Homodyne and heterodyne detection of squeezed vacuum states

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We use our interpretation of homodyne and heterodyne detection of squeezed states, viz., quantum correlation between photons as expressed in second-order correlation functions, to study various cases. In particular, we introduce the coherence time of the local oscillator and show its influence on the possible noise reduction. We also study the case in which the local oscillator is not much stronger than the squeezed state and analyze the influence of phase jitter. This approach could explain some experimental results and suggest improvements in detection schemes.

I. INTRODUCTION

Squeezed states of light[1,2] (SS's) are generally understood as states whose quantum fluctuations in one quadrature are reduced under the shot-noise limit (SNL), also called "vacuum noise." We recently showed that the SNL is not a characteristic of the field only but is indeed related to the measurement process.[3] It is the quantum noise of measurement of a coherent state (CS) and comes from the fact that the CS is not an eigenstate of the measured operator. For this reason, we should avoid speaking about the noise in the field itself but mention only the noise in the photocurrent. A CS for instance is a perfectly well-defined state that has a well-defined time evolution and does not fluctuate. However, since it is not an eigenstate of the number operator, a measurement of the number of photons gives a random result whose fluctuation is the SNL. Should we perform another kind of measurement, we should get a completely different noise.

By analyzing the influence of a beam splitter on photon statistics,[4] we explained why people consider the SNL as "vacuum noise." It is in fact related to the zero-temperature Langevin operator needed to conserve the commutation relations of the field operator, when photons are randomly deleted from a beam. Equivalent treatments of the measured uncertainties can be made either with normally ordered operators[4-6] or with vacuum fluctuation fields interfering with the local oscillator. We adopt here the first approach. SS's are states with some kind of quantum correlation between the photons. These correlations in the field can be expressed as a noise reduction in the photocurrent in homodyne or heterodyne experiments.[7-10] They also account for noise reduction in the differenced photocurrent on twin laser beams[11,12] and it has also been suggested that they could be detected by cross correlation between two photodetectors,[13] or in interference experiments.[14]

In our work,[4] we analyzed the two-port heterodyne detection scheme, and showed that the relevant functions of the field are the various second-order correlation functions at the detectors. However, we restricted ourselves to a perfect local oscillator (LO) much stronger than the SS, and we assumed a given squeezing, without relating it to its sources. We now want to remove these limitations, and analyze the cases of homodyne detection of a degenerate squeezed vacuum (as created by a degenerate parametric amplifier under threshold[8,10]), and of heterodyne detection of a nondegenerate squeezed vacuum (as created by four-wave mixing[7,9]). Since the creation of SS's by these two processes has been studied in great detail in various papers,[15-19] we shall use the simplest models (classical nonlinear susceptibility of the medium in a cavity with losses) that still give squeezing, and emphasize the differences between our results and others for the detection process. In the usual approach,[20-24] homodyne and heterodyne detection is considered to measure the quadratures of the SS and therefore the influence of the LO is neglected. Our approach,[3,4] relating all experimental quantities to the second-order correlation functions at the detectors, enables us to analyze this influence and to get new results, in particular with respect to the bandwidth of the noise reduction. We recently applied it to study the photodetection statistics of a number state.[25]

In Sec. II we write the noise in the differenced photocurrent as a function of a certain correlation function of both the LO and the SS. This enables us to analyze the case where the LO is not much stronger than the SS. In Sec. III we shall demonstrate the influence of the LO on the photodetection noise spectrum in two simple examples. In Sec. IV we introduce phase jitter and suggest ways of reducing its influence. We summarize our results and conclude in Sec. V.

II. TWO-PORT HOMODYNE AND HETERODYNE DETECTION

The well-known experimental scheme is shown in Fig. 1. One mixes the LO field and the SS at a beam splitter (BS), gets current $\langle i_1 \rangle$ and $\langle i_2 \rangle$ at the two photomultipliers (PM1 and PM2), subtracts them, and sends them to a frequency analyzer in order to get the noise spectrum. Following our previous approach, we write all experimental quantities in terms of normally ordered (NO) averages. Therefore, as explained in our previous work,[4] we do not need to introduce any "vacuum field" at the BS, and the relevant part of the fields after splitting is

$$E_1^{(+)}(t)=SE^{(+)}(t)+R\varepsilon^{(+)}(t)\ , \tag{1a}$$

$$E_2^{(+)}(t)=-RE^{(+)}(t)+S\varepsilon^{(+)}(t)\ , \tag{1b}$$

where S (respectively, R) is the real transmission (reflection) coefficient. $E(t)$ is the squeezed field before splitting and $\varepsilon(t)$ is the coherent LO. Let us again emphasize that we can use these fields only in NO expressions. In all this work, we shall deal only with steady states, for both the LO and the SS, and therefore the time enters only via the time differences in the various correlation functions.

In order to find the noise in the current, we need the three correlation functions $g_1^{(2)}(\tau)$, $g_2^{(2)}(\tau)$, $g_{12}^{(2)}(\tau)$:

$$g_1^{(2)}(\tau)=\frac{\langle E_1^{(-)}(t)E_1^{(-)}(t+\tau)E_1^{(+)}(t+\tau)E_1^{(+)}(t)\rangle}{\langle E_1^{(-)}(t)E_1^{(+)}(t)\rangle\langle E_1^{(-)}(t+\tau)E_1^{(+)}(t+\tau)\rangle} \tag{2}$$

[and similarly for $g_2^{(2)}(\tau)$],

$$g_{12}^{(2)}(\tau)=\frac{\langle E_1^{(-)}(t)E_2^{(-)}(t+\tau)E_2^{(+)}(t+\tau)E_1^{(+)}(t)\rangle}{\langle E_1^{(-)}(t)E_1^{(+)}(t)\rangle\langle E_2^{(-)}(t+\tau)E_2^{(+)}(t+\tau)\rangle}\ . \tag{3}$$

Then, from Eqs. (13) and (15) of Ref. 4, we get the spectrum of the noise in the current

$$\begin{aligned}N^2(\omega)=&\frac{eG}{2\pi}(\langle i_1\rangle+\langle i_2\rangle)\\&+\frac{\langle i_1\rangle^2}{2\pi}\int_{-\infty}^{\infty}[g_1^{(2)}(\tau)-1]e^{-i\omega\tau}d\tau\\&+\frac{\langle i_2\rangle^2}{2\pi}\int_{-\infty}^{\infty}[g_2^{(2)}(\tau)-1]e^{-i\omega\tau}d\tau\\&-2\frac{\langle i_1\rangle\langle i_2\rangle}{2\pi}\int_{-\infty}^{\infty}[g_{12}^{(2)}(\tau)-1]e^{-i\omega\tau}d\tau\end{aligned} \tag{4}$$

where $\langle i_1\rangle$ and $\langle i_2\rangle$ are the currents at detectors 1 and 2, and G is the gain of both detectors.

To calculate the various correlation functions, we now specialize to the case of a 50/50 beam splitter ($R=S=1/\sqrt{2}$), and of a squeezed vacuum (the coherent part of the SS is zero). This type of SS is the one that has been created experimentally.[7-10] In this case all the odd powers of the SS operators have zero expectation value (this result is valid for all models of creation of SS's[15-19]). After simple calculations, we get

$$g_1^{(2)}(\tau)-1=g_2^{(2)}(\tau)-1=[g_0^{(2)}(\tau)-1]+g_I^{(2)}(\tau)\ , \tag{5a}$$

$$g_{12}^{(2)}(\tau)-1=[g_0^{(2)}(\tau)-1]-g_I^{(2)}(\tau)\ , \tag{5b}$$

where

$$\begin{aligned}g_0^{(2)}(\tau)-1=&\frac{\langle\varepsilon^{(-)}(t)\varepsilon^{(-)}(t+\tau)\varepsilon^{(+)}(t+\tau)\varepsilon^{(+)}(t)\rangle-\langle\varepsilon^{(-)}(t)\varepsilon^{(+)}(t)\rangle^2}{(\langle\varepsilon^{(-)}(t)\varepsilon^{(+)}(t)\rangle+\langle E^{(-)}(t)E^{(+)}(t)\rangle)^2}\\&+\frac{\langle E^{(-)}(t)E^{(-)}(t+\tau)E^{(+)}(t+\tau)E^{(+)}(t)\rangle-\langle E^{(-)}(t)E^{(+)}(t)\rangle^2}{[\langle\varepsilon^{(-)}(t)\varepsilon^{(+)}(t)\rangle+\langle E^{(-)}(t)E^{(+)}(t)\rangle]^2}\end{aligned} \tag{6}$$

and

$$g_I^{(2)}(\tau)=\frac{\langle\varepsilon^{(-)}(t)\varepsilon^{(-)}(t+\tau)\rangle\langle E^{(+)}(t+\tau)E^{(+)}(t)\rangle+\langle\varepsilon^{(-)}(t)\varepsilon^{(+)}(t+\tau)\rangle\langle E^{(-)}(t+\tau)E^{(+)}(t)\rangle}{[\langle\varepsilon^{(-)}(t)\varepsilon^{(+)}(t)\rangle+\langle E^{(-)}(t)E^{(+)}(t)\rangle]^2}+\text{c.c.} \tag{7}$$

An important point here is that, since all odd powers of the SS vanish, there are no interference (phase dependent) terms in the intensity, so that for a 50/50 BS we get

$$\langle i_1\rangle=\langle i_2\rangle\equiv\frac{\langle i\rangle}{2} \tag{8}$$

therefore there is no contribution of $g_0^{(2)}(\tau)-1$ in the noise:

$$N^2(\omega)=\frac{eG}{2\pi}\langle i\rangle+\frac{1}{2\pi}\langle i\rangle^2\left[\int_{-\infty}^{\infty}[g_I^{(2)}(\tau)-1]e^{-i\omega\tau}d\tau\right]\ . \tag{9}$$

Since $g_I^{(2)}(\tau)$ is normalized, it does not depend on the normalization constant of the field, and we choose it in such a way that the field represents a flux. Then

$$\langle i\rangle=eG\alpha[\langle\varepsilon^{(-)}(t)\varepsilon^{(+)}(t)\rangle+\langle E^{(-)}(t)E^{(+)}(t)\rangle]\ , \tag{10}$$

where α is the dimensionless quantum efficiency of the detectors, and

$$N^2(\omega)=\frac{eG}{2\pi}\langle i\rangle\left[1+\alpha c\int_{-\infty}^{\infty}g_s(\tau)e^{-i\omega\tau}d\tau\right]\ , \tag{11}$$

where $C=\langle\varepsilon^{(-)}\varepsilon^{(+)}\rangle/(\langle\varepsilon^{(-)}\varepsilon^{(+)}\rangle+\langle E^{(-)}E^{(+)}\rangle)$ is a correction factor that enters when the LO is not much stronger than the SS, and

$$g_s(\tau)=\frac{\langle\varepsilon^{(-)}(t)\varepsilon^{(-)}(t+\tau)\rangle\langle E^{(+)}(t+\tau)E^{(+)}(t)\rangle+\langle\varepsilon^{(-)}(t)\varepsilon^{(+)}(t+\tau)\rangle\langle E^{(-)}(t+\tau)E^{(+)}(t)\rangle}{\langle\varepsilon^{(-)}(t)\varepsilon^{(+)}(t)\rangle}+\text{c.c.} \tag{12}$$

is the relevant correlation function of the fields. $g_s(\tau)$ is normalized with respect to the LO strength, so that Eq. (11) enables us to study its influence on the possible noise reduction. We write

$$N^2(\omega)=N_0^2[1+\alpha C y(\omega)] \tag{13}$$

where $N_0^2=eG\langle i\rangle(2\pi)^{-1}$ corresponds to the so-called zero line or "vacuum noise," $y(\omega)\equiv\int_{-\infty}^{\infty} g_s(\tau)e^{-i\omega\tau}d\tau$ characterizes the squeezing spectrum, and is independent of the LO intensity. However, let us already mention that the LO still enters via its first-order correlation functions, as we shall emphasize in Secs. III and IV. As mentioned in our previous works,[3,4] and as we shall see again in Sec. III, the variation domain of $y(\omega)$ is $[-1,\infty]$, so that the noise can decrease till zero for a strong LO ($C=1$) and perfect detection ($\alpha=1$).

We now summarize the results of this section.

The two-port detection scheme with 50/50 BS enables one to subtract away all the LO and all the SS noise, since the corresponding correlation function $g_0(\tau)$ does not contribute.

The only remaining noise is due to the interference between the LO and the SS, and is therefore dependent on the relative phase of the two.

The so-called zero-line N_0^2 depends on the intensities of both the LO and the SS. Therefore the standard way of experimentally determining it, by blocking the SS, is correct only when the LO is much stronger than the SS.

The best possible noise reduction is obtained with a strong LO ($C=1$). A relatively weak LO degrades it according to the correction factor C defined in Eq. (11) ($0<C\leq 1$).

This section has shown us that, in some cases, the LO intensity has to be kept as an important parameter in the detection process. We now turn to calculating the function $y(\omega)$ in two simple cases, and we shall show that the coherence time of the LO can also be a relevant parameter.

III. CALCULATION OF NOISE SPECTRUM

Our aim in this section is to show the influence of the coherence time of the LO on the noise spectrum. For this purpose, we make use of well-known methods developed by others, and we omit the detailed calculations.

A. The LO

To calculate $y(\omega)$ or $g_s(\tau)$, we need two kinds of first-order correlation function (for both the LO and the SS): $\langle \varepsilon^{(-)}(t)\varepsilon^{(+)}(t+\tau)\rangle$ and $\langle \varepsilon^{(-)}(t)\varepsilon^{(-)}(t+\tau)\rangle$, where all the fields are for the moment taken in free space, before the BS. We assume that the LO is a laser much above threshold, and neglect the intensity fluctuations. We also make all our calculations in the rotating frame of the LO so as to cancel all dependence in the frequency of the LO: Ω. Therefore we find[26]

$$\langle \varepsilon^{(-)}(t)\varepsilon^{(+)}(t+\tau)\rangle=e^{-\gamma_0|\tau|}\langle \varepsilon^{(-)}(t)\varepsilon^{(+)}(t)\rangle , \tag{14a}$$

$$\langle \varepsilon^{(-)}(t)\varepsilon^{(-)}(t+\tau)\rangle=e^{-\gamma_0|\tau|}e^{2i\phi(t)}\langle \varepsilon^{(-)}(t)\varepsilon^{(+)}(t)\rangle , \tag{14b}$$

where $\gamma_0=(T_c)^{-1}$, T_c is the coherence time of the LO, and $\phi(t)$ is its phase. If we neglect for the moment phase jitter, and remember that the same laser is used for the pumping of the nonlinear medium and for the LO, we can assume that $\phi(t)$ is constant. We choose it to be zero in the squeezing cavity, so as to get a real pumping field.

B. Degenerate SS created by parametric amplification

We take the simple model of a parametric amplifier in a cavity below threshold, pumped by a nondepleted classical field, and allow for losses, so as to get a steady state.

The Hamiltonian in the cavity is[27]

$$H=\hbar\Omega a^\dagger a+\frac{i\hbar}{2}\chi[\mathcal{E}(t)a^{\dagger 2}-\mathcal{E}(t)^* a^2]+a\Gamma^\dagger+a^\dagger\Gamma+H_R , \tag{15}$$

where χ is the real nonlinear susceptibility, $\mathcal{E}(t)$ is the pumping field at frequency Ω that we assume real in accordance with Sec. II (this choice amounts to fixing the origin of the phases), Γ and $\Gamma^\dagger$ are the loss operators, and H_R is the Hamiltonian of the reservoir.

The mathematical techniques leading to the two-time averages that we need in order to find $g_s(\tau)$ [Eq. (12)] are standard.[15,26–28] We first use the complex P representation developed by Drummond and Gardiner[29] to transform the equation of evolution of the density matrix into a c-number Fokker-Planck equation (FPE), and, by writing the corresponding stochastic equations of motion, obtain the diffusion and drift matrices. For a cavity under threshold ($p\equiv\kappa\mathcal{E}<\gamma$, where p is the pumping rate and γ is the overall damping rate of the field) we get the steady-state solution

$$\langle a\rangle=\langle a^\dagger\rangle=0 \tag{16}$$

[this result has already been used in Sec. II in order to get Eq. (5)].

If $p>\gamma$, our simple model is no longer correct, since the pumping field cannot be considered as an external parameter, but is fixed by the interactions in the cavity. This case has been treated by Collet and Walls,[16] but since we are interested only in the squeezed vacuum (that has been produced experimentally) we shall not treat it here. We now follow the approach of Lax[30] and Holm and Sargent[18] [Eqs. (29) and (35) of Ref. 18] and find the two-times averages inside the cavity:

$$\langle a(t+\tau)a(t)\rangle=\frac{e^{-\gamma\tau}}{4}\left[\frac{e^{p\tau}}{\gamma/p-1}+\frac{e^{-p\tau}}{\gamma/p+1}\right] , \tag{17a}$$

$$\langle a^\dagger(t+\tau)a(t)\rangle=\frac{e^{-\gamma\tau}}{4}\left[\frac{e^{p\tau}}{\gamma/p-1}-\frac{e^{-p\tau}}{\gamma/p+1}\right] . \tag{17b}$$

These equations generally do not appear in the literature, since most authors are directly interested in the squeezing spectrum outside the cavity. Here we want to

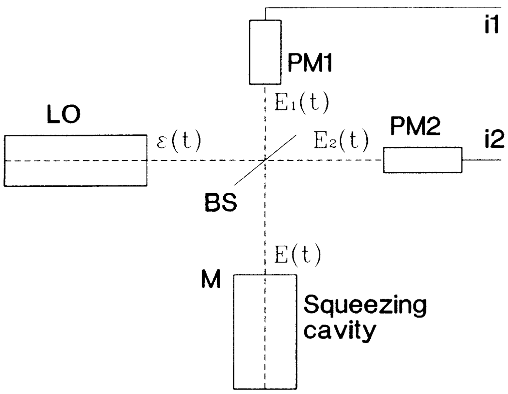

FIG. 1. Detection of a squeezed state by two-port homodyning (or heterodyning). $\varepsilon(t)$ is the output field of the coherent local oscillator (LO). $E(t)$ is the output squeezed field (M is the semireflecting output mirror). These fields are mixed by the beam splitter (BS). The two resulting fields $E_1(t)$ and $E_2(t)$ are detected by two photomultipliers (PM1 and PM2). The difference in the two currents i_1 and i_2 is sent to a spectrum analyzer.

study the influence of the LO coherence time, and therefore we should perform the Fourier transform only after adding the LO. Since the two expressions are NO, we just have to multiply every operator by the transmission coefficient of the semireflecting mirror between the cavity and the free space to get the operators outside the cavity.[4,25] Remembering that $\langle a^\dagger a\rangle$ is a number of photons, and that we choose $\langle E^{(-)}E^{(+)}\rangle$ to represent a flux, we get

$$\langle E^{(-)}E^{(+)}\rangle = 2\gamma_{\text{out}}\langle a^\dagger a\rangle \ , \tag{18}$$

where γ_{out} is the field loss rate through the mirror ($2\gamma_{\text{out}}$ is the photon loss rate), $2\gamma_{\text{out}} \equiv (cA/V)t^2$ (A, area of the beam; V, volume of the cavity; t transmission coefficient of the semireflecting mirror, chosen as real). It is now straightforward to calculate $g_s(\tau)$ and $y(\omega)$ from Eqs. (12), (13), (14), (17), and (18). We get

$$y(\omega) = y_-(\omega)\sin^2\phi + y_+(\omega)\cos^2\phi \ , \tag{19}$$

where ϕ is the phase of the LO (relative to the SS) and

$$y_-(\omega) = \frac{-4\gamma_{\text{out}}}{\gamma/p+1}\left[\frac{\gamma_0+\gamma+p}{(\gamma_0+\gamma+p)^2+\omega^2}\right] , \tag{20a}$$

$$y_+(\omega) = \frac{4\gamma_{\text{out}}}{\gamma/p+1}\left[\frac{\gamma_0+\gamma-p}{(\gamma_0+\gamma-p)^2+\omega^2}\right] . \tag{20b}$$

These expressions are correct under threshold: $\gamma/p > 1$, and with the condition that $\gamma_0 < \gamma$: the phase of the pump has to be constant for times longer than the lifetime of a squeezed photon inside the cavity.

The noise in the photocurrent oscillates according to the phase of the LO and is minimum for $\phi = \pi/2$, as is well known. Its value is

$$N^2_{\min}(\omega) = N_0^2[1+\alpha C y_-(\omega)] \ . \tag{21}$$

The noise reduction is a Lorentzian centered at $\omega=0$ and its width is $\gamma_0+\gamma+p$. The noise can be reduced till zero for perfect detection ($\alpha=1$); strong LO ($C=1$); no losses inside the squeezing cavity except through the mirror ($\gamma_{\text{out}}=\gamma$); pump strength approaching threshold ($p\approx\gamma$); long coherence time of the LO ($\gamma_0 \ll \gamma$). When the last approximation is valid ($\gamma_0 \ll \gamma$), our expression of $y_-(\omega)$ is similar to the one obtained by other authors.[15,16] If we now consider the maximum of the noise ($\phi=0$), we find

$$N^2_{\max}(\omega) = N_0^2[1+\alpha C y_+(\omega)] \tag{22}$$

so that the noise is still a Lorentzian centered at zero, but its width is $\gamma_0+\gamma-p$ so that it reduces to γ_0 close to threshold. This shows that even if $\gamma_0 \ll \gamma$, it can still have an influence on the detection process. A more detailed analysis of our results is given in Sec. III D.

C. Nondegenerate SS created by four-wave mixing

We take again a simple model, but we now need to consider two modes at frequencies $\Omega+\mu$ (created by operator a_+) and $\Omega-\mu$ (created by operator a_-). The Hamiltonian is therefore[19]

$$\begin{aligned} H = {} & \hbar(\Omega+\mu)a_+^\dagger a_+ + \hbar(\Omega-\mu)a_-^\dagger a_- \\ & + i\hbar\frac{\chi}{2}[\mathcal{E}^2(t)a_+^\dagger a_-^\dagger - \mathcal{E}^2(t)^* a_+ a_-] \\ & + [a_+\Gamma_+^\dagger + a_-\Gamma_-^\dagger + \text{H.c.}] + H_R \ . \end{aligned} \tag{23}$$

By using the same methods as in Sec. III B, we can write a FPE, find the diffusion and drift matrices, and use them to calculate the required expressions. This calculation gives exactly the same results as Sec. III B with the only difference that one has to replace ω by $\omega-\mu$ in $y_-(\omega)$ and $y_+(\omega)$ [Eq. (20)] and define the pump rate p by $p \equiv (\chi/2)\mathcal{E}^2$. Therefore we from now on concentrate on the degenerate case, knowing that all the results are equally valid for the nondegenerate case, once we translate the frequency about μ.

D. Analysis of the results

We first analyze the noise power at $\omega=0$ (degenerate case): $N^2_{\min}(0)$ and $N^2_{\max}(0)$ [Eqs. (21) and (22)] as a function of the pump strength for various values of the LO coherence time (Fig. 2). We see that a small coherence time (with respect to the squeezed photons' lifetime in the cavity) does indeed decrease in a significant way the best possible noise reduction. Therefore it seems preferable to use a strong pump in a low-quality squeezing cavity (shorter photon lifetime), as has been done by Wu *et al.*[8,10] for the degenerate case. Let us emphasize that the dependence on the LO coherence time is only for the noise reduction in the photocurrent and not for the squeezing itself. Another interesting point is that the noise reduction is practically constant for pumping power larger than half of the threshold power, so that there is no need to increase it further above this point (see also Sec. IV).

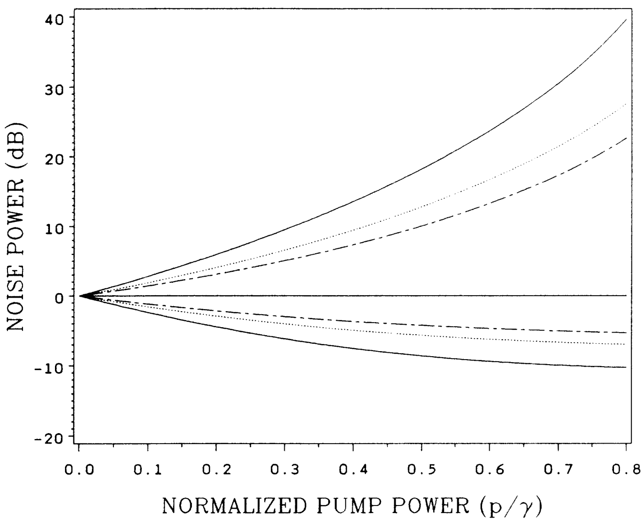

FIG. 2. Noise power as a function of the pump power (at $\omega=0$). The curves under the zero line represent $N_{\min}^2$ [Eq. (21)], the curves over the zero line represent $N_{\max}^2$ [Eq. (22)] for three cases: ——, $\gamma_0/\gamma=0$ (infinite coherence time of the LO); — — —, $\gamma_0/\gamma=0.5$; —-—-, $\gamma_0/\gamma=1$, where $(\gamma_0)^{-1}$ is the LO coherence time, γ is the squeezed cavity overall damping rate, and p is the pumping rate ($p<\gamma$ under threshold). The experimental parameters are strong LO, $C=1$ [Eq. (11)]; detection efficiency $\alpha=0.65$ [Eq. (10)]; no inside losses in the cavity, $\gamma_{\text{out}}=\gamma$ [Eq. (18)].

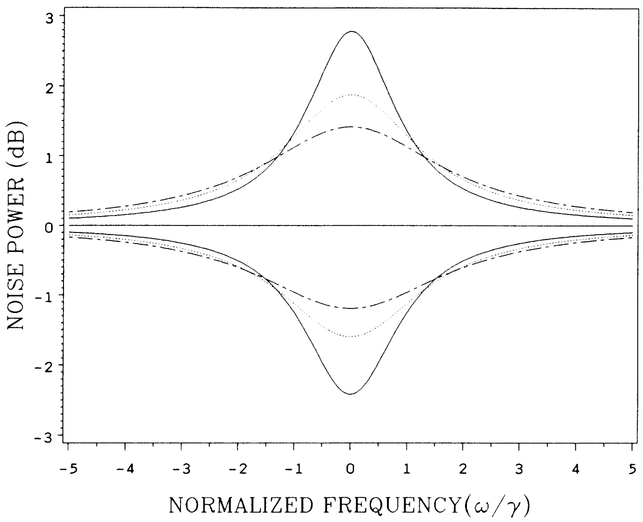

FIG. 3. Noise power as a function of the frequency, for a pumping rate $p/\gamma=0.1$ (all parameters are as in Fig. 2).

We now turn to the noise reduction bandwidth, for two values of the pumping (Fig. 3, $p=0.1$ and Fig. 4, $p=0.8$). Once again, we see the strong dependence on the LO coherence time. Its main influence is to smooth the curves, especially at high pumping, so that we have less noise reduction at the center of the bandwidth, but a better one (or at least as good) at its wings. This larger bandwidth could be experimentally interesting especially for the degenerate case, since the $1/f$ noise prevents us from investigating at very low frequencies.

IV. INFLUENCE OF PHASE JITTER

Until now, we assumed that the phase of the LO was fixed with respect to the phase of the SS. Since we use the same laser as a pump for the SS (with a frequency doubler in the degenerate parametric amplifier) and as the LO, this approach is justified, even though the absolute phase of this laser diffuse in time. However, this is not fully correct if the lifetime of the squeezed photons in the cavity is too long, in which case one has to allow for some phase diffusion. Since in the experiment of Slusher *et al.*,[7,9] the squeezing cavity has a much higher-quality factor than in the experiment of Wu *et al.*,[8,10] this problem should be more acute in the first case.

Following Klauder *et al.*,[31] we take this phase jitter into account by simply assuming that the phase of the LO varies randomly within a given interval, and performing an average over it, e.g., (for small phase jitter),

$$\bar{y}_-(\omega)=\frac{1}{2\Delta\phi}\int_{\pi/2-\Delta\phi}^{\pi/2+\Delta\phi} y(\omega)d\phi \tag{24}$$

therefore

$$\bar{y}_-(\omega)=y_-(\omega)+[y_+(\omega)-y_-(\omega)]\frac{\Delta\phi^2}{3}\ , \tag{25a}$$

$$\bar{y}_+(\omega)=y_+(\omega)-[y_+(\omega)-y_-(\omega)]\frac{\Delta\phi^2}{3}\ . \tag{25b}$$

The analytic expressions can be derived immediately from Eq. (20), but they are quite cumbersome, and we prefer to demonstrate the effects by drawing some curves. Qualitatively we know that when we arrive close to threshold, the noise increase $N_{\max}^2$ is much higher than

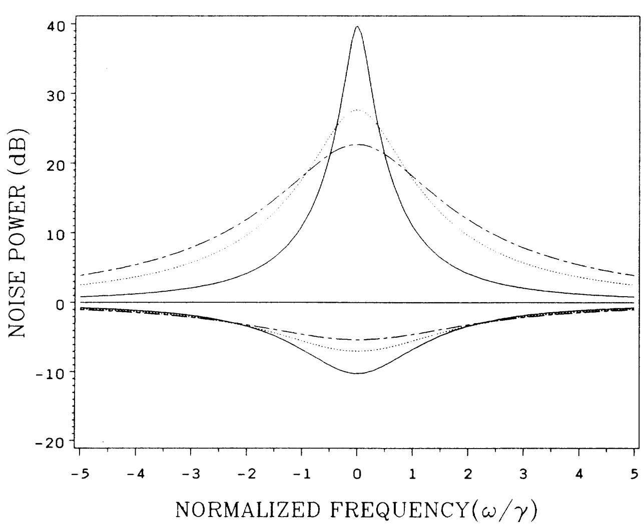

FIG. 4. Noise power as a function of the frequency, for a pumping rate $p/\gamma=0.8$ (all parameters are as in Fig. 2).

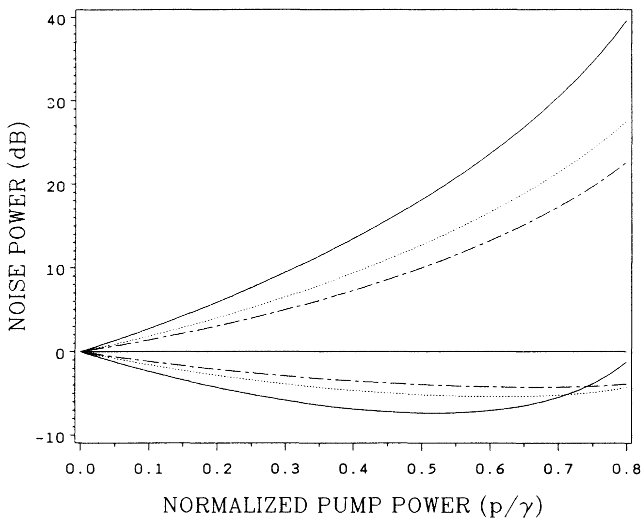

FIG. 5. Noise power as a function of the pump power (at $\omega=0$), when we introduce a certain amount of phase jitter: $\Delta\phi=10°$ (all parameters are as in Fig. 2).

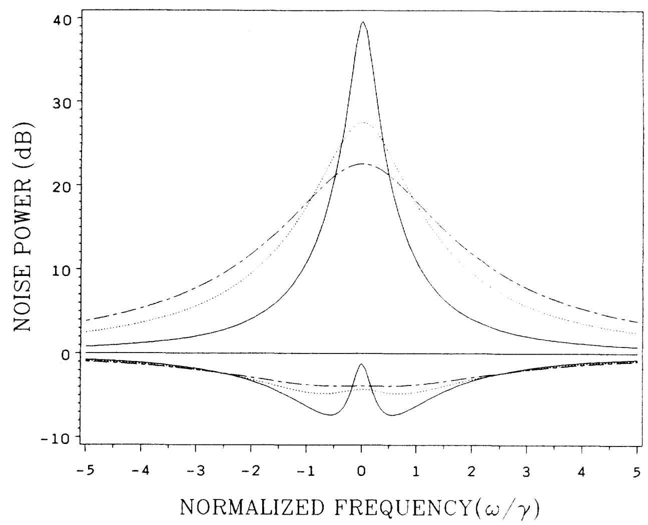

FIG. 6. Noise power as a function of the frequency, for a pumping rate $p/\gamma=0.8$, when we introduce a certain amount of phase jitter: $\Delta\phi=10°$ (all parameters are as in Fig. 2).

the noise decrease $N_{\rm min}^2$ (Fig. 2). Therefore the mixing of $y_+(\omega)$ and $y_-(\omega)$ due to phase jitter introduces a real maximum in the noise reduction (Fig. 5). If we also look at the noise reduction bandwidth, we see that, close to threshold (Fig. 4), it is much wider than the bandwidth of the corresponding noise increase. Therefore the best noise reduction is now obtained at a certain frequency shift from the center (which is $\omega=0$ for the degenerate case) (Fig. 6).

V. CONCLUSION

The usual interpretation of squeezing as the reduction of the noise in one of the quadratures of the field[1] is not complete. By considering homodyne and heterodyne detection only as a way of measuring the quadratures, most authors have not paid enough attention to the role of the LO. Our approach of the detection process, advocating the use of second-order correlation functions of the mixed field (LO plus SS), enables us to introduce straightforwardly the properties of the LO, and get the following two kinds of new results.

The influence of the strength of the LO: we showed that the "zero line" in the experiments is fixed by the sum of the LO and of the SS power, and that the best possible noise reduction is degraded when the LO is not much stronger than the SS.

The influence of the LO coherence time: we have shown that the noise reduction depends strongly on the LO coherence time, both in its maximum (Fig. 2) and in its bandwidth (Figs. 3 and 4). If it is much longer than the coherence time of the SS ($\gamma_0 \ll \gamma$), our approach reproduces the well-known results[15,16] for most cases, and the influence of a finite γ_0 can be neglected. However, once we arrive close to threshold ($p \simeq \gamma$), this is not correct anymore, since even for this case ($\gamma_0 \ll \gamma$) γ_0 fixes the bandwidth and enters in the magnitude of the noise increase $N_{\rm max}$ [Eqs. (20b) and (22)]. This effect becomes important experimentally if we introduce phase jitter that mixes $N_{\rm min}$ and $N_{\rm max}$ (Sec. IV). This phase jitter reduces the best noise reduction, but also has the interesting property of flattening the noise spectrum, and shifting the maximum from $\omega=0$ (or $\omega=\mu$ for nondegenerate SS). This effect occurs only for high pumping power, so that it should be difficult to detect it in the present experiments. However, it could become important if we arrive at a very good squeezing. The effect of the coherence time of the LO has been described here with the simplest consistent model for squeezing, but could also be derived with more detailed models in Secs. III B and III C.

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