Theory of electromagnetic absorption in strongly correlated plasmas

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We report a derivation of the frequency-dependent conductivity for high-temperature plasmas that takes into account the collective dynamics as well as the short-range correlations. This treatment rests on the joint solution of the first two members of the Bogoliubov-Born-Green-Kirkwood-Yvon hierarchy when both are able to change on the same time scale. For arbitrary frequencies this leads to an integral equation for the one-particle distribution function that can be solved by conventional variational or numerical procedures. For frequencies that are higher than the collision frequency, an explicit expression for conductivity is obtained. Effects of the short-range correlations are included by introducing an effective interaction. The high-frequency limit is further examined for an infinite ion mass and numerical results are presented.

I. INTRODUCTION

The problem of the absorption of electromagnetic waves in plasma was studied by many authors in the 1960s.[1–5] It has become since then a well-understood problem. Here one solves the response of the electron-ion (hole) system taking into account correctly the effect of the self-consistent field generated via the fluctuations of the charges while considering the electron-ion (hole) correlation effects only within the Born approximation. The leads to a correct result of the absorption in the limit when the plasma parameter r_s approaches zero. This plasma parameter r_s is, in principle, given by the ratio of the average potential energy of, say the electron, to its average kinetic energy. This theory met with considerable success in comparing it to experiments in a wide variety of problems, such as optical absorption in semiconductors, metals and nondegenerate or classical plasmas.

Similarly, much effort was directed to understanding the response of plasmas to longitudinal electric fields. To lowest order in the plasma parameter the dielectric response depends solely on the self-consistent fields. The calculation of the dielectric function is given by the well-known random-phase approximation (RPA). However, the RPA gives the correct result for the dielectric response for finite values of r_s only when small wave numbers (i.e., large distances) are considered. For large wave numbers (i.e., short-range phenomena), one finds the RPA results unacceptable. Here one must take into account the short-range effects of the charged particles due to exchange and correlations which deviate remarkably from the RPA calculations.

The problem of strongly coupled electron plasmas has been attacked by many authors[6–14] through different approaches with a considerable success. The methods can be classified according to whether they rely on the first Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) or on the second BBGKY equation. The former can be characterized as considering the dielectric function as the central object, deriving an expression for it from the first BBGKY equation, and then guaranteeing self-consistency through the use of fluctuation-dissipation-theorem-type relations. Hubbard,[6] Singwi, Tosi, Sjolander and Land[7] (STSL) and Golden, Kalman, and Silevitch[8] followed this approach in different ways. In the second BBGKY equation approach the central object is the equilibrium pair correlation function for which the equation is made self-consistent by the introduction of a decomposition of the triplet correlation function into clusters of pair correlation functions. Ichimaru[11] and Totsuji and Ichimaru[12] have pursued this method. All these different approaches have achieved important results. The equilibrium pair correlation function, equation of state, and condition for phase transition have been calculated[6,12,15,16] and the theory has been refined to the point that the original inconsistencies concerning the satisfaction of the sum-rule requirements can be removed.[8,12,16]

In the light of the important progress made in the field of one-component strongly coupled plasmas, the investigation of the equivalent two-component plasma system is much less developed. Nevertheless, many physical systems cannot be approximated as a one-component system. Laser compressed plasmas,[17] electron-hole liquids,[18] and high-Z stellar interiors[19] are in the category. Golden and Kalman[20] have proposed an approximation scheme for strongly coupled two-component plasmas. Sjolander and Scott[21] have generalized the STSL method to the two-component system and obtained the positron-electron pair correlation function.

Both STSL and Ichimaru's[11] approaches were restricted to collisionless plasmas. A complete classical derivation of the frequency-dependent conductivity for a two-component system including collision between charged particles was due to Oberman, Ron, and Dawson.[3] They used the joint solution of the first two BBGKY equations to express the dynamical conductivity. In their formalism, the triplet correlation function was totally omitted and therefore their result is only valid for weakly coupled plasmas (i.e., for small values of r_s).

In this paper we will focus our attention on the effects of short-range correlations on the conductivity and the absorption properties of a strongly coupled two-component plasma. Our purpose in this paper is to develop a theory and an overall calculation scheme which adequately treats the correlations in the dynamical conductivity. We shall present a calculation of the current response to a weak external field for a strongly correlated plasma based on a joint solution of the first two members of the BBGKY hierarchy. In our second equation of BBGKY hierarchy for two-body correlation functions, we approximate the three-body correlations functions by the products of one- and two-particle correlation function. We introduce an effective interaction term instead of the Coulomb matrix element into this equation. The short-range correlation effect will be included in this effective interaction.

II. GENERAL FORMALISM OF THE PROBLEM

We consider a gas of charged particles interacting only through a Coulomb potential. An arbitrary number of particle types is assumed, with n_s particles of types s (charge e_s, mass m_s). The system is described in general by the Liouville equations, or the hierachy of equations derived from it by integration over the coordinates and momenta of all but one particle, two particles, etc.

Let the plasma be contained in a volume V and let the position and velocity coordinates of the ith particle be given by $(\mathbf{x}_i,\mathbf{v}_i)$. For an ensemble of such plasmas, the density in phase space $D(\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_n,\mathbf{v}_1,\mathbf{v}_2,\ldots,\mathbf{v}_n,t)$, in the presence of an external electrical field $\mathbf{E}(\mathbf{x},t)$, satisfies the Liouville equation

$$\left[\frac{\partial}{\partial t}+\sum_{i=1}^{n}\mathbf{v}_i\cdot\frac{\partial}{\partial \mathbf{x}_i}+\sum_i\frac{e_i}{m_i}\mathbf{E}(\mathbf{x}_i,t)\cdot\frac{\partial}{\partial \mathbf{v}_i}-\sum_{i,j=1}^{n}\frac{e_ie_j}{m_j}\frac{\partial}{\partial \mathbf{x}_i}\cdot\frac{1}{|\mathbf{x}_i-\mathbf{x}_j|}\right]D=0\;. \tag{2.1}$$

Here we only consider the Coulomb forces of interaction between the particles and assume all external fields are zero.

The one-body function is defined as

$$f_l(\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_l,\mathbf{v}_1,\mathbf{v}_2,\ldots,\mathbf{v}_l,t)=V^l\int d\mathbf{x}_{l+1}\cdots d\mathbf{x}_n\,d\mathbf{v}_{l+1}\cdots d\mathbf{v}_n D(\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_n,\mathbf{v}_1,\mathbf{v}_2,\ldots,\mathbf{v}_n,t)\;. \tag{2.2}$$

By taking the moments of the Liouville equation, we generate the BBGKY chain equations,

$$\left[\frac{\partial}{\partial t}+\sum_{i=1}^{l}\mathbf{v}_i\cdot\frac{\partial}{\partial \mathbf{x}_i}+\sum_i\frac{e_i}{m_i}\mathbf{E}(\mathbf{x}_i,t)\cdot\frac{\partial}{\partial \mathbf{v}_i}-\sum_{i,j=1}^{l}\frac{e_ie_j}{m_j}\frac{\partial}{\partial \mathbf{x}_i}\cdot\frac{1}{|\mathbf{x}_i-\mathbf{x}_j|}\right]f_l-\sum_{i=1}^{l}\frac{e_ie_{l+1}}{m_{l+1}}\frac{\partial}{\partial \mathbf{x}_i}\frac{1}{|\mathbf{x}_i-\mathbf{x}_{l+1}|}\cdot\frac{\partial f_{l+1}}{\partial \mathbf{v}_i}=0\;. \tag{2.3}$$

We can rewrite this chain of equations by expression f_l in terms of the Mayer cluster expansion,

$$\begin{aligned}F_l=&\prod_{i=1}^{l}f(\mathbf{x}_i,\mathbf{v}_i)+\sum_{P}\left[\prod_{i=1}^{l}f(\mathbf{x}_i,\mathbf{v}_i)\right]P(\mathbf{x}_j,\mathbf{v}_j,\mathbf{x}_k,\mathbf{v}_k,t)\\&+\sum_{(P,P)}\left[\prod_{i=1}^{l}f(\mathbf{x}_i,\mathbf{v}_i)\right]P(\mathbf{x}_j,\mathbf{v}_j,\mathbf{x}_k,\mathbf{v}_k,t)P(\mathbf{x}_m,\mathbf{v}_m,\mathbf{x}_n,\mathbf{v}_n,t)+\left[\prod_{i=1}^{l}f(\mathbf{x}_i,\mathbf{v}_i)\right]T(\mathbf{x}_j,\mathbf{v}_j,\mathbf{x}_k,\mathbf{v}_k,\mathbf{x}_m,\mathbf{v}_m,t)+\cdots\;,\end{aligned} \tag{2.4}$$

where the second term is summed over pairs, the third term over pairs of pairs and the fourth over triplets, etc. $P(\mathbf{x}_{,}\mathbf{v}_m,\mathbf{x}_n,\mathbf{v}_n,t)$ is called the pair correlation function and $T(\mathbf{x}_j,\mathbf{v}_j,\mathbf{x}_k,\mathbf{v}_k,\mathbf{x}_m,\mathbf{v}_m,t)$ is called the triple correlation function. The quadrupole- and higher-order correlation functions have not been explicitly shown, because they make negligibly small contributions in all our calculations and are dropped at the outset. Using this expression, we may write down the first two equations for a multicomponent system. The equation for the one-body function becomes

$$\begin{aligned}&\left[\frac{\partial}{\partial t}+\mathbf{v}_s\cdot\frac{\partial}{\partial \mathbf{x}_s}+\frac{e_s}{m_s}\mathbf{E}(\mathbf{x},t)\cdot\frac{\partial}{\partial \mathbf{v}_s}-\frac{e_s}{m_s}\sum_s e_{s'}\int d\mathbf{x}_{s'}d\mathbf{v}_{s'}\frac{\partial V^{ss'}}{\partial \mathbf{x}_s}f(s')\cdot\frac{\partial}{\partial \mathbf{v}_s}\right]f(s)\\&\qquad=\frac{e_s}{m_s}\sum_{s'}e_{s'}n_{s'}\int d\mathbf{x}_{s'}d\mathbf{v}_{s'}\frac{\partial}{\partial \mathbf{x}_s}V^{ss'}\cdot\frac{\partial}{\partial \mathbf{v}_s}(s,s')\;,\end{aligned} \tag{2.5}$$

where

$$V^{ss'}=\frac{1}{|\mathbf{x}_s-\mathbf{x}_{s'}|}\;.$$

The equation for the two-body function becomes

$$\left[\frac{\partial}{\partial t}+\boldsymbol{v}_s\cdot\frac{\partial}{\partial \mathbf{x}_s}+\boldsymbol{v}_{s'}\cdot\frac{\partial}{\partial \mathbf{x}_{s'}}-\frac{e_s}{m_s}\sum_{s''}e_{s''}\int d\mathbf{x}_{s''}d\boldsymbol{v}_{s''}\frac{\partial V^{ss''}}{\partial \mathbf{x}_s}f(s'')\cdot\frac{\partial}{\partial \boldsymbol{v}_s}\frac{e_s}{m_s}\mathbf{E}(\mathbf{x},t)\cdot\frac{\partial}{\partial \boldsymbol{v}_s}\right.$$
$$\left.-\frac{e_{s'}}{m_{s'}}\sum_{s''}e_{s''}\int d\mathbf{x}_{s''}d\boldsymbol{v}_{s''}\frac{\partial V^{s's''}}{\partial \mathbf{x}_{s'}}f(s'')\cdot\frac{\partial}{\partial \boldsymbol{v}_{s'}}+\frac{e_{s'}}{m_{s'}}\mathbf{E}(\mathbf{x},t)\cdot\frac{\partial}{\partial \boldsymbol{v}_{s'}}\right]P(s,s')$$
$$-\frac{e_s}{m_s}\frac{\partial f(s)}{\partial \boldsymbol{v}_s}\cdot\sum_{s''}e_{s''}n_{s''}\int d\mathbf{x}_{s''}d\boldsymbol{v}_{s''}\frac{\partial}{\partial \mathbf{x}_s}V^{ss''}P(s',s'')$$
$$-\frac{e_{s'}}{m_{s'}}\frac{\partial f(s')}{\partial \boldsymbol{v}_{s'}}\cdot\sum_{s''}e_{s''}n_{s''}\int d\mathbf{x}_{s''}d\boldsymbol{v}_{s''}\frac{\partial}{\partial \mathbf{x}_{s'}}V^{s's''}P(s,s'')-\frac{e_s}{m_s}\sum_{s''}e_{s'}n_{s''}\int d\mathbf{x}_{s''}d\boldsymbol{v}_{s''}\frac{\partial}{\partial \mathbf{x}_s}V^{ss''}\cdot\frac{\partial T(s,s',s'')}{\partial \boldsymbol{v}_s}$$
$$-\frac{e_{s'}}{m_{s'}}\sum_{s'}e_{s''}n_{s''}\int d\mathbf{x}_{s''}d\boldsymbol{v}_{s''}\frac{\partial}{\partial \mathbf{x}_{s'}}V^{s's''}\cdot\frac{\partial T(s,s',s'')}{\partial \boldsymbol{v}_{s'}}=e_se_{s'}\frac{\partial}{\partial \mathbf{x}_s}V^{ss'}\cdot\left[\frac{1}{m_s}\frac{\partial}{\partial \boldsymbol{v}_s}-\frac{1}{m_{s'}}\frac{\partial}{\partial \boldsymbol{v}_{s'}}\right][f(s)f(s')+P(s,s')]\ , \tag{2.6}$$

where $\mathbf{f(s)}=\mathbf{f}(\mathbf{x}_s,\boldsymbol{v}_s,t)$ and so for P and T. These set of equations was solved by Oberman, Ron, and Dawson[3] in an external field but they completely neglect the effect of $T(1,2,3)$.

III. EFFECTIVE INTERACTION

To obtain the dominant contribution for "good" plasmas (i.e., weakly coupled plasmas), as presented in Ref. 4, one neglects entirely the three-body correlations $T(s,s',s'')$ and also discards the term proportional to $P(s,s')$ in the right-hand side (rhs) of Eq. (2.6). We first review the meaning of this approximation.

The approximation treats the self-consistent field exactly (i.e., to all contributions which are independent of r_s) but considers large-angle scattering [see rhs of Eq. (2.6)] within the Born approximation. This extended RPA treatment which is a first-order correction in the plasma parameter r_s would be called in the work also RPA, following the RPA treatment of the self-consistent field. Calculations of the conductivity, and for that matter other observables, give good agreement with experiments for plasmas having $r_s \ll 1$. However, it was determined that even for plasmas having r_s of order unity, say for metals and semiconductors, the RPA results agree fairly well with experimental observations. This is due to the fact that the dominant part of the Coulomb interaction is the dynamical screening which results from the consistent treatment of the local field. However, it was also noticed that RPA tends to do poorly for situations when the short-range part of the correlation or large momentum transfers are involved. The reason is that RPA completely neglects short-range correlations between the charged particles.

To rectify this deficiency, we develop a theory which takes into account the short-range correlation effects within RPA. That is to say, we include in our theory these correlation terms which keep the structure of the RPA integral equation unchanged. However, in our theory we consider the interaction of the charged particles beyond the random-phase and the Born approximations to include short-range correlations. We therefore will obtain a RPA-like result with renormalized dynamical screening which includes effects of short-range correlations. Our theory also takes into account the short-range correlations in the scattering process between the charged particles. To say it differently, we take into account multiple scattering, i.e., we incorporate the effects of positive and negative correlations on the scattering process.

To consider the collisional plasma together with the effect of short-range correlation, we must solve Eqs. (2.5) and (2.6) simultaneously and include the two-particle short-range correlations effects. Therefore we must retain the effect of $T(1,2,3)$. Let us first analyze the RPA terms in the left-hand side (lhs) of Eq. (2.6), which is an integral equation for $P(s,s')$. A simple pictorial way is to represent the four terms arising from the Coulomb interaction in Figs. 1(a)–1(d). Here a noncorrelated particle is represented by a line and a correlated pair is given by two lines with a shaded area between them. All the diagrams in Fig. 1 represent a single Coulomb scattering of a correlated pair [say, $P(s,s')$] from a third particle (say, s'') (Born approximation). O'Neil and Rostoker[22] and Ichimaru[11] used an ansatz for the three-particle correlation with good success. Their ansatz reads

$$T(s,s',s'')=\frac{P(s,s')P(s,s'')}{f(s)}+\frac{P(s',s'')P(s'',s)}{f(s'')}+\frac{P(s',s'')P(s,s')}{f(s')}\sum_{s_4}n_{s_4}\int d\mathbf{x}_{s_4}\frac{P(s,s_4)P(s',s_4)P(s'',s_4)}{f(s_4)^3}\ . \tag{3.1}$$

Note that their result was given for the equilibrium situation where $P(s,s')=f(s)f(s')g(s,s')$, etc. This ansatz for $T(s,s',s'')$ is given in terms of a sum over all possible pairs of two-particle correlations plus a term including four-particle correlations which is of high order and we shall omit here. To incorporate the contribution from $T(s,s',s'')$, we follow the work of O'Neil and Rostoker.[22] We generalize their result to the time-independent case and write $T(s,s',s'')$ in terms of all possible interacting pairs, in which only one of the pairs in each term is time dependent (i.e., field dependent) and the other one is assumed to be in equilibrium. This is consistent with the linear-response theory

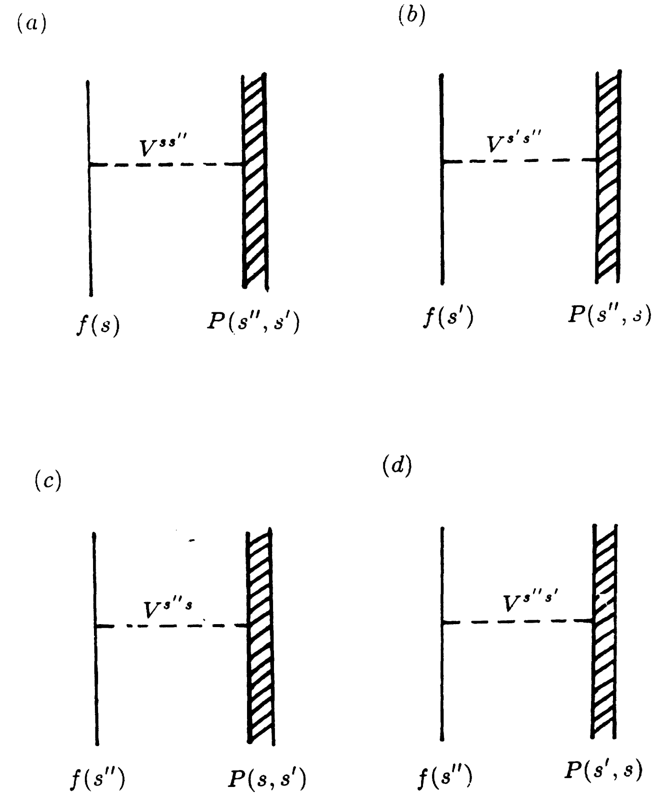

FIG. 1. The class of diagram that contributes to the conductivity in the random-phase approximation, where $f(s)$ is a one-particle propagator, $P(s,s')$ is a two-particle propagator, and $V^{ss'}$ is the two-body Coulomb interaction.

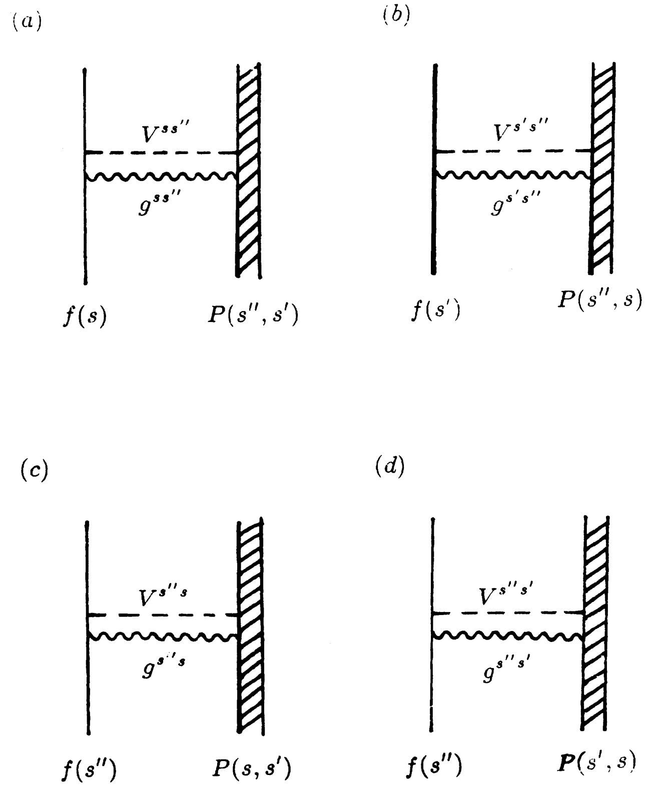

FIG. 2. The class of diagram that contributes to the conductivity in the random-phase approximation, where $f(s)$ is a one-particle propagator, $P(s,s')$ is a two-particle propagator, $g^{ss'}$ is the pair correlation function, and $V^{ss'}$ is the two-body Coulomb interaction.

where only one photon is being absorbed. Thus our ansatz includes six terms which follow from the expression of $T(s,s',s'')$ as given in Ref. 22:

$$T(s,s',s'')=f(s')g(s,s')P(s,s'')+f(s')g(s',s'')P(s'',s)+f(s'')g(s',s'')P(s,s') \\ +f(s'')g(s,s'')P(s,s')+f(s)g(s,s'')P(s'',s')+f(s)g(s',s)P(s'',s')\ , \tag{3.2}$$

where $g(s,s')=g(\mathbf{x}_s-\mathbf{x}'_s)$ is the static pair correlation function (field independent). The contribution of the three-particle correlations to Eq. (2.6) will be given by

$$-\frac{e_s}{m_s}\sum_{s''}e_{s''}n_{''}\int d\mathbf{x}_{s''}d\mathbf{v}_{s''}d\mathbf{v}_{s''}\frac{\partial}{\partial\mathbf{x}_s}V^{ss''}\cdot\frac{\partial}{\partial\mathbf{v}_s} \\ \times[f(s')g(s,s')P(s,s'')+f(s')g(s',s'')P(s'',s)+f(s'')g(s',s')P(s,s') \\ +\underline{f(s'')g(s,s'')P(s,s')}+\underline{f(s)g(s,s'')P(s'',s')}+f(s)g(s',s)P(s',s'')] \\ -\frac{e_{s'}}{m_{s'}}\cdot\sum_{s''}e_{s''}n_{s''}\int d\mathbf{x}_{s''}d\mathbf{v}_{s''}\frac{\partial}{\partial\mathbf{x}_{s'}}V^{s's''}\cdot\frac{\partial}{\partial\mathbf{v}_{s'}} \\ \times[f(s')g(s,s')P(s,s'')+\underline{f(s')g(s',s'')P(s'',s)}+\underline{f(s'')g(s',s'')P(s,s')} \\ +f(s'')g(s,s'')P(s,s')+f(s)g(s,s'')P(s'',s')+f(s)g(s',s)P(s'',s')]\ . \tag{3.3}$$

There are 12 terms altogether. Four of these twelve terms on the rhs of Eq. (3.3) are underlined. These underlined terms do not change the structure of the RPA integral equation. However, the inclusion of these terms leads to a RPA-like integral equation where each Coulomb matrix element $V^{ss'}$ (Born approximation) is replaced by a scattering matrix represented by an effective potential $U^{ss'}$ which is defined as

$$\frac{\partial}{\partial\mathbf{x}}U^{ss'}(\mathbf{x})=[1+g(\mathbf{x}_s-\mathbf{x}_{s'})]\frac{\partial}{\partial\mathbf{x}}V^{ss'}(\mathbf{x}) \tag{3.4}$$

and whose Fourier transformation in momentum space can be written as

$$U_q^{ss'}=V_q^{ss'}\left[1+\int d\mathbf{q}'\frac{\mathbf{q}'\cdot\mathbf{q}'}{q'^2}[S^{ss'}(\mathbf{q}-\mathbf{q}')-1]\right]$$
$$=V_q^{ss}(1+W_q^{ss'})\ , \tag{3.5}$$

where $V_q=4\pi/q^2$ and $S^{ss'}(q)$ is the static structural factor and is related to the pair correlation function through

$$S^{ss'}(q)=\int d\mathbf{x}\,e^{i\mathbf{q}\cdot\mathbf{x}}[g(\mathbf{x}_s-\mathbf{x}_{s'})-1]\ . \tag{3.6}$$

We now illustrate the four underlined terms of $T(s,s',s'')$ in Figs. 2(a)–2(d). It is clear that Fig. 2(a) when added to Fig. 1(a) can be interpreted in a perturbation series as a multiple scattering process.

Our last point of approximation is the inhomogeous part in the rhs of Eq. (2.6) which is the source term for $P(s,s')$. Here, RPA neglects $P(s,s')$ in comparison with $f(s)f(s')$. To be consistent with our approximation we approximate $P(s,s')$ as $f(s)f(s')g(s,s')$. In this approximation we consider the correlations to be time independent, thus we do not allow the electric field to interfere with the scattering process. This is a good approximation when short-range interaction or large momentum transfer is being considered.

We can now write the approximate integral equation which is a RPA-like equation given in terms of the effective interaction $U^{ss'}$ as

$$\Bigg[\frac{\partial}{\partial t}+\boldsymbol{v}_s\cdot\frac{\partial}{\partial\mathbf{x}_s}+\boldsymbol{v}_{s'}\cdot\frac{\partial}{\partial\mathbf{x}_{s'}}-\frac{e_s}{m_s}\sum_{s''}e_{s''}\int d\mathbf{x}_{s''}d\boldsymbol{v}_{s''}\frac{\partial}{\partial\mathbf{x}_s}U^{ss''}f(s'')\cdot\frac{\partial}{\partial\boldsymbol{v}_s}$$
$$-\frac{e_{s'}}{m_{s'}}\sum_{s''}e_{s''}\int d\mathbf{x}_{s''}d\boldsymbol{v}_{s''}\frac{\partial U^{ss''}}{\partial\mathbf{x}_{s'}}f(s'')\cdot\frac{\partial}{\partial\boldsymbol{v}_{s'}}+\frac{e_s}{m_s}\mathbf{E}(\mathbf{x},t)\cdot\frac{\partial}{\partial\boldsymbol{v}_s}+\frac{e_{s'}}{m_{s'}}\mathbf{E}(\mathbf{x},t)\cdot\frac{\partial}{\partial\boldsymbol{v}_{s'}}$$
$$-\frac{e_s}{m_s}\sum_{s''}e_{s''}\int d\mathbf{x}_{s''}d\boldsymbol{v}_{s''}g(\mathbf{x}_s-\mathbf{x}_{s''})\frac{\partial}{\partial\mathbf{x}_s}U^{ss''}f(s'')\cdot\frac{\partial}{\partial\boldsymbol{v}_s}$$
$$-\frac{e_{s'}}{m_{s'}}\sum_{s''}e_{s''}\int d\mathbf{x}_{s''}d\boldsymbol{v}_{s''}g(\mathbf{x}_{s'}-\mathbf{x}_{s''})\frac{\partial}{\partial\mathbf{x}_{s'}}U^{s's''}f(s'')\cdot\frac{\partial}{\partial\boldsymbol{v}_{s'}}\Bigg]P(s,s')$$
$$-\frac{e_s}{m_s}\frac{\partial f(s)}{\partial\boldsymbol{v}_s}\cdot\sum_{s''}e_{s''}n_{s''}\int d\mathbf{x}_{s''}d\boldsymbol{v}_{s''}\frac{\partial}{\partial\mathbf{x}_s}U^{ss''}P(s',s'')-\frac{e_{s'}}{m_{s'}}\frac{\partial f(s')}{\partial\boldsymbol{v}_{s'}}\cdot\sum_{s''}e_{s''}n_{s''}\int d\mathbf{x}_{s''}d\boldsymbol{v}_{s''}\frac{\partial}{\partial\mathbf{x}_{s'}}U^{s's''}P(s,s'')$$
$$=e_se_{s'}\frac{\partial}{\partial\mathbf{x}_s}U^{ss'}\cdot\left[\frac{1}{m_s}\frac{\partial}{\partial\boldsymbol{v}_s}-\frac{1}{m_{s'}}\frac{\partial}{\partial\boldsymbol{v}_{s'}}\right]f(s)f(s')\ . \tag{3.7}$$

We thus find out that Eq. (3.7) behaves like the RPA case. The only difference between Eq. (3.7) and the RPA is that in our case all the Coulomb interaction terms of RPA are replaced by effective interactions. We may point out that our approach is similar to that given by STSL. In STSL the chain was truncated in the first member and the pair correlation function becomes a part of the effective interaction. Here we truncate the chain in the second member and the triple correlation function becomes a part of effective interaction. Our next step is to solve Eq. (3.7) using some perturbation procedure.

IV. EVALUATION OF THE CURRENT

We consider our system to be under the influence of a small spatially uniform electric field $\mathbf{E}(\mathbf{x},t)=\mathbf{E}e^{-i\omega t}$. For small departures from thermal equilibrium and for a spatially homogenous system, the first two members of BBGKY hierarchy can be written, after linearization, as

$$\frac{\partial f_1(s)}{\partial t}-\frac{e_s}{m_s}\sum_{s'}e_{s'}n_{s'}\int d\mathbf{x}_{s'}d\boldsymbol{v}_{s'}\frac{\partial}{\partial\mathbf{x}_s}V^{ss'}\cdot\frac{\partial}{\partial\boldsymbol{v}_s}P_1(s,s')=-\frac{e_s}{m_s}\mathbf{E}\cdot\frac{\partial f_0(s)}{\partial\boldsymbol{v}_s}e^{-i\omega t} \tag{4.1}$$

and

$$\left[\frac{\partial}{\partial t}+(\boldsymbol{v}_s-\boldsymbol{v}_{s'})\cdot\frac{\partial}{\partial\mathbf{x}_s}\right]P_1(s,s')$$
$$-\frac{e_s}{m_s}\frac{\partial f_0(s)}{\partial\boldsymbol{v}_s}\cdot\sum_{s''}e_{s''}n_{s''}\int d\mathbf{x}_{s''}d\boldsymbol{v}_{s''}\frac{\partial}{\partial\mathbf{x}_s}U^{ss''}P_1(s',s'')-\frac{e_s}{m_{s'}}\frac{\partial f_0(s')}{\partial\boldsymbol{v}_{s'}}\cdot\sum_{s''}e_{s''}n_{s''}\int d\mathbf{x}_{s''}d\boldsymbol{v}_{s''}\frac{\partial}{\partial\mathbf{x}_{s'}}U^{s's''}P_1(s,s'')$$
$$=\frac{e_s}{m_s}\frac{\partial f_1(s)}{\partial\boldsymbol{v}_s}\cdot\sum_{s''}e_{s''}n_{s''}\int d\mathbf{x}_{s''}d\boldsymbol{v}_{s''}\frac{\partial}{\partial\mathbf{x}_s}U^{ss''}P_0(s',s'')+\frac{e_{s'}}{m_{s'}}\frac{\partial f_1(s')}{\partial\boldsymbol{v}_{s'}}\cdot\sum_{s''}e_{s''}n_{s''}\int d\mathbf{x}_{s'}d\boldsymbol{v}_{s''}\frac{\partial}{\partial\mathbf{x}_{s'}}U^{s's''}P_0(s,s'')$$
$$+e_se_{s'}\frac{\partial}{\partial\mathbf{x}_s}U^{ss'}\cdot\left[\frac{1}{m_s}\frac{\partial}{\partial\boldsymbol{v}_s}-\frac{1}{m_{s'}}\frac{\partial}{\partial\boldsymbol{v}_{s'}}\right][f_1(s)f_0(s')+f_0(s)f_1(s')]-\mathbf{E}\cdot\left[\frac{e_s}{m_s}\frac{\partial}{\partial\boldsymbol{v}_s}-\frac{e_{s'}}{m_{s'}}\frac{\partial}{\partial\boldsymbol{v}_{s'}}\right]P_0(s,s')e^{-i\omega t}\ , \tag{4.2}$$

where f_1 and P_1 are, respectively, the perturbed one- and two-body distribution function, while f_0 and P_0 represent the equilibrium functions

$$f_0(s)=\left[\frac{m_s}{2\pi T}\right]^{3/2}\exp\left[\frac{-m_s v^2}{2T}\right] \tag{4.3}$$

and

$$P_0(s,s')=f_0(s)f_0(s')g_0(\mathbf{x}_s-\mathbf{x}_{s'}) , \tag{4.4}$$

where T is the absolute temperature in energy units. There is no simple analytical expression for g_0 for strongly correlated plasma. If we define a spatially independent function $f(\omega)$ by

$$f(t)=f(\omega)e^{-i\omega t}$$

and a spatially dependent function $f(q,\omega)$ by

$$f(\mathbf{x},t)=\int d\mathbf{q}\, f(q,\omega)e^{-i\omega t-i\mathbf{q}\cdot\mathbf{x}}$$

then we can write the steady-state forms of Eqs. (4.1) and (4.2)

$$(-i\omega+\delta)f_1(\mathbf{v}_s,\omega)\frac{e_s}{m_s}\sum_{s'}e_{s'}n_{s'}\int d\mathbf{q}\,V_q^{ss}\mathbf{q}\cdot\frac{\partial}{\partial\mathbf{v}_s}G^{ss'}(\mathbf{q},\mathbf{v}_s,\omega)=-\frac{e_s}{m_s}\mathbf{E}\cdot\frac{\partial f_0(s)}{\partial\mathbf{v}_s} \tag{4.5}$$

and

$$\begin{aligned}[-\omega+\delta-i\mathbf{q}\cdot(\mathbf{v}_s-\mathbf{v}_{s'})]P_1^{ss'}(q,\omega)&+\frac{ie_s}{m_s}\mathbf{q}\cdot\frac{\partial f_s}{\partial\mathbf{v}_s}\sum_{s''}e_{s''}n_{s''}U_q^{ss''}G^{s's''}(-\mathbf{q},\mathbf{v}_{s'},\omega)-\frac{ie_{s'}}{m_{s'}}\mathbf{q}\cdot\frac{\partial f_{s'}}{\partial\mathbf{v}_{s'}}\sum_{s''}e_{s''}n_{s''}U_q^{s's''}G^{ss''}(\mathbf{q},\mathbf{v}_s,\omega)\\
&=-\frac{ie_s}{m_s}\mathbf{q}\cdot\frac{\partial f_1(s)}{\partial\mathbf{v}_s}f_{s'}\sum_{s''}e_{s''}n_{s''}U_q^{ss''}g_0^{s's''}(q)+\frac{ie_{s'}}{m_{s'}}\mathbf{q}\cdot\frac{\partial f_1(s')}{\partial\mathbf{v}_{s'}}f_s\sum_{s''}e_{s''}n_{s''}U_q^{s's''}g_0^{ss''}(q)\\
&\quad+iU_q^{ss'}\mathbf{q}\cdot\left[\frac{e_s}{m_s}\frac{\partial}{\partial\mathbf{v}_s}-\frac{e_{s'}}{m_{s'}}\frac{\partial}{\partial\mathbf{v}_{s'}}\right][f_1(s)f_{s'}+f_sf_1(s')]-\mathbf{E}\cdot\left[\frac{e_s}{m_s}\frac{\partial}{\partial\mathbf{v}_s}-\frac{e_{s'}}{m_{s'}}\frac{\partial}{\partial\mathbf{v}_{s'}}\right]f_sf_{s'}g_0^{ss'}(q) ,\end{aligned} \tag{4.6}$$

where $f_s=f_0(s)$. In Eqs. (4.5) and (4.6) we have defined

$$G^{ss'}(\mathbf{q},\omega)=\int d\mathbf{v}'\,P_1(s,s',\mathbf{v},\mathbf{v}',\mathbf{q},\omega) . \tag{4.7}$$

We take δ to be a small positive number, which decides the contour and then is put to zero. We have made explicit use of the fact that for a spatially homogeneous system $P(s,s')$ is only a function of $(\mathbf{x}_s-\mathbf{x}_{s'})$.

It is clear that for the determination of the current density

$$\mathbf{j}(\omega)=\sum_s e_s n_n\int d\mathbf{v}\,\mathbf{v}f_1(s,\mathbf{v},\omega) . \tag{4.8}$$

Making use of Eq. (4.5), the current can be written as

$$\mathbf{j}(\omega)=\mathbf{j}_0(\omega)+\mathbf{j}_1(\omega) , \tag{4.9}$$

where

$$\mathbf{j}_0(\omega)=\frac{\omega_e^2+\omega_i^2}{4\pi\omega}\mathbf{E}=\sigma_0\mathbf{E} . \tag{4.10}$$

Here ω_s is the plasma frequency of the s component. For $j_1(\omega)$ we have

$$\mathbf{j}_1(\omega)=\frac{1}{\omega}\sum_{s,s'}\frac{e_s^2}{m_s}e_{s'}n_{s'}\int d\mathbf{q}\,V_q\int d\mathbf{v}\,\mathbf{v}\mathbf{q}\cdot\frac{\partial}{\partial\mathbf{v}}G^{ss'}(\mathbf{q},\mathbf{v},\omega) . \tag{4.11}$$

Integrating by parts with respect to $\mathbf{v}$, we obtain

$$\mathbf{j}_1(\omega)=\frac{1}{\omega}\sum_{s,s'}\frac{e_s^2}{m_s}e_{s'}n_{s'}\int d\mathbf{q}\,\mathbf{q}V_q\int d\mathbf{v}\,G^{ss'}(\mathbf{q},\mathbf{v},\omega) . \tag{4.12}$$

Or more explicitly (we assign e the charge of the electron and e_i the charge of the ion)

$$\mathbf{j}_1(\omega)=\mathbf{j}_1^e(\omega)+\mathbf{j}_1^i(\omega)$$
$$=\frac{e^2}{\omega m_e}\int d\mathbf{q}\,\mathbf{q}V_q\int d\boldsymbol{v}[en_e G^{ee}(\mathbf{q},\boldsymbol{v},\omega)+e_i n_i G^{ei}(\mathbf{q},\boldsymbol{v},\omega)]$$
$$+\frac{e_i^2}{\omega m_i}\int d\mathbf{q}\,\mathbf{q}V_q\int d\boldsymbol{v}[e_i n_i G^{ii}(\mathbf{q},\boldsymbol{v},\omega)+\mathrm{en_e}\,\mathrm{G}^{\mathrm{ie}}(\mathbf{q},\boldsymbol{v},\omega)]\ . \quad (4.12')$$

Therefore only $G^{ss'}(\mathbf{q},\boldsymbol{v},\omega)$ and $f_1(\omega)$ are of interest. We convert Eq. (4.6) into an integral equation for $G^{ss'}(\mathbf{q},\boldsymbol{v},\omega)$

$$\sum_{s'}\epsilon_{s''s'}(q,\boldsymbol{v},w)G^{ss''}(\mathbf{q},\boldsymbol{v},w)+\sum_{s''}X_{s''s}(q,\boldsymbol{v})\int d\boldsymbol{v}'\,\frac{G^{s's''}(-\mathbf{q},\boldsymbol{v}',w)}{\hat{\mathbf{q}}\cdot\boldsymbol{v}'-(\hat{\mathbf{q}}\cdot\boldsymbol{v}+w)-i\alpha}=Q_{ss'}(\mathbf{q},\boldsymbol{v},w)\ , \quad (4.13)$$

where $\hat{\mathbf{q}}=\mathbf{q}/q$, $w=\omega/q$, $\alpha=\delta/q$. The quantities $\epsilon_{ss'}$, $X_{ss'}$, and $Q_{ss'}$ are defined as

$$\epsilon_{s''s'}(q,\boldsymbol{v},w)=\delta_{s',s''}-\int d\boldsymbol{v}'\frac{X_{s''s'}(q.\boldsymbol{v}')}{\hat{\mathbf{q}}\cdot\boldsymbol{v}'-(\hat{\mathbf{q}}\cdot\boldsymbol{v}+w)-i\alpha} \quad (4.14)$$

and

$$X_{s''s'}(q,\boldsymbol{v}')=\frac{e_{s'}}{m_{s'}}\hat{\mathbf{q}}\cdot\frac{\partial f_{s'}}{\partial\boldsymbol{v}'}e_{s''}n_{s''}U_q^{s's''} \quad (4.15)$$

and

$$Q_{ss'}(\mathbf{q},\boldsymbol{v},w)=\int\frac{d\boldsymbol{v}'}{\hat{\mathbf{q}}\cdot\boldsymbol{v}'-(\hat{\mathbf{q}}\cdot\boldsymbol{v}+w)-i\alpha}$$
$$\times\left[-\frac{\mathbf{E}}{q}\cdot\left[\frac{e_s}{m_s}\frac{\partial}{\partial\boldsymbol{v}}-\frac{e_{s'}}{m_{s'}}\frac{\partial}{\partial\boldsymbol{v}}\right]f_sf_{s'}g_0^{ss'}(q)\right.$$
$$-\frac{ie_s}{m_s}\hat{\mathbf{q}}\cdot\frac{\partial f_1(s)}{\partial\boldsymbol{v}}f_{s'}\sum_{s''}e_{s''}n_{s''}U_q^{ss''}g_0^{ss''}(q)+\frac{ie_{s'}}{m_{s'}}\hat{\mathbf{q}}\cdot\frac{\partial f_1(s')}{\partial\boldsymbol{v}'}f_s\sum_{s''}e_{s''}n_{s''}U_q^{s's'}g_0^{s's''}(q)$$
$$\left.+iU_q^{ss'}\hat{\mathbf{q}}\cdot\left[\frac{e_s}{m_s}\frac{\partial}{\partial\boldsymbol{v}}-\frac{e_{s'}}{m_{s'}}\frac{\partial}{\partial\boldsymbol{v}'}\right][f_1(s)f_{s'}+f_sf_1(s')]\right]. \quad (4.16)$$

Equation (4.13) can be solved in terms of the unknown function f_1 and by substitution of the result in Eq. (4.5) one has a kinetic-type equation for f_1 in the presence of an external field **E** and including the short-range correlation. It was emphasized in Ref. 4 that this kinetic equation is different from the conventional one—a Fokker-Planck-type kinetic equation with an inhomogeneous electric field term $\mathbf{E}\cdot(\partial f_0/\partial\boldsymbol{v})$—due to the fact that Eq. (4.13) has a corresponding inhomogeneous terms too. This twofold appearance of the electric field in our kinetic equation occurs because for high-frequency fields the time scales of the changes of the one-body distribution function and pair correlation function are not distinctly different in Bogoliubov procedure.[23,24] We should emphasize here that our integral equation (4.13) is different from that in Ref. 4. We include the effect of short-range correlations. This effect breaks the symmetry between the Coulomb interactions ($U_q^{ei}\neq U_q^{ee}$). Therefore all coefficients, kernels, and unknown functions in our integral equation are tensors in nature. In Sec. V we shall consider the solution of the usual Hilbert problem[25] in our tensor case.

V. SOLUTION OF INTEGRAL EQUATION

When all quantities in Eq. (4.13) are scalars, the method of solution is due to Guernsey.[26] Oberman *et al.*[4] solved a modified problem in the case with an external electric field. Here we will generalize their method to the case where the solutions are tensors. We start by introducing a function

$$\bar{F}(\mathbf{q},u)=\int d\boldsymbol{v}\,\delta(u-\hat{\mathbf{q}}\cdot\boldsymbol{v})F(\mathbf{q},\boldsymbol{v}) \quad (5.1)$$

in terms of which Eq. (4.13) can be written as

$$\sum_{s''}\epsilon_{s''s'}(q,\boldsymbol{v},w)G^{ss''}(\mathbf{q},\boldsymbol{v},w)+\sum_{s''}X_{s''s}(q,\boldsymbol{v})\int du'\frac{G^{s's''}(-\mathbf{q},u',w)}{u'-(\hat{\mathbf{q}}\cdot\boldsymbol{v}+w)-i\alpha}=Q_{ss'}(\mathbf{q},\boldsymbol{v},w)\ . \quad (5.2)$$

In our present problem, the system consists of electrons and ions. Thus $G^{ss'}$ has four different elements. Equation (5.2) is still difficult to solve due to coupling between different elements. Here, $G^{ss'}$, $X_{ss'}$, and $\epsilon_{ss'}$ are all 2×2 matrices. To make progress, we write G as a 4×1 column, or a vector, then $X_{ss'}$ and $\epsilon_{ss'}$ must be written as 4×4 operators. One can see later this change will reserve the correct form of the original equations for each element, i.e., Eq. (5.2). We may easily show that correct assignments for each new element are the following:

$$G=\begin{bmatrix}G^{ee}\\G^{ei}\\G^{ie}\\G^{ii}\end{bmatrix},\quad Q=\begin{bmatrix}Q_{ee}\\Q_{ei}\\Q_{ie}\\Q_{ii}\end{bmatrix}, \quad (5.3)$$

and

$$D=\begin{bmatrix} e_{ee} & \epsilon_{ie} & 0 & 0 \\ e_{ei} & \epsilon_{ii} & 0 & 0 \\ 0 & 0 & \epsilon_{ee} & \epsilon_{ie} \\ 0 & 0 & \epsilon_{ei} & \epsilon_{ii} \end{bmatrix} \quad (5.4)$$

and

$$X=\begin{bmatrix} X_{ee} & X_{ie} & 0 & 0 \\ 0 & 0 & X_{ee} & X_{ie} \\ X_{ei} & X_{ii} & 0 & 0 \\ 0 & 0 & X_{ei} & X_{ii} \end{bmatrix} . \quad (5.5)$$

Now Eq. (5.2) can be written as

$$D(q,\mathbf{v},w)G(\mathbf{q},\mathbf{v},w)+X(q,\mathbf{v})\int du'\frac{G(-\mathbf{q},u',w)}{u'-(\hat{\mathbf{q}}\cdot\mathbf{v}+w)+i\alpha} =Q(\mathbf{q},\mathbf{v},w) . \quad (5.6)$$

The solution for Eq. (5.6) can be obtained by the standard method of singular integral equation.[27] However, in generalizing the method to operators, special care must be taken in keeping the right orders of each operator. Our result can be written as

$$G(\mathbf{q},\mathbf{v})=D^{-1}(\mathbf{q},\hat{\mathbf{q}}\cdot\mathbf{v})Q(\mathbf{q},\mathbf{v}) - \int du'\frac{1}{2\pi i}\frac{B(u'+w,\mathbf{q},\mathbf{v},u)}{u'-\hat{\mathbf{q}}\cdot\mathbf{v}-i\eta} \times\left[\int dx\frac{Q(\mathbf{q},x)}{x-u'+i\eta} - \int dx\frac{Q(-\mathbf{q},-x-w)}{x-u'-i\eta}\right] , \quad (5.7)$$

where D^{-1} is the inverse of the dielectric tensor D. The matrix element of D^{-1} can be obtained as

$$D^{-1}=\begin{bmatrix} \epsilon_{ee}^{-1} & \epsilon_{ie}^{-1} & 0 & 0 \\ \epsilon_{ei}^{-1} & \epsilon_{ii}^{-1} & 0 & 0 \\ 0 & 0 & \epsilon_{ee}^{-1} & \epsilon_{ie}^{-1} \\ 0 & 0 & \epsilon_{ei}^{-1} & \epsilon_{ii}^{-1} \end{bmatrix} , \quad (5.8)$$

where

$$\epsilon_{ee}^{-1}=\frac{\epsilon_{ii}}{\tilde{\epsilon}},\quad \epsilon_{ei}^{-1}=\frac{-\epsilon_{ie}}{\tilde{\epsilon}},\quad \epsilon_{ie}^{-1}=\frac{-\epsilon_{ei}}{\tilde{\epsilon}},\quad \epsilon_{ii}^{-1}=\frac{\epsilon_{ee}}{\tilde{\epsilon}} . \quad (5.9)$$

Here $\tilde{\epsilon}$ is defined as

$$\tilde{\epsilon}=\epsilon_{ee}\epsilon_{ii}-\epsilon_{ei}\epsilon_{ie} . \quad (5.10)$$

It should be noted that $\tilde{\epsilon}$ is not the dielectric function, but the modes of collective excitation are given by the zeros of $\tilde{\epsilon}$. The matrix

$$B=D^{-1}(u'+w)X(\mathbf{q},\mathbf{v})[D^*(u')]^{-1}$$

can be written as

$$B=\begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} , \quad (5.11)$$

where each element can be easily worked out through simple multiplication of matrices, for example, the second column can be written as (in our latter evaluation of current, only the second and third columns are needed)

$$\begin{aligned} b_{12}&=X_{ee}(\mathbf{q},\mathbf{v})\epsilon_{ee}^{-1}(u'+w)[\epsilon_{ie}^*(u')]^{-1} +X_{ie}(\mathbf{q},\mathbf{v})\epsilon_{ee}^{-1}(u'+w)[\epsilon_{ii}^*(u')]^{-1} ,\\ b_{22}&=X_{ee}(\mathbf{q},\mathbf{v})\epsilon_{ei}^{-1}(u'+w)[\epsilon_{ie}^*(u')]^{-1} +X_{ie}(\mathbf{q},\mathbf{v})\epsilon_{ei}^{-1}(u'+w)[\epsilon_{ii}^*(u')]^{-1} ,\\ b_{32}&=X_{ei}(\mathbf{q},\mathbf{v})\epsilon_{ee}^{-1}(u'+w)[\epsilon_{ie}^*(u')]^{-1} +X_{i}(\mathbf{q},\mathbf{v})\epsilon_{ee}^{-1}(u'+w)[\epsilon_{ii}^*(u')]^{-1} ,\\ b_{42}&=X_{ei}(\mathbf{q},\mathbf{v})\epsilon_{ei}^{-1}(u'+w)[\epsilon_{ie}^*(u')]^{-1} +X_{ie}(\mathbf{q},\mathbf{v})\epsilon_{ei}^{-1}(u'+w)[\epsilon_{ii}^*(u')]^{-1} , \end{aligned} \quad (5.12)$$

and similarly the third column can be written as

$$\begin{aligned} b_{13}&=X_{ee}(\mathbf{q},\mathbf{v})\epsilon_{ie}^{-1}(u'-w)[\epsilon_{ee}^*(u')]^{-1} +X_{ie}(\mathbf{q},\mathbf{v})\epsilon_{ie}^{-1}(u'-w)[\epsilon_{ei}^*(u')]^{-1} ,\\ b_{23}&=X_{ee}(\mathbf{q},\mathbf{v})\epsilon_{ii}^{-1}(u'-w)[\epsilon_{ee}^*(u')]^{-1} +X_{ie}(\mathbf{q},\mathbf{v})\epsilon_{ii}^{-1}(u'-w)[\epsilon_{ei}^*(u')]^{-1} ,\\ b_{33}&=X_{ei}(\mathbf{q},\mathbf{v})\epsilon_{ie}^{-1}(u'-w)[\epsilon_{ee}^*(u')]^{-1} +X_{ii}(\mathbf{q},\mathbf{v})\epsilon_{ie}^{-1}(u'-w)[\epsilon_{ei}^*(u')]^{-1} ,\\ b_{43}&=X_{ei}(\mathbf{q},\mathbf{v})\epsilon_{ii}^{-1}(u'-w)[\epsilon_{ee}^*(u')]^{-1} +X_{ie}(\mathbf{q},\mathbf{v})\epsilon_{ii}^{-1}(u'-w)[\epsilon_{ei}^*(u')]^{-1} . \end{aligned} \quad (5.12a)$$

Now that we have solved G in terms of f_1 and $\mathbf{E}$, Eq. (4.5) is a linear integral equation for f_1 valid for arbitrary frequency ω. In the general case this equation is of a similar nature to those usually encountered in transport theory and can be solved, in principle, by conventional techniques.

VI. HIGH-FREQUENCY LIMIT

An explicit representation of the conductivity is possible if one is interested in the frequency regime $\omega\tau\gg 1$ where τ is the cumulative 90° deflection time.[22] It is clear from a study of the time behavior of the terms in Eq. (4.5) that in the low-frequency limit, $\omega\tau\ll 1$, the first term on the left-hand side is of order $\omega\tau$ compared to the other terms, while in the high-frequency limit, $\omega\tau \gg 1$, the second term on the left-hand side is of order $(\omega\tau)^{-1}$ compared to the other terms. Thus, to lowest order the first (second) term can be neglected in the low (high-) frequency case. Here we only consider the high-frequency situation. In this limit, to zero order in $(\omega\tau)^{-1}$, f_1 is simply given by

$$f_1(s,\boldsymbol{\nu},\omega) = -\frac{i}{\omega+i\delta}\frac{e_s}{m_s}\mathbf{E}\cdot\frac{\partial f_s}{\partial \boldsymbol{\nu}} = -\frac{i}{\omega+i\delta}\frac{e_s}{T}f_s\mathbf{E}\cdot\boldsymbol{\nu} . \tag{6.1}$$

In this order of approximation there is no resistivity and one must proceed to next order in $(\omega\tau)^{-1}$. To this end, the first iteration is obtained by substituting Eq. (6.1) into Eq. (4.16). We obtain for Q_{ss}

$$\begin{aligned} Q_{ss}(\mathbf{q},\boldsymbol{\nu},w) = \int d\boldsymbol{\nu}' \frac{1}{\hat{\mathbf{q}}\cdot\boldsymbol{\nu} - (\hat{\mathbf{q}}\cdot\boldsymbol{\nu}+w) - i\alpha} \\ \times \Bigg[-\frac{\mathbf{E}}{q}\cdot\left[\frac{e_s}{m_s}\frac{\partial}{\partial\boldsymbol{\nu}} - \frac{e_{s'}}{m_{s'}}\frac{\partial}{\partial\boldsymbol{\nu}}\right] f_s f_{s'} g_0^{ss'}(q) \\ -\frac{e^2}{T}\frac{\mathbf{E}\cdot\hat{\mathbf{q}}}{\omega+i\delta}\sum_{s''} e_{s''}n_{s''}\left[\frac{U_q^{ss''}g_0^{s's''}(q)}{m_{s'}} - \frac{U_q^{s's''}g_0^{ss''}(q)}{m_{s''}}\right] f_s(\nu)f_{s'}(\nu') \\ -\frac{U_q^{ss'}}{T}\hat{\mathbf{q}}\cdot\left[\frac{e_s}{m_s}\frac{\partial}{\partial\boldsymbol{\nu}} - \frac{e_{s'}}{m_{s'}}\frac{\partial}{\partial\boldsymbol{\nu}'}\right](e_{s'}\mathbf{E}\cdot\boldsymbol{\nu} + e_{s'}\mathbf{E}\cdot\boldsymbol{\nu})f_s(\nu)f_{s'}(\nu')\Bigg] . \end{aligned} \tag{6.2}$$

We see immediately that the elements of $Q_{ss'}$ having nonzero contribution to the absorptive process are those with $s \neq s'$, i.e., Q_{ei} and Q_{ie}. Now we use this approximated Q in Eq. (5.7), then substitute it in Eq. (4.12), and we can obtain the high-frequency expression for the absorptive part of the current

$$\begin{aligned} \mathbf{j}_1^e(\omega) = \frac{e^4}{m_e\omega}\int d\mathbf{q}\,\mathbf{q}V_q \int d\boldsymbol{\nu}\Bigg[[en_e\epsilon_{ei}^{-1}(\mathbf{q},\hat{\mathbf{q}}\cdot\boldsymbol{\nu}+\omega) + e_i n_i \epsilon_{ii}^{-1}(\mathbf{q},\hat{\mathbf{q}}\cdot\boldsymbol{\nu}+\omega)]Q_{ei}(\mathbf{q},\boldsymbol{\nu}) \\ - \int du' \frac{1}{2\pi i}\frac{en_e b_{12} + e_i n_i b_{22}}{u' - \hat{\mathbf{q}}\cdot\boldsymbol{\nu} - i\delta}\left[\int dx \frac{Q_{ei}(\mathbf{q},x)}{x-u'+i\eta} - \int dx \frac{Q_{ei}(-\mathbf{q},-x-w)}{x-u'-i\eta}\right] \\ - \int du' \frac{1}{2\pi i}\frac{en_e b_{13} + e_i n_i b_{23}}{u' - \hat{\mathbf{q}}\cdot\boldsymbol{\nu} - i\delta}\left[\int dx \frac{Q_{ie}(\mathbf{q},x)}{x-u'+i\eta} - \int dx \frac{Q_{ie}(-\mathbf{q},-x-w)}{x-u'-i\eta}\right]\Bigg] \end{aligned} \tag{6.3a}$$

and

$$\begin{aligned} \mathbf{j}_1^i(\omega) = \frac{e^4}{m_i\omega}\int d\mathbf{q}\,\mathbf{q}V_q \int d\boldsymbol{\nu}\Bigg[[en_i\epsilon_{ie}^{-1}(\mathbf{q},\hat{\mathbf{q}}\cdot\boldsymbol{\nu}+\omega) + e_e n_e \epsilon_{ee}^{-1}(\mathbf{q},\hat{\mathbf{q}}\cdot\boldsymbol{\nu}+\omega)]Q_{ie}(\mathbf{q},\boldsymbol{\nu}) \\ - \int du' \frac{1}{2\pi i}\frac{en_i b_{32} + e_e n_e b_{42}}{u' - \hat{\mathbf{q}}\cdot\boldsymbol{\nu} - i\delta}\left[\int dx \frac{Q_{ie}(\mathbf{q},x)}{x-u'+i\eta} - \int dx \frac{Q_{ie}(-\mathbf{q},-x-w)}{x-u'-i\eta}\right] \\ - \int du' \frac{1}{2\pi i}\frac{en_i b_{33} + e_i n_i b_{43}}{u' - \hat{\mathbf{q}}\cdot\boldsymbol{\nu} - i\delta}\left[\int dx \frac{Q_{ei}(\mathbf{q},x)}{x-u'+i\eta} - \int dx \frac{Q_{ei}(-\mathbf{q},-x-w)}{x-u'-i\eta}\right]\Bigg] . \end{aligned} \tag{6.3b}$$

From Eqs. (6.3a) and (6.3b) we find that the only absorptive process is due to the electron-ion collision. Equations (6.3a) and (6.3b) are our general result for the current at high frequencies. This expression is exact in the lowest order of plasma parameter and including properly the short-range correlation effect. In the rest of this paper we will rewrite this result more explicitly in terms of the density fluctuations of the electrons and mobile ions. Then we shall examine our result in the limiting case in which the mass of the ions becomes much heavier compared to the mass of the electrons.

The quantity $en_e\epsilon_{ei}^{-1}(\mathbf{q},\hat{\mathbf{q}}\cdot\boldsymbol{\nu}+\omega) + e_i n_i\epsilon_{ii}^{-1}(\mathbf{q},\hat{\mathbf{q}}\cdot\boldsymbol{\nu}+\omega)$ can be rewritten, after making use of Eqs. (4.14), (4.15), and (5.9), as

$$[en_e\epsilon_{ie}(\mathbf{q},\hat{\mathbf{q}}\cdot\boldsymbol{\nu}+\omega) + e_i n_i \epsilon_{ee}(\mathbf{q},\hat{\mathbf{q}}\cdot\boldsymbol{\nu}+\omega)]\tilde{\epsilon}^{-1}(\mathbf{q},\hat{\mathbf{q}}\cdot\boldsymbol{\nu}+\omega) = \frac{e_i n_i}{\tilde{\epsilon}(\mathbf{q},\hat{\mathbf{q}}\cdot\boldsymbol{\nu}+\omega)}[1 + (W_q^{ei} - W_q^{ee})\alpha_e(\mathbf{q},\hat{\mathbf{q}}\cdot\boldsymbol{\nu}+\omega)] , \tag{6.4}$$

where Eq. (3.2) has been used and $\alpha_e(\mathbf{q},w)$ is defined as

$$\alpha_e(\mathbf{q},w) = \frac{n_e e^2}{m_e}V_q \int d\boldsymbol{\nu}\frac{\mathbf{q}\cdot\partial f_e/\partial\boldsymbol{\nu}}{u - \mathbf{q}\cdot\boldsymbol{\nu} - i\delta} = V_q\Pi^e(q,u) \tag{6.5}$$

and we use $\alpha_i(\mathbf{q},w)$ for the corresponding expression for the ions.

If we introduce the notation

$$f^{\pm}(u)=\int du'\frac{f(u')}{u'-u\mp i\delta} \tag{6.6}$$

and use the fact that $f_s(u)$ is the one-dimensional Maxwellian distribution, we obtain, after considerable algebraic manipulation,

$$\int d\mathbf{v}[en_e\epsilon_{ei}^{-1}(\mathbf{q},\hat{\mathbf{q}}\cdot\mathbf{v}+\omega)+e_in_i\epsilon_{ii}^{-1}(\mathbf{q},\hat{\mathbf{q}}\cdot\mathbf{v}+\omega)]Q_{ei}(\mathbf{q},\mathbf{v})$$
$$=\frac{e_in_i\mathbf{E}\cdot\mathbf{q}}{\omega T}\int du\frac{f_e(u)f_i^+(u+w)}{\tilde{\epsilon}(\mathbf{q},\hat{\mathbf{q}}\cdot\mathbf{v}+\omega)}[1+(W_q^{ei}-W_q^{ee})\alpha_e(\mathbf{q}\cdot\hat{\mathbf{q}}\cdot\mathbf{v}+\omega)]$$
$$\times\left\{\frac{e}{m_e}[U_q^{ei}(1-n_ig_0^{ii})-n_eg_0^{ie}U_q^{ee}]-\frac{e_i}{m_i}[U_q^{ie}(1-n_eg_0^{ee})-n_ig_0^{ei}U_q^{ii}]\right\}. \tag{6.7}$$

Now let us consider the second term in Eq. (6.3). We first regroup the quantity

$$en_eb_{12}+e_in_ib_{22}=X^{ee}(\mathbf{q},\mathbf{v})[\epsilon_{ee}^{-1}(u'+w)+\epsilon_{ie}^{-1}(u'+w)][\epsilon_{ei}^*(u')]^{-1}+X^{ie}(\mathbf{q},\mathbf{v})[\epsilon_{ee}^{-1}(u'+w)+\epsilon_{ie}^{-1}(u'+w)][\epsilon_{ii}^*(u')]^{-1}$$
$$=\{[\epsilon_{ei}^*(u')]^{-1}X^{ee}(\mathbf{q},\mathbf{v})+[\epsilon_{ii}^*(u')]^{-1}(\mathbf{q},\mathbf{v})\}[\epsilon_{ee}^{-1}(u'+w)+\epsilon_{ie}^{-1}+w)]$$
$$=\frac{1+(W^{ei}-W^{ee})\alpha_e(\mathbf{q},\mathbf{v})}{\tilde{\epsilon}(u'+w)}\{[\epsilon_{ei}^*(u')]^{-1}X^{ee}(\mathbf{q},\mathbf{v})+[\epsilon_{ii}^*(u')]^{-1}X^{ie}(\mathbf{q},\mathbf{v})\}$$
$$=\frac{1+(W^{ei}-W^{ee})\alpha_e(\mathbf{q},\mathbf{v})}{\tilde{\epsilon}(u'+w)\tilde{\epsilon}^*(u')}X^{ie}(\mathbf{q},\mathbf{v})\ . \tag{6.8}$$

Then by making the repeated use of the Poincaré-Bertrand formula (P stands for principal value)

$$\mathrm{P}\int du'\frac{1}{u'-u}\mathrm{P}\int du''\frac{1}{u''-u'}[F(u',u'')+F(u'',u')]=\mathrm{P}\int du'\frac{1}{u'-i}\mathrm{P}\int du''\frac{1}{u''-u}F(u'',u)-\pi^2F(u,u) \tag{6.9}$$

we may obtain the following expression:

$$\int dx\frac{Q_{ei}(\mathbf{q},x)}{x-u'+i\eta}-\int dx\frac{Q_{ei}(-\mathbf{q},-x-w)}{x-u'-i\eta}=\left[\frac{e}{m}-\frac{e_i}{m_i}\right]\{[U_q^{ei}(1-n_ig_0^{ii})-n_eg_0^{ie}U_q^{ee}]-[U_q^{ie}(1-n_eg_0^{ee})-n_ig_0^{ei}U_q^{ii}]\}$$
$$\times[f_e^-(u')f_i^+(u'+w)+f_e^+(u'+w)f_i^-(u')]\ . \tag{6.10}$$

Now our result for complex conductivity at high frequencies $\sigma_1(\omega)$ can be written as

$$\sigma_1(\omega)=\frac{e^5n_e^2}{6\pi^2T\omega^2}\left[\frac{1}{m}-\frac{1}{m_i}\right]$$
$$\times\int dq\,q^4V_qY_{ei}(q)$$
$$\times\int du\left\{\frac{f_e(u)f_i^+(u+w)}{\tilde{\epsilon}(u+w)}[1+(W_q^{ei}-W_q^{ee})\alpha_e(u+w)]+\frac{f_i(u)f_e^+(u+w)}{\tilde{\epsilon}(u+w)}[1+(W_q^{ie}-W_q^{ii})\alpha_i(u+w)]\right.$$
$$-\frac{1}{2\pi i}\int du'\frac{1}{u'-u-i\delta}\frac{[f_e^-(u')f_i^+(u'+w)+f_e^+(u'+w)f_i^-(u')]}{\tilde{\epsilon}(u'+w)\tilde{\epsilon}^*(u')}$$
$$\left.\times\{1-(W_q^{ei}-W_q^{ee})\alpha_e(u'+w)]\Lambda^{ei}(\mathbf{q},u,u')-[1+(W_q^{ie}-W_q^{ii})\alpha_i(u'+w)]\Lambda^{ie}(\mathbf{q},\mathrm{u},\mathrm{u}')\}\right\}, \tag{6.11}$$

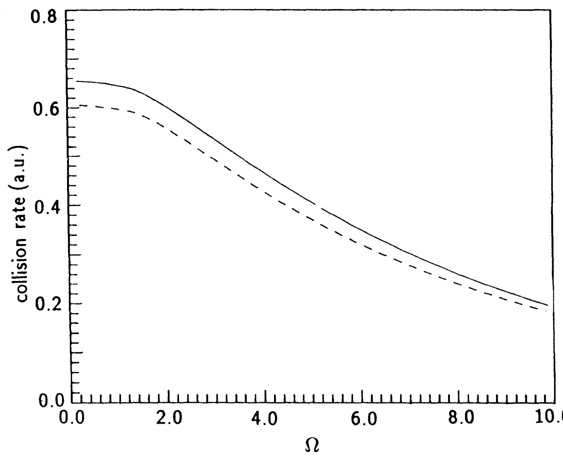

FIG. 3. Calculated collision rate vs the normalized frequency $\Omega=\omega/\omega_p$ for an electron–heavy ion system with $k_BT/E_F=1.0$ and $(e^2/k_BT)(4\pi n/3)^{1/3}=0.1$. The dashed line is calculated within RPA and the solid line is calculated with short-range correlations.

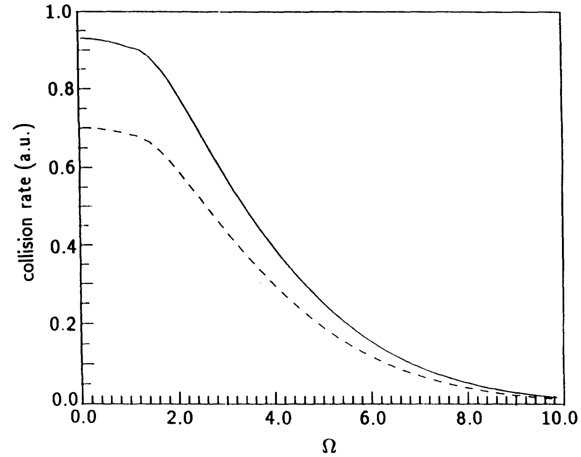

FIG. 4. Calculated collision rate vs the normalized frequency $\Omega=\omega/\omega_p$ for an electron–heavy ion system with $k_BT/E_F=0.1$ and $(e^2/k_BT)(4\pi n/3)^{1/3}=1.0$. The dashed line is calculated within RPA and the solid line is calculated with short-range correlations.

where

$$Y_{ei}(q)=\frac{e}{m_e}[U_q^{ei}(1-n_ig_0^{ii})-n_ig_0^{ie}U_q^{ee}] -\frac{e_i}{m_i}[U_q^{ie}(1-n_eg_0^{ee})-n_ig_0^{ei}U_q^{ii}] \quad (6.12)$$

and

$$\Lambda^{ei}(\mathbf{q},u,u')=X_{ee}(\mathbf{q},u)+X_{ie}(\mathbf{q},u) + \left[U_q^{ee}-\frac{U_q^{ei}U_q^{ie}}{U_q^{ii}}\right]\Pi_e(q,u')X_{ii}(q,u) \quad (6.13)$$

and $\Lambda_{ie}(\mathbf{q},u,u')$ can be obtained by interchange $e\leftrightarrow i$ in Eq. (6.13). Equation (6.11) represents our general expression for the conductivity in the high-frequency limit. The absorption properties of the plasmas can be best described by the resistivity which can be obtained from our result of complex conductivity by taking the real part of the inverse of the conductivity, i.e.,

$$R(\omega)=\mathrm{Re}\left[\frac{1}{\sigma(\omega)}\right]. \quad (6.14)$$

It is manifestly symmetric in the electrons and the mobile ions.

From our result Eq. (6.11), if we use the RPA result for screened interaction ($W_q^{ss'}=0$) and for $g_0^{ss'}$ which is given as

$$n_{s'}g_{\mathrm{RPA}}^{ss'}=\frac{q_s^2}{q^2+q_s^2}=\frac{1}{\epsilon_{\mathrm{RPA}}(q,0)}-1, \quad (6.15)$$

where $q_s=(4\pi n_s e^2/T)^{1/2}$ is the Debye wave number, we immediately recover the previous result obtained by Oberman, Ron, and Dawson[3] which reads

$$\sigma_1(\omega)=\frac{e^6n_e^2}{6\pi^2T\omega^2}\left[\frac{1}{m}-\frac{1}{m_i}\right]^2 \times\int dq\,q^4\frac{V_q^2}{\epsilon(q,0)} \times\int du\left[\frac{f_e(u)f_i^+(u+w)}{\tilde\epsilon(u+w)}+\frac{f_i(u)f_e^+(u+w)}{\tilde\epsilon(u+w)} -\frac{1}{2\pi i}\int\frac{du'}{u'-u-i\delta}\frac{[f_e^-(u')f_i^+(u'+w)+f_e^+(u'+w)f_i^-(u')]}{\tilde\epsilon(u'+w)\tilde\epsilon^*(u')}[X_{ee}(\mathbf{q},u)+X_{ii}(\mathbf{q},\mathbf{u})]\right]. \quad (6.16)$$

Our result is a generalization of theirs to the case where mobile carriers are strongly correlated through Coulomb interaction. This result can be evaluated numerically for a specific problem for any temperature and arbitrary masses and densities of the two components. We may note here that to evaluate Eq. (6.11) one must first solve the integral equation for effective interaction Eqs. (3.6) and (3.3) together with the fluctuation-dissipation theorem

$$S^{ss'}(q)=\frac{1}{V_q}\int d\omega \coth(\beta\omega/2)\,\mathrm{Im}[\epsilon_{ss'}^{-1}(q,\omega)] \,, \tag{6.17}$$

where $\beta=T^{-1}$. In the classical limit $T\gg\omega$, this integral becomes an algebraic relation through the dispersion relation

$$S^{ss'}(q)=\frac{T}{V_q}[\,\mathrm{Re}\epsilon_{ss'}^{-1}(q,0)-\delta_{ss'}] \,. \tag{6.18}$$

We shall use these relations in Sec. VII for a specific problem in which the ions have much heavier mass than the electrons.

VII. COLLISION OF THE ELECTRONS WITH HEAVY IONS

As mentioned above Eq. (6.11) can only be evaluated numerically. But it is useful and desirable to develop the asymptotic expressions when the electron to ion mass ratio is small. It is clear that in this limit the direct contribution of the ions to the current is negligible. Equation (6.11) may be written after we omit the terms proportional to the inverse of the mass of the ions:

$$\sigma_1(\omega)=\frac{e^6 n_e^2}{6\pi^2 T\omega^2 m_e^2}\int dq\, q^4 V_q[U_q^{ei}(1-n_i g_0^{ii})-n_e g_0^{ie} U_q^{ee}]$$
$$\times\int du\left\{\frac{f_e(u)f_i^{+}(u+w)-f_i(u)f_e^{-}(u+w)}{\tilde{\epsilon}(u+w)}[1+(W_q^{ei}-W_q^{ee})\alpha_e(u+w)]\right.$$
$$-\frac{X^{ee}(q,u)}{2\pi i}\int\frac{du'}{u'-u-i\delta}\,\frac{[f_e^{-}(u')f_i^{+}(u'+w)+f_e^{+}(u'+w)f_i^{-}(u')]}{\tilde{\epsilon}(u'+w)\tilde{\epsilon}^*(u')}$$
$$\left.\times\{[1+(W_q^{ei}-W_q^{ee})\alpha_e(u'+w)]\}\right\} . \tag{7.1}$$

For infinite ion mass, the distribution function for ions can be written as

$$f_i(u)\rightarrow\delta(u) \,, \tag{7.2}$$

consequently

$$f_i^{\pm}(u)\rightarrow -\mathrm{P}\left[\frac{1}{u}\right]\pm i\pi\delta(u) \,, \tag{7.3}$$

where P stands for the principal value. Under this limit, the screening is solely contributed by the electrons

$$f_e(u)=\frac{2q^2}{uk_e^2}[\epsilon_{ee}(u)-\epsilon_{ee}(0)][1+W^{ee}(q)]^{-1} \,, \tag{7.4}$$

where $k_e^2=4\pi ne^2/T$ is the screening wave number for electrons and

$$\epsilon_{ee}(0)=1+\frac{k_e^2}{q^2}[1+W^{ee}(q)] \,. \tag{7.5}$$

We now substitute Eqs. (7.2)–(7.5) in Eq. (7.1) to obtain

$$\sigma_1(\omega)=\frac{1}{24\pi^3}\frac{n_e e^4}{\omega^3 m_e^2}\int dq\, q^6 V_q\frac{[U_q^{ei}(1-n_i g_0^{ii})-n_e g_0^{ie}U_q^{ee}]}{1+W^{ee}(q)}\,\frac{\epsilon_{ee}(q,\omega)-\epsilon_{ee}(q)}{\tilde{\epsilon}(q,\omega)\tilde{\epsilon}(q,0)}\{1+[W^{ei}(q)-W^{ee}(q)]\alpha_e(\omega)\} \,. \tag{7.6}$$

In Eq. (7.6) $\tilde{\epsilon}(q,\omega)$ is still the full dielectric function including the contribution of electrons and ions. For heavy but finite ion mass, the ions not only provide momentum relaxation for electron scattering, but they also contribute to the electric current in two other ways. First, they carry current themselves, which results in a negligible contribution at $m_e/m_i\ll 1$. Second, they participate in the screening which, for large ion mass, manifests itself in retaining the contribution to the static screening which influences the conductivity at low and intermediate frequencies. Therefore we retain the ion contribution in the screening in Eq. (7.6). However, if the ions have random distribution and are not in equilibrium with the electrons (e.g., as in crystal where ions are fixed in lattice positions regardless of their interaction with electrons), these fixed ions do not participate in screening but only provide the electrons with a momentum relaxation mechanism and $\epsilon(\omega)=\epsilon_{ee}(\omega)$. Equation (7.6) reduces to a simpler form

$$\sigma_1(\omega)=\frac{1}{24\pi^3}\frac{n_e e^4}{\omega^3 m_e^2}\int dq\, q^6 V_q \frac{\epsilon_{ee}(q,0)}{\epsilon(q,0)}\frac{[U_q^{ei}(1-n_i g_0^{ii})-n_e g_0^{ie} U_q^{ee}]}{1+W^{ee}(q)}\left[\frac{1}{\epsilon_{ee}(q)}-\frac{1}{\epsilon_{ee}(q,\omega)}\right]\{1+[W^{ei}(q)-W^{ee}(q)]\alpha_e(\omega)\}. \tag{7.7}$$

Equation (7.7) is very similar to that of quantum plasmas we have studied recently.[28]

VIII. DISCUSSION

In this paper we have derived an expression for the conductivity and the collision frequency for plasmas including short-range correlations. We found that the short-range correlations will affect the dynamical conductivity in two ways. The self-consistent field which contributes to the screening of the electrons and the ions changes dramatically from the RPA result at large wave numbers when short-range correlations are incorporated. Moreover, by considering short-range correlations in our theory, we calculated the large-angle electron-ion scattering (which is responsible for the absorption process) including the positive correlations between the electrons and the ions. This makes our theory a more realistic one for cases when $r_s=e^2n^{1/3}/k_BT$ is of order of unity. In comparison we note that RPA considers the large-angle electron-ion scattering only in the Born approximation.

In order to see the effects of short-range correlation on conductivity explicitly, we consider the collision rate for an electron-heavy ion system. The collision rate can be obtained by comparing our result for conductivity $(n_e e^2/m\omega)+\sigma_1(\omega)$ with the standard Drude formula for conductivity $n_e e^2/m(\omega+i\nu)$. In Figs. 3 and 4 we have plotted the calculated ν as a function of frequencies for two different sets of parameters. It can be seen that the effect of short-range correlation is more important at the low-frequency region than at high frequencies.

In conclusion, our modified RPA theory, which includes short-range correlations, should yield a more realistic result for large r_s plasmas. It is valid as long as electron-ion bound-state effects do not dominate the two- and three-particle correlation function.

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