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Atomic-level shift in a generalized squeezed coherent state

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The reaction field theory is used to calculate the frequency shift in the generalized squeezed coherent state. The result for the squeezed vacuum state is a special case.

Recently Milburn published a paper on atomic-level shifts in a squeezed-vacuum state.[1] The energy-level shifts for a multilevel atom interacting with a squeezed vacuum have been calculated. It is found also that the level shifts are made up of two contributions: (i) the ordinary Lamb shift and (ii) a shift due to the squeezed-vacuum intensity spectrum (similar to the case of blackbody-radiation shifts). In this Comment we shall consider the effects of a squeezed vacuum on atomic-level shifts by another method that seems to us to be more direct than that used by Milburn. It emphasizes the expected role of the squeezed mode in the reaction field theory.[2]

In 1978 a theory for the interaction between electromagnetic radiation and a two-level atom in the electric- and magnetic-dipole approximation was presented by one of us[3] in the framework of the reaction field theory.[4] The dynamical equations for the fields were discussed when the field is supposed to be an unpolarized and isotropic field. It was observed also that near resonance the fields are comprised of resonant and off-resonant parts.

For an isotropic unpolarized field, near resonance, it was found that the frequency shift is given by[3]

$$\Delta=(\mu_{0s}^2+m_{0s}^2)\mathrm{Re}J^{\dagger}(\omega)\ , \tag{1}$$

where μ_{0s} and m_{0s} are, respectively, electric and magnetic expectation values between the states 0 and s of the atom, and $\mathrm{Re}J^{\dagger}(\omega)$ is given by[3,4]

$$\begin{aligned}\mathrm{Re}J^{\dagger}(\omega)=&(4\pi/3)(\omega/c)^3\ln(m_e c^2/h\omega)\\&+(4/3\pi)(\omega/c)\int_0^{k_c}k\,dk\\&+(8/3\pi)\int_0^{\infty}n_k\frac{k^3}{k^2-(\omega/c)^2}dk\ ,\end{aligned} \tag{2}$$

where n_k is the photon number of the wave vector $\mathbf{k}$, while k_C is the Compton wavelength $k_C=m_e ch^{-1}$. It should be noted that the first logarithmic term is precisely twice the nonrelativistic Bethe[5] formula for the Lamb shift. This is to be expected, since both energy levels move toward one another by an amount given by the Bethe single-level shift. The second term can be identified as precisely twice the static electromagnetic mass renormalization term which appears in the second-order perturbation theory for level shifts.[6,7] A fuller discussion of this term and its relation to the renormalization terms is found in Ref. 4.

The last term, which is field dependent through the appearance of n_k, has been used to investigate the cases of the thermal field distribution[3,8] and a rectangular pulse-type distribution.[3,4] A temperature-dependent shift has been obtained for the first distribution, while a Lamp shift[4] has been given for the latter.

Using Eq. (11) of Milburn,[1] the last term of Eq. (2) reduces identically to Eq. (25) of the same Ref. 1 for the case of a two-level atom.[9]

The photon number n_k appears in Eq. (2). Several distributions for the photon number were used in (2) to give specific formulas for the field-dependent shift.[3,4,8] We take another example when the state of the field is a generalized squeezed coherent state defined as[10]

$$|\alpha,z,m\rangle=D(\alpha)S(z)|m\rangle\ , \tag{3}$$

where $D(\alpha)$ is the displacement operator given by

$$D(\alpha)=\exp(\alpha a^{\dagger}-\alpha^{*}a)\ , \tag{4}$$

$S(z)$ is the squeezing operator

$$S(z)=\exp(\tfrac{1}{2}za^{\dagger}a^{\dagger}-\tfrac{1}{2}z^{*}aa)\ , \tag{5}$$

$z=re^{i\theta}$, and $|m\rangle$ is the mth state of the harmonic oscillator. It is found in this case that

$$n_k=\langle a_k^{\dagger}a_k\rangle=|\alpha|^2+\sinh^2 r+m\cosh^2 r\ , \tag{6}$$

where r may be a function of ω. Introducing this distribution in the last term of Eq. (2) we get

$$(8/3\pi)\int_0^{\infty}\frac{(|\alpha|^2+\sinh^2 r+m\cosh^2 r)k^3}{k^2-(\omega/c)^2}dk\ .$$

When we take $m=0$, $\alpha=0$ we get the squeezed-vacuum Lamb shift of Ref. 1.

It is important to note that there is no reason to suppose that the field is initially taken in vacuum state in order to reach Eq. (2). Hence the results presented here are more general than those obtained in Ref. 1.

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