Photodetachment spectra of H^- in parallel electric and magnetic fields

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(Received 6 March 1989)

Simple analytic formulas are presented for the photodetachment cross section of H^- in the presence of parallel electric and magnetic fields. A rich array of photodetachment spectra are displayed that illustrate the strong dependence of the cross section on the polarization of the photons and on the electric and magnetic fields.

I. INTRODUCTION

In a recent experimental study of the photodetachment cross section of H^- in static external electric fields of a few kV/cm,[1] it was observed that for photons polarized parallel to the electric field, oscillations in the cross section occur as the photon energy is varied near threshold, but with the polarization perpendicular to the field, the effect of the field is small, and the cross section differs little from the cross section in the absence of the field. These observations were briefly analyzed using a time-dependent wave-packet model.[1] Theoretical discussions of the measurements have been given by Rau and Wong[2] and by Du and Delos.[3] The oscillations in the photodetachment cross section for parallel polarization and the disappearance of the oscillation for perpendicular polarization were explained in terms of the interference of the initial outgoing wave and the returning electron wave reflected by the potential barrier caused by the electric field. A similar physical picture was used earlier to interpret the photon absorption spectrum of atoms in a magnetic field.[4]

No photodetachment experiment has been carried out for H^- in a static magnetic field. However, Blumberg, Jopson, and Larson[5(a)] observed oscillations in the photodetachment cross section of S^- in the presence of a magnetic field. The oscillations were attributed to the Landau levels for an electron moving in a magnetic field.[5(b)] Detailed theories have been given by Clark,[6] Greene,[7] and Crawford.[8]

In discussions of photodetachment of the negative ion in an electric field, a magnetic field, or parallel electric and magnetic fields, Reinhardt[9] has employed a time-dependent wave-packet model. The model generally predicts different patterns of oscillations at different fields and is helpful in a qualitative understanding of the phenomena.

Our purpose here is to obtain simple and accurate expressions for the photodetachment cross section of H^- in parallel electric and magnetic fields. We regard H^- as effectively a one-electron system with the electron loosely bound by a short-range potential.[3] Because the strength of the electric and magnetic fields is small compared to the atomic binding force, the effect of the fields on the wave function of the initial state of H^- may be ignored. The wave functions of the electron after detachment may be taken as the wave functions of an electron moving under the influence of only the parallel electric and magnetic fields, the effect of the short-range potential of the atomic core being ignored. Similar approximations have been used to derive the zero-field cross section[10] and the cross section in the presence of an electric field.[3]

The cross section is directly related to the dipole matrix elements, which depend on a region of configuration space determined by the size of the initial localized wave function. When the applied fields are present but small, then in a small region around the nucleus of the size of the initial state of H^-, the field-dependent terms in the Hamiltonian can be neglected compared to the total energy. Thus the wave functions in the small region in the presence of the fields are related to the wave functions in the absence of the fields.

In computing the dipole matrix elements between the initial wave function of H^- and the final wave functions of an electron in parallel electric and magnetic fields, the final wave functions are most conveniently written in cylindrical coordinates. We then use local approximations to relate the final wave functions in cylindrical coordinates in parallel electric and magnetic fields to the wave functions in cylindrical coordinates in the absence of any field. The local approximations are valid in the region around the nucleus. A further orthogonal transformation is then used to relate the zero-field wave functions in cylindrical coordinates to the wave functions in spherical coordinates. Finally, when the dipole matrix elements are evaluated, they are expressed as products of the zero-field dipole matrix elements and some field-dependent transformation matrix elements. Many ideas described above may also be found in papers by Harmin,[11] Greene,[7] and Rau *et al.*[2]

The remainder of the paper is organized as follows. In Sec. II, the derivation of the analytic formulas for the photodetachment cross section in parallel electric and magnetic field is presented. In Sec. III, we display and discuss the spectrum for different combinations of electric and magnetic fields. The relationship between the present more general formulas and the previous formulas in the presence of only the electric or the magnetic field is also explored. We conclude in Sec. IV. Atomic units are used throughout, unless otherwise noted.

II. THEORY

Let the binding energy of H^- be denoted by $E_b(=0.754$ eV) and the photon energy by $E_p=E+E_b$, where E is the electron energy after detachment in the absence of any field. The photodetachment cross section in the presence of parallel electric field F and magnetic field B, $\sigma(E,F,B)$, is given in terms of the dipole matrix elements by the expression

$$\sigma(E,F,B)=\left[\frac{4\pi^2 E_p}{c}\right]\sum_n |\langle f_{E,n}|D|i\rangle|^2 , \tag{1}$$

where c is the speed of light, D is the dipole operator, $|i\rangle$ is the initial state of wave function of H^-, and $|f_{E,n}\rangle$ are the final-state wave functions after detachment. The final states are labeled by the energy E and a quantum number n which distinguishes the discrete energy band in which E falls. The final wave functions are normalized according to

$$\langle f_{E,n}|f_{E',n'}\rangle=\delta(E-E')\delta_{n,n'} . \tag{2}$$

The dipole operator depends on the polarization of photons. For photons polarized along the direction z of the fields $D=z$; for circularly polarized photons $D=(x+iy)/\sqrt{2}$ and the final P state has a magnetic quantum number $m_f=1$. We use the subscript z and $+$ in $\sigma(E,F,B)$ to distinguish the linear and circular polarizations.

A. Initial state

We neglect the small effects of the electric and magnetic fields on the initial bound state of H^-. In the calculation of the photodetachment cross section of H^-, a good approximation to the initial-state wave function is given by[10,3]

$$\psi_i(r)=C_N\frac{e^{-k_b r}}{r} , \tag{3}$$

where $k_b=\sqrt{2E_b}$ has the value 0.235 588 3 and C_N is a scale factor with the value 0.315 52. Expression (3) is a good approximation to the exact wave function when r is greater than an effective radius r_{eff} but not at a smaller r. For computing the photodetachment cross section, it has been shown[12,10,3] that the representation (3) gives accurate results. (This is expected because the dipole operator and the final p wave function weigh the large-r region more heavily, and the small-r region makes an unimportant contribution to the dipole matrix elements.)

B. Final states

The effects of the short-range potential of the atomic core on the detached electron can be neglected. The final wave functions describing motion of the electron after detachment may be taken to be the wave functions of an electron in parallel electric and magnetic fields. These wave functions are conveniently written in cylindrical coordinates (ρ,Φ,z),

$$\psi_{E,n,m}(\mathbf{r})=\phi^B_{n,m}(\rho,\Phi)\phi^F_{q_n}(z) , \tag{4a}$$

where

$$\phi^F_{q_n}(z)=\left[\frac{4}{F}\right]^{1/6}\text{Ai}\left[(2F)^{1/3}\left[z-\frac{q_n^2}{2F}\right]\right] \tag{4b}$$

is the energy normalized wave function of an electron in the direction of the electric field.[13,2] Ai is the standard Airy function[14] and

$$\phi^B_{n,m}(\rho,\Phi)=(2\pi)^{-1/2}e^{im\Phi}N_{nm}e^{-\alpha\rho^2/2}(\alpha\rho^2)^{|m|/2} \times L_n^{(|m|)}(\alpha\rho^2) , \tag{4c}$$

with

$$N_{nm}=[2\alpha n!/(n+|m|)!]^{1/2} \tag{4d}$$

is the normalized wave function of an electron in a magnetic field;[14,7] 2α is the cyclotron frequency B/c. The parameters in Eqs. (4a)–(4d) are constrained by the energy relation,

$$E=\frac{k^2}{2}=E_n^m+\frac{q_n^2}{2}=\alpha(2n+m+|m|+1)+\frac{q_n^2}{2} . \tag{4e}$$

The parameter q_n defined in (4e) depends on the quantum number m. We omit m to simplify the notation. The normalizations of $\phi^F_{q_n}(z)$ and $\phi^B_{n,m}(\rho,\Phi)$ are consistent with the requirement of Eq. (2).

For laboratory field strengths, the field-dependent terms in the Hamiltonian governing the motion of the electron after detachment are small in a region of a few tens of a_0 around the nucleus, and can be neglected. Thus the wave function $\psi_{E,n,m}(\mathbf{r})$ resembles the corresponding wave function in the absence of any field around the nucleus. We now describe the precise connection of the wave functions.

The energy-normalized solution for the zero field corresponding to $\phi^F_{q_n}(z)$ is

$$\phi^{F=0}_{q_n,\pm1}=\frac{1}{\sqrt{\pi q_n}}\begin{Bmatrix}\cos(q_n z)\\ \sin(q_n z)\end{Bmatrix}, \quad \Pi_z=\pm1 \tag{5}$$

where Π_z is the parity of the solution. In a region for which $|z|$ is not too large, $\phi^F_{q_n}(z)$ can be expanded as a linear combination of $\phi^{F=0}_{q_n,+1}$ and $\phi^{F=0}_{q_n,-1}(z)$,

$$\phi^F_{q_n}(z)=A_{+1}(q_n,F)\phi^{F=0}_{q_n,+1}(z)+A_{-1}(q_n,F)\phi^{F=0}_{q_n,-1}(z) . \tag{6a}$$

The coefficients $A_{\pm1}(q_n,F)$ are independent of the variable z. They are obtained by evaluating (6a) and the derivative of Eq. (6a) with respect to z at $z=0$,[2] with the results

$$A_{+1}(q_n,F)=(\pi q_n)^{1/2}\left[\frac{4}{F}\right]^{1/6}A_i\left[\frac{-q_n^2}{(2F)^{2/3}}\right] , \tag{6b}$$

$$A_{-1}(q_n,F)=\left[\frac{\pi}{q_n}\right]^{1/2}\left[\frac{4}{F}\right]^{1/6}A_i'\left[\frac{-q_n^2}{(2F)^{2/3}}\right] . \tag{6c}$$

The wave function in the zero field corresponding to $\phi^B_{n,m}(\rho,\Phi)$ is

$$\phi^{B=0}_{n,m}(\rho,\Phi)=(2\pi)^{-1/2}e^{im\Phi}J_m(k_\rho,\rho) , \tag{7a}$$

with

$$k_\rho=(2E_n^m)^{1/2}=[2\alpha(2n+m+|m|+1)]^{1/2} . \quad (7b)$$

In a region for which ρ is not too large, $\phi^B_{n,m}(\rho,\Phi)$ is proportional to $\phi^{B=0}_{n,m}(\rho,\Phi)$,

$$\phi^B_{n,m}(\rho,\Phi)=P(n,m,B)\phi^{B=0}_{n,m}(\rho,\Phi) . \quad (7c)$$

Equation (7c) is valid provided $\alpha^2\rho^2/2 \ll E_n^m$ where $P(n,m,B)$ is a function independent of ρ and Φ. By combining Eqs. (5)–(7), we arrive at the result

$$\psi_{E,n,m}(\mathbf{r})= A_{+1}(q_n,F)P(n,m,B)\phi^{F=0}_{q_n,+1}(z)\phi^{B=0}_{n,m}(\rho,\Phi) + A_{-1}(q_n,F)P(n,m,B)\phi^{F=0}_{q_n,-1}(z) \times\phi^{B=0}_{n,m}(\rho,\Phi) . \quad (8)$$

We emphasize again that Eq. (8) is a satisfactory approximation near the atom. With increasing distance from the nucleus, the equality becomes less accurate.

In Eq. (8) the right-hand side is a linear combination of zero-field wave functions in cylindrical coordinates. The zero-field wave functions can also be written in spherical coordinates, which are more convenient in the calculations of the dipole matrix elements.

The energy normalized wave functions of an electron escaping from H are given in spherical coordinates (r,θ,Φ) by

$$f_{lm}(\mathbf{r})=(2\pi)^{-1/2}N_{lm}e^{im\Phi}P_{lm}(\cos\theta)\left[\frac{2k}{\pi}\right]^{1/2} j_l(kr) . \quad (9)$$

The zero-field wave functions in cylindrical coordinates are connected to the zero-field wave functions in spherical coordinates by a transformation,

$$\phi^{F=0}_{q_n,+1}(z)\phi^{B=0}_{n,m}(\rho,\Phi)=\sum_l{}' U_{q_n,l}(n,m)f_{lm}(\mathbf{r}) \quad (10a)$$

and similarly

$$\phi^{F=0}_{q_n,-1}(z)\phi^{B=0}_{n,m}(\rho,\Phi)=\sum_l{}'' U_{q_n,l}(n,m)f_{lm}(\mathbf{r}) . \quad (10b)$$

The summation $\sum'$ should include all the l's such that $(-1)^l=(-1)^m$, whereas the summation $\sum''$ includes all the l such that $(-1)^l=(-1)^{m+1}$.

Inserting Eq. (10) in Eq. (8) we finally have

$$\psi_{E,n,m}(\mathbf{r})=\sum_l{}' A_{+1}(q_n,F)P(n,m,B)U_{q_n,l}(n,m)f_{lm}(\mathbf{r})+\sum_l{}'' A_{-1}(q_n,F)P(n,m,B)U_{q_n,l}(n,m)f_{lm}(\mathbf{r}) . \quad (11)$$

Greene[7] has given the following matrix elements:

$$P(n,m,B)U_{q_n,l}(n,m)=\left[\frac{2l+1}{kq_n}\right]^{1/2}\left[\frac{(l-m)!}{(l+m)!}\right]^{1/2}(-1)^{[(l-1-m)/2]}P_{lm}(q_n/k) \times(2\alpha)^{1/2}[n!/(n+|m|)!]^{1/2}\left[\frac{(2n+m+|m|+1)}{2}\right]^{|m|/2} \quad (12)$$

in the phase factor $(-1)^{[x]}$, $[x]$ denotes the smallest integer greater than or equal to x. q_n and k are defined in Eq. (4e).

C. Photodetachment cross sections

The photodetachment cross sections can now be calculated by substituting $\Psi_{E,n,m}(\mathbf{r})$ in Eq. (11) for the final state $f_{E,n}$ in Eq. (1). Because of the dipole selection rule, only the $l=1$ terms in Eq. (11) contribute to the cross sections.

For photons polarized along the field direction, the $l=1$ term in $\sum''$ of Eq. (11) contributed to the cross section, $\sigma_z(E,F,B)$. Then

$$\sigma_z(E,F,B)=H_z(E,F,B)\sigma_0(E) , \quad (13a)$$

where

$$H_z(E,F,B)=\sum_n[A_{-1}(q_n,F)]^2\left[\frac{6\alpha q_n}{k^3}\right] \quad (13b)$$

and $\sigma_0(E)$ is the field-free cross section

$$\sigma_0(E)=\left[\frac{4\pi^2E_p}{c}\right]|\langle f_{10}|z|\Psi_i\rangle|^2 =\left[\frac{4\pi^2E_p}{c}\right]\left|\left\langle f_{11}\left|\frac{1}{\sqrt{2}}(x+iy)\right|\Psi_i\right\rangle\right|^2 =0.054\,08\frac{E^{3/2}}{(E_b+E)^3}a_0^2 . \quad (14)$$

For circularly polarized photons, only the $l=1$ term in $\sum'$ of Eq. (11) contributes to the cross section, $\sigma_+(E,F,B)$. Then

$$\sigma_+(E,F,B)=H_+(E,F,B)\sigma_0(E) , \quad (15a)$$

where

$$H_+(E,F,B)=\sum_n[A_{+1}(q_n,F)]^2\left[1-\frac{q_n^2}{k^2}\right]\left[\frac{3\alpha}{kq_n}\right] \times\left[1+\frac{1}{2(n+1)}\right] . \quad (15b)$$

The summation in Eq. (13b) includes all the Landau levels defined in (4e) in which m is set to zero; similarly the summation in Eq. (15b) includes all the Landau levels defined in (4e) in which m is set to 1.

In our derivation of the cross sections, we have assumed q_n is always real and non-negative. That implies that the summation in Eq. (13b) and Eq. (15b) should include only those Landau levels with energy below E. However, when the expressions in (6b) and (6c) are used in Eqs. (13b) and (15b), respectively, we find that the q_n factor cancels, and the term in the sum actually depends only on q_n^2. We therefore assume that the q_n^2-dependent term may be extrapolated to a few levels above E. Those levels far above E do not contribute to the sum because of the exponential damping of the Airy function. We therefore include all the Landau levels in the sum. We present evidence supporting the same arguments when considering the weak magnetic field limits of Eqs. (13b) and (15b) in the next section.

III. PHOTODETACHMENT SPECTRA

The zero-field cross section $\sigma_0(E)$ is a smooth function of energy E.[10,3] The H functions in Eqs. (13) and (15) contain almost all of the structure that occurs in the photodetachment cross section of H^- in parallel electric and magnetic fields. In Figs. 1(a)–1(h) we show $H_z(E,F,B)$ as a function of E for various combinations of electric and magnetic fields (see Table I) and in Figs. 2(a)–2(h) we show $H_+(E,F,B)$ as a function of E for the same set of electric and magnetic fields. These results should be valid as long as the electric field is limited to a few hundred kV/cm and the magnetic field does not exceed a few tens of tesla. The structure of the spectrum is determined primarily by the relative strength of the electric and magnetic fields.

The photodetachment process of H^- in the presence of an electric field and no magnetic field has been studied experimentally[1] and theoretically.[2,3,9] The H function shown in Figs. 1(a) and 2(a) for the two polarizations with a weak magnetic field present are close to that of the zero-magnetic field limit. The weak magnetic field produces closely spaced small-amplitude oscillations and the electric field produces the large-amplitude oscillation envelope.

The cross-section formulas in the presence of only an electric field may be obtained by taking the magnetic field to be zero in Eqs. (13) and (15). In the weak magnetic field limit, the sum in Eq. (13b) may be replaced by an integral,

$$H_z = \int (A_{-1})^2 \left[\frac{6\alpha q_n}{k^3}\right] dn \ . \tag{16a}$$

Now inserting A_{-1} from (6c) to cancel q_n, and using the relationship $E = k^2/2 = 2\alpha(n+\frac{1}{2}) + q_n^2/2$ in (16a), we find that

$$H_z(E,F,B=0) = \left[\frac{6\pi F}{(2F)^{3/2}}\right] D_{\parallel}\left[\frac{2E}{(2F)^{2/3}}\right] , \tag{16b}$$

where

$$D_{\parallel}(z) \equiv \int_{-\infty}^{z} \left[\frac{dA_i(-x)}{dx}\right]^2 dx \ . \tag{16c}$$

Exactly the same procedure gives, for H_+,

$$H_+(E,F,B=0) = \left[\frac{6\pi F}{(2F)^{3/2}}\right] D_+\left[\frac{2E}{(2F)^{2/3}}\right] , \tag{17a}$$

where

$$D_+(z) \equiv \tfrac{1}{2}\int_{-\infty}^{z} (z-x) A_i^2(-x) dx \ . \tag{17b}$$

These results agree with our earlier derivations.[3] There is an interesting point here. Had we included only those Landau levels with energy below E in (13b) and (15b), the lower limit in the integrals of (16c) and (17b) would be zero instead of $-\infty$. Although the contribution to the integral from $-\infty$ to zero is small, a more accurate result is obtained by including this region, consistent with our earlier argument that all the Landau levels should be included in the sum in Eqs. (13b) and (15b).

The case when only the magnetic field is present has been studied by Greene[7] and Crawford.[8] The H functions for the linear and circular polarizations are shown in Figs. 1(h) and 2(h), respectively, in the limit of zero electric fields.

The cross-section formulas in the presence of only a magnetic field may be obtained by setting the electric-field-dependent factor $A_{\pm 1}$ in Eqs. (13b) and (15b) to unity for Landau levels below E and to zero for Landau levels above E.

The most noticeable features in Figs. 1(h) and 2(h) are the equally spaced Landau levels and the threshold behavior above each level. For the linear polarization "above each level," $\Delta H \propto (E-E_{\rm th})^{1/2}$, whereas for the circular polarization, $\Delta H \propto (E-E_{\rm th})^{-1/2}$ "above each level." These threshold behaviors are different from the normal Wigner law. Clark[6] has shown, however, that any final-state interaction removes the infinity in the immediate vicinity of each Landau level. The apparent contradiction is resolved by noting that the restored region is extremely small for H^-. In practice, the $(E-E_{\rm th})^{1/2}$ or $(E-E_{\rm th})^{-1/2}$ threshold laws will dominate.

If both the electric and magnetic fields are present, the

TABLE I. The combinations of the electric and magnetic fields used to plot the spectrum in Figs. 1 and 2. Also listed is the dimensionless quantity R measuring the relative effects of the two fields on the spectrum. If $R > 1$ the electric field has a dominating influence on the spectrum and if $R < 1$, the magnetic field.

Figure index	F (V/cm)	B (10^3 G)	R
(a)	10	1	3.7
(b)	150	10	2.3
(c)	60	10	1.2
(d)	30	10	0.8
(e)	20	10	0.6
(f)	5	10	0.2
(g)	1	10	0.1
(h)	0	10	0.0

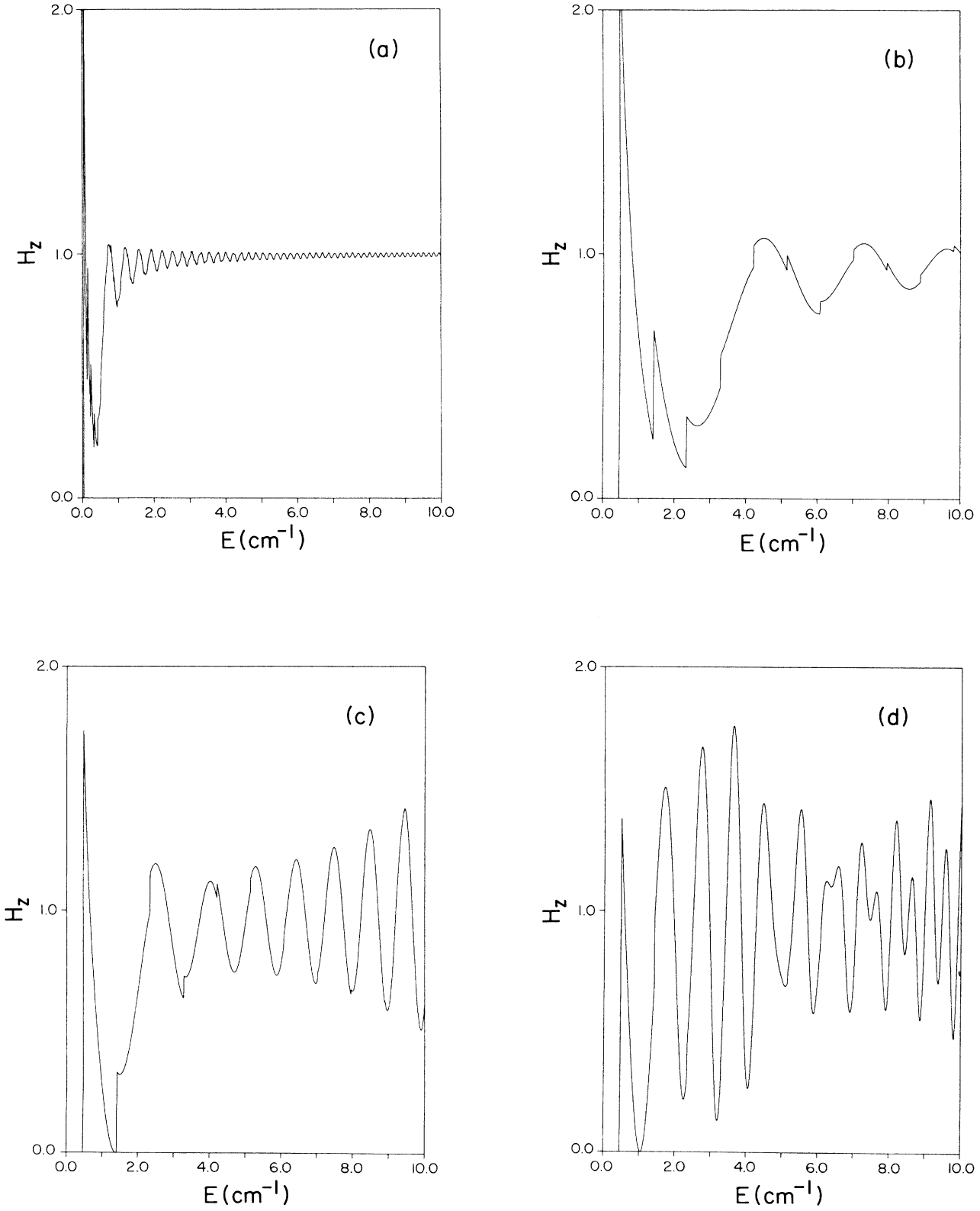

FIG. 1. The factor $H_z(E,F,B)$ defined in Eq. (13b) plotted as a function of E for the electric and magnetic fields listed in Table I. H_z multiplied by the smooth zero-field cross section $\sigma_0(E)$ in Eq. (14) gives the photodetachment cross section of H^- in parallel electric and magnetic fields with photons linearly polarized along the fields. Note H_z fluctuates around a mean value of unity.

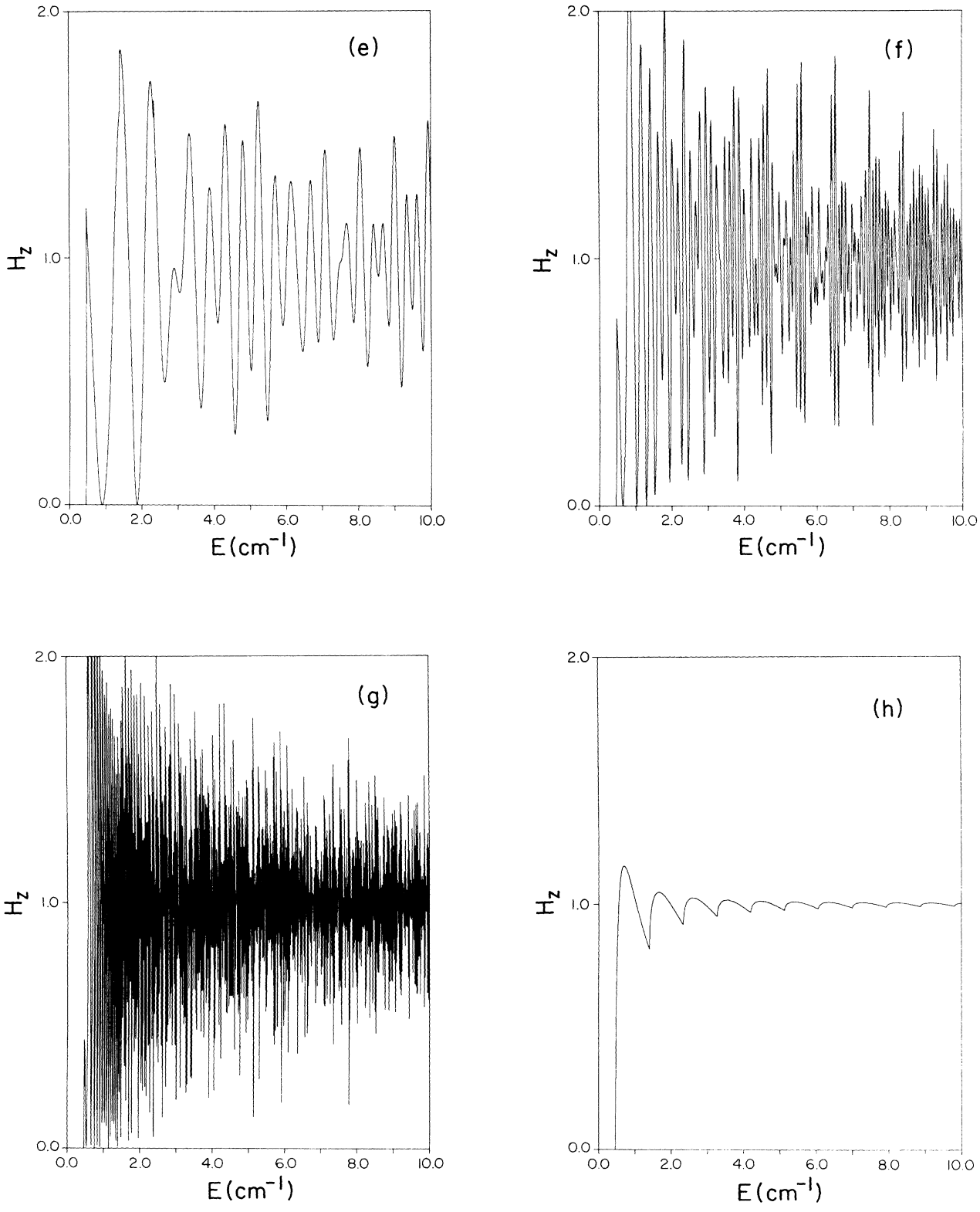

FIG. 1. (*Continued*).

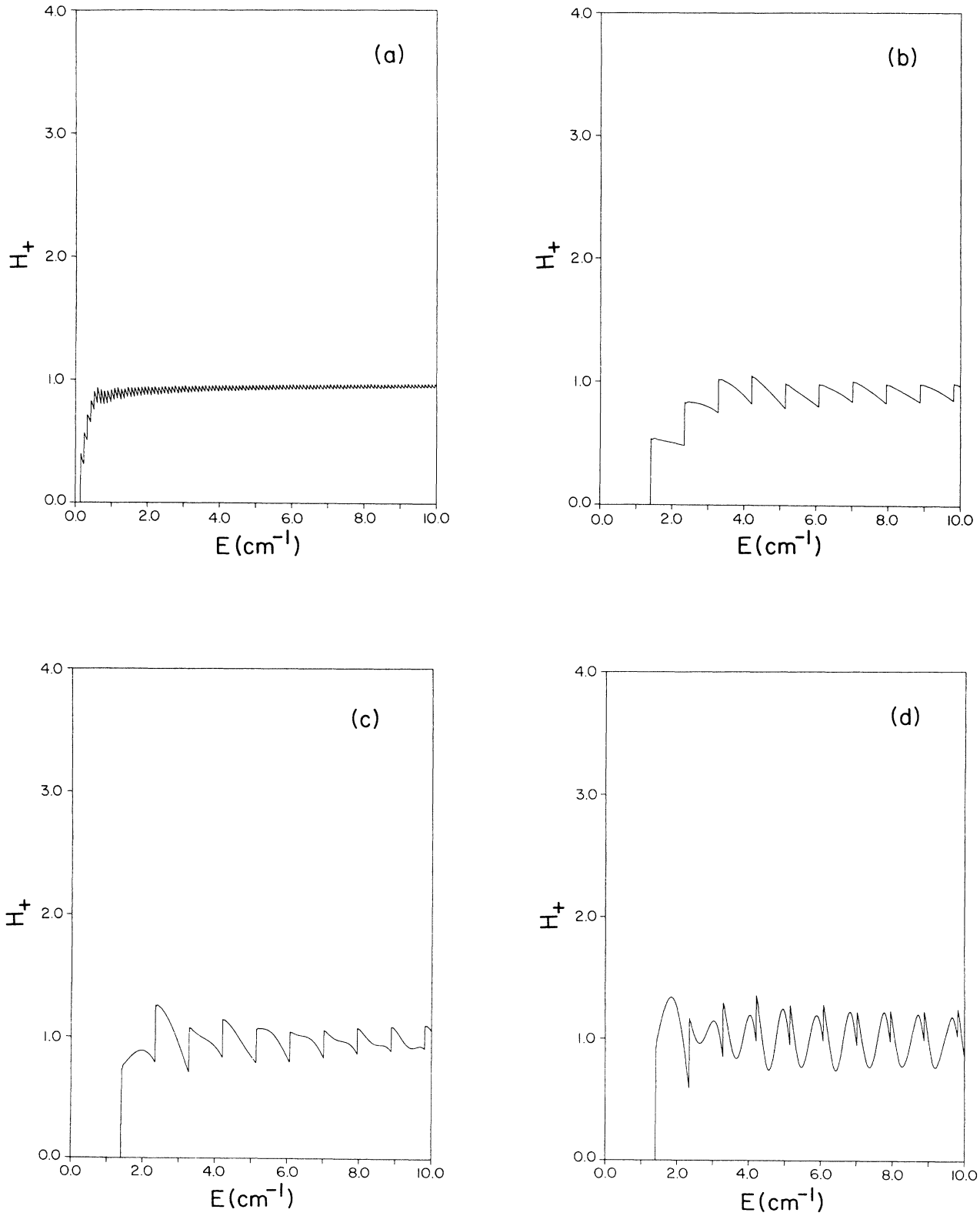

FIG. 2. The factor $H_+(E,F,B)$ defined in Eq. (15b) plotted as a function of E for the electric and magnetic fields listed in Table I. H_+ multiplied by the smooth zero-field cross section $\sigma_0(E)$ in Eq. (15) gives the photodetachment cross section of H^- in parallel electric and magnetic fields with photons circularly polarized. Note H_+ fluctuates around a mean value of unity.

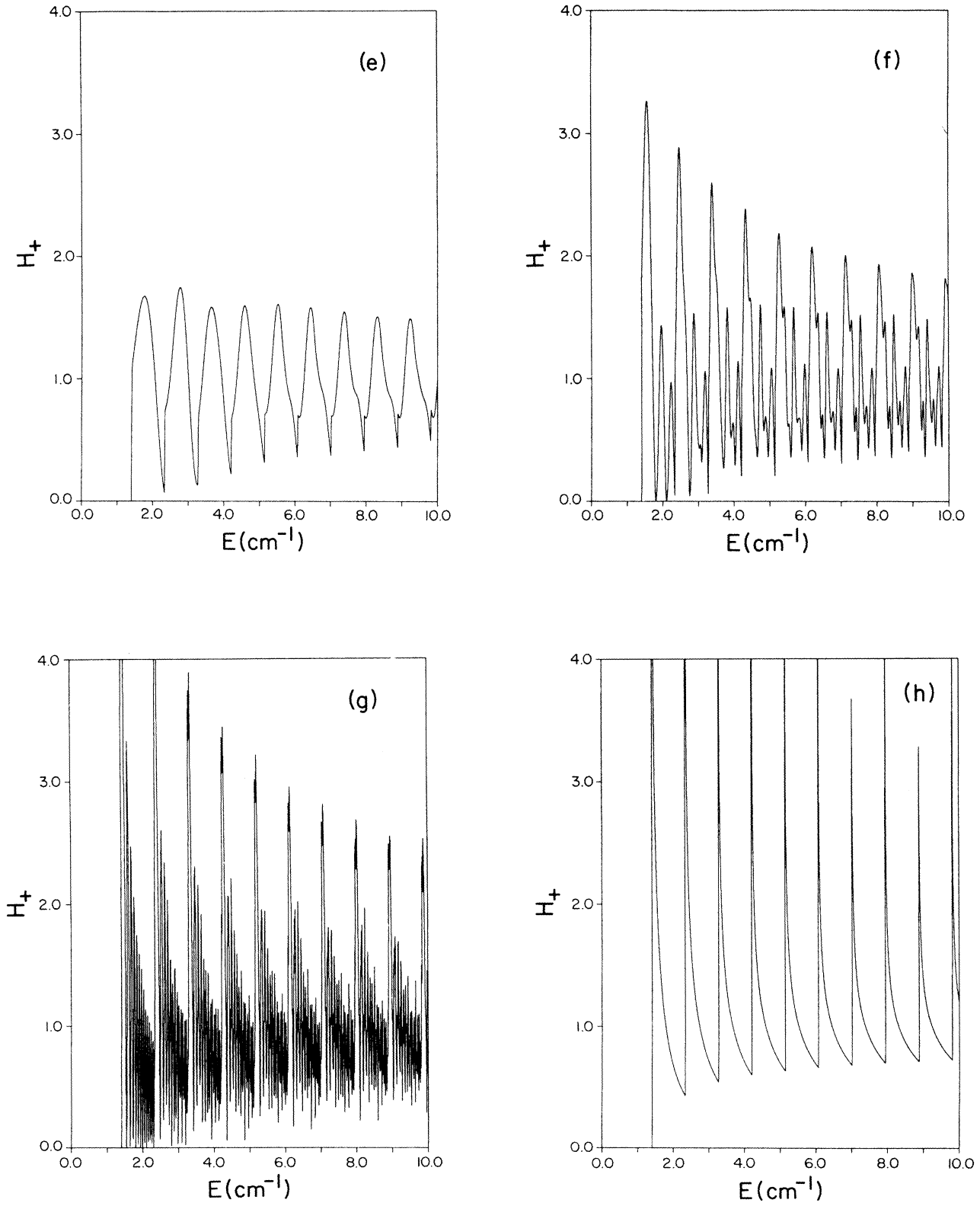

FIG. 2. (*Continued*).

pattern of the spectrum changes as the relative strength of the two fields changes. A convenient quantity which measures the relative strength of the two fields is the ratio of the energy spacing induced separately by the magnetic and electric fields. In a magnetic field, the spacing is the Landau level spacing B/c; in an electric field, the spacing of the oscillation near threshold is approximately $F^{2/3}$. Therefore we define the dimensionless ratio $R \equiv F^{2/3}/(B/c)$ [if F is measured in V/cm and B is measured in gauss, then $R = (8.0\times10^{2})F^{2/3}/B$].

In Figs. 1(a) and 2(a) the electric field is much stronger than the magnetic field. The large oscillations seen in these figures correspond to the previously calculated and observed "ripples" that occur for the electric field only. The weak magnetic field induces a closely spaced small-amplitude fine structure. This fine structure is due to the quantization of the motion of electrons in the direction perpendicular to the magnetic field. The amplitude of these oscillations is in fact proportional to B, as seen from the formulas in (13b) and (15b). The overall pattern of the spectrum when the electric field dominates is similar to the pattern in the presence of only the electric field.

In Figs. 1(b) and 2(b) the electric field is much larger. Again the large structure oscillations in Fig. 1(b) are due to the electric field, and the small sharp structures are the results of Landau levels. Figures 1(c)–1(h) and Figs. 2(c)–2(h) show the effects of decreasing the value of R (decreasing the electric field compared to the magnetic field). The wavelength (on the energy axis) of oscillations associated with the electric field steadily decreases, but the amplitude increases.

In Figs. 1(e) and 2(e) for which R is close to unity, the fields are intermingled and it is not possible to identify separately the electric and magnetic field effects.

In Figs. 1(f), 1(g), 2(f), and 2(g) R is less than unity, and the magnetic field is dominated. The spectrum is characterized by the broad Landau level envelope. As shown in Figs. 1(g) and 2(g) as the electric field approaches zero, the wavelength of the fast oscillation approaches zero like $F^{2/3}$, whereas the amplitude of the fast oscillation remains fixed.

Figures 1(h) and 2(h) show the spectra when the electric field is exactly zero. Comparing these figures with 1(g) and 2(g), for which the electric field is very small but nonzero, we find the striking result that the absorption spectrum is not continuous as F goes to zero. In the weak electric field limit in a magnetic field, because of the quantization in a magnetic field of the energy in the direction perpendicular to the fields, the electron motion is effectively one-dimensional. The electric field affects one-dimensional motion in a quite different way from three-dimensional motion. Because the final wave function $\phi^{F}_{q_n}(z)$ in the neighborhood of the nucleus changes between a symmetric function and an antisymmetric function as the photon energy is varied, and the dipole selection rule permits only a contribution from either the symmetric or antisymmetric part of the wave function of the cross section, the factor $A_{\pm1}$ in Eqs. (13b) and (15b) in the weak electric field limit oscillates violently as a function of energy with a nearly fixed amplitude. In a finite resolution measurement, however, the fast oscillation will be averaged to zero, thereby recovering the spectrum obtained in the presence of a magnetic field alone.

The electric field effect is stronger for linear polarization and the magnetic field effect for circular polarization. This is a consequence of the angular distribution of the outgoing waves.[3] For linear polarization, waves propagate up and down the electric field, and their interference produces large-amplitude oscillations.

IV. CONCLUSIONS

We have presented analytic formulas for the photodetachment cross section of H^- in parallel and magnetic fields. To a good approximation, the cross sections are a product of the cross section $\sigma_0(E)$ of H^- in the absence of any field and another function depending on the field and the polarization of the photon. For linear and circular polarizations the function may be written as a sum over Landau levels.

The resulting spectra are complicated because of the mixing of the two fields. The major features in the spectra can be understood in terms of limiting cases.

ACKNOWLEDGMENTS

I would like to thank Professor A. Dalgarno for his interest in this work and for his help during the preparation of the manuscript. I am also indebted to Professor J. B. Delos for his encouragement and for his helpful suggestions. This work was supported by the Division of Chemical Sciences, Office of Basic Energy Sciences, Office of Energy Research of the U.S. Department of Energy.

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