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Energy loss of fast nonthermal electrons in plasmas

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Simple analytical expressions for equilibration times of nonrelativistic monoenergetic electrons in plasmas are evaluated in the "weak"-beam approximation when the density of the monoenergetic electrons is much smaller than the plasma density. The equilibration time is defined as the time needed by the beam of monoenergetic electrons to lose most of its energy as a result of collisions with plasma particles having a Maxwellian energy distribution. The process of the energy equilibration is treated as a statistical superposition of both elastic (electron-electron, electron-ion, and electron–neutral-particle) and inelastic (electron–neutral-particle) collisions in the plasma. The possibility of collisionless equilibration is also discussed. Comparison of the equilibration times with the Spitzer relaxation times indicates that the former times are more appropriate for an estimate of the energy loss of the "weak" electron beams in highly ionized plasmas. The approach of this work can be generalized in a straightforward way to beam-plasma and beam-gas systems with ionic and neutral-particle beams.

INTRODUCTION

Kinetic analyses of plasmas containing electrons, ions, and neutral particles are critically dependent on a knowledge of the frequencies of the electron-impact elastic and inelastic processes in the plasmas. Since in some applications the presence of fast nonthermal electrons (entering the plasma as, for example, an externally generated beam) can increase these frequencies substantially, it is important to estimate the "lifetime" of the nonthermal electrons in the plasma. A good measure of this is the equilibration time, defined here as the time needed by the nonthermal, monoenergetic electrons (a "cold" beam) to lose most of their energy, as a result of collisions with plasma particles. The typical mean energy of particles in a partially ionized plasma is less than 2–3 eV. The initial energy of the nonthermal electrons is assumed to be much greater than the mean energies of the plasma particles, but it can be as low as several tens of eV. The question of equilibration of the nonthermal electrons with such a relatively low energy is of significant importance in many applications, but it is a difficult question, especially in the case of equilibration by inelastic collisions (electronic and vibrational excitation, ionization, etc.).[1]

The process of equilibration is treated as a statistical superposition of elastic and inelastic collisions of the nonthermal, monoenergetic electrons (hereafter called beam particles) with the particles of uniform plasma (electrons, ions, and neutral particles—hereafter called plasma particles), with the possibility of equilibration by collisionless effects. The density and initial energy of the beam particles are n_b and ε_b^0, respectively, while the densities and temperatures of the plasma particles are $n_p^{(j)}$ and $T_p^{(j)}$, respectively; the superscript (j) denotes the jth component of the plasma. (We will skip the superscript when discussing equilibration of the electron beam by Coulomb collisions.) The initial energy of the beam particles is dispersed through collisions with plasma particles, and after the equilibration time most of the initial beam energy will be lost.

The effective equilibration time of the electron beam passing through the plasma can be calculated as

$$\frac{1}{\tau_{ep}}=\frac{1}{\tau_{e\text{-}e}}+\sum_k\frac{1}{\tau_{ei}^{(k)}}+\sum_l\frac{1}{\tau_{en}^{(l)}}+\sum_j\frac{1}{\tau_{in}^{(j)}}+\sum_n\frac{1}{\tau_h^{(n)}}\ , \tag{1}$$

where $\tau_{e\text{-}e}$ is the equilibration time for the beam equilibration resulting from interactions of the beam electrons with the plasma electrons, $\tau_{ei}^{(k)}$ is the equilibration time for equilibration by Coulomb collisions with the plasma ions of the kth kind, $\tau_{en}^{(l)}$ is the equilibration time resulting from elastic collisions with the plasma neutrals (atoms, molecules) of the lth kind, $\tau_{in}^{(j)}$ is the equilibration time resulting from inelastic collisions with the plasma neutral particles of the jth kind, and $\tau_h^{(n)}$ is the beam equilibration time resulting from collisionless effects of the nth kind. In this work we estimate the equilibration times $\tau_{e\text{-}e}$, τ_{ei}, τ_{en}, τ_{in}, and τ_h for interaction of fast nonrelativistic monoenergetic electrons with plasma of particles having Maxwellian distributions.

In general, energy equilibration of the beam-plasma system can be investigated either by applying the Fokker-Planck equation (when the equilibration occurs via Coulomb collisions) or from the Boltzmann transport equation (when electron–neutral-particle collisions dominate the equilibration process). Solution of the Fokker-Planck equation is difficult and requires advanced numerical techniques. Therefore we use here an approach based on statistical superposition of the energy losses resulting from all the binary collisions of the nonthermal electrons with the electrons and ions of the plasma. Such an approach leads to simple, analytical expressions for the corresponding equilibration times. Solution of the Boltzmann transport equation, including electron–neutral-particle interactions, is also difficult. The difficulty is caused by the complexity of the numerical techniques required for the solution and by difficulties in formulation of the Boltzmann integrals for inelastic interactions. Therefore, instead of solving the Boltzmann equation, we use here the statistical superposition of the energy losses of the nonthermal electrons resulting from all binary electron–neutral-particle inelastic collisions. Again, such a procedure leads to analytical expressions for the corresponding equilibration times.

We consider in this work the case (see Fig. 1) when the initial energy distributions of the beam and plasma particles are, respectively,

$$f_b(\varepsilon_b)=\delta(\varepsilon_b-\varepsilon_b^0), \quad f_p^{(j)}(\varepsilon_p)=f_M^{(j)}(\varepsilon_p) , \tag{2}$$

where ε_b and ε_p are energies of the beam and plasma particles, respectively, ε_b^0 is the initial energy of the beam electrons, $f_b(\varepsilon_b)$ is the energy distribution of the beam electrons, $f_p^{(j)}(\varepsilon_p)$ is the energy distribution of particles belonging to the jth component of the plasma, and the subscript M denotes Maxwellian distribution.

We will assume, in what follows, absence of a meaningful magnetic field in the beam-plasma system and that the beam diameter is considerably less than the plasma diameter. Also, we use the "weak"-beam approximation requiring that $n_b \ll n_p^{(j)}$ (a common case in beam-plasma applications). If $n_b \ll n_p^{(j)}$, then to the first order in n_b the particles of the beam interact only with particles of the plasma (i.e., the beam electrons behave like noninteracting particles). Therefore, the beam particles "see" the plasma as practically a uniform gas of particles in equilibrium at all times. (A comprehensive analysis of typical assumptions made in theoretical analyses in relation to the conditions existing in real beam-plasma systems—the effects of inhomogeneity, lack of electrical neutrality, finite geometry, magnetic fields, etc.—can be found in Refs. 2 and 3.)

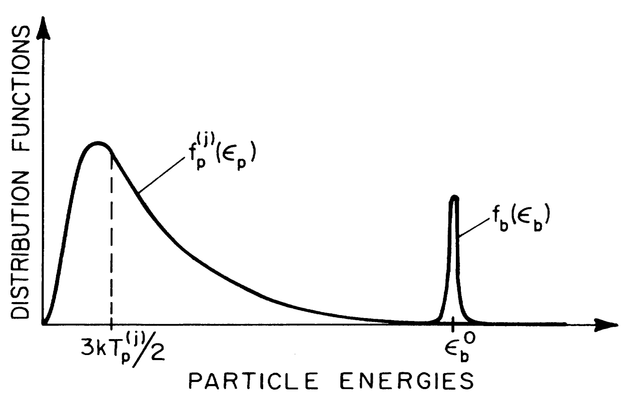

FIG. 1. Initial energy distributions for beam $[f_b(\varepsilon_b)]$ and plasma $[f_p^{(j)}(\varepsilon_p)]$ particles. The plasma particles of the jth kind have Maxwellian energy distribution with temperature $T_p^{(j)}$, while the incident-beam electrons are almost monoenergetic around the energy ε_b^0.

EQUILIBRATION BY ELASTIC COLLISIONS

The average rate of energy loss, per beam particle and per unit volume, resulting from collisions with the particles of the jth component of the uniform plasma is

$$\frac{\partial \varepsilon_b^{(j)}}{\partial t}=n_p^{(j)}\int_0^{\infty}\Delta\varepsilon_{bp}^{(j)}(v_b,v_p)v_b\sigma_{bp}^{(j)}(v_b,v_p)f_p^{(j)}(v_p)dv_p , \tag{3}$$

where $\varepsilon_b=m_b v_b^2/2$ is the energy of the beam particles, $\varepsilon_p=m_p v_p^2/2$ is the energy of the plasma particles, $\Delta\varepsilon_{bp}^{(j)}$ is the average energy loss of a beam particle in a single collision, and $\sigma_{bp}^{(j)}$ is the cross section for the elastic scattering. In general, $\Delta\varepsilon_{bp}^{(j)}$ is a function of velocities $\mathbf{v}_b$ and $\mathbf{v}_p$, the scattering angle χ, and the initial angle θ between the vectors $\mathbf{v}_b$ and $\mathbf{v}_p$ (Ref. 4)

$$\begin{aligned}\Delta\varepsilon_{bp}^{(j)}(v_b,v_p,\chi,\theta)&\\ =\kappa_{bp}^{(j)}(1-\cos\chi)[m_b v_b^2-m_p^{(j)}v_p^2&\\ +(m_p^{(j)}-m_b)v_b v_p\cos\theta] ,&\end{aligned} \tag{4}$$

where

$$\kappa_{bp}^{(j)}=\frac{m_b m_p^{(j)}}{(m_b+m_p^{(j)})^2} , \tag{5}$$

and m_b and $m_p^{(j)}$ are masses of the beam particles and the particles of the jth component of the plasma, respectively.

Assuming that the particles of the jth component of the plasma have a Maxwellian distribution (i.e., their spatial distribution of velocity vectors is isotropic), the distribution $p(\theta)$ of the angle θ between the vectors $\mathbf{v}_b$ and $\mathbf{v}_p$ is

$$p(\theta)d\theta=\tfrac{1}{2}\sin\theta\, d\theta . \tag{6}$$

Taking the above into account, the average energy loss $\Delta\varepsilon_{bp}^{(j)}$ is

$$\begin{aligned}\Delta\varepsilon_{bp}^{(j)}(v_b,v_p)&=\pi\int_0^{\pi}\int_0^{\pi}\Delta\varepsilon_{bp}^{(j)}(v_b,v_p,\chi,\theta)\sin\theta\sin\chi d\theta\, d\chi\\ &=4\pi\kappa_{bp}(m_b v_b^2-m_p^{(j)}v_p^2) .\end{aligned} \tag{7}$$

Coulomb collisions

The scattering cross section for Coulomb collision of a beam particle (with charge $Z_b e$ and mass m_b) and a plasma particle (with charge $Z_p e$ and mass m_p) is[5]

$$\sigma_{bp}(w)=4\pi\left[\frac{Z_b Z_p e^2}{\mu_{bp}w^2}\right]^2\ln\Lambda , \tag{8}$$

where

$$\Lambda=\frac{3(kT_p)^{3/2}}{2Z_b Z_p e^3(\pi n_p)^{1/2}} , \tag{9}$$

n_p and T_p are the density and temperature of the charged particles in the plasma, respectively, $\mathbf{w}=\mathbf{v}_b-\mathbf{v}_p$ (w^2

$=v_b^2+v_p^2$ because $\langle\cos\theta\rangle=0$), and μ_{bp} is the reduced mass of the collision system,

$$\mu_{bp}=\frac{m_b m_p}{m_b+m_p} . \tag{10}$$

Introducing Eqs. (7) and (8) into Eq. (3) and using a Maxwellian distribution for plasma charges, leads to the following equilibration rate for the incident electron beam:

$$\frac{\partial \varepsilon_e}{\partial t}=\frac{64 n_p \pi^{3/2} Z_p^2 e^4 \ln\Lambda}{m_e v_e}\frac{m_e}{m_p}\beta(\rho^2) , \tag{11}$$

where m_e is the electron mass, and v_e and ε_e are velocity and energy of the incident electrons, respectively,

$$\beta(\rho^2)=\int_0^\infty \frac{(1-x^2\rho^{-2})x^2\exp(-x^2)}{(1+x^2\rho^{-2})^2}dx , \tag{12}$$

$$x^2\equiv\frac{m_p v_p^2}{2kT_p} , \tag{13}$$

and

$$\rho^2\equiv\frac{m_p v_b^2}{2kT_p} . \tag{14}$$

The values of the integral (12) are given in Table I. The integral is very weakly dependent on the parameter ρ when $\rho\gg1$ (a common situation in applications). Then, the integral $\beta(\rho^2)$ can be replaced by its asymptotic value $\beta_\infty=\beta(\rho^2\to\infty)=\sqrt{\pi}/4$, so that

$$\frac{\partial \varepsilon_e}{\partial t}=\frac{64 n_p \pi^{3/2} Z_p^2 e^4 \ln\Lambda}{\sqrt{2m_e\varepsilon_e}}\frac{m_e}{m_p}\beta_\infty . \tag{15}$$

Integrating the function (15) over the energy ε_e (from the initial energy of the beam ε_e^0 to its final energy ε_e^f one obtains the time necessary for the beam electrons to decrease their energy, through the Coulomb collisions with plasma charges, to ε_e^f. This time is

$$\tau_{ep}(\varepsilon_e^0,\varepsilon_e^f)=\frac{m_p}{m_e}\frac{(2m_e)^{1/2}}{24\pi^2 Z_p^2 e^4 n_p \ln\Lambda}[(\varepsilon_e^0)^{3/2}-(\varepsilon_e^f)^{3/2}] . \tag{16}$$

(Note that these equilibration times are weakly dependent on the temperature T_p of the plasma charges.)

Relationship (16) can now be used to determine the time (the equilibration time) after which the energy of the beam will decrease, as a result of Coulomb collisions of the beam electrons with plasma charges (electrons or ions), more than, let us say, 80%. Then one obtains, neglecting the second term in the square brackets in Eq. (16), the following equilibration times for equilibration of the electron-beam energy by Coulomb collisions:

$$\tau_{e\text{-}e}(\varepsilon_e^0)=\frac{(2m_e)^{1/2}}{24\pi^2 Z_p^2 e^4 \ln\Lambda}\frac{(\varepsilon_e^0)^{3/2}}{n_p} \tag{17}$$

(n_p is the density of the plasma electrons), and

$$\frac{\tau_{ei}}{\tau_{e\text{-}e}}=\frac{m_i}{m_e} \tag{18}$$

(m_i is the mass of the plasma ions), if the plasma is electrically neutral, that is, if $n_e\approx n_i$. Equation (17) can be rewritten in a more practical form as

$$\tau_{e\text{-}e}(\varepsilon_e^0)=\frac{6.87\times10^3}{Z_p^2\ln\Lambda}\frac{(\varepsilon_e^0)^{3/2}}{n_p} , \tag{19}$$

where $\tau_{e\text{-}e}$ is in seconds, ε_e^0 in eV, and n_p in cm^{-3}. In most partially ionized plasmas ($5000\lesssim T_p\lesssim 30\,000$ K and $10^9\lesssim n_p\lesssim10^{18}$ cm^{-3}) $5\lesssim\ln\Lambda\lesssim10$, so that $\ln\Lambda\approx7$ can be assumed in Eq. (19). At higher temperatures in fully ionized plasmas, $10\lesssim\ln\Lambda\lesssim25$ and $\ln\Lambda\approx15$ seems to be a reasonable value.

It should be noted that the expressions (17)–(19) give reliable estimates of the equilibration times of the incident beam (with at least 80% of the beam energy transferred to the plasma) if the following conditions are met: (1) the final energy ε_e^f of the electron beam is much less than the beam initial energy ε_e^0, but at the same time ε_e^f should be distinctively greater than the average energy of the plasma charges $\langle\varepsilon_p\rangle=3kT_p/2$; (2) the parameter ρ^2 should be large at $\varepsilon_e\approx\varepsilon_e^f$ (see Table I); and (3) the final energy of the beam particles must be greater than some critical energy $\varepsilon_{\rm cr}$. The last requirement is a result of the fact that the beam of fast particles will heat the plasma, whereas a beam of slow particles will cool the plasma; this is in contrast to the momentum of the beam particles, which is always transferred from the beam to the plasma. The energy of the beam is transferred (collisionally) to the plasma particles of mass m_p and temperature T_p when the beam energy ε_b is greater than some critical energy $\varepsilon_{\rm cr}$; when $\varepsilon_b<\varepsilon_{cr}$ the energy is transferred to the beam from the plasma particles. The critical energy of the beam-plasma system is[6]

$$\varepsilon_{\rm cr}=\frac{m_b}{m_p}x_{\rm cr}^2 kT_p , \tag{20}$$

where $x_{\rm cr}$ can be obtained from the numerical solution of the equation

$$1+\frac{m_p}{m_b}-\frac{\pi^{1/2}\exp(x_{\rm cr}^2)}{2x_{\rm cr}}\times\left[1-\frac{2}{\pi^{1/2}}\int_{x_{\rm cr}}^\infty\exp(-y^2)dy\right]=0 . \tag{21}$$

An approximate solution of Eq. (21) gives

$$x_{\rm cr}^2\approx1 \tag{22}$$

(for electron-electron collisions), and

TABLE I. Values of the integral $\beta(\rho^2)$ [Eq. (12)].

ρ^2	5	10	50	100	1000	∞
$\beta(\rho^2)$	0.219	0.300	0.406	0.424	0.441	$\sqrt{\pi}/4=0.443$

$$x_{\rm cr}^2 \approx \ln(m_p/m_b) \approx 10 \tag{23}$$

(for electron-ion collisions), so that $\varepsilon_{\rm cr} < 3kT_p$ always. Thus the third requirement is always fulfilled in the beam-plasma systems considered in this work. Summarizing the above, one can say that the physical assumptions of this work require that

$$\frac{\varepsilon_e^f}{\varepsilon_e^0} \lesssim \frac{1}{5}, \quad \varepsilon_e^f > \langle \varepsilon_p \rangle = \tfrac{3}{2}kT_p, \quad \rho^2(\varepsilon_e^f) \gtrsim 50 \ . \tag{24}$$

These three conditions are easy to meet in the case of the electron-beam equilibration by plasma ions [then the first of the requirements (24) is sufficient]. In the case of equilibrium by plasma electrons the conditions (24) lead to the requirement

$$\varepsilon_e^0 \gtrsim 100\langle \varepsilon_p \rangle \ , \tag{25}$$

which is also easy to fulfill in real beam-plasma systems.

One should mention that we treated, when evaluating the times $\tau_{e\text{-}e}$ and τ_{ei}, the equilibration process as a statistical superposition of electron binary encounters with charges of the plasma. Several analyses showed that such a representation is indeed valid in most plasmas if the magnetic field is not too strong.[7-11]

Comparison of the equilibration times with Spitzer's relaxation times

We compare in this section the equilibration times $\tau_{e\text{-}e}$ and τ_{ei} of the present work with the commonly used energy-exchange relaxation times introduced by Spitzer[12] to characterize the rate of the energy exchange, between incident electrons and the plasma electrons and ions, that produces a large alternation in original energy distribution of the incident electrons. These relaxation times were obtained by using the Fokker-Planck approximation (only the first two diffusion coefficients in the phase space were considered). Spitzer's relaxation time $\tau_{e\text{-}e}^S$ for the energy-exchange (through collisions between the beam electrons and the plasma electrons) was defined as

$$\tau_{e\text{-}e}^S = \frac{\varepsilon_e^2}{\langle (\Delta\varepsilon_e)^2 \rangle} \ , \tag{26}$$

where $\Delta\varepsilon_e$ is the change of the energy of the beam electrons during their passage through the plasma. Although this time is not totally clear physically, it is a useful estimate of the order of magnitude of the time needed for the beam electrons to change significantly their energy distribution. One may add that it would be quite inconvenient to define the relaxation time $\tau_{e\text{-}e}^S$ as $\langle \Delta\varepsilon_e \rangle = -\varepsilon_e/\tau_{e\text{-}e}^S$ [so that $\langle \Delta\varepsilon_e \rangle = -\varepsilon_e^0 \exp(-t/\tau_{e\text{-}e}^S)$], because the incident electrons are being slowed down (i.e., lose energy) in the forward direction and accelerated (i.e., gain energy) in the perpendicular direction.

To evaluate the relaxation times $\tau_{e\text{-}e}^S$ and τ_{ei}^S, Spitzer used his general formula[12]

$$\tau_{bp}^S = \frac{(v_b^0)^3}{4A_D G(s_0)} \ , \tag{27}$$

where

$$A_D = 8\pi Z_p^2 e^4 n_p m_b^{-2} \ln\Lambda \ , \tag{28}$$

$$G(s_0) = \frac{\Phi(s_0) - s_0^{1/2}\Phi'(s_0)}{2s_0} \ , \tag{29}$$

$$s_0 = \frac{m_p}{m_b}\frac{\varepsilon_b^0}{kT_p} \ , \tag{30}$$

and $\Phi(s_0)$ is the error function

$$\Phi(s_0) = \frac{2}{\pi^{1/2}} \int_0^{(s_0)^{1/2}} \exp(-y^2)dy \tag{31}$$

that can be approximated with a good accuracy as

$$G(s_0) = \frac{1}{2s_0} \quad \text{when } s_0 \gtrsim 4 \ . \tag{32}$$

Using the approximation (32) one obtains from Eq. (27)

$$\tau_{e\text{-}e}^S(\varepsilon_e^0) = \frac{(2m_e)^{1/2} s_0'}{8\pi Z_p^2 e^4 \ln\Lambda} \frac{(\varepsilon_e^0)^{3/2}}{n_p} \ , \tag{33}$$

and

$$\frac{\tau_{ei}^S}{\tau_{e\text{-}e}^S} = \frac{m_i}{m_e} \ , \tag{34}$$

with

$$s_0' = \frac{\varepsilon_b^0}{kT_p} \ . \tag{35}$$

However, at $s_0 \gg 1$ the Fokker-Planck approximation loses its reliability and the expression (27) cannot be used when A_D and $G(s_0)$ are given by relationships (28) and (29). In such a case, one can use the partial correction proposed by Chandrasekhar,[13] that makes the following replacement in the expression (27):

$$G(s_0)\ln\Lambda = \frac{1}{2(1+m_p/m_b)^2} \ . \tag{36}$$

This replacement leads to the following relaxation times for the energy exchange:

$$\tau_{e\text{-}e}^C(\varepsilon_e^0) = \frac{(2m_e)^{1/2}}{2\pi Z_p^2 e^4} \frac{(\varepsilon_e^0)^{3/2}}{n_p} \tag{37}$$

and

$$\frac{\tau_{ei}^C}{\tau_{e\text{-}e}^C} = \left[\frac{m_i}{2m_e} \right]^2 , \tag{38}$$

if the plasma is electrically neutral. [It should be noted that even though the relationship (38) seems to be quantitatively inaccurate, the relaxation times $\tau_{e\text{-}e}^C$ and τ_{ei}^C are independent, similarly to our equilibration times (17) and (18), of the plasma temperature T_p.]

Assuming that the requirements (24) are met, the ratios of the Spitzer relaxation times $\tau_{e\text{-}e}^S$ [Eq. (33)] and $\tau_{e\text{-}e}^C$ [Eq. (37)] to the equilibrium time $\tau_{e\text{-}e}$ [Eq. (17)] of the present work are

$$\tau^S_{e\text{-}e} : \tau^C_{e\text{-}e} : \tau_{e\text{-}e} = 3\pi s'_0 : 12\pi \ln\Lambda : 1 \ . \tag{39}$$

One should keep in mind several facts when comparing the equilibration times of this work with the relaxation times of Spitzer. First, that the Spitzer energy-exchange relaxation time was formulated as a measure of the time required for relaxation to Maxwellian distribution with temperature close to T_p. In this work we deal with beams that are not required to be approximated by a relaxation process (that is why the term "equilibration" is used throughout this paper). Secondly, as discussed in Ref. 12, Spitzer's formula (33) is not reliable at high values of the ratio ε_b^0/kT_p considered in this work. This is because this formula was evaluated within the frame of the Fokker-Planck approximation neglecting diffusion coefficients of higher orders; these coefficients become important at high values of the ratio ε_b^0/kT_p. Spitzer also pointed out that the Chandrasekhar correction given in Eq. (36) does not take into account all important terms contributing to $\langle(\Delta\varepsilon_e)^2\rangle$. Even though this correction leads to a qualitatively acceptable result (lack of the dependence on the plasma temperature), it seems to be quantitatively inaccurate because it leads to a rather incorrect relationship (38). In addition, the formulas (37) and (33) give *relaxation* times (in the sense used by Spitzer[12]) not the *equilibration* time (in the sense used in this paper). Evaluation of our equilibration time $\tau_{e\text{-}e}$ does not use the Fokker-Planck approximation; instead, it is based on straightforward integration of a well-defined rate [Eq. (3)] for the energy loss. In summary, one can say that the times $\tau_{e\text{-}e}$ and τ_{ei} [Eqs. (17) and (18), respectively] seem to be reliable, and more appropriate than the Spitzer relaxation times, measures of efficiency of the energy loss during collisional interaction of a fast, "weak" electron beam with plasma electrons and ions having Maxwellian energy distributions.

Electron–neutral-particle collisions

The rate of the electron-beam energy equilibration by electron–neutral-particle elastic collisions is much lower[14] (that is, the corresponding equilibration time τ_{en} is relatively much longer) than the rate of equilibration by electron-electron collisions and electron-neutral inelastic collisions (see below), if the plasma ionization degree is higher than about $10^{-6}-10^{-5}$. Thus the contribution of the electron-neutral elastic collisions to the process of equilibration of the beam electrons can be neglected in most plasma applications.

COLLISIONLESS EQUILIBRATION

The system shown in Fig. 1 is subject to the beam-plasma instability if the plasma is highly ionized. During the first stage of the beam-plasma interaction the beam electrons are pretty much monoenergetic and the instability exciting plasma waves can be described by the hydrodynamic theory.[15] The duration of this period, hereafter called the growth time τ_h for the hydrodynamic instability, can be estimated as

$$\tau_h \sim \gamma_h^{-1} \ , \tag{40}$$

where the growth rate γ_h for this instability is

$$\gamma_h = \frac{3^{1/2}}{2^{4/3}}(\omega_b^2\omega_p)^{1/3} = 4\times 10^4 n_b^{1/3} n_p^{1/6} \ , \tag{41}$$

where

$$\omega_{b,p} = \left[\frac{4\pi e^2 n_{b,p}}{m_e}\right]^{1/2} = 5.64\times 10^4 n_{b,p}^{1/2} \tag{42}$$

are plasma frequencies (in $\sec^{-1}$) of the beam and plasma, respectively, and n_b and n_p (in cm^{-3}) are, as before, the electron densities in the beam and plasma, respectively. One should note that the growth rate γ_h can be very large and it is time independent, so that the hydrodynamic instability cannot be stabilized during its development. However, the energy loss of the beam during the hydrodynamic stage is, in most cases, not large. During the time τ_h, part of the beam energy,

$$\delta_h = \frac{\Delta\varepsilon_b}{\varepsilon_b^0} \approx \left[\frac{n_b}{2n_p}\right]^{1/3} , \tag{43}$$

is transferred into the plasma oscillation energy and a smaller part of the energy,

$$\delta'_h \approx 2\left[\frac{n_b}{2n_p}\right]^{2/3} , \tag{44}$$

is spent on heating of the beam.

The heating of the beam during the hydrodynamic stage causes a spread of the beam electron velocities, and after the time τ_h the instability acquires a kinetic character. The growth time τ_k for the kinetic instability can be estimated, using the quasilinear approximation,[5] as

$$\tau_k \sim \frac{n_p}{n_b}\tau_h \ , \tag{45}$$

so that the collisionless equilibration of the beam during the kinetic stage is usually much slower than during the hydrodynamic stage (however, some additional collisionless effects; for example, those associated with the presence of ions, can occur in some plasmas during the growth of the kinetic instability). The beam relative energy loss during the time τ_k is

$$\delta_k \sim \left[\frac{n_b}{n_p}\right]^{1/3} . \tag{46}$$

Taking the above into account and introducing the density ratio,

$$\alpha = \frac{n_b}{n_p} \ , \tag{47}$$

one can say that the collisionless equilibration can be faster than the collisional e-e equilibration if two following conditions are met: (1) the relative loss of the beam energy during the hydrodynamic stage is large (at least a half of the beam initial energy), and (2) the growth time for the hydrodynamic instability is much shorter than the collisional equilibration time $\tau_{e\text{-}e}$. In other words, the collisionless equilibration can be faster than the collisional equilibration if

$$\alpha \gtrsim \tfrac{1}{4}, \quad \text{then } \delta_h = \tfrac{1}{2} , \tag{48}$$

and

$$\frac{\tau_h}{\tau_{e\text{-}e}} \ll 1 . \tag{49}$$

Using relationships (17) and (40) the requirement (49) can be rewritten as

$$n_p \ll 10^{15} \alpha^{2/3} (\varepsilon_b^0)^3 , \tag{50}$$

where ε_b^0 is in eV and n_p in cm^{-3}. One may note that the condition (50) is easy to fulfill in most beam-plasma systems with large values of α. Thus in these systems the collisionless equilibration of the beam can be faster than, or comparable with, the collisional equilibration through Coulomb collisions. In quantitative analyses one should remember, however, that (1) the condition (50) was obtained using the time $\tau_{e\text{-}e}$ evaluated under the assumption that α is much less than 1, and (2) it may be necessary, when α is large, to consider additional instabilities, especially during the kinetic stage.

A qualitative comparison of collisionless and collisional (Coulomb) energy loss of the electron beam, with a relatively large value of α, is given in Fig. 2. We show there the time dependence of an energy beam (of initial energy ε_b^0) passing through a fully ionized plasma of density n_p and temperature T_p. Two cases (denoted by primes and double primes, respectively) are considered: (1) $\alpha' = n_b'/n_p = 0.25$ (then $\delta_h' \sim 0.5$, $\delta_k' \sim 0.6$, $\tau_k' \sim 4\tau_h'$, $\varepsilon_h' \sim 0.5\varepsilon_b^0$, and $\varepsilon_k' \sim 0.2\varepsilon_b^0$), and (2) $\alpha'' = n_b''/n_p = 0.05$ (then $\delta_h'' \sim 0.3$, $\delta_k'' \sim 0.4$, $\tau_k'' \sim 20\tau_h''$, $\varepsilon_h'' \sim 0.7\varepsilon_b^0$, and $\varepsilon_k'' \sim 0.4\varepsilon_b^0$), where ε_h and ε_k are beam energies after the hydrodynamic stage and kinetic stage, respectively. The collisional equilibration times are the same in both cases ($\tau_{e\text{-}e}' = \tau_{e\text{-}e}'' = \tau_{e\text{-}e}$) and the beam-plasma system meets the requirement (50) in both cases; consequently, the times τ_h' and τ_h'' are much shorter than $\tau_{e\text{-}e}$. As can be seen from Fig. 2, the collisionless effects become important when $\alpha \gtrsim 0.1$ and $\tau_h \ll \tau_{e\text{-}e}$. However, the contribution of the collisionless equilibration of the beam decreases rapidly with decreasing of the ratio α; at $\alpha \lesssim 0.05$ this contribution becomes negligible.

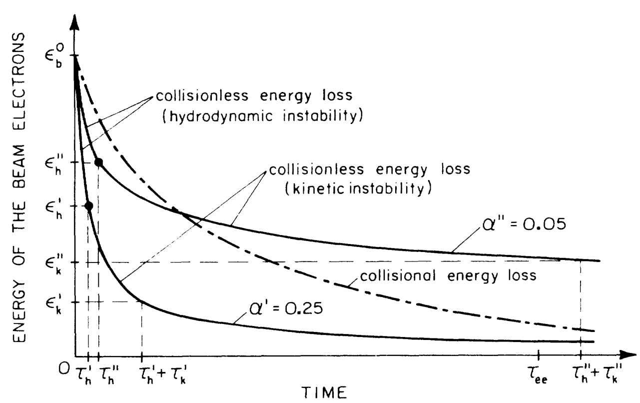

FIG. 2. Time dependence of the energy loss of the fast monoenergetic electron beam, with initial energy ε_b^0, passing through a uniform, fully ionized plasma when the value of $\alpha = n_b/n_p$ is high (n_b and n_p are beam and plasma particle densities, respectively). In one case (denoted by a prime) $\alpha = \alpha' = n_b'/n_p = 0.25$, while in the other case (denoted by a double prime) $\alpha = \alpha'' = n_b''/n_p = 0.05$. The time $\tau_{e\text{-}e} = \tau_{e\text{-}e}' = \tau_{e\text{-}e}''$ for collisional equilibration of the beam-plasma system is much longer than the times τ_h' and τ_h'' for the collisionless growth of the hydrodynamic instabilities. τ_k' and τ_k'' are times for the collisionless growth of the kinetic instabilities; ε_h' and ε_h'' are energies of the beam after the hydrodynamic stage, and ε_k' and ε_k'' are energies of the beam after kinetic stage of the collisionless instabilities. The dash-dotted curve represents the beam energy losses resulting from Coulomb interactions with the electrons and ions of the plasma. The solid curves represent the beam energy losses resulting from the collisionless effects.

EQUILIBRATION BY INELASTIC COLLISIONS

Electron-neutral inelastic collisions can play an important role in the beam energy loss because of the relatively large amount of energy that can be transferred during the collisions. Determination of the equilibration time for the electron beam interacting inelastically with plasma neutral particles is difficult because of the large number of possible channels and because of the uncertainties in collision strengths for many inelastic transitions. An estimate of the order of magnitude of this time can be done by superposing the energy losses in the binary encounters of the beam electrons with the plasma particles. The average energy loss of an incident electron along the direction h of its propagation can be given by[1]

$$\frac{d\varepsilon_b}{dh} = \sum_j \left[\frac{d\varepsilon_b}{dh} \right]^{(j)} , \tag{51}$$

where the average energy loss of the incident electron in collisions with the neutral particles of the jth component of the plasma is

$$\left[\frac{d\varepsilon_b}{dh} \right]^{(j)} = -n^{(j)}(h) S^{(j)}(\varepsilon_b) , \tag{52}$$

and where $n^{(j)}$ is the particle density of the jth component of the plasma and $S^{(j)}$ is the energy-loss (stopping-power) cross section for a collision of a beam electron with a neutral particle of the jth component.

The high-energy cross section for the energy loss in an electron-neutral inelastic collision can be given as[16]

$$S^{(j)}(\varepsilon_b) = \pi A^{(j)} e^4 \varepsilon_b^{-1} \left[\ln \left[\frac{(2+u_b) u_b^2 m_e^2 c^4}{2 (I^{(j)})^2} \right] + F(u_b) \right] , \tag{53}$$

where

$$F(u_b) = 1 - \beta_b^2 + (u_b+1)^{-2} \left[\left[\frac{u_b}{8} \right]^2 - (2u_b+1)\ln 2 \right] , \tag{54}$$

$$u_b = \frac{\varepsilon_b}{m_e c^2} , \tag{55}$$

$$\beta_b = (u_b + 1)^{-1}[u_b(u_b+2)]^{1/2} , \tag{56}$$

c is the speed of light, and $A^{(j)}$ and $I^{(j)}$ are atomic (molecular) weight and mean excitation energy, respectively, of the neutral particles of the jth component of the plasma.

The expression (53) is valid under the assumption of a "continuous-slowing-down" approximation when the incident-beam electron changes its energy with the rate of energy loss which does not fluctuate along the particle pathway. This assumption fails when the speed of the incident particles becomes comparable with the mean speed of the electrons in the neutral particles of the plasma. Then, a linear approximation proposed by Nelms[17] can be used for the cross section $S^{(j)}$ when $\varepsilon_b \le \varepsilon_0$. Although Nelms proposed value of $\varepsilon_0 = 5$ keV, it seems that the value $\varepsilon_0 = 10$ keV (see Refs. 1 and 18) is more appropriate. Assuming $\varepsilon_0 = 10$ keV and neglecting small quantities, one obtains the value of the cross section at $\varepsilon_b = \varepsilon_0$

$$S_0^{(j)} = S^{(j)}(\varepsilon_b = \varepsilon_0)$$

$$= \pi A^{(j)} e^4 \varepsilon_0^{-1} \left\{ \ln \left[\left(\frac{\varepsilon_0}{I^{(j)}} \right)^2 \right] + 1 - \ln 2 \right\} , \tag{57}$$

or

$$S_0^{(j)} \approx a A^{(j)} \ln[b/(I^{(j)})^2] , \tag{58}$$

where $a = 6.51 \times 10^{-18}$ cm^{2}eV, $b = 1.36 \times 10^8$, $I^{(j)}$ is given in eV, and $S_0^{(j)}$ is in cm^2 eV. The relationships (57) and (58) can be used below if the cross section (53) is accurate at $\varepsilon_0 = 10$ keV (note that this cross section is inaccurate at small energies). The results of Ref. 1 indicate that these relationships can indeed be considered as a reliable measure of the energy-loss cross section at energy $\varepsilon_0 = 10$ keV.

Determination of the particle mean excitation energy $I^{(j)}$, is difficult, especially in the case when the plasma particles are molecules and many inelastic channels are available. In general, this energy can be calculated either from the absorption frequencies and oscillator strengths,[19,20] or, empirically, through analysis of stopping-power and range experiments.[18] (If the neutral species are atoms, the mean excitation energy can be taken as half of the ionization potential for the atoms.)

Using Eq. (58) and the linear approximation discussed above, one obtains the energy-loss cross section for the incident electron having energy below 10 keV and colliding inelastically with a neutral particle of the jth component of the plasma,

$$S^{(j)}(0 \le \varepsilon_b \le 10 \text{ keV}) = c_0 S_0^{(j)} \varepsilon_b , \tag{59}$$

where $c_0 = 10^{-4}$ eV^{-1}, ε_b is in eV, and $S^{(j)}$ and $S_0^{(j)}$ are in cm^2 eV. (In the case where ε_b^0 is greater than 10 keV, the energy-loss cross section can be obtained by using the approach of Refs. 1 and 21.)

One can write for a monoenergetic electron beam

$$\frac{d\varepsilon_b}{dh} = v_b^{-1} \frac{d\varepsilon_b}{dt} , \tag{60}$$

and the time τ_{in} necessary for the beam of initial energy ε_b^0 to get equilibrated by inelastic collisions with the neutrals of the plasma is[1]

$$\tau_{in}(\varepsilon_b^0) = -\left[\frac{m_b}{2}\right]^{1/2} \int_{\varepsilon_b^0}^{\varepsilon_b^f} \left[\varepsilon_b^{1/2} \sum_j n^{(j)}(h) S^{(j)}(\varepsilon_b) \right]^{-1} d\varepsilon_b , \tag{61}$$

so that the equilibration time for the beam electrons colliding inelastically with the neutrals of one (the jth) component of the uniform plasma is

$$\tau_{in}^{(j)}(\varepsilon_b^0) = 3.37 \times 10^{-4} (n^{(j)} S_0^{(j)})^{-1} \left[\frac{1}{(\varepsilon_b^f)^{1/2}} - \frac{1}{(\varepsilon_b^0)^{1/2}} \right] , \tag{62}$$

where $\tau_{in}^{(j)}$ is in seconds, $n^{(j)}$ in cm^{-3}, $S_0^{(j)}$ in cm^2 eV, and ε_b^f and ε_b^0 in eV. It should be emphasized that the final energy ε_b^f of the electron beam cannot be related to $\langle \varepsilon_p \rangle = 3kT_p/2$ (the mean energy of the plasma particles) because the *inelastic* energy exchange between the beam and the neutrals of the plasma is practically independent of kinetic energies of the neutrals; under the assumptions of this work only a very small part of the plasma particles have energy comparable to, and greater than, the energy of the beam electrons. Therefore, ε_b^f is just the energy of the incident beam required at the end of the equilibration process. Consequently, the time needed for the electron beam to lose, as a result of inelastic interactions with plasma neutrals, most (let's say 80%) of its initial energy is

$$\tau_{in}^{(j)}(\varepsilon_b^0) = 4 \times 10^{-4} (n^{(j)} S_0^{(j)})^{-1} (\varepsilon_b^0)^{-1/2} . \tag{63}$$

Relationships (62) and (63) show an interesting feature of the collisional equilibration of the electron beams (with $\varepsilon_b^0 < 10$ keV) by inelastic processes; the equilibration time τ_{in} decreases with an increase of the initial beam energy ε_b^0. This results from the fact that the cross section $S^{(j)}$ $(0 < \varepsilon_b < 10$ keV) is an increasing function of the energy of the incident electrons.

Ending, one should add that some combinations of the beam density n_b and initial energy ε_b^0 can lead to a substantial increase in plasma ionization degree in the time interval shorter than the equilibration time τ_{in}. In such a case, the density of plasma neutrals $n^{(j)}$ cannot be considered constant in the expression (61).

SUMMARY AND CONCLUSIONS

The main results of this work are given by the expressions (17), (18), and (62). As discussed above, the equilibration times $\tau_{e\text{-}e}$ and τ_{ei} [Eqs. (17) and (18), respectively] of the present work seem to be more appropriate than Spitzer's relaxation times to estimate the efficiency of the energy loss of the fast electron beam during its interaction, through Coulomb collisions, with plasma charges.

The possibility of equilibration of the electron beam energy through collisionless effects practically does not exist under the assumption of the weak-beam approximation ($n_b \ll n_p$). Such effects can become important only when $n_b / n_p \gtrsim 0.1$ and when the requirement (50) is met in the beam-plasma system.

In most applications of partially ionized gases (where $T \approx 1-3$ eV and $n \approx 10^{14}-10^{20}\ \mathrm{cm}^{-3}$) the role of electron-neutral elastic collisions in energy equilibration of fast nonthermal electrons is negligible in comparison with the role of electron-electron and electron–neutral-particle inelastic collisions. If the plasma ionization degree is small, then equilibration of the nonthermal electrons can be dominated by electron–neutral-particle inelastic collisions. If the ionization degree of the plasma is moderate, then the electron-electron collisions begin to dominate equilibration of the nonthermal electrons.

A significant advantage of the approach used in this work is the fact that the final expressions for the equilibration times can be obtained in simple analytical forms. In addition, the rate for the energy loss and the equilibration times of the present work are well defined and have clear physical meaning. Finally, the approach of this work can be extended in a straightforward way to equilibration of nonthermal particles other than electrons (ions, neutrons, atoms, etc.), in plasmas and gases.

ACKNOWLEDGMENTS

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