Experimental study of the x-ray transitions in the heliumlike isoelectronic sequence

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The wavelengths of the transition $1s2p\ ^1P_1 \rightarrow 1s^2\ ^1S_0$ in heliumlike potassium, scandium, titanium, vanadium, chromium, and iron have been measured with an experimental precision of $\Delta\lambda/\lambda \approx 1/20\,000$. The data typically agree within error bars with recent theoretical predictions that include quantum electrodynamical corrections. However, the comparison shows that there are systematic differences between data and predictions, which indicate a need to include additional corrections in the calculations. In particular, it is found that the experimental wavelengths are consistently shorter than the calculated values by amounts which depend on the particular theoretical prediction used in the comparison. Measurements of the wavelengths of $\Delta n \geq 2$ x-ray transitions in heliumlike argon, scandium, titanium, vanadium, chromium, and iron are also presented.

I. INTRODUCTION

It is generally assumed that the energy levels of one-electron ions can be accurately predicted by the Dirac theory supplemented by nuclear and quantum electrodynamical (QED) corrections. By contrast there exists a variety of different approaches for calculating the energy levels of ions in the heliumlike isoelectronic sequence[1] which differ in their treatment of relativistic effects, electron interactions, or QED corrections. For example, energy levels of heliumlike ions have been calculated based on the Z-expansion method,[2] the nonrelativistic variational technique,[3] or using multiconfigurational Dirac-Fock (MCDF) orbitals.[4] Due to the different approximations used in each calculation the results may vary significantly. Therefore it is desirable to make detailed comparisons of theoretical results with experimental values and to study differences as a function of atomic number Z.

A large body of experimental values exists for $n=2\rightarrow 2$ transitions in heliumlike ions, which has been used for systematic comparisons with theory.[3,4] On the other hand, only a few data exist for $\Delta n \neq 0$ transitions to the ground state. Such transitions are of particular interest because the QED corrections as well as electron-electron correlation energies affect the ground state most strongly. Hence observations of $\Delta n \neq 0$ transitions to the ground state can provide a check of the accuracy with which these corrections are calculated. Accurate wavelength measurements of the $n=2\rightarrow 1$ resonance transitions $1s2p\ ^1P_1 \rightarrow 1s^2\ ^1S_0$ have been reported for sulfur,[5] argon,[6] iron,[7] and krypton[8] from beam-foil experiments and for argon[9] using the recoil-ion technique. The measurements have been successfully used as a test for the accuracy of the QED corrections, as agreement between experiment and theory has been found within the experimental error limits.

In this paper we report on the $n=2\rightarrow 1$ x-ray transitions of heliumlike potassium, scandium, titanium, vanadium, chromium, and iron ($Z=19$–26) measured in tokamak plasmas from the Princeton Large Torus (PLT). The heliumlike transitions have been observed concurrently with a variety of hydrogenlike lines which are used as wavelength references. Together with the measurements of Refs. 5–9, the data are used to make systematic comparisons with theoretical predictions.

We also report on a large number of $\Delta n \geq 2$ transitions to the ground state in heliumlike argon, scandium, titanium, vanadium, chromium, and iron. The $n \geq 3$ levels are less affected by electron-electron interactions, relativistic and QED effects. Hence transitions from these levels to the ground state can provide additional tests of the accuracy of the theoretical predictions for the ground-state energy. Precision wavelength measurements of the x-ray transitions in heliumlike ions from levels $n \geq 3$ to the ground state have previously been reported for argon and potassium from solar-flare observations[10] and for iron from beam-foil experiments.[11]

II. EXPERIMENT

The measurements have been performed with the PLT high-resolution Johann spectrometer[12] and cover the wavelength regions 1.4–3.8 Å. In particular, the wavelength region 3.3–3.8 Å was monitored using a $(11\bar{2}0)$ quartz crystal with a $2d$ spacing of 4.913 Å (Ref. 13) and bent to a radius of curvature of 276 cm. The details of the setup have been described earlier in Ref. 14. The same setup was used to make measurements in second-order Bragg reflection covering the wavelength region 1.6–1.9 Å.

The wavelength region 2.0–3.0 Å was monitored by a (220) silicon crystal with a $2d$ spacing of 3.840 Å (Ref. 15) and a radius of curvature of 365 cm. The experimental

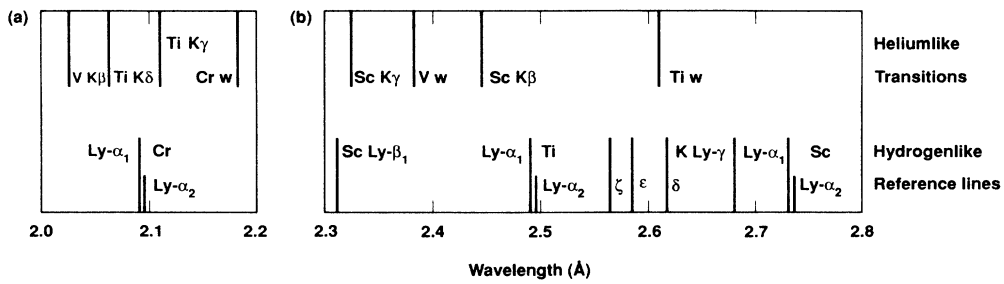

FIG. 1. Overview of the transitions observed in the wavelength range 2.0–2.8 Å showing the location of the heliumlike resonance transitions with respect to the reference lines from hydrogenic ions. All transitions have been observed in first order.

arrangement for this wavelength region is the same as described in Ref. 16. This setup also allowed us to observe the $n=4\rightarrow 1$ and $n=5\rightarrow 1$ transitions of heliumlike iron in second order which fall into the wavelength range 1.4–1.5 Å. We note that iron and chromium are the dominant high-Z impurities in PLT plasmas[17,18] so that their x-ray line line emission is most intense and easily observed even in second-order Bragg reflection.

An overview of the measurements indicating the location of the various observed transitions is given in Figs. 1 and 2. Here the heliumlike resonance transitions $1s2p\ ^1P_1\rightarrow 1s^2\ ^1S_0$ are labeled w according to the notation by Gabriel.[19] The corresponding x-ray transitions from levels with principal quantum number $n=3,4,5$ are labeled $K\beta$, $K\gamma$, and $K\delta$, respectively. The figures also show the location of the x-ray transitions $np\rightarrow 1s$ from hydrogenic argon, potassium, scandium, titanium, chromium, and iron. Using standard notation, these transitions are labeled Ly-α through Ly-ζ for $n=2$ through $n=7$, respectively. A subscript has been added to distinguish between the upper states $^2P_{3/2}$ (subscript 1) and $^2P_{1/2}$ (subscript 2) provided the two lines could be resolved in the observations. The hydrogenlike lines were recorded concurrently with the heliumlike transitions in a given spectrum. The hydrogenlike transitions are used as references with respect to which the wavelengths of the heliumlike lines have been determined. A similar procedure for determining the wavelengths of the heliumlike transitions was used in Refs. 8 and 10. The hydrogenlike transitions can be used as references because their wavelengths are theoretically well known.[20–23] Several hydrogenic transitions have been measured absolutely.[24–32] An overview of some of the measurements is given by Mohr[33] and Delattes.[34] On average these measurements agree with theory very well, and no systematic differences as a function of Z are evident from these data. For example, the Ly-α lines of argon have been measured by Beyer *et al.*[29] with a precision of 5 ppm, which is the highest precision attained so far, and their measurements agree with calculations[22] to within 2 ppm.

A typical spectrum showing the $n=2\rightarrow 1$ transitions of titanium is presented in Fig. 3(a). The spectrum was recorded at an electron temperature $T_e\leq 5$ keV. The $1s2p\ ^1P_1\rightarrow 1s^2\ ^1S_0$ resonance line is easily identified as the most intense feature in the spectrum. Also seen are the intercombination lines x and y, and the forbidden line z, as well as the lithiumlike resonance line q. Dielectronic satellite lines are very weak in the spectrum. For example, the dielectronic satellite transition $1s2p^2\ ^2D_{5/2}\rightarrow 1s^22p\ ^2P_{3/2}$, labeled j in the notation of Gabriel,[19] is almost nonexistent as a result of the high electron temperature of the measurement. In Fig. 3(b) the spectrum of heliumlike titanium is shown together with five lines of the Lyman series of hydrogenic potassium. The hydrogenic lines represent some of the lines used as references for the wavelength determination of the heliumlike lines.

The wavelength interval which can be observed in one setting of the spectrometer is typically less than 0.2 Å. Thus several settings of the spectrometer were used to

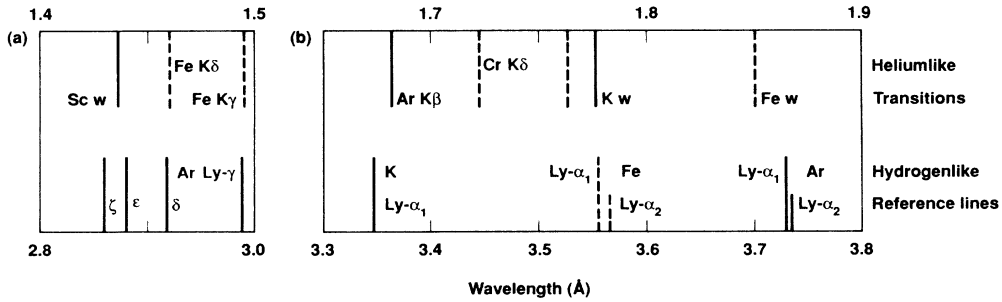

FIG. 2. Overview of the transitions observed in the wavelength range 2.8–3.8 Å in first-order Bragg reflection (solid lines) and in the wavelength range 1.4–1.9 Å in second order (dashed lines).

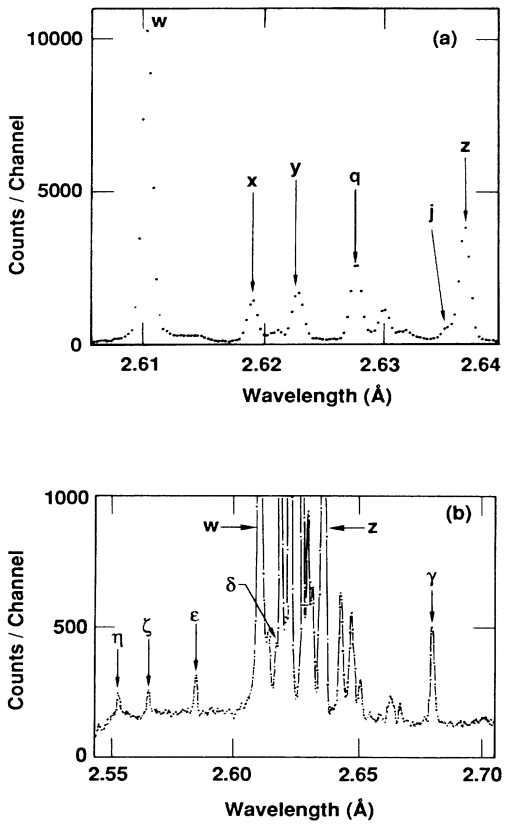

FIG. 3. Typical spectra observed with the high-resolution PLT crystal spectrometer. (a) $n=2\rightarrow 1$ spectrum of titanium showing the heliumlike resonance line w, the intercombination lines x and y, and the forbidden line z, as well as the lithiumlike resonance line q and the dielectronic satellite line j. (b) Spectrum of the Lyman series of hydrogenlike potassium recorded concurrently with the spectrum of heliumlike titanium (off scale). The hydrogenlike lines are used as wavelength references.

cover each region. Spectra obtained with different spectrometer settings in a given wavelength region were made to overlap with each other. This allowed us to relate the wavelengths of two lines which are far apart with the help of lines which are positioned in between, as described earlier in Ref. 14. It was not possible to achieve an overlap between all wavelength regions. For example, no common lines were observed which could be used to interconnect the wavelength region 2.3–2.8 Å [cf. Fig. 1(b)] and the wavelength region 2.8–3.0 Å [cf. Fig. 2(a)]. As a result, our measurements fall into the four separate wavelength regions shown in Figs. 1 and 2.

The wavelength of any heliumlike line in a given wavelength region has been determined with respect to all hydrogenlike lines observed in the same region. This determination is solely based on instrumental considerations, i.e., the detector-crystal distance, the crystal spacing, and the conversion gain of the detector electronics. As an example consider the resonance line w of heliumlike scandium, which falls into the wavelength interval 2.8–3.0 Å. Its wavelength was determined with respect to the lines Ly-γ, Ly-δ, Ly-ϵ, and Ly-ζ of the Lyman series of hydrogenic argon [cf. Fig. 2(a)]. The four experimental values for the wavelength of line w of scandium, which have been obtained this way, have been averaged, and the resulting value is given in Table I. Similarly, the experimental value listed in Table I for the wavelength of line w of titanium represents an average of individual determinations with respect to the lines Ly-α_1, Ly-α_2, and Ly-β_1 of hydrogenic scandium, the lines Ly-α_1 and Ly-α_2 of hydrogenic titanium, and the lines Ly-γ through Ly-ζ of hydrogenic potassium.

The above procedure in which the wavelength of a given heliumlike line is determined from instrumental considerations and by averaging the results was chosen because it provided us with a measure of the accuracy of the measurements. The largest difference of an individual wavelength determination from the average is found to be less than one part in 20 000. This difference represents an

TABLE I. Comparison of the experimental wavelengths of the transition $1s2p\ ^1P_1\rightarrow 1s^2\ ^1S_0$ in heliumlike ions with theory. All values are in angstroms.

Element	Z	λ_{expt}	Error (ppm)	Reference	λ_{theor}[a]	$\Delta\lambda$	λ_{theor}[b]	$\Delta\lambda$	λ_{theor}[c]	$\Delta\lambda$
Sulfur	16	5.038 68	37	Ref. 5	5.038 90	−0.000 22	5.038 73	−0.000 05	5.038 66	+0.000 02
Argon	18	3.949 08	80	Ref. 6	3.949 21	−0.000 13	3.949 07	+0.000 01	3.949 02	+0.000 06
		3.949 11	12	Ref. 9	3.949 21	−0.000 10	3.949 07	+0.000 04	3.949 02	+0.000 09
Potassium	19	3.531 73	35	This work	3.531 98	−0.000 25	3.531 86	−0.000 13	3.531 80	−0.000 07
Scandium	21	2.872 97	35	This work	2.873 17	−0.000 20	2.873 06	−0.000 09	2.873 02	−0.000 05
Titanium	22	2.610 34	35	This work	2.610 49	−0.000 15	2.610 40	−0.000 06	2.610 36	−0.000 02
Vanadium	23	2.381 90	40	This work	2.382 04	−0.000 14	2.381 95	−0.000 05	2.381 92	−0.000 02
Chromium	24	2.181 93	70	This work	2.182 11	−0.000 18	2.182 03	−0.000 10	2.182 00	−0.000 07
Iron	26	1.850 31	30	This work	1.850 48	−0.000 17	1.850 40	−0.000 09	1.850 37	−0.000 06
		1.850 28	37	Ref. 7	1.850 48	−0.000 20	1.850 40	−0.000 12	1.850 37	−0.000 09
Krypton	36	0.945 34	23	Ref. 8	0.945 50	−0.000 16	0.945 41	−0.000 07	0.945 39	−0.000 05

[a]Vainshstein and Safronova, Ref. 2.
[b]Drake, Ref. 3.
[c]Indelicato, Ref. 39.

estimate for the precision with which a wavelength can be assigned to a given heliumlike line. The uncertainty in the average, however, is likely to be lower (≤ 0.1 mÅ) because errors caused by uncertainties in the dispersion of the spectrometer as well as random errors are reduced in the average over the individual measurements.

Uncertainties in determining the wavelengths of the heliumlike lines arise also as a consequence of blends with close-by lines. In order to eliminate the effect of blends the measurements are made with high instrumental resolving power $\lambda/\Delta\lambda$. In the present measurements $\lambda/\Delta\lambda = 2500$–$8000$, except for measurements in the wavelength region 2.0–2.2 Å [cf. Fig. 1(a)] where $\lambda/\Delta\lambda = 1000$–$1200$. Thus the instrumental resolving power is in all but one region comparable to the linewidth caused by Doppler broadening, which for titanium equals $\lambda/\Delta\lambda \approx 3400$ at an ion temperature $T_i = 1$ keV typical for the plasma conditions of the present measurements.

As a consequence of the high resolution of the measurements only blends of satellite lines produced by dielectronic capture into states with $n > 3$ (Refs. 35 and 36) affect the accuracy with which the center position of a given $n = 2 \rightarrow 1$ heliumlike resonance line can be determined. The effects of high-n satellites in the x-ray spectra of heliumlike ions have been investigated extensively for the parameters prevalent in low-density tokamak plasmas. The high-n satellites tend to cause a red shift of the position of the apparent heliumlike resonance line w by less than 0.03 mÅ for the plasma conditions of the present experiments.[37] Unresolved satellites may also shift the position of a given hydrogenic reference line. This effect has been investigated in less detail than in the case of heliumlike resonance lines; however, we expect any shifts of the Ly-α_1 and Ly-α_2 lines to be no larger than those of the heliumlike lines based on an analysis of the $n = 2 \rightarrow 1$ spectrum of hydrogenic titanium in Ref. 38. In the latter case the spectrum was observed at a peak electron temperature of 2.1 keV;[38] the hydrogenlike spectra measured in the present experiment have been obtained with electron temperatures between 4 and 6 keV so that contamination of the resonance lines by high-n satellites is expected to be smaller than suggested by the spectrum in Ref. 38.

The wavelengths of the hydrogenic reference lines Ly-α_1 and Ly-α_2 were taken from calculations of Johnson and Soff,[23] the wavelengths of the Ly-β_1 lines are those of Erickson,[21] and the wavelengths of the Ly-γ, Ly-δ, Ly-ϵ, and Ly-ζ lines have been calculated by Garcia and Mack.[20]

III. COMPARISON WITH THEORY

In Table I the measured wavelengths of the heliumlike resonance transitions $1s2p\ ^1P_1 \rightarrow 1s^2\ ^1S_0$ are compared to values calculated by Vainshstein and Safronova using the Z-expansion method,[2] to values calculated by Drake using a nonrelativistic variational approach,[3] and to values from MCDF calculations by Indelicato.[4,39] All three calculations take into account QED corrections.

A systematic difference between data and theoretical values can be noted for all three calculations. In general, we find that our experimental wavelengths are shorter than the calculated values. The difference is largest for the calculations of Vainshstein and Safronova[2] where the calculated wavelengths are too large by 0.14–0.25 mÅ. The calculated wavelengths of Drake[3] agree better with our data and differ by only 0.05–0.13 mÅ. The MCDF results of Indelicato[39] give even better agreement with our data. In this case the calculated wavelengths are too long by only 0.02–0.07 mÅ.

For comparison, Table I includes the experimental values of the heliumlike transitions w found in previous experiments.[5–9] One of the six heliumlike transitions which we have measured, i.e., line w of iron, has been measured previously by Briand *et al.* using beam-foil techniques,[7] and we find excellent agreement between Briand's measurement and ours.

The fractional differences $\Delta\lambda/\lambda$ between the data and the calculated wavelengths, defined as $\Delta\lambda/\lambda = (\lambda_{\text{expt}} - \lambda_{\text{theor}})/\lambda_{\text{expt}}$, are plotted in Fig. 4. The figure also includes the values of $\Delta\lambda/\lambda$ based on the experimental values reported earlier in Refs. 5–9.

Figure 4 shows that there are systematic differences between the data and the results of each of the three calculations. Since all wavelengths which we have measured are shorter than any of the calculated values, the values of $\Delta\lambda/\lambda$ are negative for each of the three calculations. The same is found for the values of $\Delta\lambda/\lambda$ of iron and krypton measured earlier.[7,8] In the case of argon[5,9] and sulfur,[6] which represent the lowest-Z elements in our comparison, the fractional differences are found to be positive when the calculations of Indelicato[39] are used, while they are negative for the calculations of Vainshstein and Safronova.[2] In the comparison with the calculations of Drake[3] only the values of $\Delta\lambda/\lambda$ for argon are positive.

The experimental uncertainties in the fractional differences of $\Delta\lambda/\lambda$ are listed in Table I. As mentioned above, the uncertainties in our measurements are typically less than 50 ppm. If the differences between the experimental and theoretical values were merely due to the experimental uncertainties, i.e., if the theoretical values were absolutely accurate, we would expect that the values of $\Delta\lambda/\lambda$ were randomly scattered around zero. This, however, is not the case, as seen from Fig. 4. While the figure shows that there is some scatter within the values of $\Delta\lambda/\lambda$, which in part appears to be due to experimental uncertainties, the figure also shows that the values of $\Delta\lambda/\lambda$ systematically decrease as a function of Z for all three calculations. This effect is largest in Fig. 4(c) where the data are compared to calculations by Vainshstein and Safronova.[2] It is less pronounced in Figs. 4(a) and 4(b) where the data are compared to calculations by Indelicato[39] and by Drake,[3] respectively. Here the values of $\Delta\lambda/\lambda$ do not decrease as rapidly for the highest-Z elements iron and krypton as in Fig. 4(c). The decrease for the lower-Z elements between argon and chromium, however, is very similar in all three cases. In this range of Z the main difference between Figs. 4(a), 4(b), and 4(c) is that the values of all $\Delta\lambda/\lambda$ are shifted by a common offset with respect to each other. In particular, the values of $\Delta\lambda/\lambda$ in Fig. 4(b) are shifted on average by -15 ppm with respect to the values in Fig. 4(a). Those in Fig. 4(c)

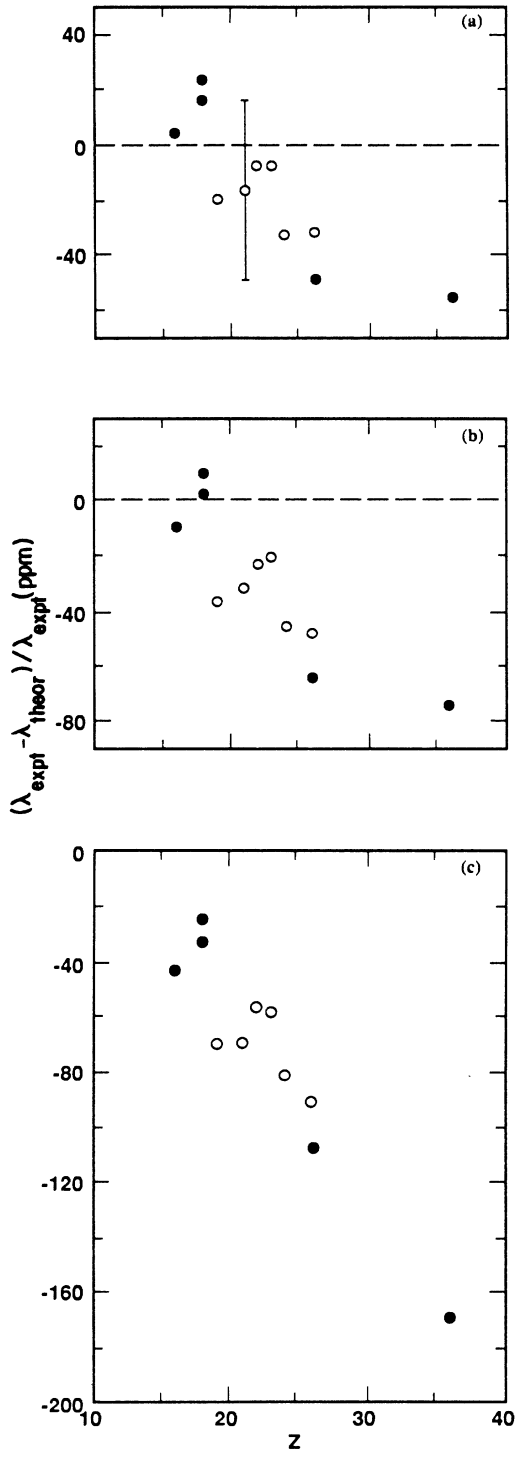

FIG. 4. Relative differences between measured and calculated wavelengths of the heliumlike resonance line $1s^2p\ ^1P_1 \rightarrow 1s^2\ ^1S_0$ for elements between $Z=18$ and 36. Open circles represent present data; solid circles represent previously published data. (a) Wavelengths calculated by Indelicato, Ref. 39; (b) wavelengths calculated by Drake, Ref. 3; (c) wavelengths calculated by Vainshstein and Safronova, Ref. 2. The error bar in (a) is representative of the experimental uncertainties of the data (cf. Table I).

are shifted by about -45 ppm. These shifts, as well as the systematic decrease of $\Delta\lambda/\lambda$ with Z, are clearly the result of the respective calculations. They are an indication that correction terms need to be included in the calculations to remove the differences between theory and data.

IV. $\Delta n \geq 2$ TRANSITIONS

Our measurements of wavelengths of the heliumlike resonance transitions also include x-ray transitions from quantum levels $n>2$ to ground. In particular, we have observed the resonance transition $1s3p\ ^1P_1 \rightarrow 1s^2\ ^1S_0$ in argon, scandium, and vanadium, the transition $1s4p\ ^1P_1 \rightarrow 1s^2\ ^1S_0$ in scandium, titanium, chromium, and iron, and the transition $1s5p\ ^1P_1 \rightarrow 1s^2\ ^1S_0$ in titanium, chromium, and iron. The experimental values of the wavelengths of these transitions are given in Table II.

It should be noted that the resonance lines from levels $n \geq 4$ are blended with the corresponding intercombination lines, i.e., transitions of the type $1snp\ ^3P_1 \rightarrow 1s^2\ ^1S_0$, since in this case the intercombination line has a wavelength very close to that of the resonance line. Only in the case of the $3p \rightarrow 1s$ transitions was it possible to resolve the resonance line from the intercombination line (cf. the $K\beta$ spectrum of argon in Fig. 5). The $K\beta$ spectra have shown that the intercombination line is much weaker than the resonance line. We assume that this is also true for the $n \geq 4$ transitions. Hence we presume that a possible red shift of the center position of the apparent resonance line caused by a blend of the resonance line with the close-by, weak intercombination line is very small. In the case of the line $K\beta_1$ of argon, shown in Fig. 5, the shift was determined to be less than 0.02 mÅ, when the line $K\beta_2$ was included in the fitting limits of a single Voigt profile to the data. We have, however, made no attempt to model the shift for the $n \geq 4$ lines, and further investigations are needed to determine a bound on the shift of the apparent resonance lines, especially for high-Z heliumlike ions. The effect of high-n satellites on the center position of the apparent resonance lines also need further study. Their effect, however, is expected to be negligible, because excited configurations $1s3lnl'$ $(n \geq 3)$ can autoionize to excited configurations $1s2l''$ in addition to the usual radiative and autoionizing decay channels so that the $n=3 \rightarrow 1$ x-ray shield from the decay of these levels is greatly reduced.

Measurements of $\Delta n > 1$ transition in the heliumlike isoelectronic sequence have been reported previously for the $3p \rightarrow 1s$ transition in potassium and the $5p \rightarrow 1s$ transition in argon from solar-flare spectra.[10] In addition, the $3p \rightarrow 1s$ and $4p \rightarrow 1s$ transitions have been measured in heliumlike iron in beam-foil observations.[11] These values have also been listed in Table II. In the beam-foil measurement,[11] the intercombination lines were well resolved from the resonance lines, while in the solar observations[10] the transitions appear blended.

Because the electron-correlation and QED corrections are virtually negligible for the excited state compared to the corresponding corrections for the ground state, high-n x-ray transitions can provide a direct measure of the ac-

TABLE II. Comparison of the experimental wavelengths of the transitions $1snp\ ^1P_1 \rightarrow 1s^2\ ^1S_0$ ($n=3,4,5$) with theory. All values are in angstroms.

Transition	Element	λ_{expt}	Error (ppm)	Reference	λ_{theor}[a]	$\Delta\lambda$
$1s3p\ ^1P_1 \rightarrow 1s^2\ ^1S_0$	Argon	3.365 51	35	This work	3.365 71	−0.000 20
	Potassium	3.008 60	37	Ref. 10	3.008 72	−0.000 12
	Scandium	2.445 51	35	This work	2.445 65	−0.000 14
	Vanadium	2.026 27	60	This work	2.026 40	−0.000 13
	Iron	1.573 14	25	Ref. 11	1.573 22	−0.000 08
$1s4p\ ^1P_1 \rightarrow 1s^2\ ^1S_0$	Scandium	2.324 18	35	This work	2.324 28	−0.000 10
	Titanium	2.110 90	45	This work	2.110 94	−0.000 04
	Chromium	1.763 42	35	This work	1.763 38	+0.000 04
	Iron	1.494 63	40	This work	1.494 63	0.000 00
		1.494 57	27	Ref. 11	1.494 63	−0.000 06
$1s5p\ ^1P_1 \rightarrow 1s^2\ ^1S_0$	Argon	3.128 47	64	Ref. 10	3.128 35	+0.000 12
	Titanium	2.063 33	50	This work	2.063 45	−0.000 12
	Chromium	1.723 57	50	This work	1.723 60	−0.000 03
	Iron	1.460 81	35	This work	1.460 85	−0.000 04

[a]Vainshstein and Safronova, Ref. 2.

curacy of the theoretical predictions of such corrections affecting the ground state, as pointed out by Indelicato.[11] Thus measurements of the x-ray transitions from high-n levels complement measurements of the $n=2\rightarrow 1$ transitions by placing additional constraints on the theoretical results.

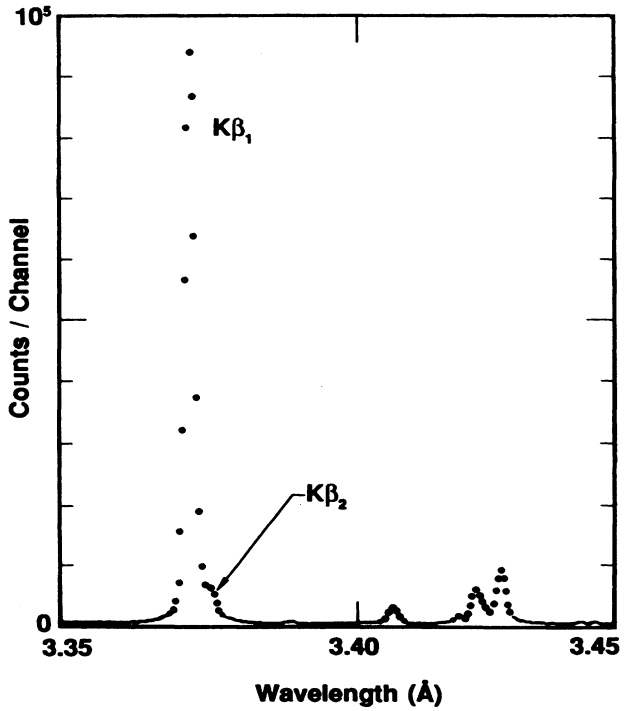

FIG. 5. Spectrum of $n=3\rightarrow 1$ transitions in heliumlike argon. The transitions $1s3p\ ^{1,3}P_1 \rightarrow 1s^2\ ^1S_0$ have been labeled $K\beta_1$ and $K\beta_2$, respectively. The unmarked features are satellite lines arising from the decay of excited levels of the type $1s2l3l'$.

A comparison between published calculations for $\Delta n>2$ transitions by Vainshstein and Safronova[2] and the data are given in Table II. The relative differences between the data and the theoretical values, $\Delta\lambda/\lambda=(\lambda_{expt}-\lambda_{theor})/\lambda_{expt}$, are plotted in Figs. 6–8 for lines $K\beta$, $K\gamma$, and $K\delta$, respectively. In the figures we have also included the data measured in Refs. 10 and 11.

A systematic difference between the measured and calculated wavelengths is found in the case of the $K\beta$ lines. Here the calculated wavelengths are generally too long by 0.08–0.20 mÅ, and the corresponding values of $\Delta\lambda$ are all negative. The values of $\Delta\lambda$ for the $K\beta$ line of potassium reported in Ref. 10 and of iron reported in Ref. 11 are comparable in magnitude to the present data and are also negative. The values of $\Delta\lambda/\lambda$ are plotted in Fig. 6. The figure shows that the fractional difference between the data and the calculation is constant and is about −50 ppm. Thus the values of $\Delta\lambda/\lambda$ for the $K\beta$ lines do not exhibit the systematic decrease as a function of Z which has been found for the $K\alpha$ lines in Fig. 4(c).

The calculated values for the lines $K\gamma$ and $K\delta$ appear to agree well with the data. However, we note that there are six values of $\Delta\lambda$, including the value of the line $K\delta$ of iron reported in Ref. 11, which are negative compared to only two which are positive. This may indicate that the calculated wavelengths are slightly too long, although there are not yet enough data to allow us to say this with confidence. The data indicate, however, that the differences, if they indeed existed, would most likely be smaller than those found for the $K\alpha$ and $K\beta$ lines in Figs. 4(c) and 6, respectively.

V. SUMMARY AND DISCUSSION

In summary we note that systematic differences between calculated and theoretical wavelengths of the x-ray transitions in heliumlike ions have been observed. The differences are very small and range between 10 and 90

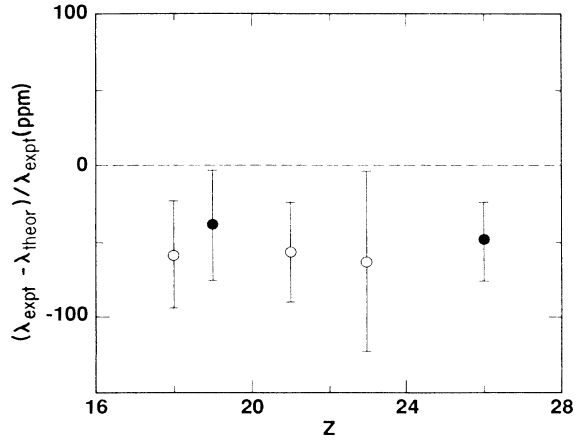

FIG. 6. Relative differences between measured and calculated wavelengths of heliumlike resonance line $1s3p\ ^1P_1 \rightarrow 1s^2\ ^1S_0$ for various elements. Open circles represent present data; solid circles represent previously published data. The theoretical wavelengths are from Vainshstein and Safronova, Ref. 2.

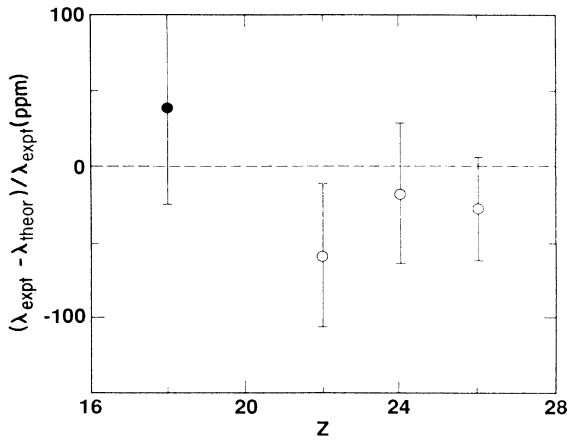

FIG. 8. Relative differences between measured and calculated wavelengths of heliumlike resonance line $1s5p\ ^1P_1 \rightarrow 1s^2\ ^1S_0$ for various elements. Open circles represent present data, solid circles previously published data. The theoretical wavelengths are from Vainshstein and Safronova, Ref. 2.

ppm. These differences correspond to less than a few percent of the QED corrections so that our data agree with and affirm previous measurements of the QED corrections. The differences depend on the type of calculations performed and are smallest for those calculations which are most *ab initio*. They represent evidence for corrections missing in the calculations. A correction term has recently been added[40,41] to the ground-state energy computed by Vainshstein and Safronova,[2] which results in much better agreement with our data in the case of $K\alpha$ transitions.[40] The term, however, worsens the agreement in the case of the $K\beta$, $K\gamma$, and $K\delta$ transitions. A suggestion has also been made that the difference may arise from the treatment of the nuclear size.[42]

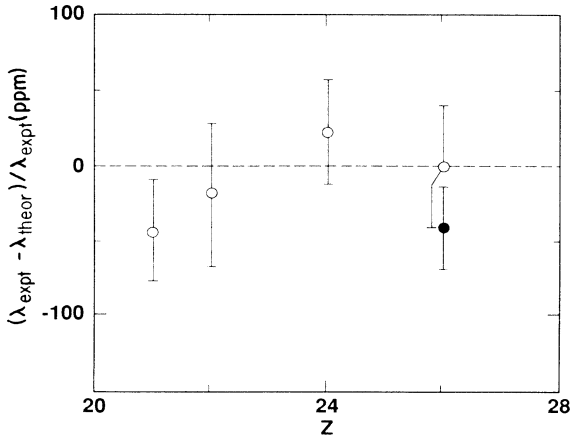

FIG. 7. Relative differences between measured and calculated wavelengths of heliumlike resonance line $1s4p\ ^1P_1 \rightarrow 1s^2\ ^1S_0$ for various elements. Open circles represent present data, solid circles previously published data. The theoretical wavelengths are from Vainshstein and Safronova, Ref. 2.

In most cases the differences between the measured and calculated wavelengths lie within or just outside the experimental error limits. Therefore we had to rely on a large number of data to demonstrate systematic differences. Our study thus emphasizes the importance of measurements along isoelectronic sequences in order to study such systematic effects.

Experimental measurements which are at least an order of magnitude more precise than the present measurements are necessary, in order to probe differences between data and calculations of the transition energies of medium-Z elements directly. For this purpose, it may be necessary to measure the x-ray transitions under conditions where dielectronic satellites are completely negligible. Such measurements may be possible in very-high-electron-temperature and low-ion-temperature tokamak plasmas or in ion traps.

Finally, we note that additional calculations of the $\Delta n \geq 2$ x-ray transition energies are needed as a function of Z. The comparisons presented in this paper are based on computations using the Z-expansion method.[2] To our knowledge comparable calculations based on other theoretical approaches have not yet been published as a function of Z.

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