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Atomic dipole in front of a phase-conjugate mirror

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We study the effect of the presence of a phase-conjugate mirror on the evolution of a classical radiating dipole and of a quantum-mechanical two-state atom. In both cases the modified evolution equations are solved and explicit solutions are presented. The evolution of the classical dipole and of the atomic coherences depends on the initial relative phase of the atomic dipole and the phase-conjugate mirror.

I. INTRODUCTION

For many years much theoretical and experimental work has been devoted to the question in what way the evolution of a radiating system is affected by the presence of macroscopic media. Especially, the spontaneous emission of an atom in front of dielectrics[1–3] or conductors (ordinary mirror),[4–7] in the neighborhood of other atoms[8,9] or in cavities[10–14] has been studied. The advent of the phase-conjugate mirror[15] (PCM) has opened new, interesting ways to manipulate the matter-radiation interaction, since it has several new features.

Firstly, the PCM reverses the propagation direction of the incident radiation independent of the angle of incidence. Therefore the effect of the PCM on the atomic evolution does not depend on the distance between the atom and the PCM. This is true for distances up to the coherence length of the atom, since then we have to account for the finite speed of light. Since the PCM consists of a nonlinear medium driven by pump fields, the generated phase-conjugate reflection can gain energy at the expense of the energy of the pump fields. Consequently, for sufficiently high pump-fields intensities, the reflectivity of the PCM can exceed unity. Finally, the PCM has an inherent phase due to the driving pump fields. Therefore, we expect that the decay properties of the atom must depend on the initial phase of the atomic dipole. Due to these properties, we expect that the presence of the PCM will give rise to new interesting effects in the atomic evolution.

Several authors[16–18] have discussed the damping of a classical dipole in front of a PCM. These papers concentrated mainly on finding the modified damping rates of the dipole, while no attention was given to the phase dependences of the evolution. Furthermore, the results for the damping rates in these papers seem to diverge. The problem of a quantum-mechanical atom in front of a PCM has also received some attention.[18] However, this treatment did not account for the phase dependence, since it focused on the populations rather than the coherences. Recently, Cook and Milonni[19] studied the evolution of a sample of N identical atoms in front of a PCM. In this treatment, the number of atoms taken was large and confined in a small volume compared to the radiation wavelength, and the semiclassical theory of radiation was applied. With these approximations, the intensity of the collective fluorescence was studied.

In the present paper, we consider the evolution of a classical dipole and of an atom in front of a PCM based on nearly degenerate four-wave mixing. We describe the PCM in Sec. II both classically and quantum-mechanically. The classical description is used in Sec. III. where we obtain an explicit expression for the time evolution of a classical dipole in front of the PCM. This evolution depends on the initial phase φ of the dipole and on the phase ψ of the PCM. In Sec. IV we present the quantum-mechanical description of the PCM and obtain an explicit result for the evolution of the density matrix of the two-state atom. The evolution of the coherences is found to be similar to the classical dipole evolution. These results illustrate the importance of the phase in the evolution of a dipole or atom in front of a PCM.

II. DESCRIPTION OF THE PHASE-CONJUGATE MIRROR (PCM)

When a radiating system, such as a classical dipole or a quantum-mechanical atom radiates in front of a PCM, this system couples with a radiation field that in turn is coupled to the PCM. In this section, we discuss the evolution of the radiation field due to its coupling with the PCM. The coupling of the dipole with the local radiation field will be studied in subsequent sections.

The model of the PCM that we shall use is based on nearly degenerate four-wave mixing. This process occurs when an incident beam with frequency ω_i intersects a standing light wave of frequency ω_0 in a nonlinear medium with third-order susceptibility $\chi^{(3)}$, and it leads to the coherent generation of radiation counterunning and phase conjugate to the incident beam with frequency

$$\omega_r = 2\omega_0 - \omega_i \ . \qquad (2.1)$$

The setup is sketched in Fig. 1(a). In terms of photon processes, a photon of the incident beam leads to stimulated emission of an additional photon into this beam, and to creation of a photon in the phase-conjugate reflected beam, at the expense of absorption of two photons, one from each of the opposite-running pump waves that make up the standing wave [Fig. 1(b)].

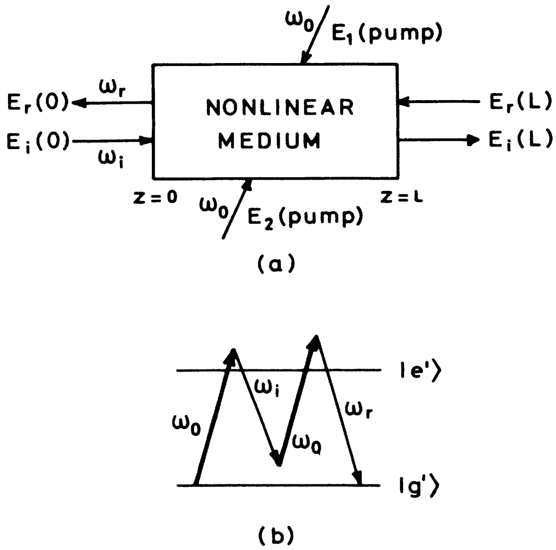

FIG. 1. (a) Nearly degenerate four-wave mixing geometry. E_1 and E_2 are strong pump fields, while E_i and its phase-conjugate reflection E_r are weak. (b) Scheme of four-wave mixing in terms of atomic transitions. Absorptions are indicated by double arrows and stimulated emissions by single arrows.

Throughout this paper, we position the nonlinear medium, with length L, along the z axis [see Fig. 1(a)]. The distance between the atomic dipole and the PCM is D. The atomic dipole is located at $z=-D$. The boundary of the nonlinear medium at $z=0$ as indicated in Fig. 1(a) is then the actual mirror surface of the PCM. We assume that the distance D is much larger than the wavelength of radiation, and we take the diameter of the surface of the PCM to be small compared to this distance D. Due to these assumptions, the radiation emitted by the atomic dipole has normal incidence on the PCM and passes through a layer of the nonlinear medium of thickness L. The only geometrical parameter that enters in the description of the PCM is, then, the solid angle $4\pi\alpha$ subtended by the PCM at the atomic dipole. Throughout this paper we ignore the polarizations of the fields. Furthermore, we assume that the depletion of the standing wave is negligible and that the resonance frequency of the dipole is equal to the frequency of the pump photons. Finally, we neglect losses in the nonlinear medium. For later use we separately consider a classical and quantum-mechanical description of the fields in the PCM.

A. Classical description of the PCM

The four classical monochromatic fields that are involved in the four-wave mixing can be written in the general form

$$E_\xi(\mathbf{r},t)=\mathrm{Re}[E_\xi(\mathbf{r})e^{i\mathbf{k}_\xi\cdot\mathbf{r}-i\omega_\xi t}] , \tag{2.2}$$

with $\xi=1$, 2, i, and r. The counterrunning pump fields $E_1(\mathbf{r},t)$ and $E_2(\mathbf{r},t)$ have frequencies $\omega_1=\omega_2=\omega_0$ and wave vectors $\mathbf{k}_1=-\mathbf{k}_2$. The incident field $E_i(\mathbf{r},t)$ is propagating along the positive z direction, so that $\mathbf{k}_i=(0,0,k_i)$. The wave numbers k_ξ and the frequencies ω_ξ are related by $k_\xi=\omega_\xi/c$. These three incident input waves generate a third-order nonlinear polarization in the medium. We are only interested in the polarization which generates a field at a frequency near resonance. The contribution obeying the phase-matching condition is then[15]

$$P^{(3)}(\omega_r=2\omega_0-\omega_i)$$
$$=\mathrm{Re}\{\epsilon_0\chi^{(3)}E_1(\mathbf{r})E_2(\mathbf{r})[E_i(\mathbf{r})]^*e^{-i\mathbf{k}_i\cdot\mathbf{r}-i\omega_r t}\} . \tag{2.3}$$

The wave propagation in the nonlinear medium is governed by the Maxwell wave equation

$$\left[\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]E(\mathbf{r},t)=\frac{1}{\epsilon_0 c^2}\frac{\partial^2}{\partial t^2}P(\mathbf{r},t) , \tag{2.4}$$

where $P(\mathbf{r},t)$ is the total polarization in the medium. Throughout this paper, we neglect effects due to the linear polarization. The nonlinear $P^{(3)}$ [Eq. (2.3)] generates a phase-conjugate field E_r with frequency ω_r and propagating in the $-z$ direction. Here we shall employ the slowly varying amplitude approximation (SVAA), which assumes that the amplitudes of the fields do not change much over a distance that is large compared with the wavelength. Hence we have

$$\left|\frac{\partial^2 E_\xi}{\partial z^2}\right| \ll \left|k_\xi\frac{\partial E_\xi}{\partial z}\right| . \tag{2.5}$$

Furthermore, we shall assume that

$$|\omega_i-\omega_r| \ll \omega_0 , \tag{2.6}$$

and that $\chi^{(3)}$ varies negligibly in the relevant frequency range. Substituting (2.2) and (2.3) in (2.4), while using (2.5) as well as the fact that the depletion of the pump fields is negligible, yields the differential equation between incident and phase-conjugate field[20]

$$\frac{d}{dz}E_r(z)=-i\eta[E_i(z)]^*e^{i\delta z} , \tag{2.7}$$

where we have defined, using (2.6),

$$\eta=\frac{\omega_0}{2c}\chi^{(3)}E_1E_2 \tag{2.8}$$

and

$$\delta=k_r-k_i . \tag{2.9}$$

Conversely, the pump fields together with the phase-conjugate field give rise to a third-order nonlinear polarization that couples to the incident field, according to

$$\frac{d}{dz}[E_i(z)]^*=-i\eta^*E_r(z)e^{-i\delta z} , \tag{2.10}$$

with η and δ defined in (2.8) and (2.9), respectively. We assume that the PCM extends between $z=0$ and $z=L$. Solving (2.7) and (2.10) for given boundary values $E_r(L)$

and $E_i(0)$ yields for the reflected phase-conjugate field at $z=0$[20,15]

$$E_r(0)=\nu_\delta[E_i(0)]^*+\mu_\delta E_r(L) \tag{2.11}$$

and for the transmitted field at $z=L$

$$[E_i(L)]^*=\mu_\delta[E_i(0)]^*+\nu_\delta^* e^{-i\delta L}E_r(L) , \tag{2.12}$$

where

$$\nu_\delta=i\eta\frac{\sin(\Lambda L)}{\Lambda\cos(\Lambda L)-\frac{1}{2}i\delta\sin(\Lambda L)}\equiv|\nu_\delta|e^{i\psi_\delta} , \tag{2.13}$$

$$\mu_\delta=\frac{\Lambda e^{-i\delta L/2}}{\Lambda\cos(\Lambda L)-\frac{1}{2}i\delta\sin(\Lambda L)} , \tag{2.14}$$

and with

$$\Lambda=(|\eta|^2+\tfrac{1}{4}\delta^2)^{1/2} . \tag{2.15}$$

Note that μ_δ and ν_δ satisfy the relation

$$|\mu_\delta|^2-|\nu_\delta|^2=1 , \tag{2.16}$$

showing that $E_r(0)$ and $[E_i(L)]^*$ are related to $E_r(L)$ and $[E_i(0)]^*$ by a hyperbolic rotation. This is a direct consequence from the fact that the relation coupling $[E_i(L)]^*$ and $E_r(L)$ to $[E_i(0)]^*$ and $E_r(0)$ is unitary. For the classical PCM the conjugate field is zero at $z=L$. Hence we put $E_r(L)=0$ in Eq. (2.11). The conjugate field at the surface $z=0$ of the PCM is then proportional to the complex conjugate of the incoming probe field. Note that the amplitude reflectivity $|\nu_\delta|$ can exceed unity as can be seen in (2.13). The amplitude of the response field decreases when the detuning between the frequency of the incoming field and the pump fields becomes larger. Furthermore, the PCM has an inherent phase ψ_δ that changes with the phases of the two pump fields E_1 and E_2. When the incoming field has the phase φ at the surface $z=0$, so that

$$E_i(0)=|E_i(0)|e^{i\varphi} , \tag{2.17}$$

then the reflected conjugate field at this same plane has the phase difference $\psi_\delta-2\varphi$ with the incident field. This phase difference is invariant for a time translation of all the fields. On the other hand, when only the phase of the probe field is changed, this phase difference between the incident field and the reflected field is modified.[21]

B. Quantum-mechanical description of the PCM

Now we describe the PCM for a quantum-mechanical incoming probe field and phase-conjugate reflected field. For the pump fields, we retain a classical description.

The electric field operator outside the nonlinear medium can be written in the form

$$E_{\mathbf{k}}(\mathbf{r},t)=i\hbar\left[\frac{\omega_k}{2\mathcal{V}\hbar\epsilon_0}\right]^{1/2}(a_{\mathbf{k}}e^{i\mathbf{k}\cdot\mathbf{r}-i\omega_k t}-\text{H.c.}) , \tag{2.18}$$

with $\mathcal{V}$ the quantization volume and where $a_{\mathbf{k}}$ and $a_{\mathbf{k}}^\dagger$ are the annihilation and creation operators which obey the usual commutation relations

$$[a_{\mathbf{k}},a_{\mathbf{k}}^\dagger]=1 . \tag{2.19}$$

To describe the four-wave mixing, we introduce separate creation and annihilation operators for the fields on either side of the nonlinear medium by $a_i^\dagger(\beta)$ and $a_i(\beta)$ for the incident field and by $a_r^\dagger(\beta)$ and $a_r(\beta)$ for the reflected phase-conjugate field, where the index $\beta=0,L$ indicates that the operators describe the fields for $z<0$ ($\beta=0$) or for $z>L$ ($\beta=L$). This idea of locally defined field operators is common in quantum-mechanical nonlinear optics.[22-24] In the four-wave mixing process the input fields are independent. Therefore we have the relations

$$[a_i(0),a_r(L)]=0,\quad [a_i(0),a_r^\dagger(L)]=0 . \tag{2.20}$$

Since in our simplified model we have neglected the losses in the nonlinear medium, we can directly obtain the relation between the output field operators $a_i^\dagger(L)$ and $a_r(0)$ and the input field operators $a_i^\dagger(0)$ and $a_r(L)$ from the classical results (2.11) and (2.12) by replacing the classical amplitudes $E_{i,r}$ by the corresponding field operators $i\hbar(\omega_0/2\hbar\epsilon_0\mathcal{V})^{1/2}a_{i,r}$.[22] This yields for the annihilation operator of the reflected field at $z=0$ (Ref. 22)

$$a_r(0)=-\nu_\delta a_i^\dagger(0)+\mu_\delta a_r(L) \tag{2.21}$$

and for the creation operator of the transmitted incident field at $z>L$ (Ref. 22)

$$a_i^\dagger(L)=\mu_\delta a_i^\dagger(0)-\nu_\delta^* e^{-i\delta L}a_r(L) , \tag{2.22}$$

with ν_δ and μ_δ defined in (2.12) and (2.13). Note that due to the hyperbolic rotation the output field operators obey the commutation relations

$$[a_r(0),a_r^\dagger(0)]=1,\quad [a_i(L),a_i^\dagger(L)]=1 , \tag{2.23}$$

as they should, according to (2.19). When losses in the medium are taken into account the above quantization procedure no longer holds, since additional noise operators also appear in the relations (2.21) and (2.22).[23,24] Since $a_r(L)$ is now a field operator, it cannot be omitted in (2.21) in the quantum-mechanical description of the PCM, in contrast to the classical case. This operator describes the quantum fluctuations of the vacuum field at $z>L$. These vacuum fluctuations contribute to the reflected phase-conjugate field after being amplified by the PCM. The annihilation operator of the reflected field for $z<0$ is thus a mixture of the annihilation operator $a_r(L)$ of the field incident on the opposite side of the mirror and the creation operator $a_i^\dagger(0)$ of the incident field at $z=0$.

III. CLASSICAL DIPOLE IN FRONT OF A PCM

In free space the evolution equation of a classical oscillating dipole $p(t)$, which is damped due to emission of radiation, is described by the Abraham-Lorentz equation[25]

$$\dddot{p}(t)+\gamma\dot{p}(t)+\omega_0^2 p(t)=0 , \tag{3.1}$$

with ω_0 the oscillation frequency. The radiative reaction term $\gamma\dot{p}(t)$ in (3.1) accounts for the radiative damping of the classical dipole due to the emission of radiation. This damping rate γ follows from the requirement that the emitted energy of radiation is compensated by the loss of oscillator energy. When γ is assumed to be small compared with the oscillator frequency ($\gamma \ll \omega_0$), Eq. (3.1) gives rise to two well-known solutions which are given by

$$p(t)=\mathrm{Re}(p_0 e^{i\varphi_0-\gamma t/2-i\omega_0 t}) , \tag{3.2}$$

and its complex conjugate. Throughout this section we shall only consider solutions of the evolution equation with positive-frequency parts $\exp(-i\omega_0 t)$.

In the presence of the PCM, the emitted field is reflected back towards the dipole, and this reflected field exerts an additional force on the dipole. The positive-frequency component of the reflected field is equal to $\alpha\nu$ times the complex conjugate of the emitted field, with

$$\nu=|\nu|e^{i\psi}\equiv\nu_0 , \tag{3.3}$$

with ν_0 defined in (2.13), since only a fraction α ($4\pi\alpha$ solid angle of the PCM) of the field emitted by the dipole reaches the PCM. Here we implicitly assume that the quantity ν_δ does not vary appreciably over the width of the spectrum emitted by the dipole. This requires that the coupling strength η is large compared with γ/c. Hence the positive-frequency part of the additional damping term also will be $\alpha\nu$ times the complex conjugate of the positive-frequency part of the radiative reaction term.[18,19] The presence of the PCM gives, therefore, rise to an additional damping term and the evolution equation (3.1) is replaced by

$$\ddot{p}(t)+\gamma\dot{p}(t)+\alpha\nu\gamma[\dot{p}(t)]^* e^{-2i\omega_0 t}+\omega_0^2 p(t)=0 \tag{3.4}$$

for the positive-frequency part of the dipole, where ν is defined in (3.3). Note that this evolution equation is no longer linear in the complex dipole $p(t)$, due to the presence of its complex conjugate: a linear combination of two solutions of (3.4) with complex coefficient is not necessarily a solution. On the other hand, (3.4) may be viewed as a coupled set of real linear differential equation for the real and imaginary part of $p(t)$. In the case that $\gamma,\gamma|\nu|\ll\omega_0$, the second-order differential equation has two independent solutions,

$$p_+(t)=i\exp[-\tfrac{1}{2}\gamma(1+\alpha|\nu|)t-i\omega_0 t+\tfrac{1}{2}i\psi] \tag{3.5}$$

and

$$p_-(t)=\exp[-\tfrac{1}{2}\gamma(1-\alpha|\nu|)t-i\omega_0 t+\tfrac{1}{2}i\psi] . \tag{3.6}$$

The general solution of (3.4) is an arbitrary linear combination of (3.5) and (3.6) *with real coefficients*. The general physical solution for the oscillating dipole in front of the mirror is, therefore,

$$p(t)=\mathrm{Re}[A_+p_+(t)+A_-p_-(t)] , \tag{3.7}$$

with A_+ and A_- arbitrary real coefficients. The values of A_+ and A_- are determined by the initial amplitude and phase of the dipole.

Equation (3.7) demonstrates that the dipole in front of the PCM decays with two different damping rates $\gamma(1+\alpha|\nu|)$ and $\gamma(1-\alpha|\nu|)$. These different rates were already obtained by Bochove,[18] but we believe that the present treatment clarifies the essential part played by the phase of the dipole. The solutions (3.5) and (3.6) diverge from the results of Refs. 16 and 17, where the phase dependence of the evolution was ignored. Our result demonstrates that the phase plays an essential part.

In the case of solution (3.5), the damping force $-\gamma\dot{p}$ due to the self-field is in phase with the damping force due to the reflected field, which yields an enhanced decay rate. In the case of solution (3.6), the damping force due to the self-field and the reflected field have opposite phase, and the decay is obstructed. For a reflectivity $\alpha|\nu|>1$, the dipole oscillation will be enhanced.

The ratio of A_+ and A_- determines the phase difference between the damping force of the self-field and that of the reflected field. This phase dependence is not discussed in earlier work.[16–18] References 16 and 17 did not yield these different damping rates, since these works did not consider various initial phases of the dipole compared with the phase of the pump field. In contrast to what is found in Refs. 16 and 17, the damping rate does not vary within a continuous range. It is the initial phase that selects a linear combination of the solutions (3.5) and (3.6), each with its specific decay rate.

As a special case of (3.7), we consider the solution that reduces to (3.2) in the limit of $|\nu|$ approaching zero. Hence we obtain

$$\begin{aligned}p(t)=\mathrm{Re}\{&p_0 e^{-i\omega_0 t+i\psi/2}\\ &\times[\cos(\varphi_0-\tfrac{1}{2}\psi)e^{-\gamma(1-\alpha|\nu|)t/2}\\ &\quad+i\sin(\varphi_0-\tfrac{1}{2}\psi)e^{-\gamma(1+\alpha|\nu|)t/2}]\} ,\end{aligned} \tag{3.8}$$

where φ_0 is the initial phase of the dipole. An interesting feature of Eq. (3.8) is that the relative phase $\varphi_0-\frac{1}{2}\psi$ determines the contribution of the different damping rates to the evolution of the dipole. This is not surprising, since from Sec. II it follows that $2\varphi_0-\psi$ determines the phase difference between the incident and the reflected field at the surface of the PCM. When $\varphi_0-\frac{1}{2}\psi=0$, we obtain from (3.4) that the damping forces due to the self-field and the reflected self-field have opposite phase giving rise to obstructed decay. For $\varphi_0-\frac{1}{2}\psi=\frac{1}{2}\pi$, both damping forces are in phase giving rise to enhanced decay as is shown by solution (3.8).

IV. ATOM IN FRONT OF PCM

We consider a two-state atom, with ground state $|g\rangle$ and excited stated $|e\rangle$ and with transition frequency ω_0. The interaction Hamiltonian in the interaction picture for the atom coupled to the vacuum field in the dipole approximation is given by

$$\begin{aligned}V_0=-i\hbar\sum_{\mathbf{k}} p&\left[\frac{\omega_k}{2\mathcal{V}\hbar\epsilon_0}\right]^{1/2}\\ &\times[a_{\mathbf{k}}e^{-i\omega_k t}(S_+e^{i\omega_0 t}+S_-e^{-i\omega_0 t})-\mathrm{H.c.}] ,\end{aligned} \tag{4.1}$$

with raising and lowering operators $S_\pm$,

$$S_+ = |e\rangle\langle g|, \quad S_- = |g\rangle\langle e| \ . \tag{4.2}$$

p is the atomic transition dipole and $\mathcal{V}$ is the quantization volume. The coupling to the vacuum field causes spontaneous radiative decay of the atom. We now position the two-state atom in front of a PCM. Due to the particular form of the PCM, we can divide the atom-field interaction into two parts. Let θ be the set of field modes $\mathbf{k}$ which couple to the atom and the PCM, and are pointing in the direction from the PCM towards the atom. The field modes with $\mathbf{k}\notin\theta$ are not affected by the PCM. The coupling of the atom to these modes is given by (4.1). The field modes with $\mathbf{k}\in\theta$ that couple to the atom are affected by the PCM. These field modes contain the reflected field generated by the PCM. The nonlinear polarization $P^{(3)}$ in the PCM is induced by the vacuum fluctuations in the modes incident on both sides of the PCM. This polarization modifies the field at the position of the atom. This modification is accounted for by making the substitution (2.21) for the annihilation operator of the modes with $\mathbf{k}\in\theta$ at the position of the atom. This demonstrates that the field modes with $\mathbf{k}\in\theta$ become excited due to the presence of the PCM. The atom now couples to a reservoir which is modified by the PCM as expressed by the substitution of (2.21). The modified interaction Hamiltonian describing the atom-field interaction in the presence of a PCM is given by

$$\begin{aligned} V = & -i\hbar \sum_{\substack{\mathbf{k}\\ \mathbf{k}\notin\theta}} p \left[\frac{\omega_k}{2\mathcal{V}\hbar\epsilon_0}\right]^{1/2} [a_{\mathbf{k}} e^{-i\omega_k t}(S_+ e^{i\omega_0 t} + S_- e^{-i\omega_0 t}) - \text{H.c.}] \\ & -i\hbar \sum_{\substack{\mathbf{k}\\ \mathbf{k}\in\theta}} p \left[\frac{\omega_k}{2\mathcal{V}\hbar\epsilon_0}\right]^{1/2} \{[\mu_\delta a_{\mathbf{k}}(L) e^{-i\omega_k t} - \nu_\delta a^\dagger_{-\mathbf{k}} e^{-i(2\omega_0-\omega_k)t}](S_+ e^{i\omega_0 t} + S_- e^{-i\omega_0 t}) - \text{H.c.}\} \end{aligned} \tag{4.3}$$

with ν_δ and μ_δ defined in (2.13) and (2.14), and where we used (2.1). Since the modification of the field at the position of the atom are described by a modification of the field operators, the state vector of the modified vacuum is unchanged.

The evolution equation for the atomic density matrix can be obtained by using reservoir theory,[26] which we briefly outline. The radiation field is treated as a large reservoir whose evolution is negligible influenced by the interaction with the atom. The radiation field is initially in the vacuum state $|0\rangle_R\langle 0|$. The evolution equation for the density matrix $\Pi(t)$ for the atom-field system, in the interaction picture, is given by

$$\frac{d}{dt}\Pi(t) = -\frac{i}{\hbar}[V(t),\Pi(t)] \ . \tag{4.4}$$

In order to express this evolution in the rapidly decaying correlation functions of the radiation field, we formally integrate the evolution equation (4.4), which yields

$$\Pi(t) = -\frac{i}{\hbar}\int_0^t [V(\tau),\Pi(\tau)]d\tau \ . \tag{4.5}$$

Substituting this result back into Eq. (4.4) and taking the trace over the states of the radiation field then gives

$$\frac{d}{dt}\rho(t) = -\frac{1}{\hbar^2}\text{Tr}_R \int_0^t d\tau [V(t),[V(t-\tau),\Pi(t-\tau)]] \ , \tag{4.6}$$

where $\rho = \text{Tr}_R(\Pi)$ is the reduced density matrix for the atom alone. Next, we apply the Born-Markov approximation, in which the buildup of correlations between atom and field are neglected. This is justified when a characteristic evolution time of the atomic system is assumed to be large compared with the atom-field correlation time. Then Eq. (4.6) is approximated by

$$\frac{d}{dt}\rho(t) = -\frac{1}{\hbar^2}\text{Tr}_R \int_0^\infty d\tau [V(t),[V(t-\tau),\rho(t)|0\rangle_R\langle 0|]] \ . \tag{4.7}$$

Substituting (4.3) into (4.7) and evaluating the trace and integral in the standard way, while omitting nonsecular terms, gives for the evolution equation for the atomic density matrix in the interaction picture

$$\begin{aligned} \frac{d}{dt}\rho(t) = & -\tfrac{1}{2}A(1+\alpha|\nu|^2)[S_+S_-\rho(t) + \rho(t)S_+S_- - 2S_-\rho(t)S_+] \\ & -\tfrac{1}{2}\alpha A|\nu|^2[S_-S_+\rho(t) + \rho(t)S_-S_+ - 2S_+\rho(t)S_-] + \alpha A\nu S_+\rho(t)S_+ + \alpha A\nu^* S_-\rho(t)S_- \ , \end{aligned} \tag{4.8}$$

with ν defined in (3.3), and α the fraction of the solid angle occupied by the PCM and where we have used (2.16). Furthermore, we have defined

$$A=\frac{\omega_0^3 p^2}{2\pi\epsilon_0 \hbar c^3}\ , \tag{4.9}$$

which is the Einstein coefficient for spontaneous emission in vacuum. In Eq. (4.8) we have omitted a divergent imaginary contribution describing a level shift. This level shift turns out to be equal to the Lamb shift in free space. We shall absorb this level shift in ω_0.

The result (4.8) can be interpreted as follows. The presence of the PCM excites the vacuum field. The atom in front of a PCM decays, therefore, in a heat bath, which gives rise to stimulated transitions between the two states with probability $\alpha A|\nu|^2$. An atom initially in the ground state can be excited in this modified vacuum field. A special feature of the heat bath in the presence of the PCM is that it has an inherent phase. This gives rise to two additional phase-dependent terms in the evolution equation of the atom. In an ordinary incoherent heat bath, such as a broadband radiation field, these terms average to zero. The property of generating a heat bath with an inherent phase is characteristic for conjugating media. This phase of the heat bath gives rise to a phase-dependent coupling between the coherences. A similar effect was found for the classical dipole, due to a phase-dependent coupling between the dipole field and its complex conjugate. These results are in contradiction with the results for the two-level atom in Ref. 18, where inhibited decay of the populations was found.

A phase-dependent evolution of the atomic coherences is also found for an atom in a squeezed vacuum produced by a degenerate parametric amplifier as has been discussed by Gardiner.[27] The evolution equation of an atom in such a squeezed vacuum is similar to (4.8).[27]

The evolution equation (4.8) for the density matrix ρ can readily be solved yielding

$$\rho_{ee}(t)=\frac{\alpha|\nu|^2}{1+2\alpha|\nu|^2}+\left[\rho_{ee}(0)-\frac{\alpha|\nu|^2}{1+2\alpha|\nu|^2}\right]\exp[-A(1+2\alpha|\nu|^2)t]\ , \tag{4.10a}$$

$$\rho_{gg}(t)=1-\rho_{ee}(t)\ , \tag{4.10b}$$

$$\begin{aligned}\rho_{eg}(t)=|\rho_{eg}(0)|\{&\sin(\varphi-\tfrac{1}{2}\psi)i\exp[-\tfrac{1}{2}A(1+2\alpha|\nu|+2\alpha|\nu|^2)t+\tfrac{1}{2}i\psi]\\&+\cos(\varphi-\tfrac{1}{2}\psi)\exp[-\tfrac{1}{2}A(1-2\alpha|\nu|+2\alpha|\nu|^2)t+\tfrac{1}{2}i\psi]\}\ ,\end{aligned} \tag{4.10c}$$

$$\rho_{ge}(t)=[\rho_{eg}(t)]^*\ , \tag{4.10d}$$

with

$$\rho_{eg}(0)=|\rho_{eg}(0)|e^{i\varphi}\ . \tag{4.11}$$

We notice that $\rho_{ee}(t)$ decays at an enhanced rate to the nonzero stationary value $\alpha|\nu|^2/(1+2\alpha|\nu|^2)$, reflecting that absorption from the ground state to the excited state also takes place. Furthermore, we find that (4.10c) has the form of Eq. (3.8). Note that there is no term $\exp(-i\omega_0 t)$ in (4.10c), since ρ describes the atomic evolution in the interaction representation. Again we obtain two different damping rates whose contribution to the evolution of the coherences depends on the phase of the coherence. A change in the relative phase $\psi-2\varphi$ gives rise to a nontrivial change to the evolution of the coherences. In contrast to the classical dipole case, the damping rates now have the correlation factors $1\pm2\alpha|\nu|+2\alpha|\nu|^2$ rather than $1\pm\alpha|\nu|$. This difference basically arises since a harmonic oscillator may have an arbitrarily large internal energy, whereas the energy of a two-state atom is bounded by the excited state energy.

V. CONCLUSIONS

In this paper, we discussed the evolution of a classical radiating dipole and of a two-state atom in front of a phase-conjugate mirror. We derived explicit expressions for the time evolution of the classical dipole and of the density matrix for the atom. Due to the inherent phase of the PCM, these results depend, in a nontrivial manner, on the relative phase of the initial atomic dipole and the PCM.

The classical dipole evolution is described by Eq. (3.7). There are two decay rates whose contribution to the evolution are determined by the relative phase of the initial dipole and the PCM. The atomic evolution, described by a density matrix, is given in Eq. (4.10). Apart from extra effects due to the stimulated transitions caused by the excited vacuum field, we find that the evolution of the coherences exhibit a similar behavior as the classical radiating dipole. Furthermore, the presence of the PCM enables the atom to make transitions from the ground state to the excited state.

The essential phase dependence displayed in the evolution equations opens new interesting ways to manipulate the phase and the spectral properties of the emitted radiation. Since we have control over the phase of the two driving pump fields that determine the phase of the PCM, we expect that experimental verification of this phase dependence is possible in principle.

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