

Dicke narrowing in strong fields: The atom-as-antenna analogy

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It is well known that when a radiator (or absorber) suffers velocity-changing collisions that do not perturb its internal state, the oscillator's successive Doppler shifts are motionally averaged. This process (Dicke narrowing) gives rise to a homogeneously broadened sub-Doppler resonance, which is often astride a broad, apparently Doppler-width, pedestal. Though it has long been held that this pedestal is an inhomogeneous remnant of Doppler broadening, no evidence has ever been obtained to substantiate this expectation. In the present study we investigate the behavior of Dicke-narrowed line shapes in strong fields in an evacuated wall-coated cell. Our experimental results indicate that the hypothesis of an inhomogeneous Doppler remnant pedestal is invalid. Rather, the pedestal is homogeneously broadened and is best understood in terms of the oscillator's response to the electromagnetic field's power spectral density as observed in its rest frame. This conclusion suggests an intimate relationship between the process of Dicke narrowing and the behavior of quantum systems in the presence of stochastic fields. By envisioning the quantum system as a narrow-band antenna that only absorbs a small portion of the field's energy, we develop a simple intuitive model to account for the interaction of the atom with the nonmonochromatic field; the appeal of this simple theory is that it removes the need for extensive computation. Agreement between this "atom-as-antenna" analogy and experiment is very good.

I. INTRODUCTION

Roughly 30 years ago Dicke^{1,2} considered the problem of a quantum-mechanical oscillator emitting (or absorbing) radiation of wavelength λ in a static, one-dimensional potential well of spatial extent a . He recognized that if a was less than or equal to λ , then the radiation spectrum of the oscillator would consist of a sub-Doppler central resonance (spike), sitting on top of a broad pedestal as illustrated in Fig. 1. For his simple illustrative situation, Dicke found that the width of the narrow central resonance was equal to the radiator's natural dephasing rate, while the broad pedestal had a line contour nearly identical with the normal Doppler line.¹ In practice this "Dicke (or collision) narrowing" is typically achieved by employing an inert buffer gas to reduce an atom or molecule's mean free path so that it is less than the radiation wavelength. Under these conditions, however, the walls of Dicke's potential well move, and there is a high probability for forward scattering. As a consequence, though narrowed line shapes are obtained, broad pedestals are neither predicted nor observed when buffer-gas collisions dominate the narrowing process.³

Recently, however, there has been a renewed interest in the process of collision narrowing as originally envisioned by Dicke, partly because it corresponds to the manner in which sub-Doppler resonances are obtained in certain atomic clocks and masers.^{4,5} In these devices the atomic storage vessel forms the static potential well, and

the vessel's inner walls are coated with a nonperturbing material so that velocity-changing collisions do not disturb the internal state of the atom. Unfortunately, though broad pedestals have been experimentally observed and theoretically predicted under these conditions,

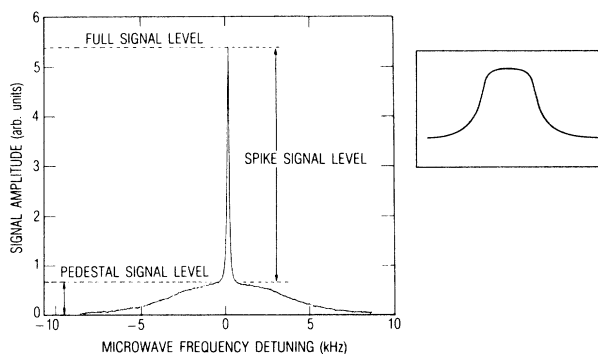


FIG. 1. This figure shows a typical Dicke-narrowed line shape as obtained in our experiment: a narrow central resonance (spike) sitting on top of a broad pedestal. Various signal levels are indicated in the figure. Rabi frequencies were determined in our experiment by measuring the spike width in the regime of power broadening, and using these values to calibrate our microwave attenuator settings. The figure in the inset shows the distorted line shapes that were observed when the pedestal amplitude saturated; the spike in this inset is of negligible amplitude.

striking ambiguities have arisen with regard to the pedestal's width.

In storage vessels (i.e., resonance cells) coated with long-chain alkanes (paraffins), Robinson and Johnson⁵ found a Gaussian pedestal with a width of about 9 kHz for the ^{87}Rb 0-0 hyperfine transition in a 200 cm³ resonance cell. The normal Doppler linewidth under their experimental conditions (approximately room temperature) was 9.1 kHz. Robinson and Johnson's observation is consistent with Dicke's original calculations, and supports the notion that these pedestals are a remnant of Doppler broadening. Additionally, Camparo and Frueholz have observed pedestal widths [full width at half maximum (FWHM)] of 9.2 kHz for this transition in room-temperature paraffin-coated cells which were attached to a vacuum system.⁶ However, in both classical⁷ and quantum-mechanical⁸ three-dimensional Monte Carlo calculations of the Dicke-narrowing process, ^{87}Rb 0-0 hyperfine transition pedestal widths substantially narrower than the Doppler width have been predicted (i.e., pedestal widths on the order of 5 kHz). Furthermore, these predictions have been verified in Dicke-narrowing experiments which used atomic resonance cells coated with siloxane materials.⁸

To reconcile these differing experimental results, Frueholz and Camparo⁸ have argued that when approximately Doppler width pedestals are observed, the ensemble of oscillators has a nonthermal speed distribution. Since polarized atoms yield the observed signal in the above experiments, these authors suggest that high-speed atoms scatter off the paraffin surface through a direct-elastic channel, which preserves the atomic polarization of these fast atoms as a result of small atom-surface interaction times. Slower atoms trap on the paraffin surface, are depolarized to a greater degree, and consequently contribute less to the observed resonance. These authors then argue that in the siloxane coated cells all atoms scatter off the surface through a trapping-desorption channel, so that the speed distribution of the oscillators contributing to the observed (albeit lower) signal is thermal. Though this hypothesis⁹ concerning the role of the surface scattering channels reconciles the various observations and calculations of Dicke pedestal widths, a major question regarding the nature of the pedestal nonetheless remains: if the Dicke pedestal is an inhomogeneous remnant of the Doppler broadening, why should thermal speed distributions yield pedestal widths less than the Doppler width?

To address this question we undertook an experimental study of Dicke-narrowed line shapes in the presence of strong resonant fields. As will be shown below, our results are in striking disagreement with expectations based on the hypothesis that the pedestal is an inhomogeneous remnant of Doppler broadening. Rather, the evidence that we have obtained suggests that the pedestal is best understood in terms of the oscillator's response to the electromagnetic field's power spectral density as observed in its rest frame.¹⁰ This conclusion then suggests that one can separate the problem of collision narrowing into two distinct parts. First, one recognizes that as a result of the oscillator's motion the stochastically varying Doppler

shifts (i.e., discrete frequency jumps) give rise to a non-monochromatic electromagnetic field-power spectral density in the oscillator's rest frame. In the process originally envisioned by Dicke, this spectral density is composed of a delta-function spike resting on a broad pedestal, whose width is determined by the ensemble speed distribution. To determine the shape of the resonance, one must then determine the behavior of the oscillator in the presence of this nonmonochromatic field. This view of collision narrowing has strong ties to the behavior of quantum systems in the presence of stochastic fields,¹ and suggests that an understanding of one will have consequences for an understanding of the other.

II. EXPERIMENT

The double-resonance experiment employed in our study of Dicke-narrowed line shapes has been described previously.⁸ Briefly, as illustrated in Fig. 2, light from a single-mode diode laser is tuned to the $5^2P_{3/2}(F=3,2,1)-5^2S_{1/2}(F=2)$ optical transition of ^{87}Rb at 780.2 nm, and creates a population imbalance between the alkali atom's two ground-state hyperfine levels (i.e., $F=1$ and $F=2$).¹² The Rb vapor is contained within a spherical Pyrex cell coated with a long-chain alkane (tetracontane).⁵ The cell has a diameter of ~ 4 cm,

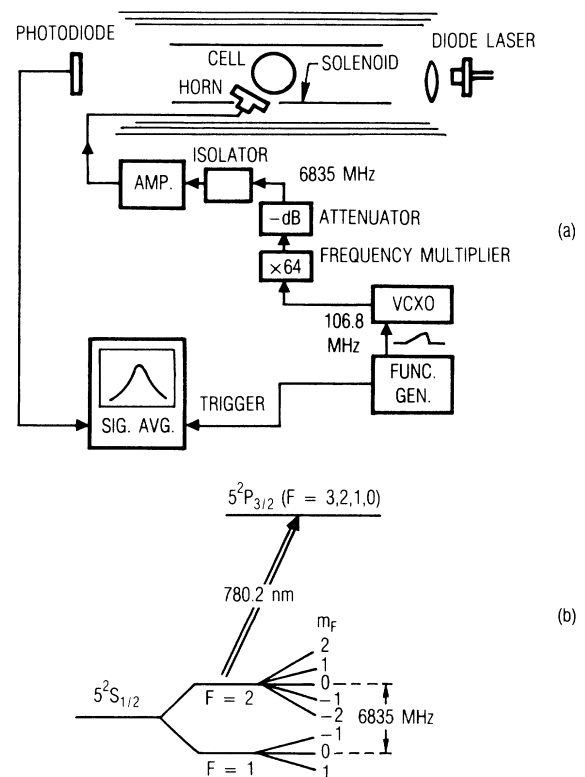


FIG. 2. (a) Optical-pumping double-resonance experimental arrangement for observing the ^{87}Rb 0-0 hyperfine transition line shape. VCXO stands for voltage-controlled crystal oscillator. (b) energy-level diagram of ^{87}Rb showing the atomic transitions of interest.

which is greater than one-half the transition wavelength, and ensures that the pedestal portion of the Dicke-narrowed line shape will have a reasonably large amplitude compared to the spike portion of the line shape. As a result of the optical-pumping process, atoms are removed from the absorbing hyperfine manifold, and the light intensity transmitted by the Rb vapor (which is proportional to the number density of atoms in the absorbing state) is maximized. To observe the Dicke-narrowed hyperfine transition line shape, the frequency of a microwave field emanating from a horn is slowly swept over the ground state ($F=2, m_F=0$)-($F=1, m_F=0$) transition at 6835 MHz, and the light intensity transmitted by the vapor is monitored.

Figure 1 is an example of two Dicke-narrowed line shapes obtained in our experiment. In the limit of low optical pumping rates and microwave Rabi frequency we find that the linewidth of the spike is 70 Hz (linewidth will always imply fullwidth at half-maximum unless otherwise noted), while the pedestal displays a linewidth of 7.3 kHz; the Doppler width of this transition under our experimental conditions (room temperature) is 9.1 kHz. Though this pedestal width is smaller than the Doppler width, in other experiments on paraffin-coated cells we have observed approximately 9 kHz pedestals. This cell was chosen for the present study, however, because it gave the best example of Dicke's standard spike and pedestal line shape. The line shape shown in the inset was observed under extreme saturation broadening conditions, and will be discussed subsequently.

To explain the sub-Doppler pedestal width, we believe that either the critical speed for trapping on this paraffin surface is low (i.e., the polarized atom speed distribution is relatively close to thermal)⁸ or there is a small trace of foreign gas in the cell due to outgassing over the cell's lifetime. Based on the Monte Carlo calculations similar to those of Ref. 8, we estimate that any foreign gas in the resonance cell would have a pressure no greater than a few millitorr, and experiments performed at Duke University would tend to substantiate this conclusion.¹³ (All Monte Carlo calculations associated with the present work described the dynamic evolution of the quantum-mechanical oscillator in terms of the density matrix.) At this foreign gas density, pressure broadening of the 0-0 hyperfine transition is expected to be less than ~ 10 Hz,¹⁴ which is negligible on the scale of the pedestal width. Further indication of insignificant foreign gas pressure broadening comes from our observation of a 70 Hz spike linewidth, which includes contributions from the optical-pumping rate and the wall relaxation rate.¹⁵ Thus, though a small amount of foreign gas could affect the Dicke-narrowing process slightly, by altering speed distributions and atomic trajectories, it could not change an inhomogeneously broadened pedestal into a homogeneous one. Similarly, a different polarized atom speed distribution, due to an altered critical speed for trapping on the paraffin surface, would have no effect on the inhomogeneous or homogeneous character of the pedestal.

We examined the spike linewidth as a function of the microwave power attenuator settings, and found it to be a linearly increasing function of field strength. Since the

spike signal shape is determined by the 0-0 hyperfine transition line shape in the absence of Doppler broadening, its saturation behavior is well understood. Specifically, if one takes care to ensure that saturated hyperfine transition linewidths are not enhanced by relatively high optical-pumping rates, then the 0-0 hyperfine transition linewidth is equal to twice the Rabi frequency.¹⁶ Thus, the saturation-broadening behavior of the spike could be used to calibrate the microwave power attenuator settings to Rabi frequency.

III. RESULTS AND ANALYSIS

A. Saturation broadening

Figure 3 illustrates the saturation behavior of the Dicke-narrowed line shape's amplitude. For nearly all the field strength values employed in our experiment, the full signal level was at or near its saturated value. Note that as the field strength increased, the pedestal amplitude also increased (until it reached the signal saturation level). For the data of Fig. 3 saturation of the pedestal amplitude occurred at a Rabi frequency of ~ 400 Hz. Beyond this critical Rabi frequency, we found that increases in field strength led to distortions in the shape of the pedestal (i.e., the pedestal appeared as if its top portion had been flattened). An example of this distortion shape is shown by the inset of Fig. 1.

The distortions observed at these high Rabi frequencies are not easily explained by saturation broadening of an inhomogeneous resonance line shape,¹⁷ which in the case of Doppler broadening is expected to be a Voigt profile (i.e., the convolution of a Gaussian with a power-broadened Lorentzian). For field strengths below the saturation value an inhomogeneous line shape will increase in amplitude, but typically will not change its general appearance. For field strengths above the saturation value each individual Lorentzian undergoes saturation broadening, and in the limit of very high fields the observed line shape takes on the form of a homogeneous

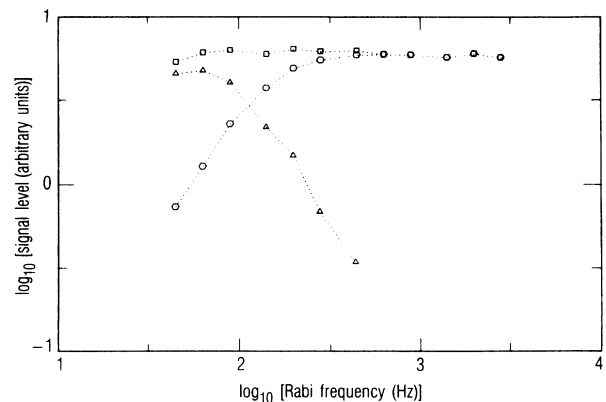


FIG. 3. Saturation behavior of (i) the full signal amplitude (squares), (ii) the spike signal amplitude (triangles), and (iii) the pedestal amplitude (circles).

Lorentzian. In general, however, the inhomogeneous line shape does not change its shape as drastically as is indicated by the inset of Fig. 1. Thus, even if one were willing to accept that inhomogeneous widths less than the 9 kHz Doppler value were possible, one would have the difficult task of explaining the anomalous shape of the saturated resonances.

Stronger evidence indicating that the Dicke pedestal is not a remnant of Doppler broadening is provided by the variation of the pedestal linewidth with Rabi frequency. These data are shown in Fig. 4, along with the results from two sets of theoretical calculations. The first set of theoretical values was obtained from Monte Carlo calculations (\times), and it is clear that in this case theory and experiment agree fairly well. We attribute the slight discrepancy between theory and experiment to the fact that the Monte Carlo calculations consider an idealized two-level quantum system, whereas the ^{87}Rb atom has a total ground-state degeneracy of eight.¹⁶ In these Monte Carlo calculations we chose a critical speed for trapping on the paraffin surface of 1.4×10^4 cm/sec,⁸ employed a dephasing rate of 35 Hz (40-Hz optical-absorption rate and 15-Hz collisional relaxation rate^{16,18}), and set the cell diameter equal to 4 cm.

In addition to obtaining linewidth data from Monte Carlo calculations, we have employed them to investigate specific characteristics of the Dicke-narrowed line shape. In the limit of very low Rabi frequencies the Dicke-narrowed resonance can be described by a convolution of the atomic line shape and the power spectral density of the radiation field as observed in the atom's rest frame. The form of this power spectral density is associated with the various Doppler shifts due to velocity-changing collisions, and our calculations indicate that in the rest frame of the atom the field appears as a delta-function spike sitting on top of a broad (~ 7.3 kHz) pedestal. (The power spectral density is generated via the Fourier transform of the time varying Doppler shifts, which are obtained from the Monte Carlo atomic trajectories.)

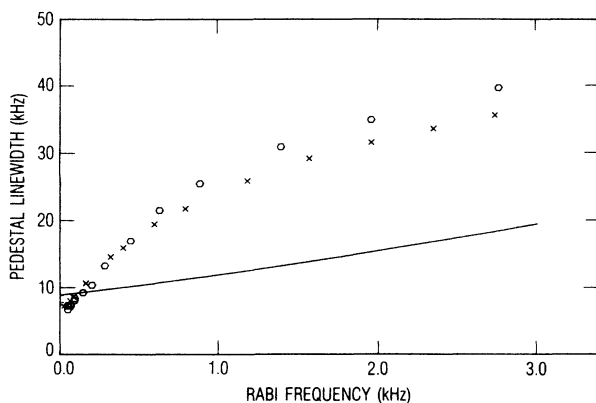


FIG. 4. Power-broadening behavior of the pedestal. Circles show the experimental data, \times 's the results from Monte Carlo calculations, and the solid line represents theoretical linewidth values for an inhomogeneous Doppler-broadened Gaussian convolved with a power-broadened Lorentzian.

Furthermore, the calculations indicate that in this particular cell geometry only $\sim 15\%$ of the field energy appears in the spike portion of the power spectral density. Since it is the spike portion of the power spectral density that gives rise to the spike portion of the observed signal, this observation would suggest that Rabi frequencies determined by saturated spike linewidths underestimate the full-field Rabi frequency by a factor of 2.6.¹⁹ This supposition is confirmed by the Monte Carlo line shapes, which show that the power-broadened spike linewidth yields an "effective" Rabi frequency a factor of 2.6 smaller than the actual Rabi frequency employed in the calculations. (In the following portions of this paper all quoted theoretical Rabi frequencies will correspond to spike linewidth measurement values, so as to facilitate comparison with experiment.) Finally, when we fit this power spectral density to a "generalized" Lorentzian shape {i.e., $\kappa/[A^2 + (\nu - \nu_f)^2]^n$, where ν_f is the field's central frequency} we find that the wings fall off approximately like ν^{-5} .

The solid line curve in Fig. 4 corresponds to a theoretical calculation of Voigt profile saturation broadening.²⁰ In the Voigt profile calculations we employed a 9.1-kHz Gaussian pedestal, composed of 70-Hz homogeneous Lorentzians which power broaden as the Rabi frequency is increased. Note that at the highest Rabi frequencies investigated (3 kHz) the linewidth of the saturated Lorentzians would be ~ 6 kHz, approximately equal to the width of the power spectral density's pedestal, but that the observed width of the Dicke line shape pedestal is nearly an order of magnitude larger. Comparing the experimental data with the saturated Voigt profile calculations, it is clear that over the entire range of Rabi frequencies the agreement is quite poor. As a result of these investigations we are forced to the conclusion that the assumption of an inhomogeneous Doppler-broadened pedestal is untenable.

B. Empirical model of the atom as antenna

Since the pedestal cannot be interpreted as the remnant of a normal Doppler-broadened line shape, we now have the task of rationalizing the pedestal's origin. In this endeavor it seems natural to associate the appearance of the Dicke pedestal with an interaction between the atom and the power spectral density's broad feature, and in this regard the saturation behavior of the pedestal would appear to have strong ties to the interaction of quantum systems with stochastic fields.¹¹ As an initial attempt at rationalizing the origin of the pedestal, we intend to view the atomic oscillator as a narrow-band antenna, which absorbs some fraction of the radiation field's total energy depending upon the overlap of the radiation field and the atomic antenna's "response function." Consequently, in this "atom-as-antenna" analogy the Rabi frequency that governs the quantum system's dynamic behavior is determined by that fraction of the field's energy which is "absorbed." At present we have no rigorous theoretical justification for this analogy; yet, as will be shown below, we have achieved excellent and somewhat surprising agreement between this empirical theory and

our experimental results.

To make the atom-as-antenna analogy quantitative, we will focus our attention on the broad feature of the field's power spectral density. We find that in the atom's rest frame the power spectral density is well described by a generalized Lorentzian

$$P(\nu)d\nu = \frac{|E_0|^2 \beta_n A^{2n-1} d\nu}{8\pi^2[(\nu - \nu_f)^2 + A^2]^n}, \quad (1a)$$

where

$$\beta_n \equiv \sqrt{\pi} \frac{\Gamma(n)}{\Gamma(n-1/2)}. \quad (1b)$$

Here, $P(\nu)d\nu$ is the field energy density in the frequency range ν to $\nu + d\nu$, $|E_0|^2/8\pi$ is the total-field energy density, ν_f is the frequency of the monochromatic field as observed in the laboratory frame, and A is related to the full width Γ [not to be confused with the gamma function $\Gamma(m)$] of the field's spectral density in the rest frame of the atom: $A = \Gamma/\{2[(2)^{1/n}-1]^{1/2}\}$. This choice of functional form has the merits that it (1) is relatively general, (2) is symmetric about $\nu = \nu_f$, and (3) reduces to the well-understood Lorentzian shape in suitable limits. The generalized Lorentzian was fit to the spectral density generated by our Monte Carlo calculations, and we found that $n \approx 2.6$.

We now invoke our antenna analogy and say that the appropriate Rabi frequency ω_1 for describing the dynamics of the field-atom interaction is determined by an overlap of the atomic antenna's (unit amplitude) response function $L(\nu - \nu_a)$ and the field's spectral density

$$\omega_1^2(\nu_a - \nu_f) \sim \int_0^\infty P(\nu - \nu_f) L(\nu - \nu_a) d\nu. \quad (2)$$

Note that in this analogy the Rabi frequency depends on the detuning between the field and the atom ($\nu_a - \nu_f$). We will assume that the atomic antenna's response function is well approximated by the atomic absorption cross section within a frequency range on the order of the full-field Rabi frequency, and that beyond this range it drops to zero²¹

$$L(\nu - \nu_a) \approx \gamma^2 / [\gamma^2 + (\nu - \nu_a)^2]. \quad (3)$$

Additionally, we will restrict our attention to the case where the radiation field's power spectral density does not vary much over the atomic antenna response function (i.e., $\Gamma \gg \gamma$). With these assumptions we then find

$$\omega_1^2(\delta) \approx \beta_n \gamma A^{2n-1} \Omega^2 / (\delta^2 + A^2)^n, \quad (4)$$

where Ω is the Rabi frequency associated with the pedestal portion of the field's spectral density, and $\delta \equiv \nu_a - \nu_f$.

This Rabi frequency is then substituted into the Bloch equations, which are solved in the steady state. Note, though, that as far as the Bloch equations are concerned the atom and electromagnetic field are in resonance; in the atom-as-antenna analogy the field-atom detuning only enters the Bloch equations through its determination of the Rabi frequency $\omega_1(\delta)$. In this way one predicts that the observed signal S will have the form

$$S \sim [1 + \gamma(\delta^2 + A^2)^n / \beta_n A^{2n-1} \Omega^2]^{-1}, \quad (5)$$

from which it is easy to show that the observed pedestal linewidth Δ_{ped} will be

$$\Delta_{\text{ped}} = 2A[(2 + \beta_n \Omega^2 / A\gamma)^{1/n} - 1]^{1/2}. \quad (6)$$

Figures 5(a) and 5(b) show the shape of these theoretical Dicke pedestals for two different values of the Rabi frequency. In Fig. 5(a) the Rabi frequency is 100 Hz, which is just above the saturation value (~ 35 Hz), and the resonance clearly displays what would be considered a "normal" shape. Figure 5(b) shows the same resonance for a Rabi frequency of 3 kHz, approximately equal to the highest Rabi frequencies used in the present experiment, and as was observed experimentally the line shape displays a flattened top. Consequently, with regard to its ability to predict qualitatively the anomalous saturated Dicke pedestal shapes, the empirical atom-as-antenna analogy does quite well.

The pedestal linewidth predicted by our empirical theory is shown in Fig. 6, where the Monte Carlo (two-

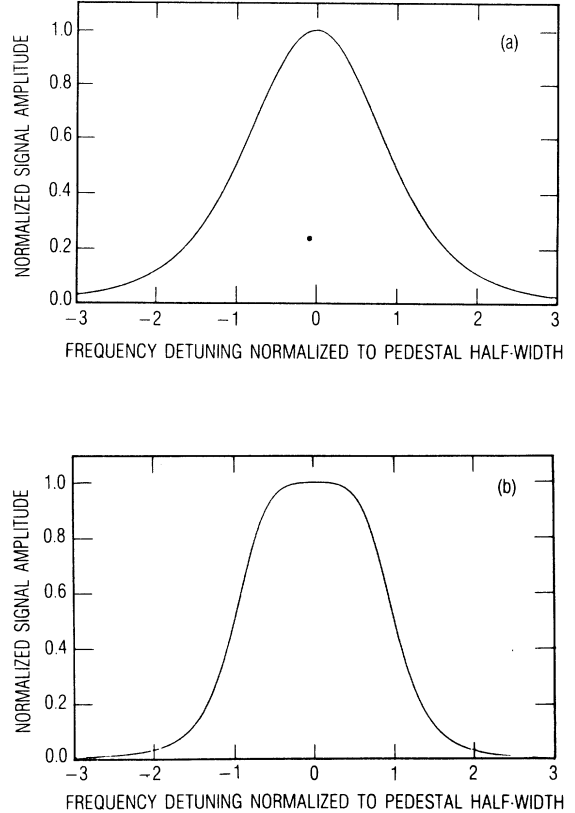


FIG. 5. Predicted line shapes in the atom-as-antenna analogy. (a) Shows the case of relatively low Rabi frequency (100 Hz): the Rabi frequency is large enough to saturate the resonance, but not so large as to seriously distort the line shape. (b) Shows the case of relatively high Rabi frequency (3 kHz); here the saturated line shape is grossly distorted, displaying the flat-top appearance that was observed experimentally. In both calculations $n = 2.62$ and $A = 6.663$ kHz.

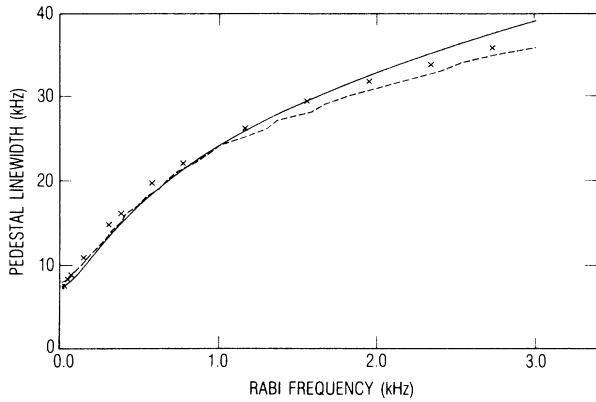


FIG. 6. Saturation broadening of the pedestal linewidth in the atom-as-antenna analogy. \times 's again correspond to the results from Monte Carlo calculations. The solid line is the prediction of pedestal linewidth based on Eq. (6) and our assumption of a generalized Lorentzian shape for the field's power spectral density. The dashed line is the result from a numerical atom-as-antenna calculation using the more accurate Monte Carlo generated spectral density.

level quantum system) pedestal data have been reproduced for comparative purposes. Here, the solid curve corresponds to the pedestal linewidth predicted by Eq. (6) and our analytical spectral density formula, while the dashed line shows the results from numerical calculations using the actual Monte Carlo generated power spectral density in the atom-as-antenna analogy. Given the simplicity of this atomic antenna view of the atom in strong stochastic fields, it is remarkable that the empirical theory does so well in predicting the pedestal linewidths. The excellent agreement supports our previous contention that the Dicke pedestal is intimately associated with the interaction between the atom and the electromagnetic field's power spectral density as observed in the atom's rest frame. More importantly, however, is the fact that the excellent agreement indicates that the paradigm of the atom-as-antenna may have more general merit for describing the interaction of quantum systems with stochastic fields. Given the importance of understanding this latter question, it is desirable to test all possible anomalous predictions that this empirical theory makes.

C. Light narrowing of Dicke pedestals

Examining Eq. (6), one finds that in the atom-as-antenna analogy the pedestal width is predicted to narrow as the atomic dephasing rate γ increases,

$$\Delta_{\text{ped}} \sim (\Omega^2 / \gamma A)^{1/2n} \quad (\Omega^2 > \gamma A). \quad (7)$$

Such behavior is quite atypical, and its observation would lend strong support to the above empirical theory. To test this prediction experimentally we exploited the fact that in an optical pumping experiment the dephasing rate is linearly related to the optical-absorption rate.¹⁵ Thus, by removing neutral density filters from the laser-beam path, illustrated in Fig. 2, we were able to experimentally

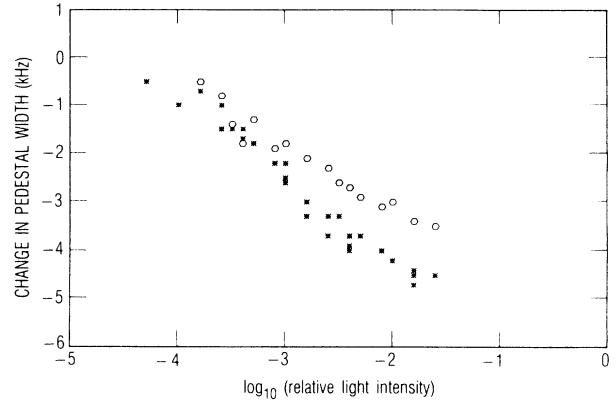


FIG. 7. Observation of anomalous pedestal narrowing as discussed in the text: circles correspond to a Rabi frequency of 170 Hz (as measured by the spike linewidth), and stars correspond to a Rabi frequency of 870 Hz. In these experiments the quantum system's dephasing rate was increased by increasing the optical-pumping rate. Note that for the 170-Hz Rabi frequency (using the data presented in Fig. 4) the narrowing effect is approximately 30% over the full light intensity range.

increase the quantum system's dephasing rate. Unfortunately, because the optical-pumping rate enhances the power broadened linewidths in multiquantum level systems,¹⁶ this procedure makes it difficult to compare quantitatively the atom-as-antenna theory with experiment. Nonetheless, the prediction can be tested for its qualitative agreement with experiment.

Experimental data for the pedestal linewidth as a function of normalized optical absorption rate are presented in Fig. 7, and data obtained at two different values of the Rabi frequency are shown. It is clear that the predicted narrowing is present, and fairly substantial. For the case of the 170-Hz Rabi frequency it can be recognized that the observed narrowing of the pedestal linewidth is $\sim 30\%$ (using the data of Fig. 4). Assuming a 40-Hz optical-absorption rate at the lowest light intensity levels and a 15-Hz collisional relaxation rate, Eq. (6) also predicts $\sim 30\%$ narrowing.²²

IV. SUMMARY AND DISCUSSION

With regard to our initial motivation for the study of Dicke narrowing in strong fields, we have found that the long-held assumption of an inhomogeneously broadened Doppler remnant pedestal is untenable. Further, the experimental data are inconsistent with inhomogeneously broadened pedestal line shapes in general. Our results suggest, instead, that the pedestal is intimately connected with the electromagnetic field's power spectral density as observed in the rest frame of the quantum-mechanical oscillator, which immediately suggests strong ties between the phenomena of Dicke narrowing and the important question of the behavior of quantum systems in stochastic fields. In this view of Dicke narrowing, the problem of understanding the narrowed line shape divides naturally into two parts. First, there is the problem of understand-

ing the electromagnetic field's power spectral density, which is related to the geometry of velocity-changing collisions, and ensemble speed distributions. Then, once this spectral density is known, there is the problem of understanding the behavior of the quantum system in the non-monochromatic field.

Concerning this latter problem, we have found that viewing the quantum system as a narrow-band antenna has merit. In the atom-as-antenna analogy one imagines that the quantum system only absorbs a fraction of the radiation field's energy. This fraction then determines the dynamic response of the system to the radiation field via the Bloch (density matrix) equations. Using this atom-as-antenna analogy we have obtained very good agreement with experiment and our Monte Carlo calculations: (1) the analogy has predicted the anomalous shape of saturated pedestals; (2) it has yielded excellent agreement with Monte Carlo calculations of the saturated pedestal width, and (3) the analogy has predicted the experimentally verified relaxation narrowing of the pedestal. We believe that this same analogy may have broader implications for understanding the behavior of the quantum systems in stochastic fields.

In this regard it is worth considering Eqs. (5) and (6) in the case that $n = 1$ (i.e., a Lorentzian spectral density). We then find that the observed resonance has a Lorentzian shape, with a linewidth given by

$$\Delta_{\text{ped}} = 2[A^2 + (A/\gamma)\Omega^2]^{1/2}. \quad (8)$$

This expression is similar to what would be obtained for a two-level quantum system in the presence of a monochromatic field, except the quantum system's dephasing rate $1/T_2$ has been replaced by the field half-width A . For comparative purposes we note that Georges and Lambropoulos (GL) have rigorously analyzed the problem of two atomic levels coupled by a phase-diffusing field (i.e., Lorentzian power spectral density) using a density-matrix formalism.²³ They found that the single-photon resonance is a Lorentzian with a linewidth Δ_{GL} ,

$$\Delta_{\text{GL}} = 2\{(A + \gamma)^2 + [(A + \gamma)/\gamma]\Omega^2\}^{1/2}. \quad (9)$$

This formula is very similar to Eq. (8), and in the limit that $A \gg \gamma$ the two expressions are equivalent.²⁴ Therefore, not only does the atom-as-antenna analogy have merit in the analysis of Dicke-narrowed line shapes, but given this result it appears that the analogy may have more general validity for describing the behavior of quantum systems in the presence of stochastic fields.

Though this analogy is intriguing, much remains to be done before its utility is truly established. First, given the agreement between this empirical theory and the Monte Carlo calculations, it should be possible to arrive at the atom-as-antenna analogy through the two-level density-matrix equations. Such an analysis would indicate the approximations that are made when invoking the analogy, and consequently the limits to which this analogy may be taken. Additionally, it would be worthwhile to experimentally investigate the behavior of quantum systems in the presence of various stochastic processes, in particular stochastic amplitude variations. Though the above experimental and theoretical work indicates the analogy's apparent validity for stochastic phase and frequency variations, whether the analogy can be made to work for stochastic amplitude fluctuations is an entirely different question. Only at the conclusion of such studies will it be possible to evaluate the significance of the atom-as-antenna analogy, either as a fortuitous model that works reasonably well for phase-fluctuating fields, or as a generally valid approximation that provides insight into the behavior of quantum systems in the presence of arbitrary stochastic fields.

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⁹Experiments are now in progress both at The Aerospace Corporation and Duke University to test the validity of this hypothesis.

¹⁰H. G. Robinson, in *Proceedings of the Sixteenth Annual Pre-*

cise Time and Time Interval (PTTI) Applications and Planning Meeting, Greenbelt, Maryland, 1984 (unpublished).

¹¹See, for example, S. N. Dixit, P. Zoller, and P. Lambropoulos, *Phys. Rev. A* **21**, 1289 (1980), and references therein.

¹²J. C. Camparo, *Contemp. Phys.* **26**, 443 (1985).

¹³In an experiment performed at Duke University a cell that was sealed under high vacuum was opened several months later, and its volatile contents examined. It was found that the cell contained approximately 1 mTorr of helium and 3 mTorr of hydrogen. Concerning the present cell, it was sealed under a pressure of 10^{-7} Torr approximately one month prior to the conclusion of these experiments.

¹⁴F. Hartmann and F. Hartmann-Boutron, *Phys. Rev. A* **2**, 1885 (1970).

¹⁵W. Happer, *Rev. Mod. Phys.* **44**, 169 (1972); J. Vanier, *Can. J. Phys.* **47**, 1461 (1969).

¹⁶J. C. Camparo and R. P. Frueholz, *Phys. Rev. A* **31**, 1440 (1985); **32**, 1888 (1985).

¹⁷A. M. Portis, Phys. Rev. **91**, 1071 (1953); T. G. Castner, *ibid.* **115**, 1506 (1959).

¹⁸Since the optical pumping radiation only interacts with one of the states coupled by the microwave field, the dephasing rate due to optical pumping is one-half the optical-photon absorption rate. See Ref. 16 for further details.

¹⁹From the Monte Carlo calculations we have $E_{\text{spike}}^2/E_{\text{tot}}^2=0.15$, where E is the field strength in the specified portion of the spectral density. Since the Rabi frequency Ω is proportional to E , $\Omega_{\text{tot}}=2.6\Omega_{\text{spike}}$.

²⁰P. Minguzzi and A. DiLieto, J. Mol. Spectrosc. **109**, 388 (1985).

²¹With regard to phase fluctuations, it is reasonable to assume that for $|\nu-\nu_a|$ greater than the full-field Rabi frequency the

atoms would not absorb any radiant energy. This is a result of adiabaticity requirements for the dynamic response of the atom to rapid phase changes as discussed by Camparo and Frueholz: Phys. Rev. A **38**, 6143 (1988).

²²Note that the Rabi frequency Ω in Eq. (6) corresponds to the field strength in the pedestal portion of the spectral density. Hence, in order to compare with experiment we set Ω equal to 92% of the total-field Rabi frequency. Thus, if the spike Rabi frequency is 170 Hz, we have $\Omega=408$ Hz; see Ref. 19.

²³A. T. Georges and P. Lambropoulos, Phys. Rev. A **18**, 587 (1978).

²⁴We note that if we define γ_1 and γ_2 as the "bare" atom longitudinal and transverse relaxation rates, then the two expressions are only equivalent when $\gamma_1=\gamma_2=\gamma$.