Transition from quantum-noise-driven dynamics to deterministic dynamics in a multimode laser

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The fluctuation behavior of the mode amplitudes in a multimode dye laser near threshold is investigated numerically. The laser is modeled by semiclassical, third-order laser theory that includes a stochastic Langevin force to simulate quantum noise. We find two different limiting types of behavior. Just above the lasing threshold the mode fluctuations are found to be dominated by quantum noise, while above a certain pump power the fluctuations are found to be essentially deterministic. The effect of pump fluctuations is also simulated. We find that a multimode laser acts as a low-pass filter on the pump fluctuations, as is found for a single-mode laser.

I. INTRODUCTION

In a continuous-wave, Fabry-Perot-cavity dye laser operating in many longitudinal modes, observation shows that the individual mode intensities fluctuate full scale, while the total intensity remains essentially constant.[1–3] The correlation time of the fluctuations has been observed to decrease with increasing pump power.[2,3] Theoretical attempts to describe the mode-intensity fluctuations as arising from quantum noise are successful at predicting full-scale mode-intensity fluctuations, but the mode-intensity correlation times are predicted to increase with increasing pump power,[4–6] in contradiction to the experiments.[2,3] It is known that deterministic nonlinear dynamics can also lead to aperiodic fluctuations in lasers,[7] and when noise is added to these systems an even richer variety of behavior can be observed.[8] A question to be resolved is—what processes determine the mode-intensity correlation times in a multimode dye laser?

This work presents solutions of the third-order semiclassical Langevin equations for a multimode, homogeneously broadened laser near threshold. We show that there is an above-threshold transition in the nature of the mode fluctuations as the operating point of the laser is increased. In the regime immediately above the lasing threshold, quantum-noise-driven fluctuations dominate, and the mode-intensity correlation time is found to increase with increasing pump power. Above a certain power, about 0.1% above threshold (or a pump parameter of approximately 100), deterministic processes dominate and the correlation time is found to decrease with increasing pump power. This illustrates that there is a maximum correlation time for the individual mode intensities, and that this maximum occurs when the effects of quantum noise and deterministic fluctuations are roughly equal. Understanding the nature and origin of the mode fluctuations is of crucial importance in the field of intracavity laser spectroscopy, since larger correlation times are thought to lead to higher sensitivity.[1]

In experiments on multimode (>2) lasers to date, the observed mode fluctuations have been dominated by deterministic processes—the quantum-noise-driven fluctuations have not been seen.[2,3,9] This is because in these experiments, the near-threshold behavior of the laser was dominated by pump fluctuations. In order to determine how pump fluctuations alter the behavior of a multimode laser, we have included them in some of our numerical simulations. We find that the noise spectrum of the total intensity of a multimode laser is the same as that derived by Yu *et al.* for a single-mode laser.[10] The dye laser acts as a low-pass filter for the pump fluctuations, with a bandpass that is determined by the cavity decay rate and the fraction above threshold that the laser is operated. The modulation of the individual mode intensities simply follows the modulation of the total intensity; thus if the pump fluctuations can be made small enough, they will not wash out the quantum-mechanical effects.

Mandel and co-workers have studied the statistical behavior of a two-mode laser, for which the third-order, semiclassical theory equations of motion for the dimensionless complex mode amplitudes E_i are

$$\frac{d}{dT}E_i=\left[a_i-\sum_{j=1}^{N}\xi_{ij}|E_j|^2\right]E_i+q_i(T)\ , \tag{1}$$

where in this case $N=2$ and $i=1,2$.[11–14] In these equations T is dimensionless time, a_i is the pump parameter of mode i, and ξ_{ij} are the mode coupling coefficients. The Langevin noise source $q_i(T)$ has Gaussian statistics, zero mean, and correlation function

$$\langle q_i(T)q_j^*(T')\rangle=2\delta_{ij}\delta(T-T')\ . \tag{2}$$

These equations have been studied in detail,[15] and have demonstrated interesting stochastic behavior, some of which has been experimentally verified.[16,17] However, no deterministic fluctuations are predicted.

Hioe has studied the generalization of Eqs. (1) to many modes (large N; $i=1,2,\ldots,N$).[4] This amounts to making the free-running approximation, in which coherent mode coupling is neglected. Hioe found that for these equations, with $a_i=a$ and $\xi_{ij}=1$, the modes all have equal average intensities and variances, and that the normalized variances approach unity in the limit of large a and N. It has been shown, however, that the equations in the free-running approximation in the absence of noise $[q_i(T)=0]$ do not display deterministic fluctuations; they quickly reach a steady state.[3,18] Hioe's stochastic model predicts that the mode-intensity correlation time will continue to increase with increasing pump power, in contradiction with experiment.[2,3] Kovalenko has obtained essentially the same results by using multimode photon rate equations with terms added to simulate quantum noise.[6] Ajvasjan *et al.* have extended the rate-equation model of Kovalenko to include phenomenologically the effects of nonlinear mode coupling, assumed to arise from stimulated Brillouin scattering.[2,19] The solutions obtained from this model show quasiperiodic behavior with fluctuation times that agree with experiment in the region where deterministic fluctuations dominate. It was also speculated in this work that sufficiently close to threshold the mode-intensity fluctuations will be dominated by quantum noise.

We have shown in Ref. 3 that deterministic fluctuations can arise in a fundamental model of a multimode dye laser, without the need to postulate the existence of other processes. In order to obtain these fluctuations it is necessary to include coherent mode coupling, which in third-order theory corresponds to four-wave mixing among the modes. It is also necessary to include the effects of spatial hole burning, which are especially important in standing-wave cavities with thin gain media. These fluctuations, which are driven by four-wave mixing, cannot be seen in free-running, linearized, or photon rate-equation treatments of the laser.

II. EQUATIONS OF MOTION

The equations that govern the evolution of the complex mode amplitudes $b_l(t)$ for a standing-wave dye laser in third-order laser theory are[3,11,12]

$$\frac{d}{dt}b_l=\left[g-\gamma_l-\frac{g}{\eta}\sum_n \xi_{lnn}|b_n|^2\right]b_l - \frac{g}{\eta}\sum_{j,j\neq n}\sum_{n,n\neq l}\xi_{lnj}b_n b_j^* b_{l-n+j}+F_l(t)\ . \quad (3)$$

The presence of the four-wave mixing terms, i.e., the terms in the double summation, is what makes the multimode problem fundamentally different from the one- or two-mode problem, and is what differentiates our model from the treatments discussed above. The mode amplitudes are defined by mode expansion of the total electric field $E(r,z,t)$, in Gaussian units,

$$E(r,z,t)=i\sum_l (2\pi\hbar\omega_l)^{1/2}b_l(t)e^{-i\omega_l t}u_l(r,z)+\text{c.c.}\ , \quad (4)$$

where ω_l is the bare cavity frequency of mode l, and the $u_l(r,z)$ are the TEM_{00} Hermite-Gaussian modes of a standing-wave cavity whose mirrors are at $z=0$ and $z=L$

$$u_l(r,z)=\left[\frac{4}{\pi L w^2(z)}\right]^{1/2}\exp\left[-\frac{r^2}{w^2(z)}\right]\times\sin\left[k_l z+\frac{k_l r^2}{2R(z)}\right]\ , \quad (5)$$

where $k_l=\omega_l/c$. We have ignored the Guoy phase shift[20] in order to simplify the mode functions. This is valid because we have assumed a single transverse mode; all of the longitudinal modes of the cavity experience the same phase shift, and their relative phases are not affected. Since the mirrors are many confocal parameters from the waist, the phase shift is $-\pi/2$ at one mirror, 0 at the waist (in the thin gain medium), and $\pi/2$ at the other mirror. Thus the effect of this phase shift is effectively to move each of the cavity mirrors by $\lambda/4$, which has no effect on our results.

Given the expansion defined by Eq. (4), $|b_l|^2$ corresponds to the number of photons in mode l. In Eqs. (3) γ_l is the cavity decay rate of mode l,g is the gain coefficient, and η is the saturation photon number, $\eta=(\hbar\gamma\beta L w_0^2)/(12\omega_0 d^2)$, where ω_0 is the center lasing frequency, d is the lasing transition dipole moment, γ is the population relaxation rate, β is the dipole relaxation rate, and w_0 is the mode radius in the gain medium. The assumption that the linewidth β is large made possible the adiabatic elimination of the atomic variables from the equations of motion.[3] The mode coupling coefficients are given by $\xi_{lll}=1$ and

$$\xi_{lnj}=\tfrac{2}{3}C_{lnj}\left[\frac{\gamma}{\gamma+i(\omega_n-\omega_l)}+\delta_{nj}\right]\quad (n\neq l)\ . \quad (6)$$

The coefficients C_{lnj} are mode spatial overlap integrals

$$C_{lnj}=\frac{4}{\delta z}\int_{z_1}^{z_1+\delta z}\sin(k_l z)\sin(k_n z)\times\sin(k_j z)\sin(k_{l-n+j}z)dz\ , \quad (7)$$

where the integrals extend over the gain medium, which is located at $z=z_1$ and has thickness δz. The effects of spatial hole burning are contained in the coefficients C_{lnn}, which have the property $\frac{1}{2}\leq C_{lnn}\leq\frac{3}{2}$. The Langevin forces $F_l(t)$ have Gaussian statistics, zero mean, and correlation function[11,12]

$$\langle F_l(t)F_{l'}^*(t')\rangle=2\gamma_l\delta_{ll'}\delta(t-t')\ . \quad (8)$$

The derivation of Eqs. (3) using the mode functions defined in Eq. (5) proceeds exactly as described in Ref. 3. In order to do the radial integration it is necessary to use the fact that the gain medium is thinner than the confocal parameter of the cavity ($\simeq 0.5$ mm) and is located at the waist. Inside the gain medium the mode functions may be approximated as

$$u_l(r,z)=\left[\frac{4}{\pi L w_0^2}\right]^{1/2}\exp\left[-\frac{r^2}{w_0^2}\right]\sin(k_l z)\ . \quad (9)$$

It must also be assumed that the excitation by the pump laser is spatially uniform over the mode volume in the gain medium, which is not completely valid. In order to obtain the equations used in Ref. 3 from Eqs. (3), it is necessary to make the substitution $A_l(t)=i(8\pi\hbar\omega_l/V)^{1/2}b_l(t)$, where the effective cavity mode volume is $V=\frac{1}{2}\pi w_0^2 L$ (the effective mode volume was given incorrectly in Ref. 3 as $V=\pi w_0^2 L$).

For comparison of our results to previous work, it is useful to rewrite Eqs. (3) in the same scaled, dimensionless variables employed in Eq. (1). Defining a scaling time as $\phi=\eta^{1/2}/\gamma_0$ where γ_0 is the loss rate of the center mode, the scaled variables are $T=t/\phi$, $E_l=b_l/(\phi\gamma_0)^{1/2}$, $a_l=\phi(g-\gamma_0)$, and $f_l(T)=F_l(t)(\phi/\gamma_0)^{1/2}$. Note that the factor $\eta^{1/2}$ appearing in the definition of the scaling time is equal to the mean number of photons in the cavity exactly at threshold; thus $\eta^{1/2}\gg 1$. Using these variables, Eq. (3) becomes

$$\frac{d}{dT}E_l=\left[a_l-\frac{g}{\gamma_0}\sum_n \xi_{lnn}|E_n|^2\right]E_l-\frac{g}{\gamma_0}\sum_{j,j\neq n}\sum_{n,n\neq l}\xi_{lnj}E_nE_j^*E_{l-n+j}+f_l(T)\ , \tag{10}$$

and the correlation function of the noise term is

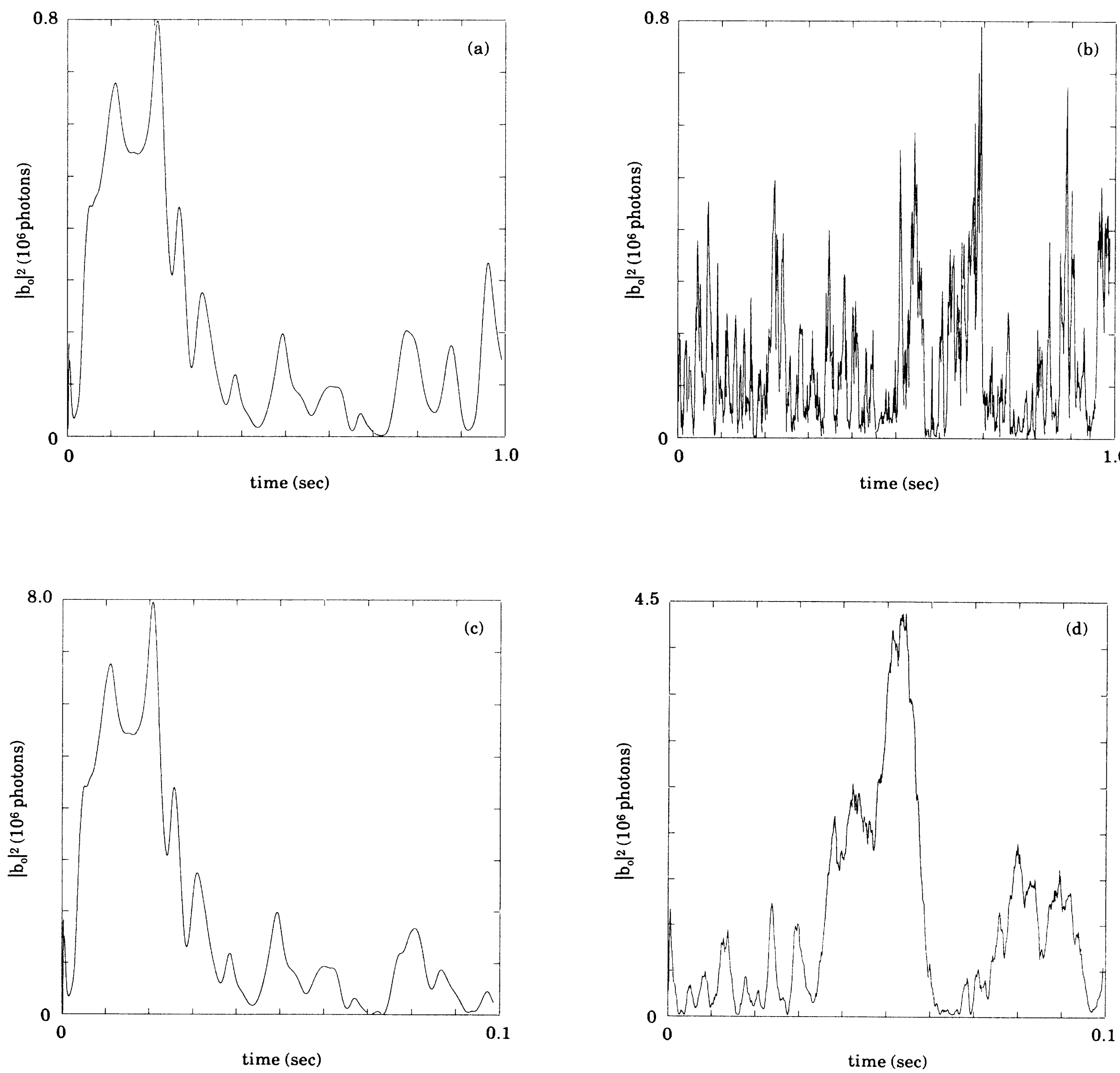

FIG. 1. The number of photons $|b_0|^2$ in the center mode of a 31-mode dye laser plotted vs time, as determined by Eqs. (3): (a) 0.03% above threshold without quantum noise, (b) 0.03% above threshold with noise, (c) 0.3% above threshold without noise, and (d) 0.3% above threshold with noise.

$$\langle f_l(T) f_{l'}^*(T') \rangle = 2\frac{\gamma_l}{\gamma_0}\delta_{ll'}\delta(T-T') \ . \tag{11}$$

In the limit that $g \simeq \gamma_0$, which is valid very near threshold, and $\gamma_l \simeq \gamma_0$, which is valid when the cavity loss profile is very broad, these are the same as Eqs. (1) and (2), except for the presence here of the four-wave mixing terms.

III. INTRINSIC MODE FLUCTUATIONS

The parameters we have used in our simulations are for a three-mirror-cavity dye laser: $\gamma_l = \gamma_0 = 8.6\times10^6$ sec^{-1},[21] $\gamma = 5\times10^8$ sec^{-1}, $\eta = 1.1\times10^{10}$, and g was varied, having a value of 8.7×10^6 sec^{-1} at 1% above threshold. The cavity parameters are $L = 52$ cm, $z_1 = 5$ cm, and $\delta z = 400$ μm. We have included 31 modes in the numerical simulations. The noise $F_l(t)$ is included by adding independent, random numbers to the mode amplitudes after each integration step Δt of the numerical calculation. The random numbers are Gaussian with zero mean and a variance of $2\gamma_0\Delta t$.

Typical solutions of Eqs. (3) are shown in Fig. 1, where 1(a) and 1(b) show the time evolution of the center mode for the laser operating 0.03% above threshold. We define $\rho = (g-\gamma_0)/\gamma_0$ to be the fraction above threshold that the laser is operated; at 0.03% above threshold $\rho = 0.0003$. The pump parameter a, which is the same for all modes ($a_l = a$), is equal to 32 at 0.03% above threshold. Figures 1(c) and 1(d) show solutions for the laser operating 0.3% above threshold. In 1(a) and 1(c) we have set the Langevin noise term $F_l(t)$ equal to zero. It can be seen from these two figures that the solutions do not scale exactly, due to the small difference between g and γ_0 in Eq. (10). At 0.03% above threshold one can see that the addition of noise dramatically affects the time evolution [Fig. 1(b)]; the solution with noise shows more rapid fluctuations. It may be surprising that noise has such a large effect when the average number of photons per mode is about 10^5, while the average number of noise photons per mode is 1. It is notable that the total intensity remains nearly constant, even in this region where the individual mode fluctuations are driven by noise. At 0.3% above threshold [Fig. 1(d)], the addition of noise perturbs the temporal evolution, but the time scales of the large fluctuations with and without noise appear to be the same.

In order to quantify the mode-fluctuation time scale, we have calculated the intensity autocorrelation function $\langle \Delta I_0(t)\Delta I_0(t+\tau)\rangle / \langle I_0\rangle^2$ where $\Delta I_0 = |b_0|^2 - \langle |b_0|^2\rangle$, and $\langle\ \rangle$ indicates a time average. To do this averaging we have used time series that are approximately seven times longer than those shown in Fig. 1. We have defined the correlation time to be the time τ_c at which

$$\langle \Delta I_0(t)\Delta I_0(t+\tau_c)\rangle = \tfrac{1}{2}\langle \Delta I_0^2\rangle \ . \tag{12}$$

In Fig. 2 we present a log-log plot of the calculated mode-intensity correlation time versus the percentage above threshold. The correlation times for simulations without quantum noise are seen to lie nearly on a straight line of slope -1, as expected from the above scaling arguments. The solid line in Fig. 2 has slope -1, and is adjusted to fit the data points, as no simple expression for it is presently known. At 0.3% above threshold ($a = 320$) and higher, the correlation time decreases with increasing power and is the same with and without quantum noise. Thus in this region the dynamics are dominated by deterministic processes. For pump powers 0.1% above threshold and lower, the correlation time is significantly smaller when quantum noise is included and increases with increasing pump power. As threshold is approached the behavior of the correlation time begins to agree with the prediction of Kovalenko's stochastic rate-equation model, which is[6]

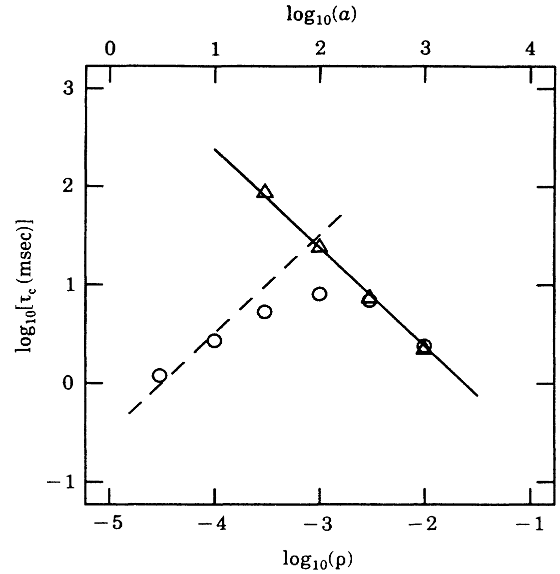

FIG. 2. A log-log plot of the mode-intensity correlation time vs ρ, the fraction that the laser is operated above threshold, for solutions with quantum noise (circles) and without quantum noise (triangles). The solid line has slope-1 and is drawn through the points without noise. The dashed line is a plot of the predictions of Kovalenko's stochastic rate-equation model [Eq. (13)]. Also plotted along the upper axis is the log of the pump parameter, a.

$$\tau_c = \langle |b_0|^2\rangle / 2\gamma_0 \ . \tag{13}$$

In Fig. 2 the dashed line is a plot of Eq. (13); the average photon number is calculated by using the approximate rate-equation expression $\langle |b_0|^2\rangle = 3\eta(g-\gamma_0)/(2Ng)$, where $N = 31$. This expression is obtained from Eq. (3) by dropping the four-wave mixing and Langevin terms, replacing ξ_{lnn} by its average value, $\frac{2}{3}$, and assuming all mode intensities $|b_n|^2$ are equal. The average photon number in Eq. (13) can also be evaluated by time averaging the numerical solution for $|b_0|^2$, with the two methods yielding the same result to within 50%. This remarkably simple behavior seems to indicate that very near threshold the effects of the coherent mode coupling become unimportant, allowing Kovalenko's treatment to be approximately correct.

IV. EFFECTS OF PUMP FLUCTUATIONS

In order to study how pump fluctuations influence the laser behavior, we have taken the laser gain to be time dependent. If we define the peak-to-peak modulation of the pump laser at frequency ω, $M_p(\omega)$, as the ratio of the peak-to-peak power excursion to the average power, we can express a sinusoidal modulation of the gain as

$$g(t)=\bar{g}[1+\tfrac{1}{2}M_p(\omega)\cos(\omega t+\phi_p)] \ . \tag{14}$$

The resulting peak-to-peak modulation $M_d(\omega)$ of the dye laser is defined analogously. This method of simulating pump fluctuations is somewhat different from most previous models[10,22,23] in that both the gain and saturation level fluctuate when we apply Eq. (14), not just the gain. The total intensity is defined as

$$I(t)=\sum_{l=1}^{N}|b_l(t)|^2 \ , \tag{15}$$

and can be approximately represented as

$$I(t)=\bar{I}[1+\tfrac{1}{2}M_d(\omega)\cos(\omega t+\phi_d)] \ . \tag{16}$$

Figure 3 shows the total intensity of the dye laser operat-

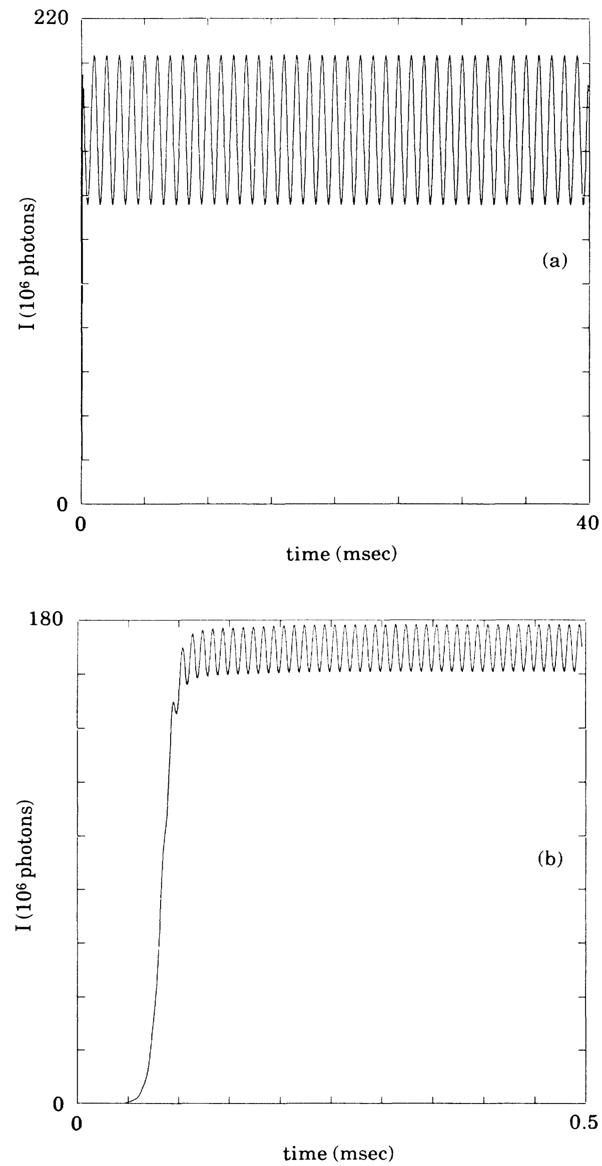

FIG. 3. The total intensity I of a multimode laser when subjected to a modulated pump. The laser is nominally 1% above threshold ($\rho=0.01$) and the peak-to-peak modulation of the pump laser is 0.4% [$M_p(\omega)=0.004$]. In (a) the modulation frequency $\omega/2\pi$ is 1 kHz, while in (b) the modulation frequency is 100 kHz.

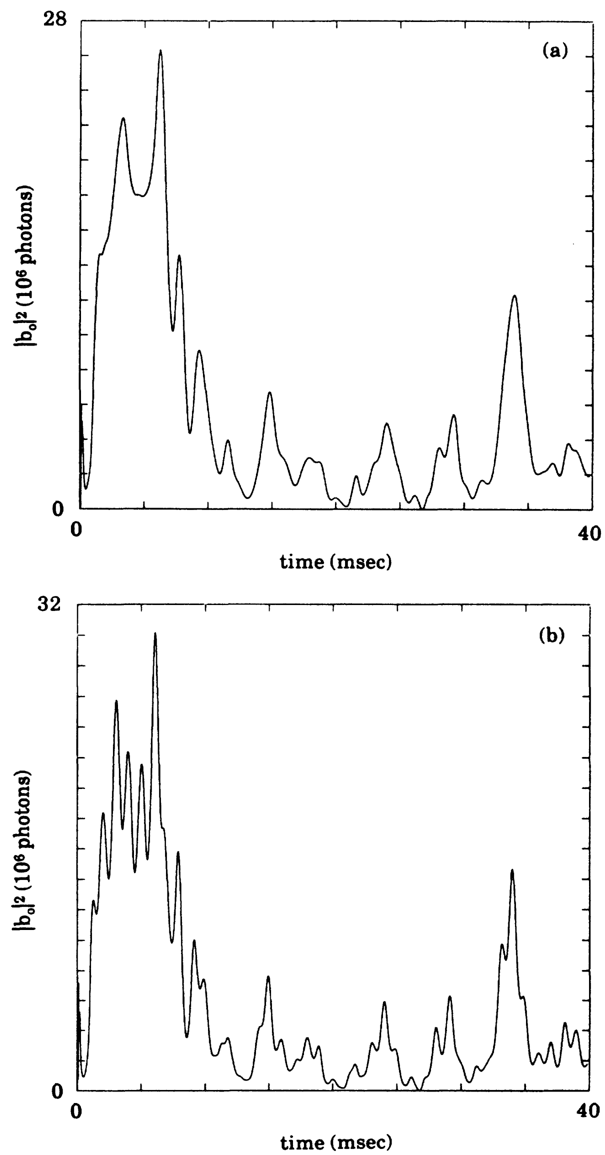

FIG. 4. The number of photons $|b_0|^2$ in the center mode of a 31-mode dye laser at 1% above threshold plotted vs time. In (a) there is no pump modulation. In (b) the parameters are the same as those in Fig. 3(a) [1 kHz modulation, $M_p(\omega)=0.004$].

ing at 1% above threshold for two different modulation frequencies ω, but the same value of the modulation depth $M_p(\omega)$. It can be seen that the dye laser cannot as easily follow the higher-frequency modulation. In addition to the decreased modulation depth of the dye laser, there is also a phase shift between the pump and dye laser intensities ($\phi_d \neq \phi_p$), showing that the dye laser is acting as a low-pass filter on the pump.

If it is assumed that the dye laser follows the pump laser linearly, i.e., the modulation frequency is small, then the peak-to-peak modulation of the dye laser is given by

$$M_d(\omega) = \frac{M_p(\omega)(1+\rho)}{\rho} , \tag{17}$$

where ρ is the fraction that the laser is operated above threshold ($\rho \ll 1$). Equation (17) is valid for $M_p(\omega) \le 2\rho/(1+\rho)$, i.e., when the pump fluctuations are not large enough to cause the laser to go below threshold. If Eq. (17) is modified to account for the frequency-dependent behavior derived by Yu *et al.* for the case of a single-mode laser,[10] the peak-to-peak modulation of the dye laser is

$$M_d(\omega) = \frac{M_p(\omega)(1+\rho)}{\rho} \frac{\Gamma_d}{(\omega^2+\Gamma_d^2)^{1/2}} , \tag{18}$$

where $\Gamma_d = 2\rho\gamma_0$ is the effective bandpass of the dye laser response. If the power spectrum of the pump fluctuations is much broader than Γ_d, the power spectrum of the dye laser fluctuations described by Eq. (18) is a Lorentzian of width Γ_d. By performing simulations at several different fractions above threshold and several different modulation depths and frequencies, we have found Eq. (18) to predict the peak-to-peak dye laser modulation to better than 0.5%.

We have also found that the individual mode intensities simply follow the modulation of the total intensity. This can be seen in Fig. 4, where we show the intensity of the center lasing mode at 1% above threshold. In Fig. 4(a) there is no pump modulation, while in Fig. 4(b) it can be seen that modulating the pump merely causes a modulation around the solution obtained with no pump modulation. This demonstrates that if the low-frequency pump noise can be kept small enough, it will be experimentally possible to observe the effects of quantum noise in a multimode laser. In order to extract accurately correlation times from the time series, it will probably be necessary to keep the dye laser fluctuations below 10%. Thus, at $\rho = 0.001$, where the noise transition occurs, one would need a pump laser whose peak-to-peak intensity stability at low frequencies is better than 0.01% in order to begin to see the effects of quantum noise in the laser we are describing.

V. CONCLUSIONS

We have demonstrated that there are two different limiting regimes of operation for a multimode, standing-wave dye laser. Just above the lasing threshold the individual mode-intensity fluctuations are driven by quantum noise. Higher above threshold ($>0.1\%$) the noise plays little role, and the resulting fluctuations are deterministic. This has a dramatic effect on the behavior of the mode-intensity correlation time as a function of power. In the noise-driven regime the correlation time increases with power, while in the region where deterministic fluctuations dominate, the correlation time decreases with power.

We have also shown, using numerical simulations, that a multimode dye laser acts as a low-pass filter for the fluctuations of the pump laser. The bandpass is determined by the cavity decay rate and how far above threshold the laser is operated. Close to threshold the laser can follow only the low-frequency fluctuations of the pump. This is an example of critical slowing down near a phase transition, and has previously been observed in a single-mode laser.[10] Pump fluctuations do not preclude seeing the effects of quantum noise in a multimode laser if they are kept to sufficiently small levels.

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