Vacuum effects on interference in two-photon down conversion

Z. Y. Ou, L. J. Wang, and L. Mandel
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627
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A proposed experiment is analyzed theoretically. In the proposed experiment two coherent pump waves fall on two identical nonlinear crystals, down-converted signal and idler beams from the two crystals are mixed by two beam splitters, and the coincidence counting rate for photons leaving the beam splitters is measured. We show that this counting rate depends on the phase difference between the two coherent pump waves, and results from the interference of the vacuum with the down-converted photons. The experiment could be used to look for locality violations along the lines recently proposed by Grangier, Potasek, and Yurke [Phys. Rev. A **38**, 3132 (1988)], but without the need for a coherent reference beam for homodyning.

I. INTRODUCTION

In recent years there has been renewed interest in the nonlinear process of parametric two-photon down-conversion, which was first observed by Burnham and Weinberg,[1] and has since served as basis for the study of several nonclassical features of light.[1-15] The theory of the process leading to correlated photon pairs has also been treated numerous times with varying approximations.[16-32] Some calculations apply specifically to the spontaneous down-conversion process, while others focus more on the parametric oscillator in which the nonlinear medium is inside a resonant cavity. Although superficially it might seem that the spontaneous process would be simpler to treat, unlike the oscillator it necessarily involves a continuum of modes that interact with each other and cannot be discarded when interest is focused on the time-resolved correlation properties.

Grangier *et al.*[31] appear to have been the first to emphasize that the down-converted photon pair carries phase information about the pump field. They suggested that this phase information might be extractable when the signal and idler photons are mixed with coherent oscillator fields and a photoelectric correlation measurement is made. An analysis of the experiment showed that it might form the basis for a new test of Bell's inequality in phase measurements. However, since the coherent reference beams have to oscillate at the down-converted frequency, rather than at the pump frequency, the suggested experiment poses practical problems.

Herewith we propose and analyze another experiment that also probes the phase coherence and could be used to test Bell's inequality, but involves no homodyning with a coherent reference beam. Instead, down-converted photons from two similar nonlinear media are mixed and one measures the rate of coincidence detection. Other, closely related, experimental possibilities have also been discussed.[33,34] Our treatment of the problem makes it clear that the vacuum field plays an essential role in the interference in a way that appears not to have been appreciated before.

II. QUANTUM STATE OF THE DOWN-CONVERTED FIELD

Because the experiment we wish to analyze depends on time-resolved photon coincidence or correlation measurements, a two- or three-mode treatment is not appropriate, even if it allows a complete time-dependent solution to be given. With emphasis on spontaneous down-conversion, we shall use a perturbative treatment in the interaction picture.

We consider a nonlinear dielectric in the form of a rectangular parallelopiped of sides l_1, l_2, l_3 and volume $\mathcal{V}$ centered at the origin, within which a quantum field interacts parametrically with a classical pump field. We take the pump to be in the form of a plane, monochromatic wave described by

$$\mathbf{E}^{(+)}(\mathbf{r},t)=\mathbf{V}e^{i(\mathbf{k}_0\cdot\mathbf{r}-\omega_0 t)} . \tag{1}$$

If the crystal has a nonlinear second-order susceptibility tensor χ_{lij}, the interaction energy $\hat{H}_I$ in the interaction picture is of the general form

$$\hat{H}_I(t)=\int_{\mathcal{V}} d\mathbf{r}\,\chi_{lij}E_l^{(+)}(\mathbf{r},t)\hat{E}_i^{(-)}(\mathbf{r},t)\hat{E}_j^{(-)}(\mathbf{r},t) + \text{H.c.} , \tag{2}$$

in which $\hat{\mathbf{E}}(\mathbf{r},t)$ is the quantum field, and all Hilbert space operators are labeled by the caret. The integral is to be taken over the volume $\mathcal{V}=l_1 l_2 l_3$ of the nonlinear dielectric. In order to avoid complications associated with refraction at the dielectric-air interface, we assume that the nonlinear dielectric is embedded in a passive linear medium of the same refractive index.

Because the nonlinear susceptibility may vary with frequency, we shall generalize Eq. (2) somewhat by making a mode expansion of the field vectors and writing instead

$$\hat{H}_I(t)=\frac{1}{L^3}\sum_{\mathbf{k}',s'}\sum_{\mathbf{k}'',s''}\chi_{lij}(\omega_0,\omega',\omega'')V_l(\epsilon^*_{\mathbf{k}'s'})_i(\epsilon^*_{\mathbf{k}''s''})_j\int_{\mathcal{V}} d\mathbf{r}\,e^{i(\mathbf{k}_0-\mathbf{k}'-\mathbf{k}'')\cdot\mathbf{r}}e^{i(\omega'+\omega''-\omega_0)t}\hat{a}^\dagger_{\mathbf{k}'s'}\hat{a}^\dagger_{\mathbf{k}''s''}+\text{H.c.} , \tag{3}$$

so that

$$\frac{1}{i\hbar}\int_0^t dt'\,\hat{H}_I(t')=\frac{1}{i\hbar}\frac{1}{L^3}\sum_{\mathbf{k}',s'}\sum_{\mathbf{k}'',s''}\chi_{lij}(\omega_0,\omega',\omega'')V_l(\boldsymbol{\epsilon}^*_{\mathbf{k}'s'})_i(\boldsymbol{\epsilon}^*_{\mathbf{k}''s''})_j \times\prod_{m=1}^{3}\left[\frac{\sin[\frac{1}{2}(\mathbf{k}_0-\mathbf{k}'-\mathbf{k}'')_m l_m]}{\frac{1}{2}(\mathbf{k}_0-\mathbf{k}'-\mathbf{k}'')_m}\right] \times e^{i(\omega'+\omega''-\omega_0)t/2}\frac{\sin[\frac{1}{2}(\omega'+\omega''-\omega_0)t]}{\frac{1}{2}(\omega'+\omega''-\omega_0)}\hat{a}^\dagger_{\mathbf{k}'s'}\hat{a}^\dagger_{\mathbf{k}''s''}+\text{H.c.} \tag{4}$$

If the initial state of the quantum field is the vacuum state, then the state after a time t is given by

$$|\psi(t)\rangle=\exp\left[\frac{1}{i\hbar}\int_0^t dt'\,\hat{H}_I(t')\right]|\text{vac}\rangle\ .$$

If first-order processes dominate, so that we need to keep only the first two terms in the expansion of the exponential, we can write

$$|\psi(t)\rangle=|\text{vac}\rangle_s|\text{vac}\rangle_i+\frac{1}{i\hbar}\frac{1}{L^3}\sum_{[\mathbf{k}',s']_s}\sum_{[\mathbf{k}'',s'']_i}\chi_{lij}(\omega_0,\omega',\omega'')V_l(\boldsymbol{\epsilon}^*_{\mathbf{k}'s'})_i(\boldsymbol{\epsilon}^*_{\mathbf{k}''s''})_j \times\prod_{m=1}^{3}\left[\frac{\sin[\frac{1}{2}(\mathbf{k}_0-\mathbf{k}'-\mathbf{k}'')_m l_m]}{\frac{1}{2}(\mathbf{k}_0-\mathbf{k}'-\mathbf{k}'')_m}\right] \times e^{i(\omega'+\omega''-\omega_0)t/2}\frac{\sin[\frac{1}{2}(\omega'+\omega''-\omega_0)t]}{\frac{1}{2}(\omega'+\omega''-\omega_0)}|\mathbf{k}',s'\rangle_s|\mathbf{k}'',s''\rangle_i\ . \tag{5}$$

In Eq. (5) it has been assumed that the modes associated with signal and idler photons are distinct and that the corresponding Hilbert spaces do not overlap. We denote the set of signal modes by $[\mathbf{k}',s']_s$ and the set of idler modes by $[\mathbf{k}'',s'']_i$.

We shall now simplify this relation in several respects, by supposing that only one polarization is present and that the directions of the down-converted signal and idler photons are well defined by the measurement apparatus. We then treat s',s'' and the directions of $\mathbf{k}',\mathbf{k}''$ as fixed, and write in place of Eq. (5) the simpler relation

$$|\psi(t)\rangle=M|\text{vac}\rangle_s|\text{vac}\rangle_i +\eta V\,\delta\omega\sum_{\omega'}\sum_{\omega''}\phi(\omega',\omega'')\frac{\sin[\frac{1}{2}(\omega'+\omega''-\omega_0)t]}{\frac{1}{2}(\omega'+\omega''-\omega_0)} \times e^{i(\omega'+\omega''-\omega_0)t/2}|\omega'\rangle_s|\omega''\rangle_i\ . \tag{6}$$

Somewhat similar expansions to represent two-photon states have been introduced before to describe the field in down-conversion[35] and in the cascade-decay process.[36,37] Here $\delta\omega$ is the mode spacing and $\phi(\omega',\omega'')$ is a function that is symmetric in ω',ω'', is peaked at $\omega'=\frac{1}{2}\omega_0=\omega''$, and is normalized so that

$$1=2\pi\delta\omega\sum_{\omega}|\phi(\omega,\omega_0-\omega)|^2\rightarrow 2\pi\int_0^\infty d\omega\,|\phi(\omega,\omega_0-\omega)|^2\ . \tag{7}$$

As the quantization volume becomes infinitely long, $\delta\omega\rightarrow 0$ and sums over frequency ω can be replaced by frequency integrals. If the pump intensity $|V|^2$ is expressed in units of photons per second, then the parameter η in Eq. (6) is dimensionless. When $\delta\omega\rightarrow 0$, normalization of $|\psi(t)\rangle$ requires that

$$1=|M|^2+|\eta V|^2\int_0^\infty d\omega'\int_0^\infty d\omega''|\phi(\omega',\omega'')|^2\left[\frac{\sin[\frac{1}{2}(\omega'+\omega''-\omega_0)t]}{\frac{1}{2}(\omega'+\omega''-\omega_0)}\right]^2\ . \tag{8}$$

Hence M cannot be strictly unity, but M is very close to unity when the down-conversion amplitude is very small.

Let us examine the second term on the right-hand side of Eq. (8) (denoted $\mathcal{T}_2$) when t is long. On putting $\omega''+\omega'-\omega_0=\Omega$ we see that

$$\mathcal{T}_2=|\eta V|^2\int_0^\infty d\omega'\int_0^\infty d\omega''|\phi(\omega',\omega'')|^2\left[\frac{\sin[\frac{1}{2}(\omega'+\omega''-\omega_0)t]}{\frac{1}{2}(\omega'+\omega''-\omega_0)}\right]^2$$

$$=|\eta V|^2\int_0^\infty d\omega'\int_{\omega'-\omega_0}^\infty d\Omega\,|\phi(\omega',\omega_0-\omega'+\Omega)|^2\left[\frac{\sin(\frac{1}{2}\Omega t)}{\frac{1}{2}\Omega}\right]^2\ . \tag{9}$$

As t becomes long, the dominant contribution to the Ω integral comes from small Ω. If $|\phi(\omega',\omega_0-\omega'+\Omega)|^2$ does not vary too rapidly with Ω, we can make the approximation of replacing it by $|\phi(\omega',\omega_0-\omega')|^2$, and since $|\phi(\omega',\omega_0-\omega')|^2$ is peaked near $\omega'=\omega_0/2$, the Ω integral then yields, to a good approximation,

$$\int_{-\infty}^{\infty} d\Omega \left| \frac{\sin(\frac{1}{2}\Omega t)}{\frac{1}{2}\Omega} \right|^2 = 2\pi t \ . \tag{10}$$

Hence Eq. (8) becomes

$$1=|M|^2+|\eta V|^2 2\pi t \int_0^{\infty} d\omega |\phi(\omega,\omega_0-\omega)|^2 \ ,$$

and with the help of Eq. (7)

$$1=|M|^2+|\eta V|^2 t \ . \tag{11}$$

The small down-conversion approximation then implies that

$$t|\eta V|^2 \ll 1, \quad |M|\approx 1 \ . \tag{12}$$

III. PRINCIPLE OF THE PROPOSED EXPERIMENT

Let us consider the experiment shown in outline in Fig. 1. A coherent laser beam of frequency ω_0 is split into two by the beam splitter BS_P, and the two beams are used to pump two identical nonlinear crystals NL1 and NL2. The two sets of down-converted signal and idler beams $s1,i1$ and $s2,i2$ emerging from the two crystals are mixed by the two beam splitters BS_A and BS_B, and the mixed signals fall on two photodetectors D_A and D_B, as shown. The photoelectric pulses from the two detectors are fed to a coincidence counter that yields the rate R_{AB} of "simultaneous" photodetections. Let us first show that for monochromatic light beams R_{AB} depends on the phase difference between the two pump beams.

Let V_1 and V_2 be the complex amplitudes of the classical pump waves falling on the two nonlinear crystals NL1 and NL2, respectively. We shall take the state $|\psi\rangle$ of the down-converted light emerging from NL1 and NL2 to be the direct-product state

$$|\psi\rangle=|\psi_1\rangle_1|\psi_2\rangle_2 \ , \tag{13}$$

with $|\psi_1\rangle_1$ and $|\psi_2\rangle_2$ given by a simplified form of Eq. (6),

$$|\psi_1\rangle_1=M_1|\text{vac}\rangle_{s1}|\text{vac}\rangle_{i1}+\eta_1 V_1 F_1|\omega_1\rangle_{s1}|\omega_1'\rangle_{i1} \ , \tag{14}$$

$$|\psi_2\rangle_2=M_2|\text{vac}\rangle_{s2}|\text{vac}\rangle_{i2}+\eta_2 V_2 F_2|\omega_1\rangle_{s2}|\omega_1'\rangle_{i2} \ . \tag{15}$$

With 50:50% beam splitters BS_A,BS_B, and with equal paths from the crystals NL1, NL2 to BS_A, BS_B, the fields at the two detectors D_A,D_B will then be of the general form

$$\hat{E}_A^{(+)} \propto \hat{a}_{s1}+i\hat{a}_{s2} \ , \tag{16}$$

$$\hat{E}_B^{(+)} \propto \hat{a}_{i1}+i\hat{a}_{i2} \ , \tag{17}$$

where $\hat{a}_{s1}$, $\hat{a}_{s2}$, $\hat{a}_{i1}$ and $\hat{a}_{i2}$ are photon-annihilation operators for the four signal and idler modes. The rate R_{AB} of detecting photons in coincidence at D_A and D_B is then proportional to

$$R_{AB} \propto \langle \psi|\hat{E}_A^{(-)}\hat{E}_B^{(-)}\hat{E}_B^{(+)}\hat{E}_A^{(+)}|\psi\rangle \ . \tag{18}$$

With the help of Eqs. (13)–(17) this yields

$$R_{AB} \propto |\eta_1 V_1 F_1 M_2-\eta_2 V_2 F_2 M_1|^2+O(|\eta_1 V_1|^2|\eta_2 V_2|^2) \ . \tag{19}$$

If we assume for simplicity that

$$|\eta_1 V_1 F_1 M_2|=|\eta_2 V_2 F_2 M_1| \ , \tag{20}$$

and we put

$$\begin{aligned} V_1&=|V_1|e^{i\theta_1} \ ,\\ V_2&=|V_2|e^{i\theta_2} \ ,\\ M_2\eta_1 F_1&=|\eta_1 F_1|e^{i\phi_1} \ ,\\ M_1\eta_2 F_2&=|\eta_2 F_2|e^{i\phi_2} \ , \end{aligned} \tag{21}$$

then up to terms of order $|\eta_1 V_1|^2$ we obtain

$$R_{AB} \propto |\eta_1 V_1 F_1 M_2|^2[1-\cos(\theta_1-\theta_2+\phi_1-\phi_2)] \ . \tag{22}$$

It follows that the phase difference $\theta_1-\theta_2$ between the two pump waves can be determined from measurements of the two-photon coincidence rate R_{AB}, without the need for any homodyning as in the scheme of Grangier *et al.*[31] The appearance of the M_1,M_2 factors [see Eqs. (14) and (15)] shows that the vacuum state plays an essential role in this interference experiment. To order η the state $|\psi\rangle$ is actually a linear superposition of vacuum and two-photon states.

We will now treat the same experiment more realistically without assuming monochromaticity, equal path lengths, or infinitesimal time resolution of the coincidence counter. We start by calculating the rate of photodetection.

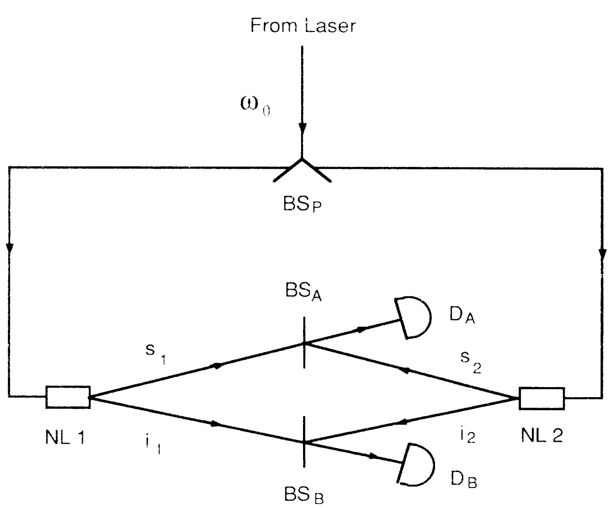

FIG. 1. Outline of the proposed experiment.

IV. RATE OF PHOTODETECTION

We now suppose that the signal photons fall on a photodetector of quantum efficiency α_s. Let us express the field $\hat{E}_s^{(+)}(t)$ at the exit face of the nonlinear medium in the form

$$\hat{E}_s^{(+)}(t)=(\delta\omega/2\pi)^{1/2}\sum_{\omega}\hat{a}_s(\omega)e^{-i\omega t}\ , \tag{23}$$

where $\hat{a}_s(\omega)$ is the photon-annihilation operator for the signal mode at frequency ω and $\hat{E}_s^{(-)}(t)\hat{E}_s^{(+)}(t)$ is in units of photons per second. The rate of signal-photon detection $R_s(t)$ by the detector is then given by

$$R_s(t)=\alpha_s\langle\psi(t)|\hat{E}_s^{(-)}(t-\xi)\hat{E}_s^{(+)}(t-\xi)|\psi(t)\rangle\ , \tag{24}$$

where ξ is the propagation time from crystal to the detector. If we make use of Eqs. (6) and (23) we obtain

$$R_s(t)=\alpha_s\frac{\delta\omega}{2\pi}|\eta V|^2\sum_{\omega'}\sum_{\omega''}e^{i(\omega'-\omega'')(t-\xi)}(\delta\omega)^2 \times\sum_{\omega_1}\sum_{\omega_2}\sum_{\omega_3}\sum_{\omega_4}\phi^*(\omega_1,\omega_2)\phi(\omega_3,\omega_4) \times\frac{\sin[\frac{1}{2}(\omega_1+\omega_2-\omega_0)t]}{\frac{1}{2}(\omega_1+\omega_2-\omega_0)}\frac{\sin[\frac{1}{2}(\omega_3+\omega_4-\omega_0)t]}{\frac{1}{2}(\omega_3+\omega_4-\omega_0)} \times e^{i(\omega_3+\omega_4-\omega_1-\omega_2)t/2}{}_i\langle\omega_2|_s\langle\omega_1|\hat{a}_s^\dagger(\omega')\hat{a}_s(\omega'')|\omega_3\rangle_s|\omega_4\rangle_i\ . \tag{25}$$

In the limit $\delta\omega\to 0$ this reduces to

$$R_s(t)=\alpha_s|\eta V|^2\frac{1}{2\pi}\int_0^\infty d\omega|F(\omega;t)|^2\ , \tag{26}$$

in which

$$F(\omega,t)\equiv\int_0^\infty d\omega'\,\phi(\omega',\omega)\frac{\sin[\frac{1}{2}(\omega'+\omega-\omega_0)t]}{\frac{1}{2}(\omega'+\omega-\omega_0)}e^{i\omega'(t/2-\xi)}\ . \tag{27}$$

A similar expression would of course hold for the rate of counting idler photons.

In the short-time limit $|F(\omega,t)|^2$ becomes proportional to t^2 and so does $R_s(t)$. However, the domain of large t is more relevant. By using the same argument as that leading to Eq. (10) above, we find that for long times t

$$F(\omega,t)\approx\int_{\omega-\omega_0}^\infty d\Omega\,\phi(\omega_0-\omega,\omega)\frac{\sin(\frac{1}{2}\Omega t)}{\frac{1}{2}\Omega}e^{i\Omega(t/2-\xi)}\times e^{i(\omega_0-\omega)(t/2-\xi)} \approx 2\pi\phi(\omega_0-\omega,\omega)e^{i(\omega_0-\omega)(t/2-\xi)}\ , \tag{28}$$

so that

$$R_s(t)\approx\alpha_s|\eta V|^2 2\pi\int_0^\infty d\omega|\phi(\omega_0-\omega,\omega)|^2\ ,$$

and with the help of Eq. (7),

$$R_s(t)\approx\alpha_s|\eta V|^2\ . \tag{29}$$

It follows from Eq. (29) that $|\eta|^2$ is the dimensionless number that converts the rate of incident pump photons into the rate of down-converted signal photons.

V. INTERFERENCE BETWEEN TWO PAIRS OF DOWN-CONVERTED LIGHT BEAMS

We are now ready to treat the theory underlying the experiment shown in outline in Fig. 1 in a more realistic way. The photoelectric pulses from the two detectors are fed to a coincidence counter that yields the rate R_{AB} of "simultaneous" detections occurring within some electronic resolving time T_R. We will show once again that even in the absence of any coherent reference beam the photon pair counting rate R_{AB} depends on the phase difference between the two pump beams.

Let V_1 and V_2 be the complex amplitudes of the two classical pump waves falling on the crystals NL1 and NL2, respectively. Then we take the quantum state $|\psi(t)\rangle$ of the down-converted light at time t to be the direct-product state

$$|\psi(t)\rangle=|\psi_1\rangle_1|\psi_2\rangle_2\ , \tag{30}$$

where, by reference to Eq. (6), we have

$$|\psi_1\rangle_1=M_1|\text{vac}\rangle_{s1}|\text{vac}\rangle_{i1}+\eta_1V_1\delta\omega\sum_{\omega_1}\sum_{\omega_1'}\phi_1(\omega_1,\omega_1')e^{i(\omega_1+\omega_1'-\omega_0)t/2}\frac{\sin[\frac{1}{2}(\omega_1+\omega_1'-\omega_0)t]}{\frac{1}{2}(\omega_1+\omega_1'-\omega_0)}|\omega_1\rangle_{s1}|\omega_1'\rangle_{i1}\ . \tag{31}$$

$$|\psi_2\rangle_2=M_2|\text{vac}\rangle_{s2}|\text{vac}\rangle_{i2}+\eta_2V_2\delta\omega\sum_{\omega_2}\sum_{\omega_2'}\phi_2(\omega_2,\omega_2')e^{i(\omega_2+\omega_2'-\omega_0)t/2}\frac{\sin[\frac{1}{2}(\omega_2+\omega_2'-\omega_0)t]}{\frac{1}{2}(\omega_2+\omega_2'-\omega_0)}|\omega_2\rangle_{s2}|\omega_2'\rangle_{i2}\ . \tag{32}$$

The sets of modes labeled $s1$, $i1$, $s2$, and $i2$ are supposed to be distinct and nonoverlapping and each corresponds to a different subsection of Hilbert space.

With the help of Eq. (23) the electromagnetic fields $\hat{E}_A^{(+)}(t)$ and $\hat{E}_B^{(+)}(t)$ incident on the two photodetectors D_A and D_B are then expressible in the form

$$\hat{E}_A^{(+)}(t)=(\delta\omega/2\pi)^{1/2}\left[\sum_{\omega}t_A\hat{a}_{s1}(\omega)e^{-i\omega(t-\tau_{s1}-\tau_A)}+r_A'\hat{a}_{s2}(\omega)e^{-i\omega(t-\tau_{s2}-\tau_A)}\right]\ , \tag{33}$$

$$\hat{E}_B^{(+)}(t)=(\delta\omega/2\pi)^{1/2}\left[\sum_{\omega} t_B\hat{a}_{i1}(\omega)e^{-i\omega(t-\tau_{i1}-\tau_B)}+r'_B\hat{a}_{i2}(\omega)e^{-i\omega(t-\tau_{i2}-\tau_B)}\right] . \tag{34}$$

Here t_A, r_A, t'_A, and r'_A are the complex transmissivity and reflectivity of beam splitter BS_A from one side and from the other side, and similarly for BS_B. τ_{s1}, τ_{i1}, τ_{s2}, and τ_{i2} are propagation times from the crystals to the respective beam splitters, which are similar but could differ slightly, and τ_A,τ_B are propagation times from BS_A to detector D_A and from BS_B to detector D_B.

The probability amplitude $P_{\rm am}$ for a transition from the initial state $|\psi(t)\rangle$ at time t to some final state $|\psi_F\rangle$ at time $t+\tau$ via a photodetection at A at time t leading to an intermediate state $|\chi\rangle$, followed by a second photodetection at B at later time $t+\tau$ is

$$P_{\rm am}=\langle\psi_F|\hat{E}_B^{(+)}(t+\tau)\exp\left[-\frac{i}{\hbar}\int_t^{t+\tau}\hat{H}_I(t')dt'\right]|\chi\rangle\langle\chi|\hat{E}_A^{(+)}(t)|\psi(t)\rangle . \tag{35}$$

If τ is very short, so that the possibility of additional down conversions occurring in the interval t to $t+\tau$ can be neglected, the unitary time evolution operator on the right-hand side of Eq. (35) can be discarded. The sum over all intermediate states χ then gives the probability amplitude for the transition irrespective of the unknown intermediate state, which is

$$P_{\rm am}=\langle\psi_F|\hat{E}_B^{(+)}(t+\tau)\hat{E}_A^{(+)}(t)|\psi(t)\rangle . \tag{36}$$

After taking the square modulus, summing over all possible final states $|\psi_F\rangle$, and multiplying by the two detector quantum efficiencies α_A,α_B, one obtains the joint probability density $\mathcal{P}_{AB}(t,t+\tau)$ for the two photon detections

$$\begin{aligned}\mathcal{P}_{AB}(t,t+\tau)&=\alpha_A\alpha_B\langle\psi(t)|\hat{E}_A^{(-)}(t)\hat{E}_B^{(-)}(t+\tau)\hat{E}_B^{(+)}(t+\tau)\hat{E}_A^{(+)}(t)|\psi(t)\rangle\\&=\alpha_A\alpha_B|\hat{E}_B^{(+)}(t+\tau)\hat{E}_A^{(+)}(t)|\psi(t)\rangle|^2 .\end{aligned} \tag{37}$$

We now calculate $\mathcal{P}_{AB}$ by substituting for $|\psi(t)\rangle$ from Eqs. (30)–(32) and for $\hat{E}_A$ and $\hat{E}_B$ from Eqs. (33) and (34). We then obtain after some manipulation

$$\begin{aligned}\mathcal{P}_{AB}(t,t+\tau)=\alpha_A\alpha_B\Bigg|&t_At_B(\eta_1V_1)\frac{(\delta\omega)^2}{2\pi}\sum_{\omega_1}\sum_{\omega'_1}\phi_1(\omega_1,\omega'_1)\frac{\sin[\frac{1}{2}(\omega_1+\omega'_1-\omega_0)t]}{\frac{1}{2}(\omega_1+\omega'_1-\omega_0)}\\&\times e^{-i\omega_1(t/2-\tau_A-\tau_{s1})}e^{-i\omega'_1(t/2-\tau_B-\tau_{i1}+\tau)}e^{-(1/2)i\omega_0t}|\text{vac}\rangle_{s1}|\text{vac}\rangle_{i1}|\psi_2\rangle_2\\&+r'_Ar'_B\eta_2V_2\frac{(\delta\omega)^2}{2\pi}\sum_{\omega_2}\sum_{\omega'_2}\phi_2(\omega_2,\omega'_2)\frac{\sin[\frac{1}{2}(\omega_2+\omega'_2-\omega_0)t]}{\frac{1}{2}(\omega_2+\omega'_2-\omega_0)}\\&\times e^{-i\omega_2(t/2-\tau_A-\tau_{s2})}e^{-i\omega'_2(t/2-\tau_B-\tau_{i2}+\tau)}e^{-i\omega_0t/2}|\psi_1\rangle_1|\text{vac}\rangle_{s2}|\text{vac}\rangle_{i2}\\&+t_Ar'_B\eta_1V_1\eta_2V_2\frac{(\delta\omega)^3}{2\pi}\sum_{\omega_1}\sum_{\omega'_1}\sum_{\omega_2}\sum_{\omega'_2}\phi_1(\omega_1,\omega'_1)\phi_2(\omega_2,\omega'_2)\\&\times\frac{\sin[\frac{1}{2}(\omega_1+\omega'_1-\omega_0)t]}{\frac{1}{2}(\omega_1+\omega'_1-\omega_0)}\frac{\sin[\frac{1}{2}(\omega_2+\omega'_2-\omega_0)t]}{\frac{1}{2}(\omega_2+\omega'_2-\omega_0)}\\&\times e^{-i\omega_1(t/2-\tau_A-\tau_{s1})}e^{i\omega'_1t/2}e^{i\omega_2t/2}e^{-i\omega'_2(t/2-\tau_B-\tau_{i2}+\tau)}\\&\times e^{-i\omega_0t}|\text{vac}\rangle_{s1}|\omega'_1\rangle_{i1}|\omega_2\rangle_{s2}|\text{vac}\rangle_{i2}\\&+r'_At_B\eta_1V_1\eta_2V_2\frac{(\delta\omega)^3}{2\pi}\sum_{\omega_1}\sum_{\omega'_1}\sum_{\omega_2}\sum_{\omega'_2}\phi_1(\omega_1,\omega'_1)\phi_2(\omega_2,\omega'_2)\\&\times\frac{\sin[\frac{1}{2}(\omega_1+\omega'_1-\omega_0)t]}{\frac{1}{2}(\omega_1+\omega'_1-\omega_0)}\frac{\sin[\frac{1}{2}(\omega_2+\omega'_2-\omega_0)t]}{\frac{1}{2}(\omega_2+\omega'_2-\omega_0)}\\&\times e^{i\omega_1t/2}e^{-i\omega'_1(t/2-\tau_B-\tau_{i1}+\tau)}e^{-i\omega_2(t/2-\tau_A-\tau_{s2})}\\&\times e^{i\omega'_2t/2}|\omega_1\rangle_{s1}|\text{vac}\rangle_{i1}|\text{vac}\rangle_{s2}|\omega'_2\rangle_{i2}\Bigg|^2 .\end{aligned} \tag{38}$$

By virtue of the weak-down-conversion assumption embodied in relation (12), the main contribution to $\mathcal{P}_{AB}(t,t+\tau)$ comes from that portion of the first two terms on the right proportional to $|\text{vac}\rangle_1|\text{vac}\rangle_2$, and when $\delta\omega\to 0$, we obtain to a good approximation

$$\begin{aligned}\mathcal{P}_{AB}(t,t+\tau)=\alpha_A\alpha_B\Bigg|&\frac{1}{2\pi}\int_0^\infty d\omega\int_0^\infty d\omega'\frac{\sin[\frac{1}{2}(\omega+\omega'-\omega_0)t]}{\frac{1}{2}(\omega+\omega'-\omega_0)}e^{-i\omega(t/2-\tau_A)}e^{-i\omega'(t/2-\tau_B+\tau)}\\&\times[t_At_B\eta_1V_1M_2\phi_1(\omega,\omega')e^{i(\omega\tau_{s1}+\omega'\tau_{i1})}\\&+r'_Ar'_B\eta_2V_2M_1\phi_2(\omega,\omega')e^{+i(\omega\tau_{s2}+\omega'\tau_{i2})}]\\&\times|\text{vac}\rangle_{s1}|\text{vac}\rangle_{i1}|\text{vac}\rangle_{s2}|\text{vac}\rangle_{i2}\Bigg|^2\\=\alpha_A\alpha_B|&t_At_B\eta_1V_1M_2f_1(t;-\tau_A-\tau_{s1},-\tau_B-\tau_{i1}+\tau)\\&+r'_Ar'_B\eta_2V_2M_1f_2(t;-\tau_A-\tau_{s2},-\tau_B-\tau_{i2}+\tau)|^2\ .\end{aligned}\tag{39}$$

We have introduced the functions f_j defined by

$$f_j(t;\tau_1,\tau_2)\equiv\frac{1}{2\pi}\int_0^\infty d\omega\int_0^\infty d\omega'\phi_j(\omega,\omega')\frac{\sin[\frac{1}{2}(\omega+\omega'-\omega_0)t]}{\frac{1}{2}(\omega_0+\omega'-\omega_0)}e^{-i\omega(t/2+\tau_1)}e^{-i\omega'(t/2+\tau_2)}\quad(j=1,2)\tag{40}$$

When t is long, Eq. (40) can be simplified. As before, we put $\omega'+\omega-\omega_0=\Omega$ so that

$$f_j(t;\tau_1,\tau_2)=\frac{e^{-i\omega_0(t/2+\tau_2)}}{2\pi}\int_0^\infty d\omega\int_{\omega-\omega_0}^\infty d\Omega\,\phi_j(\omega,\omega_0-\omega+\Omega)e^{-i\omega(\tau_1-\tau_2)}\frac{\sin(\frac{1}{2}\Omega t)}{\frac{1}{2}\Omega}e^{-i\Omega(t/2+\tau_2)}\ ,$$

and we observe that for long t the dominant contributions to the Ω integral come from small Ω. This allows us to approximate $\phi_j(\omega,\omega_0-\omega+\Omega)$ by $\phi_j(\omega,\omega_0-\omega)$, and since $\phi_j(\omega,\omega_0-\omega)$ is peaked at $\omega=\frac{1}{2}\omega_0$, to replace the lower limit on the Ω integral by $-\infty$. Provided $\tau_2<0$, the Ω integral then yields unity, and we have finally for long t

$$f_j(t;\tau_1,\tau_2)\approx e^{-i\omega_0(t/2+\tau_2)}\int_0^\infty d\omega\,\phi_j(\omega,\omega_0-\omega)e^{-i\omega(\tau_1-\tau_2)}=e^{-i\omega_0(t/2+\tau_2)}g_j(\tau_1-\tau_2)\ ,\tag{41}$$

where

$$g_j(\tau)=\int_0^\infty d\omega\,\phi_j(\omega,\omega_0-\omega)e^{-i\omega\tau}\quad(j=1,2)\ .\tag{42}$$

By making use of Eq. (41) in Eq. (39), we see that, provided $\tau<\tau_B+\tau_{i1}$ and $\tau<\tau_B+\tau_{i2}$,

$$\begin{aligned}\mathcal{P}_{AB}(t,t+\tau)=\alpha_A\alpha_B|&t_At_B\eta_1V_1M_2g_1(\tau_B-\tau_A+\tau_{i1}-\tau_{s1}-\tau)e^{+i\omega_0\tau_{i1}}\\&+r'_Ar'_B\eta_2V_2M_1g_2(\tau_B-\tau_A+\tau_{i2}-\tau_{s2}-\tau)e^{+i\omega_0\tau_{i2}}|^2\ .\end{aligned}\tag{43}$$

The presence of the factors M_1 or M_2 in the terms on the right [cf. Eqs. (31) and (32)] shows that the vacuum plays an essential role in the probability density $\mathcal{P}_{AB}(t,t+\tau)$.

As $\phi_j(\omega,\omega_0-\omega)$ is peaked at $\omega=\frac{1}{2}\omega_0$ and has some spectral width $\Delta\omega$, it follows from the definition (42) that

$$\begin{aligned}g_j(\tau)&=e^{-i\omega_0\tau/2}\int_{-\omega_0/2}^\infty d\omega'\phi_j(\omega_0/2+\omega',\omega_0/2-\omega')e^{-i\omega'\tau}\\&=e^{-i\omega_0\tau/2}|g_j(\tau)|\ ,\end{aligned}\tag{44}$$

where the integral is real and does not vary appreciably with τ over any time interval that is much smaller than $1/\Delta\omega$. Hence $|g_j(\tau)|$ is a relatively slowly varying function of τ.

Equation (43) simplifies substantially if we set

$$\begin{aligned}\tau_{i1}-\tau_{i2}&=\delta\tau_i\ ,\\\tau_{s1}-\tau_{s2}&=\delta\tau_s\ ,\end{aligned}\tag{45}$$

where $c\delta\tau_i$ and $c\delta\tau_s$ represent optical-path differences with $|\delta\tau_i-\delta\tau_s|\ll 1/\Delta\omega$, and we put

$$g_1(\tau)=g_2(\tau)=g(\tau)\ .\tag{46}$$

Then with the help of Eq. (44) we obtain

$$\begin{aligned}\mathcal{P}_{AB}(t,t+\tau)=\alpha_A\alpha_B&|g(\tau_B-\tau_A+\tau_{i1}-\tau_{s1}-\tau)|^2\\&\times\{|t_At_BM_2\eta_1V_1|^2+|r'_Ar'_BM_1\eta_2V_2|^2\\&+2|t_At_BM_2\eta_1V_1|\,|r'_Ar'_BM_1\eta_2V_2|\cos[\theta_1-\theta_2+\phi_1-\phi_2+\omega_0(\delta\tau_i+\delta\tau_s)/2]\}\ ,\end{aligned}\tag{47}$$

where we have put

$$\begin{aligned} V_1 &= |V_1| e^{i\theta_1} , \\ V_2 &= |V_2| e^{i\theta_2} , \\ t_A t_B \eta_1 M_2 &= |t_A t_B \eta_1 M_2| e^{i\phi_1} , \\ r'_A r'_B \eta_2 M_1 &= |r'_A r'_B \eta_2 M_1| e^{i\phi_2} . \end{aligned} \tag{48}$$

Finally, we integrate $\mathcal{P}_{AB}(t,t+\tau)$ with respect to τ over the resolving time T_R of the coincidence detector, in order to arrive at the expected photon-coincidence counting rate $\mathcal{P}_{AB}$. If $T_R \gg 1/\Delta\omega$, which is usually the case because of the large down-conversion bandwidth $\Delta\omega$, and if $|\tau_B - \tau_A + \tau_{i1} - \tau_{s1}| \ll T_R/2$, then

$$\begin{aligned} \int_{-T_R/2}^{T_R/2} d\tau |g(\tau_B - \tau_A + \tau_{i1} - \tau_{s1} - \tau)|^2 &\approx \int_{-\infty}^{\infty} d\tau |g(\tau)|^2 \\ &= 2\pi \int_0^{\infty} d\omega |\phi(\omega, \omega_0 - \omega)|^2 \\ &= 1 , \end{aligned} \tag{49}$$

where we have used the Plancherel theorem together with Eq. (7). Then if $|t_A t_B| = |r'_A r'_B|$ and $|\eta_1 V_1 M_2| \approx |\eta_2 V_2 M_1|$, we obtain from Eqs. (47) and (49)

$$\mathcal{R}_{AB} = 2\alpha_A \alpha_B |t_A t_B M_2 \eta_1 V_1|^2 \times \{1 + \cos[\theta_1 - \theta_2 + \phi_1 - \phi_2 + \omega_0(\delta\tau_i + \delta\tau_s)/2]\} . \tag{50}$$

For the special case of symmetric beam splitters, with $\eta_1 = \eta_2$ we have $\phi_1 - \phi_2 = \pi$, and Eq. (50) simplifies to

$$\mathcal{R}_{AB} = 2\alpha_A \alpha_B |t_A t_B M_2 \eta_1 V_1|^2 \times \{1 - \cos[\theta_1 - \theta_2 + \omega_0(\delta\tau_i + \delta\tau_s)/2]\} , \tag{51}$$

in which the dependence on the phase difference $\theta_1 - \theta_2$ is similar to that found previously in Eq. (22).

VI. DISCUSSION

The important conclusion to be drawn from Eq. (51), as from Eq. (22), is that the difference $\theta_1 - \theta_2$ between the phases θ_1, θ_2 of the two classical pump waves shows up as an interference term in the photon coincidence counting rate $\mathcal{R}_{AB}$. The phase information is therefore carried by the down-converted photons. But unlike Grangier *et al.*,[31] we have treated an experiment in which there is no homodyning against a local oscillator.

The process we have discussed could be described as due to the interference of two two-photon probability amplitudes for photon pairs produced from sources 1 and 2. But unlike the previously reported interference experiments based on down-conversion,[9–15] in this case the vacuum state plays an essential role, as is evident from the appearance of the factor M in the coincidence rate [cf. Eq. (51)]. Indeed, in a sense it is the superposition of the vacuum state of the field and the two-photon down-conversion state which is responsible for the predicted effect.

Comparison with Eq. (29) for the rate of detection R_s of signal photons shows that, apart from the transmissivity factors $|t_A t_B|^2$ and the additional detector efficiency α_B, and the fact that we have two sources instead of one, the coincidence rate $\mathcal{R}_{AB}$ is similar in magnitude to the single-photon counting rate R_s. This conclusion is perhaps surprising, because the experiment shown in Fig. 1 depends on the interference of two sets of down-converted light beams, for which one might superficially have expected a very much lower counting rate. Of course, the result reflects the fact that the vacuum makes the dominant contribution to the state of the field in Eq. (6), and that the measurement depends on the detection of just two, and not four, photons.

Finally, we note that the coincidence counting rate $\mathcal{R}_{AB}$ given by Eq. (51) depends on the phase difference $\theta_1 - \theta_2$ and on the optical path differences $c(\delta\tau_i + \delta\tau_s)$ defined by Eq. (45). It follows that the kind of locality violation experiments proposed by Grangier *et al.*[31] could also be performed with the apparatus shown in Fig. 1, without the need for a coherent reference beam at the down-converted frequency. In that case, $\delta\tau_i, \delta\tau_s$ would be adjustable parameters while θ_1, θ_2 are held constant.

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