrix-element version of Eq. (2.1). Consequently, the following relations hold:

$$d_w = d_v^*(v^*\to w,\ w^*\to v) \ , \tag{2.16a}$$

$$D_{ww} = D_{vv}^*(v^*\to w,\ w^*\to v) \ . \tag{2.16b}$$

Using the complex P representation, the expectation values of normally ordered operator products $(a^\dagger)^l a^k$ are

$$\langle(a^\dagger)^l a^k\rangle = \int_{c,c'} dv\, dw P(v,w) w^l v^k \ . \tag{2.17}$$

The equations of motion for the expectation values of the field operators a and $a^\dagger$ are found from Eqs. (2.17) and (2.15) as

$$\frac{d}{dt}\langle v\rangle = \langle d_v\rangle \ , \tag{2.18a}$$

$$\frac{d}{dt}\langle w\rangle = \langle d_w\rangle \ . \tag{2.18b}$$

Consequently, the steady-state locking values of v and w (i.e., v_0 and w_0) satisfy the equations $d_v(v_0,w_0)=0$ and $d_w(w_0,v_0)=0$. With the help of Eq. (2.16a) we conclude that

$$w_0 = v_0^* \ . \tag{2.19}$$

Toward studying laser amplitude and phase fluctuations, we define Hermitian quadrature operators as

$$a_1 = \frac{ae^{-i\varphi_0} + a^\dagger e^{i\varphi_0}}{2} \tag{2.20a}$$

and

$$a_2 = \frac{ae^{-i\varphi_0} - a^\dagger e^{i\varphi_0}}{2i} \ , \tag{2.20b}$$

which obey the commutation relation $[a_1,a_2]=\frac{1}{2}i$. Here φ_0 is a time-independent constant chosen to set $\langle a_2\rangle_{SS}=0$. Thus a_1 and a_2 are the in-phase and in-quadrature operators of the laser field in the steady state, respectively. Parallel to Eqs. (2.20), we introduce two complex variables

$$v_1=\frac{ve^{-i\varphi_0}+we^{i\varphi_0}}{2}\ , \tag{2.21a}$$

$$v_2=\frac{ve^{-i\varphi_0}-we^{i\varphi_0}}{2i}\ ; \tag{2.21b}$$

thus $\langle a_i\rangle=\langle v_i\rangle$, $i=1,2$. Converting the complex Fokker-Planck equation (2.15) to the new complex variables v_1 and v_2, we arrive at

$$\frac{\partial}{\partial t}P(v_1,v_2,t)=\left[-\frac{\partial}{\partial v_1}d_1-\frac{\partial}{\partial v_2}d_2+\frac{\partial^2}{\partial v_1^2}D_{11}+\frac{\partial^2}{\partial v_2^2}D_{22}+2\frac{\partial^2}{\partial v_1\partial v_2}D_{12}\right]P(v_1,v_2,t)\ , \tag{2.22}$$

where

$$d_1=\frac{d_v e^{-i\varphi_0}+d_w e^{i\varphi_0}}{2}\ , \tag{2.23a}$$

$$d_2=\frac{d_v e^{-i\varphi_0}-d_w e^{i\varphi_0}}{2i}\ , \tag{2.23b}$$

$$D_{11}=\tfrac{1}{4}(2D_{wv}+D_{vv}e^{-i2\varphi_0}+D_{ww}e^{i2\varphi_0})\ , \tag{2.23c}$$

$$D_{22}=\tfrac{1}{4}(2D_{wv}-D_{vv}e^{-i2\varphi_0}-D_{ww}e^{i2\varphi_0})\ , \tag{2.23d}$$

$$D_{12}=\frac{1}{4i}(D_{vv}e^{-i2\varphi_0}-D_{ww}e^{i2\varphi_0})\ . \tag{2.23e}$$

The equations of motion for the expectation values of quadrature operators are found from Eq. (2.22) as

$$\frac{d}{dt}\langle v_i\rangle=\langle d_i\rangle,\quad i=1,2\ . \tag{2.24}$$

We are now ready to calculate the variances of operators a_1 and a_2. First we find from Eqs. (2.17), (2.20), and (2.21),

$$\langle :(\Delta a_i)^2:\rangle=\langle(\Delta a_i)^2\rangle-\tfrac{1}{4}=\langle(\delta v_i)^2\rangle,\quad i=1,2 \tag{2.25}$$

where $\delta v_i=v_i-\langle v_i\rangle$ $(i=1,2)$. Then we obtain the equations of motion for $\langle\delta v_i\delta v_j\rangle$ $(i,j=1,2)$ from Eq. (2.22),

$$\frac{d}{dt}\langle(\delta v_i)^2\rangle=2\langle d_i\delta v_i\rangle+2\langle D_{ii}\rangle,\quad i=1,2 \tag{2.26a}$$

$$\frac{d}{dt}\langle\delta v_1\delta v_2\rangle=\langle d_1\delta v_2\rangle+\langle d_2\delta v_1\rangle+2\langle D_{12}\rangle\ . \tag{2.26b}$$

In the steady state, $\langle a_i\rangle_{SS}=v_{i0}$ $(i=1,2)$ are real, as can be seen from Eqs. (2.19) and (2.21). As mentioned above, φ_0 is chosen to set $v_{20}=0$. Similar to the previous discussion of steady-state photon-number and phase variances, the steady-state values of $\langle\delta v_i\delta v_j\rangle$ are readily obtained from Eqs. (2.26) by setting $d/dt=0$ on the left-hand side and expanding d_i,D_{ij} $(i,j=1,2)$ on the right-hand side up to first order in δv_1 and δv_2 around the steady-state locking point $v_{10}\neq0$, $v_{20}=0$. Usually one needs to solve three coupled first-order algebraic equations to obtain $\langle(\delta v_i)^2\rangle$ $(i=1,2)$. When $\partial d_1(v_{10},v_{20})/\partial v_2=\partial d_2(v_{10},v_{20})/\partial v_1=0$, however, these equations become uncoupled. The conditions for stable locking at point (v_{10},v_{20}) are

$$\frac{\partial d_i(v_{10},v_{20})}{\partial v_i}<0,\quad i=1,2 \tag{2.27}$$

and the steady-state variances are simply given by [see Eqs. (2.25) and (2.26a)]

$$\langle(\Delta a_i)^2\rangle_{SS}=\frac{1}{4}+\frac{D_{ii}(v_{10},v_{20})}{|\partial d_i(v_{10},v_{20})/\partial v_i|},\quad i=1,2\ . \tag{2.28}$$

The condition $D_{ii}(v_{10},v_{20})<0$ $(i=1$ or $2)$ implies squeezing, since then $\langle(\Delta a_i)^2\rangle_{SS}<\frac{1}{4}$.

In the steady state, since both v_1 and v_2 (or v and w) are locked and $\langle a_2\rangle_{SS}=v_{20}=0$, the laser amplitude (or photon-number) fluctuation is in the a_1 direction mainly while the laser-phase fluctuation is in the a_2 direction only. Consequently, when $\langle\hat n\rangle_{SS}=\langle wv\rangle_{SS}=v_{10}^2\gg1$, the laser-amplitude noise equals the variance of a_1 in the steady state [note that $r^2=n$, $2r(\delta r)=\delta n$]

$$\langle(\Delta r)^2\rangle_{SS}=\frac{1}{4\langle\hat n\rangle_{SS}}\langle(\Delta\hat n)^2\rangle_{SS}=\langle(\Delta a_1)^2\rangle_{SS}\ , \tag{2.29a}$$

and the normalized laser-phase noise is equal to the a_2's variance in the steady state

$$\langle\hat n\rangle_{SS}\langle(\Delta\varphi)^2\rangle_{SS}=\langle(\Delta\hat a_2)^2\rangle_{SS}\ . \tag{2.29b}$$

We see that, when the steady-state mean photon number is much larger than unity, squeezing in the a_1 quadrature is equivalent to sub-Poissonian light (i.e., photon antibunching) while squeezing in the a_2 quadrature means phase squeezing. In addition, the minimum uncertainty product for the photon number and phase is seen from Eqs. (2.29) and $[a_1,a_2]=\frac{1}{2}i$ to be

$$\langle(\Delta\hat n)^2\rangle_{SS}\langle(\Delta\varphi)^2\rangle_{SS}\geq\tfrac{1}{4}\ . \tag{2.30}$$

III. EFFECTS OF A SQUEEZED VACUUM ON LASERS EXHIBITING PHASE LOCKING

The vacuum in a laser arises from the leaking of outside vacuum through the laser output mirror into the laser cavity. As suggested by Gea-Banacloche,[14] one can shine a broadband squeezed-vacuum field into the cavity and, consequently, change the heat bath of the laser field from the usual vacuum to a squeezed vacuum (also plus absorption losses). It is our purpose in this section to study the effects of a squeezed vacuum on lasers exhibiting phase locking. We separate the cavity loss rate γ into two parts,[14,20] $\gamma=\gamma_t+\gamma_a$, with $\gamma_t(\gamma_a)$ representing transmission (absorption) loss of the laser field. The master equation for the intracavity field-density operator ρ can be written as the sum of the gain-related part and the loss-related part. In a squeezed vacuum, the loss-related part of the field master equation has been worked out,[21]

$$(\partial\rho/\partial t)_{\text{loss}} = -\tfrac{1}{2}[\gamma_a(\rho a^\dagger a - a\rho a^\dagger) + \gamma_t(N+1)(\rho a^\dagger a - a\rho a^\dagger) + \gamma_t N(\rho a a^\dagger - a^\dagger \rho a) + \gamma_t M(2a^\dagger \rho a^\dagger - a^\dagger a^\dagger \rho - \rho a^\dagger a^\dagger)] + \text{H.c.}, \quad (3.1)$$

where N and M satisfy the inequality $N(N+1) \geq |M|^2$. When $N=M=0$, ordinary vacuum case is recovered (assuming zero thermal photon). Following the general discussion in Sec. II, we find the loss-related parts (labeled by superscripts l) of the drift and diffusion coefficients in the Glauber-Sudarshan P representation

$$d_I^l = -\gamma I + \tfrac{1}{2}\gamma_t[N + |M|\cos(2\varphi - \psi)], \quad (3.2a)$$

$$d_\varphi^l = \frac{\gamma_t |M|}{2I}\sin(2\varphi - \psi), \quad (3.2b)$$

$$D_{II}^l = \gamma_t I[N + |M|\cos(2\varphi - \psi)], \quad (3.2c)$$

$$D_{\varphi\varphi}^l = \frac{\gamma_t}{4I}[N - |M|\cos(2\varphi - \psi)], \quad (3.2d)$$

$$D_{I\varphi}^l = -\tfrac{1}{2}\gamma_t |M|\sin(2\varphi - \psi), \quad (3.2e)$$

where $M = |M|e^{i\psi}$ has been used. Note that d_φ^l and $D_{I\varphi}^l$ vanish when $2\varphi = \psi$. When the mean photon number $n_0 = \langle I \rangle \gg 1$, as is generally the case, the loss-related part of the intensity drift coefficient can be approximated as $d_I^l = -\gamma I$, same as in the usual vacuum. In lasers with injected atomic coherence, $|d_\varphi| \gg |d_\varphi^l|$ when $n_0 \gg 1$, as shown in Sec. IV, so the loss-related part of the phase-drift coefficient can be neglected in this case, i.e., $d_\varphi^l = 0$, again the same as in the usual vacuum. Consequently, we conclude that the steady-state operation points (n_0, φ_0) of coherently pumped lasers do not change when the ordinary vacuum is replaced by a squeezed vacuum.

In the complex P representation, the loss-related parts of the drift and diffusion coefficients are found to be

$$d_1^l = -\tfrac{1}{2}\gamma v_1, \quad (3.3a)$$

$$d_2^l = -\tfrac{1}{2}\gamma v_2, \quad (3.3b)$$

$$D_{11}^l = \tfrac{1}{4}\gamma_t[N + |M|\cos(2\varphi_0 - \psi)], \quad (3.3c)$$

$$D_{22}^l = \tfrac{1}{4}\gamma_t[N - |M|\cos(2\varphi_0 - \psi)], \quad (3.3d)$$

$$D_{12}^l = -\tfrac{1}{4}\gamma_t |M|\sin(2\varphi_0 - \psi). \quad (3.3e)$$

Equations (3.3a) and (3.3b) also indicate that the steady-state locking value (v_0, w_0) is not influenced by the parameter N or M of the squeezed vacuum.

The diffusion coefficients, however, change with N and M and are phase sensitive. In the following we consider a squeezed vacuum having a minimum-uncertainty product $N(N+1) = |M|^2$. In this case we can set

$$N = \sinh^2 r, \quad |M| = \sinh r \cosh r, \quad (3.4)$$

where $r > 0$ is the usual squeezing parameter. Substituting Eq. (3.4) into Eqs. (3.3c)–(3.3e), one obtains

$$D_{11}^l = \tfrac{1}{8}\gamma_t(e^{2r} - 1), \quad D_{22}^l = \frac{\gamma_t}{8}(e^{-2r} - 1), \quad \text{when } \varphi_0 = \tfrac{1}{2}\psi, \tfrac{1}{2}\psi + \pi, \quad (3.5a)$$

$$D_{11}^l = \tfrac{1}{8}\gamma_t(e^{-2r} - 1), \quad D_{22}^l = \frac{\gamma_t}{8}(e^{2r} - 1), \quad \text{when } \varphi_0 = \tfrac{1}{2}(\psi \pm \pi). \quad (3.5b)$$

D_{12} vanishes in both situations. Depending on the phase difference $2\varphi_0 - \psi$, either D_{11}^l or D_{22}^l can be negative and thus one of the total diffusion coefficients, either D_{11} or D_{22}, can be reduced.

Although it has no effect on the first moments of a laser exhibiting phase locking, a squeezed vacuum changes the second moments of such a laser. It follows from Eqs. (2.25), (2.28), and (3.5) that the contributions to the a_i's variance from the squeezed vacuum are proportional to D_{ii}^l at the steady-state locking point and read

$$\langle(\Delta a_i)^2\rangle_{\text{SS}}^l = \frac{\gamma_t(e^{\pm 2r} - 1)}{8|\partial d_i(v_{10}, v_{20})/\partial v_i|}, \quad \text{when } \varphi_0 = \tfrac{1}{2}\psi, \tfrac{1}{2}\psi + \pi, \quad (3.6a)$$

$$\langle(\Delta a_i)^2\rangle_{\text{SS}}^l = \frac{\gamma_t(e^{\mp 2r} - 1)}{8|\partial d_i(v_{10}, v_{20})/\partial v_i|}, \quad \text{when } \varphi_0 = \tfrac{1}{2}(\psi \pm \pi). \quad (3.6b)$$

Here the upper signs are for $i=1$ and the lower ones for $i=2$. One sees that the squeezed vacuum can reduce one quadrature variance with the expense of an increase in the other quadrature. A proper choice for $2\varphi_0 - \psi$ will enable us to reduce the field's noise in either amplitude or phase quadrature.

IV. COHERENTLY PUMPED TWO-LEVEL LASER IN A SQUEEZED VACUUM

A coherently pumped two-level laser in an ordinary vacuum has been studied in Ref. 12. We consider a coherently pumped two-level laser[12] in a squeezed vacuum in this section. We are interested in the case where the jth atom is injected into the laser cavity at time t_j with its initial density matrix in the Schrödinger picture

$$\rho^j(t_j) = \begin{bmatrix} \rho_{aa} & \bar{\rho}_{ab} e^{-i\nu t_j} \\ \bar{\rho}_{ba} e^{i\nu t_j} & \rho_{bb} \end{bmatrix}, \quad (4.1)$$

where a and b refer to the upper and lower levels, respectively; ν is the laser frequency; and ρ_{aa}, ρ_{bb} and $\bar{\rho}_{ab} = \bar{\rho}_{ba}^*$ are the same for all atoms. The nonlinear master equation for the reduced field density operator ρ (in the interaction picture) and the corresponding Fokker-Planck equation in the Glauber-Sudarshan P representation, have been obtained in Ref. 12 for an ordinary vacuum. Based on the discussion in Sec. II and the properties of the squeezed vacuum given in Sec. III we get, for a squeezed vacuum now, total drift and diffusion coefficients in the complex P representation under the initial atomic condition (4.1),

$$d_v=\frac{v}{2}\left[\frac{\alpha(\rho_{aa}-\rho_{bb})(1-i\delta)}{1+wv\beta/\alpha}-\gamma+2i(\nu-\Omega)\right]$$
$$-i\left[S\bar{\rho}_{ab}\left[1+\frac{2g^2wv}{\Gamma(\Gamma-i\Delta)}\right]\right.$$
$$\left.+S^*\bar{\rho}_{ba}\frac{2g^2v^2}{\Gamma(\Gamma+i\Delta)}\right]\frac{1}{1+wv\beta/\alpha}\,, \tag{4.2a}$$

$$D_{wv}=\frac{\alpha\rho_{aa}+\frac{1}{4}\beta(1+\delta^2)(\rho_{aa}+\rho_{bb})wv}{2(1+wv\beta/\alpha)}$$
$$-\frac{\beta(\rho_{aa}-\rho_{bb})wv}{4(1+wv\beta/\alpha)^2}+\frac{(iS\bar{\rho}_{ab}w-iS^*\bar{\rho}_{ba}v)\beta/2\alpha}{2(1+wv\beta/\alpha)^2}$$
$$+\left[\frac{iS\bar{\rho}_{ab}w}{\Gamma(\Gamma-i\Delta)}-\frac{iS^*\bar{\rho}_{ba}v}{\Gamma(\Gamma+i\Delta)}\right]\frac{g^2}{1+wv\beta/\alpha}+\tfrac{1}{2}\gamma_t N\,, \tag{4.2b}$$

$$D_{vv}=\frac{\beta(\rho_{aa}-\rho_{bb})(i\delta-1)v^2}{4(1+wv\beta/\alpha)^2}-\frac{\beta(\rho_{aa}+\rho_{bb})(1+\delta^2)v^2}{8(1+wv\beta/\alpha)}$$
$$+\frac{iS\bar{\rho}_{ab}v}{1+wv\beta/\alpha}\left[\left[1+\frac{2g^2wv}{\Gamma(\Gamma-i\Delta)}\right]\frac{\beta/2\alpha}{1+wv\beta/\alpha}\right.$$
$$\left.-\frac{g^2}{\Gamma(\Gamma-i\Delta)}\right]$$
$$+iS^*\bar{\rho}_{ba}\frac{2g^2v^3}{\Gamma(\Gamma+i\Delta)}\frac{\beta/2\alpha}{(1+wv\beta/\alpha)^2}+\tfrac{1}{2}\gamma_t M\,, \tag{4.2c}$$

where

$$\alpha=\frac{2r_a g^2}{\Gamma^2+\Delta^2},\quad \beta=\frac{8r_a g^4}{(\Gamma^2+\Delta^2)^2},\quad S=\frac{r_a g}{\Gamma+i\Delta}\,,$$
$$\delta=\Delta/\Gamma,\quad \Delta=\omega_{ab}-\nu\,, \tag{4.3}$$

r_a is the atomic injection rate, g is the coupling constant, Γ is the atomic decay rates (taken to be the same for both levels), $\hbar\omega_{ab}$ is the energy difference between levels a and b, and Ω is the empty cavity mode frequency. Note that in deriving Eqs. (4.2) 1 is neglected as compared with wv. Also d_w and D_{ww} are given by Eqs. (2.16) and (4.2).

We focus on the resonant case $\Omega=\omega_{ab}$ in the following discussion. From symmetry consideration we can set $\nu=\Omega$, thus $\Delta=0$, as in an ordinary two-level laser (i.e., $\bar{\rho}_{ab}=0$). Substituting Eqs. (4.2a) and (2.16a) into Eqs. (2.23) and using Eqs. (2.21), one finds that when $\Delta=0$,

$$d_1=\frac{v_1}{2}\left[\frac{\alpha(\rho_{aa}-\rho_{bb})}{1+(v_1^2+v_2^2)\beta/\alpha}-\gamma\right]+\frac{|S\bar{\rho}_{ab}|\sin(\theta-\varphi_0)}{1+(v_1^2+v_2^2)\beta/\alpha}$$
$$+\frac{v_2|S\bar{\rho}_{ab}|\beta/\alpha}{1+(v_1^2+v_2^2)\beta/\alpha}$$
$$\times[v_1\cos(\theta-\varphi_0)+v_2\sin(\theta-\varphi_0)]\,, \tag{4.4a}$$
$$d_2=\frac{v_2}{2}\left[\frac{\alpha(\rho_{aa}-\rho_{bb})}{1+(v_1^2+v_2^2)\beta/\alpha}-\gamma\right]-\frac{|S\bar{\rho}_{ab}|\cos(\theta-\varphi_0)}{1+(v_1^2+v_2^2)\beta/\alpha}$$
$$-\frac{v_1|S\bar{\rho}_{ab}|\beta/\alpha}{1+(v_1^2+v_2^2)\beta/\alpha}$$
$$\times[v_1\cos(\theta-\varphi_0)+v_2\sin(\theta-\varphi_0)]\,, \tag{4.4b}$$

where $\theta=\arg\bar{\rho}_{ab}$. In the steady state, v_1 and v_2 are locked to real values $v_1=v_{10}$, $v_2=v_{20}$ satisfying $d_1(v_{10},v_{20})=0$ and $d_2(v_{10},v_{20})=0$ (cf. Sec. II). As discussed in Sec. II, φ_0 is chosen such that $v_{20}=0$. In the present case we find

$$\varphi_0=\theta-\tfrac{1}{2}\pi \tag{4.5}$$

(another solution $\varphi_0'=\theta+\frac{1}{2}\pi$ is not stable). The mean photon number $n_0\equiv v_{10}^2$ is determined by the equation

$$\frac{\alpha(\rho_{aa}-\rho_{bb})}{1+n_0\beta/\alpha}-\gamma+\frac{2|S\bar{\rho}_{ab}|}{\sqrt{n_0}(1+n_0\beta/\alpha)}=0\,. \tag{4.6}$$

It is easy to see from Eqs. (4.4) that at the locking point $v_{10}=\sqrt{n_0}$, $v_{20}=0$, with φ_0 given by Eq. (4.5),

$$\frac{\partial d_1(v_{10},v_{20})}{\partial v_2}=\frac{\partial d_2(v_{10},v_{20})}{\partial v_1}=0\,, \tag{4.7}$$

and

$$\partial d_2(v_{10},v_{20})/\partial v_2=-|S\bar{\rho}_{ab}|/v_{10}<0\,.$$

Substituting Eqs. (4.2b), (4.2c), and (2.16b) into Eqs. (2.23c)–(2.23e) and using Eqs. (2.21), we find at the locking point with φ_0 given by Eq. (4.5),

$$D_{11}(v_{10},v_{20})=\frac{\alpha}{4(1+n_0\beta/\alpha)^2}$$
$$\times\left[\rho_{aa}+\rho_{bb}\frac{n_0\beta}{\alpha}-2|\bar{\rho}_{ab}|\left[\frac{n_0\beta}{\alpha}\right]^{1/2}\right]$$
$$+\tfrac{1}{4}\gamma_t[N+|M|\cos(2\varphi_0-\psi)]\,, \tag{4.8a}$$
$$D_{22}(v_{10},v_{20})=\frac{\alpha}{4(1+n_0\beta/\alpha)}\left[\rho_{aa}+(\rho_{aa}+\rho_{bb})\frac{n_0\beta}{2\alpha}\right.$$
$$\left.-|\bar{\rho}_{ab}|\left[\frac{n_0\beta}{\alpha}\right]^{1/2}\right]$$
$$+\tfrac{1}{4}\gamma_t[N-|M|\cos(2\varphi_0-\psi)]\,, \tag{4.8b}$$
$$D_{12}(v_{10},v_{20})=-\tfrac{1}{4}\gamma_t|M|\sin(2\varphi_0-\psi)\,. \tag{4.8c}$$

Because of Eqs. (4.7) we can use Eqs. (2.28) to obtain the steady-state variances of the operators a_1 and a_2 in which $D_{ii}(v_{10},v_{20})$ $(i=1,2)$ is given by Eqs. (4.8) and $\partial d_i(v_{10},v_{20})/\partial v_i$ is readily obtainable from Eqs. (4.4) with the help of Eqs. (4.5) and (4.6). Moreover, $D_{ii}(v_{10},v_{20})<0$ $(i=1$ or $2)$ means squeezing. The variances depend on the squeeze angle ψ of the squeezed vacuum since $D_{ii}(v_{10},v_{20})$ does so too.

Before we proceed to give an example, it is worthwhile here to compare quadrature variances with photon-number and phase variances for $\Delta=0$. At the locking point $v_1=\sqrt{n_0}$, $v_2=0$ or, in terms of laser intensity and phase, $I=n_0$, $\varphi=\varphi_0=\theta-\frac{1}{2}\pi$. A comparison of Eqs. (4.8) with Eqs. (3.2) plus Eqs. (4.17) in Ref. 12 gives

$$D_{II}(n_0,\varphi_0)=4n_0D_{II}(v_{10},v_{20}) , \tag{4.9a}$$

$$D_{\varphi\varphi}(n_0,\varphi_0)=n_0^{-1}D_{\varphi\varphi}(v_{10},v_{20}) , \tag{4.9b}$$

$$D_{I\varphi}(n_0,\varphi_0)=2D_{12}(v_{10},v_{20}) , \tag{4.9c}$$

and a comparison of Eqs. (4.4) with Eqs. (4.5) in Ref. 12, with little algebra, shows

$$\frac{\partial d_1(v_{10},v_{20})}{\partial v_1}=\frac{\partial d_I(n_0,\varphi_0)}{\partial I} , \tag{4.10a}$$

$$\frac{\partial d_2(v_{10},v_{20})}{\partial v_2}=\frac{\partial d_\varphi(n_0,\varphi_0)}{\partial \varphi}=-\frac{|S\bar{\rho}_{ab}|}{\sqrt{n_0}} . \tag{4.10b}$$

Combining Eqs. (4.10) with Eqs. (2.12) and (2.28), one arrives at

$$\langle(\Delta\hat{n})^2\rangle_{SS}=4n_0\langle(\Delta a_1)^2\rangle_{SS} , \tag{4.11a}$$

$$\langle(\Delta\varphi)^2\rangle_{SS}=n_0^{-1}\langle(\Delta a_2)^2\rangle_{SS} , \tag{4.11b}$$

which are just Eqs. (2.29) obtained from simple physical arguments.

We now look at an interesting case examined in Ref. 12,

$$\rho_{aa}=1-\rho_{bb}=1-\frac{\gamma}{\alpha},\quad |\bar{\rho}_{ab}|=(\rho_{aa}\rho_{bb})^{1/2} , \tag{4.12}$$

and correspondingly [satisfying Eq. (4.6) and being stable]

$$n_0=\frac{\alpha}{\gamma}\left[\frac{\alpha-\gamma}{\beta}\right] . \tag{4.13}$$

Using Eqs. (4.3), (4.5), (4.12), and (4.13), we find from Eqs. (4.4)

$$\frac{\partial d_1(v_{10},v_{20})}{\partial v_1}=\frac{\partial d_2(v_{10},v_{20})}{\partial v_2}=-\gamma . \tag{4.14}$$

Substituting Eqs. (4.14), (4.8), (3.4), (4.12), and (4.13) into Eqs. (2.28), we get

$$\langle(\Delta a_1)^2\rangle_{SS}=\frac{1}{4}-\frac{\gamma_t(1-e^{-2r})}{8\gamma} ,\quad \psi=2\varphi_0+\pi=2\theta , \tag{4.15a}$$

$$\langle(\Delta a_2)^2\rangle_{SS}=\frac{1+(\alpha/\gamma)}{8}-\frac{\gamma_t(1-e^{-2r})}{8\gamma} ,\quad \psi=2\varphi_0=2\theta-\pi . \tag{4.15b}$$

The first terms in Eqs. (4.15) have been found in the case of an ordinary vacuum (see Ref. 12), whereas the second terms (proportional to γ_t) represent the effect of the squeezed vacuum, in agreement with Eqs. (3.6) and (4.14). Squeezing occurs when either $\langle(\Delta a_1)^2\rangle_{SS}<\frac{1}{4}$ or $\langle(\Delta a_2)^2\rangle_{SS}<\frac{1}{4}$. We recall that $\gamma=\gamma_t+\gamma_a$. In the limit $\gamma_a\ll\gamma_t$, $e^{-2r}\ll 1$, Eqs. (4.15) and (4.11) yield (1) for $\psi=2\varphi_0+\pi$, $\langle(\Delta a_1)^2\rangle_{SS}\approx\frac{1}{8}$, i.e., 50% squeezing in the laser amplitude and sub-Poissonian photon statistics $\langle(\Delta\hat{n})^2\rangle_{SS}=\frac{1}{2}n_0$; and (2) for $\psi=2\varphi_0$, $\langle(\Delta a_2)^2\rangle_{SS}\approx\alpha/8\gamma$, $\langle(\Delta\varphi)^2\rangle_{SS}=\alpha/(8n_0\gamma)$, i.e., phase squeezing occurs if $\alpha<2\gamma$, with a maximum of nearly 50% phase squeezing if $\alpha\approx\gamma$. For example, for a hypothetical case with $\gamma_a=0.05\gamma_t$, $e^{-2r}=0.2$ (80% squeezing in the squeezed vacuum), and $\alpha=1.2\gamma$, one obtains (1) for $\psi=2\varphi_0+\pi$, $\langle(\Delta a_1)^2\rangle_{SS}=\frac{1}{4}\times 0.62$, i.e., 38% laser amplitude squeezing; and (2) for $\psi=2\varphi_0$, $\langle(\Delta a_2)^2\rangle_{SS}=\frac{1}{4}\times 0.72$, i.e., 28% laser phase squeezing.

Thus we have shown that one can obtain a squeezed laser field in a two-level laser if the atoms are initially prepared in a coherent superposition of the upper and lower levels and the laser operation builds up from a squeezed vacuum. Without the initial atomic coherence a squeezed input vacuum is able, at best, to reduce the diffusion coefficient in one quadrature (phase or amplitude diffusion) by a factor of 2, but never produces a squeezed field (see Ref. 14). Thus the role of atomic coherence is crucial in reducing fluctuations below the vacuum level.

V. SUMMARY

The quantum theory of lasers was originally developed for incoherent pumping (i.e., with no initial atomic coherence between the lasing levels, which is reflected in the fact that the initial atomic density operator is diagonal) and for buildup of the laser field from an ordinary vacuum. In the present paper we have generalized this "standard" quantum theory to include the effects of the atomic coherence and squeezed vacuum. In so doing our main theoretical tool has been the reduced density operator for the field only. As is often the case in laser physics and quantum optics a c-number representation of the density matrix is more favorable. When squeezing is present the complex P representation turns out to be very useful, since the distribution function $P(v,w,t)$ satisfies a Fokker-Planck equation which can be obtained from the Fokker-Planck equation for the Glauber-Sudarshan representation $P(v,v^*,t)$ by just replacing v^* with w. This correspondence at the same time indicates that the Glauber-Sudarshan (or diagonal) P representation can also be used to calculate variances if the resulting $P(v,v^*,t)$ is not interpreted as a quasiprobability distribution function.

In the next step, we included the effect of a squeezed vacuum into the Fokker-Planck equation which now simultaneously accounts for initial atomic coherence and the buildup from the squeezed vacuum. The squeezed vacuum changes the diffusion coefficients and quadrature variances, but does not affect the laser intensity I_0 and phase φ_0.

As an application, we have investigated the case of a traditional one-photon laser with two-level active atoms when initial atomic coherence between the lasing levels is present and the laser oscillation builds up from the squeezed vacuum. We have found that with the help of atomic coherence a significant part of the spontaneous-emission noise can be eliminated and a correlated-emission laser operation is possible.[12] With the further inclusion of a squeezed vacuum we have found that the field can actually be in a squeezed state with stable squeezing in the amplitude or phase quadrature. With a realistic choice of parameter values, nearly 50% squeezing in either quadrature is possible in the intracavity field.

ACKNOWLEDGMENTS

We gratefully acknowledge useful discussions with Professor M. O. Scully and the support of the U.S. Office of Naval Research.

---

*On leave from the Central Research Institute for Physics, P.O. Box 49, H-1525 Budapest 114, Hungary.

[1]M. Sargent III, M. O. Scully, and W. E. Lamb, Jr., *Laser Physics* (Addison-Wesley, Reading, MA, 1974).

[2]H. Haken, in *Light and Matter,* Vol. XXV/2C of *Handbüch der Physik,* edited by L. Genzel (Springer, Berlin, 1970).

[3]W. H. Louisell, *Quantum Statistical Properties of Radiation* (Wiley, New York, 1974).

[4]For an overview see, e.g., H. Haken, *Synergetics, An Introduction,* 2nd ed. (Springer, Berlin, 1978).

[5]S. Singh and M. S. Zubairy, Phys. Rev. A **21**, 281 (1980).

[6]M. D. Reid and D. F. Walls, Phys. Rev. A **28**, 332 (1983).

[7]A. L. Schawlow and C. H. Townes, Phys. Rev. **112**, 1940 (1958).

[8]M. O. Scully, Phys. Rev. Lett. **55**, 2802 (1985); M. O. Scully and M. S. Zubairy, Phys. Rev. A **35**, 752 (1987); W. Schleich and M. O. Scully, *ibid.* **37**, 1261 (1988); J. Bergou, M. Orszag, and M. O. Scully, *ibid.* **38**, 754 (1988).

[9]I. R. Senitzky, J. Opt. Soc. Am. B **1**, 879 (1984); K. Wódkiewicz, P. L. Knight, S. J. Buckle, and S. M. Barnett, Phys. Rev. A **35**, 2567 (1987).

[10]For reviews of squeezed states see D. F. Walls, Nature **306**, 141 (1983); R. Loudon and P. L. Knight, J. Mod. Opt. **34**, 709 (1987).

[11]M. O. Scully, K. Wódkiewicz, M. S. Zubairy, J. Bergou, N. Lu, and J. Meyer ter Vehn, Phys. Rev. Lett. **60**, 1832 (1988).

[12]N. Lu and J. A. Bergou, Phys. Rev. A **40**, 237 (1989).

[13]C. W. Gardiner, Phys. Rev. Lett. **56**, 1917 (1986); H. J. Carmichael, A. S. Lane, and D. F. Walls, *ibid.* **58**, 2539 (1987).

[14]J. Gea-Banacloche, Phys. Rev. Lett. **59**, 543 (1987).

[15]P. D. Drummond and C. W. Gardiner, J. Phys. A **13**, 2353 (1980); P. D. Drummond, C. W. Gardiner, and D. F. Walls, Phys. Rev. A **24**, 914 (1981).

[16]R. J. Glauber, Phys. Rev. **131**, 2766 (1963).

[17]E. C. G. Sudarshan, Phys. Rev. Lett. **10**, 277 (1963).

[18]M. Lax and W. H. Louisell, Phys. Rev. **185**, 568 (1969).

[19]J. Bergou, M. Orszag, M. O. Scully, and K. Wódkiewicz, Phys. Rev. A **39**, 5136 (1989).

[20]Y. Yamamoto, S. Machida, and O. Nilsson, Phys. Rev. A **34**, 4025 (1986).

[21]C. W. Gardiner and M. J. Collett, Phys. Rev. A **31**, 3761 (1985).