X-Ray Production by Alpha-Particle Impact*

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Recent observations of characteristic x rays produced by higher-energy (30–80 MeV) α particles are compared to previous measurements and to theoretical calculations based on the impulse approximation. These data are found to be in agreement with theory. The effects of choice of fluorescence yield are discussed.

Previous examinations[1,2] of the cross sections for ionization of K-shell electrons by proton impact indicated that the observations agreed reasonably well with theoretical binary-encounter-approximation estimates, over a wide range of elements, cross sections, and energies. In this note we discuss the application of the impulse approximation to K-shell ionization by α-particle impact.

The binary-encounter model is based on the assumption that the dominant interaction is the energy exchange between the incident particle and the atomic electron in question. One thus calculates the laboratory-frame cross section $d\sigma/d\Delta E$ for the exchange of energy ΔE between the incident particle of mass m_1 moving with velocity $\vec{v}_1$ and the bound electron moving with velocity $\vec{v}_2$, averaged over a spherically symmetric distribution of directions for $\vec{v}_2$. This differential cross section, given by expression (1) of Ref. 1, is then integrated over all allowable ΔE, and averaged over the speed distributions of the bound electrons, as indicated in Eq. (2) of Ref. 1.

For ionization of electrons within a given subshell, the above procedure is equivalent to finding the square of the Coulomb amplitude for scattering of the incident particle and the electrons, and weighting this by a momentum distribution determined by the atom. When stated this way, we recognize this as the *impulse* approximation,[3] in which a plane wave is used for the outgoing electron. It is thus the coincidence of the classical and quantal Coulomb scattering cross section which allows the successful application of a classical binary-encounter expression in an explicitly quantal problem.

We have discussed in Ref. 1 the modification of this model to include approximately the repulsion of the incident particle by the atomic nucleus. The two effects considered are the bending of the trajectory, which is nearly always negligible for protons or α particles, and the reduction in kinetic energy of the incident particle, which is only important near threshold. Equations (5) and (6) of Ref. 1 show the corrections to the cross section when repulsion of the nucleus is taken into account.

One of the useful features of this model is the fact that the results obey a scaling law.[1,4] Perhaps the most appropriate statement of this law is that, for a given incident particle, the product of the square of binding energy and the cross section is a universal function of the ratio E/U of the incident energy to binding energy:

$$U^2\sigma = f(E/U) .$$

This statement remains true even after averaging over hydrogenic velocity distributions.[4] The advantage to this is that, since $U^2\sigma$ is a function only of E/U and not E or U separately, cross sections

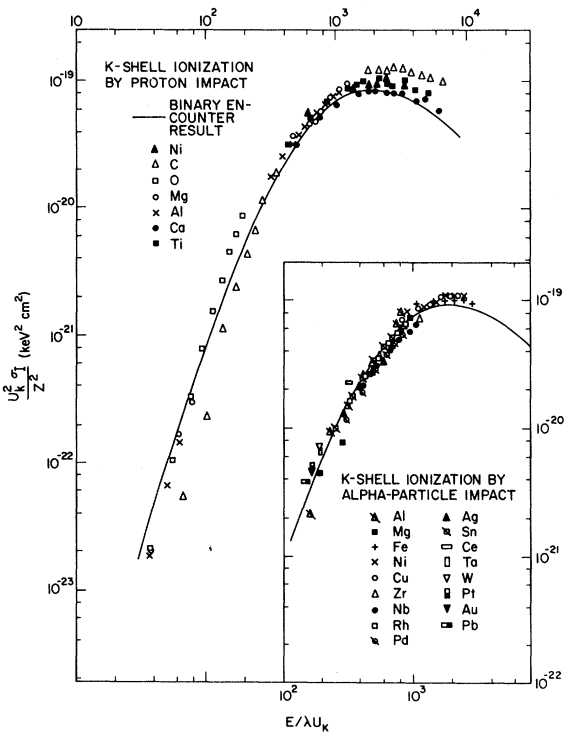

FIG. 1. Comparison of present results with experimental ionization cross sections. Upper portion shows comparison with proton impact data (Ref. 2). Lower right-hand corner shows the α-particle data (Refs. 5, 6, and 7); solid line is the binary-encounter result.

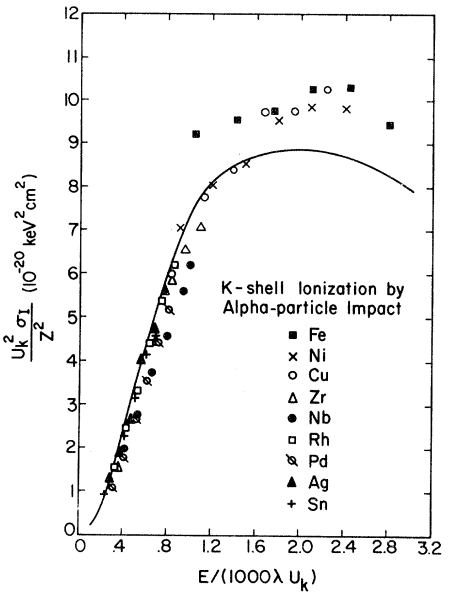

FIG. 2. Comparison of experimental x-ray production cross sections induced by α particles from Ref. 6 with theory, using theoretical values for the fluorescent yields (Ref. 9), and the present results for theoretical ionization cross sections.

for different atoms define the same function. For a given E/U, the product $U^2\sigma$ is the same independent of the separate values of E and U. (Different velocity distribution will, of course, produce different functions.)

We have calculated this universal function for α particles, using Eqs. (1), (2), (4), and (5) of Ref. 1. [Equation (1) must be multiplied by Z^2, where Z is the projectile charge.] However, as can be seen from the Appendix in Ref. 4, for very small m_e/m_1 the cross section at a given velocity depends only weakly on the incident particle mass. As a result, the quantity $U^2\sigma/Z^2$, where Z is the projectile charge, is to an excellent approximation a universal function of $E/\lambda U$, where $\lambda = m_1/m_p$ is the ratio of projectile to proton mass. The curve for α particles is practically indistinguishable from that for protons for $E/\lambda U > 100$. At $E/\lambda U = 76$, $U^2\sigma/Z^2$ for α particles is 10% higher than that for protons.

Figure 1 shows a comparison of the above results with available experimental data. The data are from Refs. 5–7. Also shown are the proton impact results, taken from Ref. 2. It can be seen that for both protons and α particles, the impulse approximation provides a reasonable estimate of the experimentally defined curve for $U^2\sigma/Z^2$ vs $E/\lambda U$. It should be noted that the α-particle data cover a range of nuclear charge extending from $Z = 12$ to $Z = 82$, and α-particle energies from 1 to 80 Mev.

All comparisons with theory of inner-shell ionizations as inferred by measurement of x-rays involve the use of the fluorescence yield for the atom in question. The comparisons in Fig. 1 are based on the fluorescent yields recommended by the individual experimenters, which are generally consistent with those quoted in Ref. 8. However, this introduces the possibility that the apparent energy dependence of the ionization cross section be influenced by choice of fluorescence yield. Note that the α-particle data for different elements occurs at different values of $E/\lambda U$. (The proton data have more overlap.)

A more direct comparison of theory with experiment is yielded by using theoretical values of the fluorescence yield. By doing this we examine directly the behavior of x-ray production cross sections as determined theoretically vs experimentally. Such a comparison is shown in Fig. 2 on linear scales. The theoretical fluorescence yields used are shown in Table I, as given in Ref. 9. The experimental x-ray production cross sections are from Ref. 5. We note the good agreement of theory and experiment, already evident in Fig. 1.

The values of fluorescence yield used in Fig. 2 differ by no more than 15% from those used in Fig. 1. Figure 2 is to be compared with Fig. 5 of Ref. 5, where the same data are compared to the Born approximation. It should be noted that the energy dependence predicted by the impulse approximation is in better agreement with experiment than the Born estimate, even allowing for the adjusted fluorescence yields.

While the dominant behavior of these ionization cross sections does appear to be in accord with an impulse-approximation model, there are trends which may be interesting to investigate further. As the nuclear charge is decreased, the experimental data seem to show a steeper rise with projectile energy than predicted. The carbon and oxygen data for proton impact and the aluminum data for α-particle impact are examples of this (see Fig. 1). Adiabatic changes in the bound

TABLE I. Fluorescent yields.

Element	ω_k
Fe	0.344
Ni	0.414
Cu	0.448
Zr	0.741
Nb	0.759
Rh	0.820
Pd	0.833
Ag	0.844
Sn	0.874

electron's energy are not accounted for in the model. These would be expected to be larger for the smaller nuclei. A more detailed examination of the formulation is required to see if one can retain the basic simplicity of the impulse model and obtain corrections for these effects.

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Interpretation of Experimental Differential Elastic Scattering Cross Section for $H^+ + Ne$†

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The result of a differential scattering experiment of H^+ by Ne at 5.71-eV collision energy is compared to the differential cross section predicted by (a) a JWKB partial-wave calculation based on a potential model, (b) a similar calculation based on the *ab initio* Peyerimhoff calculated intermolecular potential for the NeH^+ system, and (c) a new and more efficient calculational scheme developed by Remler involving Regge poles. The agreement of the Peyerimhoff prediction, (b), with the experimental data is very good. Differences between the Peyerimhoff potential and the final values of the parameters retrieved from the iterative calculation in (a), as well as implicit ambiguities of (a) in reproducing the features of a low-resolution experiment, are discussed in terms of semiclassical theory and the Remler-Regge method.

I. INTRODUCTION

The results and analysis of several experiments on the low-energy differential elastic scattering of H^+ by noble-gas atoms have been reported.[1] In Ref. 1, the experimental differential cross section $\sigma_{expt}(\theta)$ for each of the systems NeH^+, ArH^+, and KrH^+ was compared to a calculated differential cross section utilizing the JWKB method and a chosen analytic form for the interatomic potential $V(r)$. For each system the internuclear equilibrium separation r_m and the well depth U for the assumed $V(r)$ were varied in the calculations until reasonable agreement was obtained between the experimental and calculated differential cross sections.

Since then we have come to believe that the results obtained from such potential-model calculations do not necessarily yield reliable intermolecular potentials, unless the resolution in the experiment is very good. The purpose of this paper is (a) to compare the $V(r)$ found via the method described above (and in Ref. 1) to an *ab initio* calculation of the intermolecular potential for NeH^+ due to Peyerimhoff,[2] (b) to discuss why there is a rather significant difference in the two results, and (c) to apply a new method due to Remler[3] involving Regge poles which can be used to efficiently calculate the differential cross section, where the starting point for such calculations is based on one's intuition in light of the classical deflection function.

II. EXPERIMENT AND SEMIQUANTAL CALCULATIONS

The semiquantal calculation of Ref. 1 as regards the NeH^+ system will be briefly recapitulated. The JWKB phase shifts were found using the potential model

$$V(r) = (\tilde{C}/\rho)\ \exp\beta(1-\rho) - C_6/\rho^6 - C_4/\rho^4 \ , \qquad (1)$$

with

$$\rho = r/r_m \ , \qquad C_4 = \alpha e^2/2r_m^4 \ ,$$