Radiative Corrections. II. Compton Effect*

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The cross section for the double Compton effect is integrated numerically in the laboratory system, and the effect of the radiative tail to the Compton line is discussed. The double Compton effect is also studied in the c.m. system for the cases in which the energy of one of the emitted photons is much smaller than the electron rest energy or much smaller than the energy of the incident photon. Expressions for the radiative corrections to the total cross section for Compton scattering are obtained in the high-energy approximation to first order in the fine-structure constant. The total cross section for the double Compton effect is computed by numerical integration. The contribution to the absorption coefficient for high-energy photons from double Compton scattering and Compton scattering with the emission and reabsorption of a virtual photon is obtained.

I. INTRODUCTION

Brown and Feynman[1] have evaluated the differential cross section for the radiative corrections to the cross section for Compton scattering to first order in the fine-structure constant $\alpha = \frac{1}{137}$. Because of the infrared divergence the radiative corrections alone do not have any physical meaning. This is a consequence of the impossibility of experimentally separating a given process which includes the emission of photons of very low energy (below the experimental energy resolution) from the same process in the absence of emitted low-energy photons. When the effects of a virtual and a real low-energy photon both are taken into account the infrared divergence disappears, and a finite physically meaningful cross section remains. More exactly, the cross section for all processes which do not contain an infinite number of soft photons is zero, but if the energy loss by emission of soft photons is not too small (the logarithm of the energy loss in units of the rest mass of the electron should not be much larger than 1) it is, to first order in α, sufficient to assume that a single soft photon is responsible for the energy loss. Brown and Feynman therefore also calculated the cross section for double Compton scattering (in the lab system) in which the energy of one of the emitted photons is much smaller than the rest energy of the electron. This restriction ($k_{\max} \ll m$) is not easily fulfilled in actual experiments, and after quoting some known results in Sec. II we, therefore, consider in Sec. III the effect of two-photon emission for the case of more practical energy resolutions. The integrations have been done numerically, and the radiative tail to the Compton line is shown for a number of energies and scattering angles.

In Sec. IV we discuss a high-energy approximation, and we obtain approximate expressions for the cross sections.

In Sec. V we examine the double Compton effect in the c.m. system for the case in which the energy of one of the emitted photons is much smaller than the energy of the primary photon, but much larger than the electron rest energy. Combining this cross section with the formula of Brown and Feynman we obtain an expression for the corrections to first order in α to the Compton cross section at high energies.

In Sec. VI we integrate analytically in the high-energy approximation the expressions for the radiative corrections and the soft-photon double Compton effect, and we obtain the corrections to the total cross section. We compute the total cross section for the hard-photon double Compton scattering by numerical integration, and the correction to the absorption coefficient for high-energy photons due to double Compton effect and lowest-order radiative corrections to the Klein-Nishina formula is obtained. Section VII contains a discussion of the results.

The system of units and notation will follow closely the conventions used in the book of Jauch and Rohrlich.[2] We shall use units for which $\hbar = 1$, $c = 1$, and $m = 1$.

II. SOME PREVIOUS RESULTS

A. Klein-Nishina Formula

The Feynman diagrams representing Compton scattering to lowest order in α are shown in Fig. 1. The incoming photon and electron have four-momenta k_0 and p_1, and the outgoing photon and electron have four-momenta k_1 and p_2, respectively. The cross section for unpolarized particles corresponding to these diagrams was first evaluated by Klein and Nishina,[3] and may, according to Ref. 1, be written in the following invariant form:

$$d\sigma_0 = 2\pi r_0^2 (d\tau/\kappa^2)\, U, \tag{2.1}$$

where

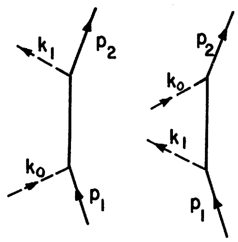

FIG. 1. Diagrams for single Compton scattering.

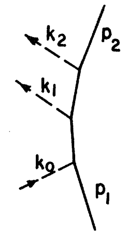

FIG. 3. Double Compton scattering diagram.

$$\kappa = 2p_1k_0 = 2p_2k_1 \, , \tag{2.2}$$

$$\tau = -\, 2p_1k_1 = -\, 2p_2k_0 \, , \tag{2.3}$$

$$U = 4\left(\frac{1}{\kappa}+\frac{1}{\tau}\right)^2 - 4\left(\frac{1}{\kappa}+\frac{1}{\tau}\right) - \frac{\kappa}{\tau} - \frac{\tau}{\kappa} \, , \tag{2.4}$$

and r_0 is the classical radius of the electron. A product p_1k_0 means $\vec{p}_1\vec{k}_0 - \epsilon_1\omega$, where $\vec{p}_1$ and ϵ_1 are the momentum and energy of the electron, and $\vec{k}_0$ and ω_0 are the momentum and energy of the photon. From the conservation laws $p_1 + k_0 = p_2 + k_1$ there follows the Compton relation

$$\omega_1 = \frac{(1-\beta_1\cos\alpha_0)\omega_0}{1-\beta_1\cos\alpha_1 + (\omega_0/\epsilon_1)(1-\cos\theta_1)} \, . \tag{2.5}$$

Here α_0 and α_1 are the angles between $\vec{p}_1$ and $\vec{k}_0$ and $\vec{p}_1$ and $\vec{k}_1$, respectively, θ_1 is the angle between $\vec{k}_0$ and $\vec{k}_1$, and $\beta_1 = |\vec{p}_1|/\epsilon_1$. In the lab system ($\vec{p}_1 = 0$) or in the c.m. system ($\vec{p}_1 + \vec{k}_0 = \vec{p}_2 + \vec{k}_1$) the relations above can be simplified.

B. Virtual-Photon Radiative Corrections

In Fig. 2 we show a typical one of the Feynman diagrams which by interference with the diagrams of Fig. 1 give the radiative corrections to first order in α. The complete set of diagrams also shown in Fig. 2(A) of Ref. 4. The corrections have been evaluated by Brown and Feynman,[1] according to whom the cross section including the corrections is $d\sigma_0(1+\delta^C_{\rm vir})$, where

$$\delta^C_{\rm vir} = -\frac{\alpha}{\pi U}\,{\rm Re}\,U^C_{\rm vir}(\kappa,\tau) \, . \tag{2.6}$$

and $U^C_{\rm vir}(\kappa,\tau)$ is given by Eq. (3.3) of Ref. 4.

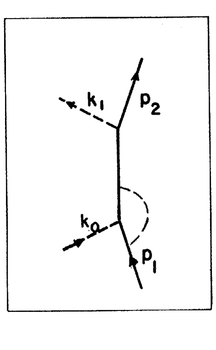

FIG. 2. Radiative correction diagram.

C. Differential Cross Section for Double Compton Effect

In Fig. 3 we show one of the diagrams for double Compton scattering. All diagrams are shown in Fig. 2(B) of Ref. 4. The two emitted photons have four-momenta k_1 and k_2. The differential cross section has been calculated by Mandl and Skyrme.[5] According to Ref. 2 their result is

$$d\sigma_{\rm real} = \alpha r_0^{\,2}\,\frac{d\Omega_1 d\Omega_2}{(4\pi)^2}\,\frac{\omega_1\omega_2 d\omega_1}{\omega_0\epsilon_1(1-\beta_1\cos\alpha_0)} \times \frac{X}{\epsilon_1(1-\beta_1\cos\theta_2') + \omega_0(1-\cos\theta_2) - \omega_1(1-\cos\theta_{12})} \, . \tag{2.7}$$

Here θ_2 is the angle between $\vec{k}_0$ and $\vec{k}_2$, θ_2' is the angle between $\vec{p}_1$ and $\vec{k}_2$, and θ_{12} is the angle between $\vec{k}_1$ and $\vec{k}_2$. The quantity X is defined in Eqs. (8.3) and (8.4) of Ref. 4 as a function of the invariant quantities

$$\kappa_1 = -p_1k_1, \quad \kappa_2 = -p_1k_2, \quad \kappa_3 = p_1k_0, \qquad \kappa_1' = p_2k_1, \quad \kappa_2' = p_2k_2, \quad \kappa_3' = -p_2k_0 \, . \tag{2.8}$$

The conservation laws $p_1 + k_0 = p_2 + k_1 + k_2$ can be used to express $d\sigma_{\rm real}$ in terms of the variables ω_1, θ_1, θ_2, and ϕ (cf. Fig. 4). In terms of these variables

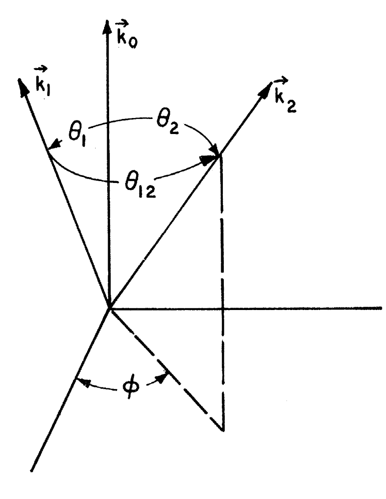

FIG. 4. Relative directions in double Compton scattering.

the quantities ω_2 and θ_{12} appearing in Eq. (2.7) are given by

$$\cos\theta_{12}=\cos\theta_1\cos\theta_2+\sin\theta_1\sin\theta_2\cos\phi \tag{2.9}$$

and

$$\omega_2=\frac{\omega_0\epsilon_1(1-\beta_1\cos\alpha_0)-\omega_1\epsilon_1(1-\beta_1\cos\theta_1')-\omega_1\omega_0(1-\cos\theta_1)}{\epsilon_1(1-\beta_1\cos\theta_2')+\omega_0(1-\cos\theta_2)-\omega_1(1-\cos\theta_{12})}\ . \tag{2.10}$$

D. Soft-Photon Double Compton Effect

For the case in which the energy of one of the emitted photons is much smaller than the electron mass,

$$\omega_2\leq\omega_{2\max}\ll 1, \tag{2.11}$$

the cross section for the double Compton effect reduces to the Klein-Nishina formula times a factor. The cross section is then

$$d\sigma_{\mathrm{real,soft}}=\frac{\alpha}{4\pi^2}d\sigma_0\frac{d^3k_2}{\omega_2}\left(\frac{p_1}{p_1k_2}-\frac{p_2}{p_2k_2}\right)^2, \tag{2.12}$$

where $d\sigma_0$ is given by Eq. (2.1). Integrating over the directions of $\vec{k}_2$ and over $|\vec{k}_2|$ from 0 to $\omega_{2\max}$ we get[4]

$$d\sigma_{\mathrm{real,soft}}=-\frac{\alpha}{4\pi}\,d\sigma_0\left[2(1-2y\coth 2y)\ln\left(\frac{\omega_{2\max}}{\lambda}\right)-\frac{1}{2\beta_1}\ln\frac{1+\beta_1}{1-\beta_1}-\frac{1}{2\beta_2}\ln\frac{1+\beta_2}{1-\beta_2}+\tfrac{1}{2}\coth 2y\,[F(\beta_1)+\theta(\beta_2{}^2-\vec{\beta}_1\vec{\beta}_2)\,F(\beta_2)]\right], \tag{2.13}$$

where $F(\beta)$ is given by Eq. (A3.7) of Ref. 4, λ is a small "photon mass", and the step function $\theta(x)$ is $\theta(x)=\pm 1$ for $x\gtrless 0$.

The function y is defined by

$$4\sinh^2 y=(p_1-p_2)^2=-(\kappa+\tau). \tag{2.14}$$

To evaluate the cross section in the laboratory system, we must let $\beta_1\to 0$. We then find the same expression for $d\sigma_{\mathrm{real,soft}}$ as was obtained by Brown and Feynman[1]:

$$d\sigma_{\mathrm{real,soft}}=-\frac{\alpha}{\pi}\,d\sigma_0\left[2(1-2y\coth 2y)\left(\ln\frac{\omega_{2\max}}{\lambda}-\frac{1}{2}\right)+4y\coth 2y[h(2y)-1]\right], \tag{2.15}$$

where

$$h(y)=(1/y)\int_0^y u\,du\coth u. \tag{2.16}$$

In the c.m. system, where $\beta_1=\beta_2$ and $\vec{\beta}_1\vec{\beta}_2=\beta_1{}^2\times\cos\theta_1$, the cross section becomes

$$d\sigma_{\mathrm{real,soft}}=-\frac{\alpha}{\pi}d\sigma_0\left[2(1-2y\coth 2y)\ln\left(\frac{\omega_{2\max}}{\lambda}\right)-\frac{1}{\beta_1}\ln\frac{1+\beta_1}{1-\beta_1}+\coth 2y\,F(\beta_1)\right], \tag{2.17}$$

where $F(\beta)$ is still given by Eq. (A3.7) of Ref. 4, and the quantities a, b, and c appearing there are $a=\beta_1\cos(\frac{1}{2}\theta_1)$, $b=(1-a)/(1+a)$, and $c=[b\tan(\frac{1}{4}\theta_1)]^{1/2}$.

The cross sections given in Eqs. (2.15) and (2.17) are valid for all initial photon energies ω_0, and if we combine them with the virtual-photon radiative corrections, Eq. (2.6), the infrared divergent term disappears, and we obtain the radiative correction to the Klein-Nishina formula.

III. RADIATIVE TAIL

We now consider a Compton scattering experiment in which photons emitted at a given angle θ_1 with the direction of the incoming photon are recorded. For single Compton scattering, the energy ω_1 of the emitted photon is fixed by the energy ω_0 of the incident photon and θ_1. According to Eq. (2.5) we find that the energy of the emitted photon in the single Compton effect $\omega_1=\omega_s$ is in the laboratory system given by

$$\omega_1\to\omega_s=\omega_0/[1+\omega_0(1-\cos\theta_1)]\ . \tag{3.1}$$

We suppose that the measuring apparatus records photons with energy in the region from $\omega_s-\Delta E$ to ω_s. Thus not only single Compton photons are counted but also some from double Compton scattering. If ΔE is much smaller than 1 the contribution from double Compton scattering is given approximately by Eq. (2.15) with $\Delta E=\omega_{2\max}$. It should be noted, however, that the condition $\omega_s-\Delta E<\omega_1<\omega_s$ is not identical to the condition $0<\omega_2<\Delta E$, since $\omega_1+\omega_2$ is not a constant. Therefore, in order to obtain the cross section corresponding to observation of photons with energy between $\omega_s-\Delta E$ and ω_s we add the double Compton cross section with the condition $\omega_2<\omega_{2\max}$, to the cross section in which ω_1 is integrated from $\omega_s-\Delta E$ to an upper limit defined by $\omega_2>\omega_{2\max}$. Calling the latter part $d\sigma_{\mathrm{real,hard}}$ we find

$$d\sigma_{\mathrm{real,hard}}=\int_{-1}^{+1}d(\cos\theta_2)\int_0^{2\pi}d\varphi\int_{\omega_s-\Delta E}^{\omega_{1\max}}d\omega_1\frac{d\sigma_{\mathrm{real}}}{d\Omega_2 d\omega_1}\ , \tag{3.2}$$

where the condition

$$\omega_2=\frac{\omega_0(\omega_s-\omega_1)}{\omega_s[1+\omega_0(1-\cos\theta_2)-\omega_1(1-\cos\theta_{12})]}\geq\omega_{2\max} \tag{3.3}$$

defines

$$\omega_{1\max} = \omega_s - \omega_{2\max}\frac{[1+\omega_0(1-\cos\theta_1)][1+\omega_0(1-\cos\theta_2)]-\omega_0(1-\cos\theta_{12})}{[1+\omega_0(1-\cos\theta_1)][1+\omega_0(1-\cos\theta_1)-(1-\cos\theta_{12})\omega_{2\max}]} . \quad (3.4)$$

Since $d\sigma_{\text{real,hard}}$ can be written in the form

$$d\sigma_{\text{real,hard}} = A_H - B\ln(\omega_{2\max})$$

we have subtracted the infrared part from the integrand in Eq. (3.2) and computed numerically the quantity A_H which is not dependent on $\omega_{2\max}$ but depends on ΔE. We have used the Monte Carlo method. That is, we choose random values for the variables of integration and take the mean value of a large number (about 10 000) of samples. We also compute the rms deviation. At high energies the accuracy is improved by use of $\{\ln[1+\omega_0 \times(1-\cos\theta_2)]\}/\ln(1+2\omega_0)$ as a variable instead of $\cos\theta_2$. The accuracy in the correction to the Compton cross section is then about 10% for 1-BeV photons, and better than that for lower energies.

The corrections to the Compton cross section from a virtual and a real soft photon can be written as $\delta_{\text{vir}} = F_V + F\ln\lambda$ and

$$\delta_{\text{real,soft}} = \frac{d\sigma_{\text{real,soft}}}{d\sigma_0} = F_S + F\ln\left(\frac{\omega_{2\max}}{\lambda}\right),$$

respectively. We write the contribution from hard photons

$$\delta_{\text{real,hard}} = \frac{d\sigma_{\text{real,hard}}}{d\sigma_0}$$

as

$$\delta_{\text{real,hard}} = F_H(\Delta E) - F\ln(\omega_{2\max}).$$

Using Eqs. (2.6) and (2.15) we have computed F_V+F_S for several values of ω_0 and θ_1. The values of F_V+F_S are shown in Table I, and F_H is plotted as a function of $\Delta E/\omega_s$ in Fig. 5. The ΔE-dependent correction to the Klein-Nishina formula is given by the sum $\delta = \delta_{\text{vir}} + \delta_{\text{real,soft}} + \delta_{\text{real,hard}}$ which is equal to

$$\delta = F_V + F_S + F_H. \quad (3.5)$$

Meister and Yennie[6] have evaluated an approximate expression for the correction δ, retaining only logarithmic terms. Their expression [Eq. (4.8) on p. 1224 of their paper] should be fairly accurate for small laboratory scattering angles but relatively poor for larger angles. Comparing with the results of this paper we find, for example, that for an initial photon energy $\omega_0 = 100$ MeV, relative energy resolution $\Delta E/\omega_s = 0.05$, and scattering angle $\theta_1 = (2/\omega_0)^{1/2} = 5.8°$, their expression gives $\delta = -1.48\%$ to be compared with $\delta = -1.1\%$ from this paper. However, for $\theta_1 = 90°$ their expression gives for the same resolution $\delta = -2.76\%$ as compared with our value $\delta = -1.6\%$.

TABLE I. Quantity F_V+F_S which is the nondivergent part of the corrections to Compton scattering due to radiative corrections and soft photons.

θ_1(deg)	30	90	150	ω_0(MeV)
$10^2(F_V+F_S)$	0.63	0.474	−0.30	0.662
$10^2(F_V+F_S)$	−0.188	−0.287	−0.152	6.14
$10^2(F_V+F_S)$	−6.22	−5.04	−4.41	10^2
$10^2(F_V+F_S)$	−14.0	−11.8	−10.9	10^3

The energy spectrum $d\sigma_{\text{real,hard}}/d\omega_1$, which is the radiative tail to Compton scattering, can be found by differentiation of $d\sigma_{\text{real,hard}}$ with respect to ΔE. The quantity

$$R(\omega_1) = \frac{d\sigma_{\text{real,hard}}}{d\Omega_1 d\omega_1}\bigg/\frac{d\sigma_0}{d\Omega_1}$$

has been tabulated in Table II for several values of ω_1, ω_0, and θ_1. These spectra are useful if one applies a detector which does not have a rectangular response function. Carrassi and Passatore[7] have calculated some such spectra before and part of their results are confirmed by ours.

When ω_1 approaches ω_s the spectrum $d\sigma_{\text{real}}/d\Omega_1 d\omega_1$ increases portionally to $1/(\omega_s-\omega_1)$. One should, however, remember that this seemingly large effect is not physically meaningful since it is compensated by the effects of virtual photons, and therefore the number of photons emitted in $d\Omega_1$ with energy between ω_s and $\omega_s - \Delta E$ is finite.

The spectrum $d\sigma_{\text{real}}/d\Omega_1 d\omega_1$ also diverges as $1/\omega_1$ if ω_1 approaches 0. This divergence does not produce any difficulties since only photons with a finite energy can be observed, and therefore the divergent region can not be reached experimentally. Some care must be taken if one also wants to integrate the spectrum over $d\omega_1$ and $d\Omega_1$. The expression (2.7) gives the probability for emission of two photons of energy ω_1 and ω_2 into solid angles $d\Omega_1$ and $d\Omega_2$, respectively. However, when we integrate over a region of angles and energies we may sometimes count identical final states twice. In order to obtain the correct integral cross section we therefore have to multiply our integrand by a factor of $\frac{1}{2}$ each time a configuration of final states is produced twice. The overlap of final states is of no importance as long as at least one of the variables stays fixed, but when the integral over the last variable is performed a factor of $\frac{1}{2}$ must be supplied. Addition of the divergence at the upper and lower end of the spectrum and division by 2 then produces a divergent term which exactly cancels the divergent term from the integral of the radiative corrections. Thus all divergences are removed in the total cross section as they should be, since the total cross section is

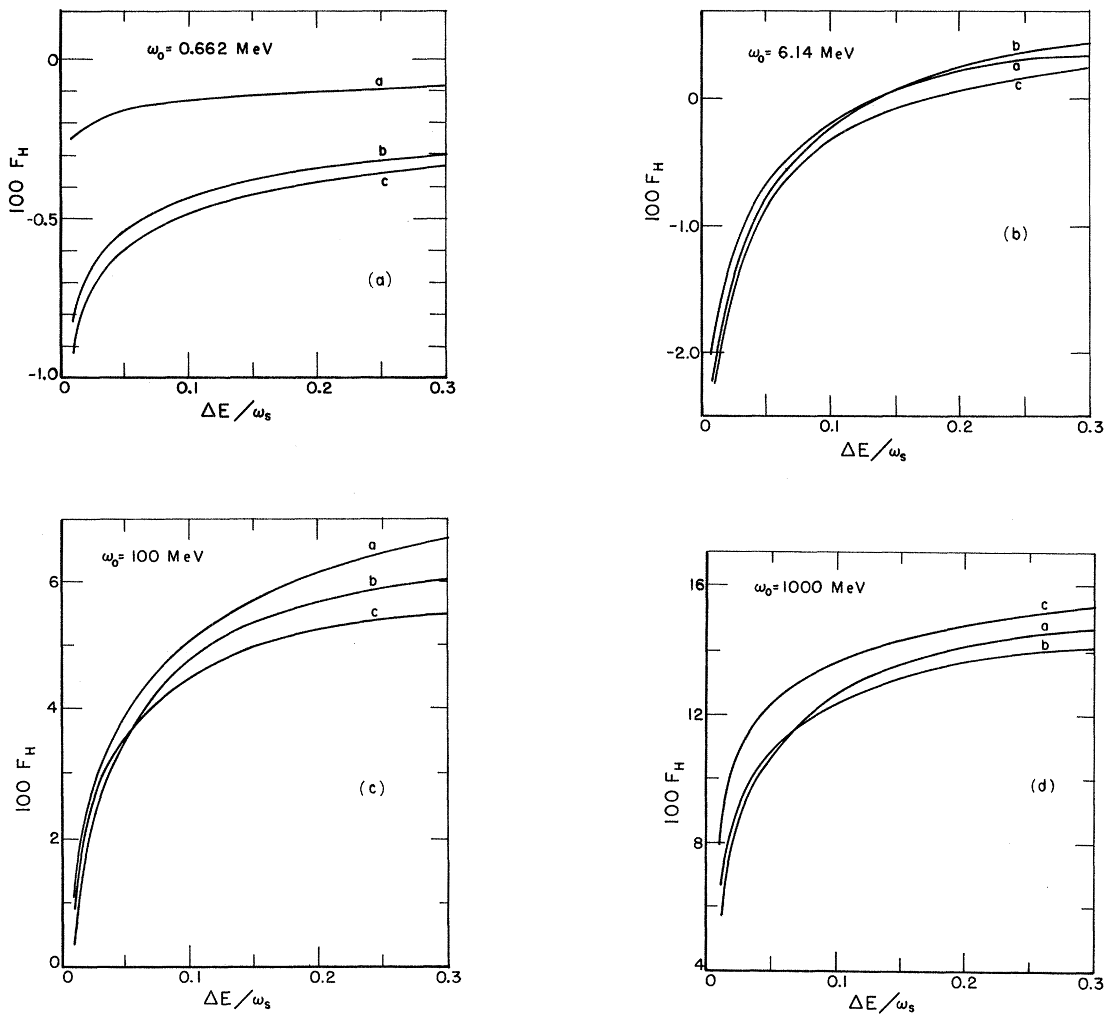

FIG. 5. (a) The quantity F_H as a function of $\Delta E/\omega_s$ for the photon energy $\omega_0=0.662$ MeV. The curves a, b, and c refer to scattering angles $\theta_1=30°$, $90°$, and $150°$, respectively. The ΔE-dependent correction to the differential Compton cross section is given by $\delta=F_V+F_S+F_H$. (b) The quantity F_H as a function of $\Delta E/\omega_s$ for $\omega_0=6.14$ MeV [cf. Fig. 5(a)]. (c) The quantity F_H as a function of $\Delta E/\omega_s$ for $\omega_0=100$ MeV [cf. Fig. 5(a)]. (d) The quantity F_H as a function of $\Delta E/\omega_s$ for $\omega_0=1000$ MeV [cf. Fig. 5(a)].

a physical quantity (e.g., measurable by an absorption experiment).

IV. HIGH-ENERGY APPROXIMATIONS

We now consider the high-energy approximation meaning that we neglect all terms in the cross section which give a vanishing contribution to the total cross section when the energy of the incident photons is large. That is, in the laboratory system we neglect all terms which at all angles are of the order $1/\omega_0$ compared to the largest terms, and we also neglect terms which are large only in a small region of the angles so that the contribution to the total cross section from these terms is negligible.

In order to obtain the total cross section for single Compton scattering we have to integrate the differential cross section over the angle θ_1 between k_0 and $\vec{\mathrm{k}}_1$. At high energies in the laboratory system, the Compton cross section, like the cross sections for all high-energy processes, has a maximum when the outgoing particles are emitted at small angles to the forward direction. We can not, however, obtain the correct integrated cross section by making a small-angle approximation since high-momentum transfers contribute significantly to the cross section. The contribution to the Compton cross section from very small angles $[\theta_1 < O(1/\omega_0)]$ is negligible, but all angles of the order $1/\omega_0^{1/2}$ or larger give a significant contribution. This is easily verified for the Klein-Nishina formula, and we have shown that it is also true for the radiative corrections and the

TABLE II. Double Compton spectrum $d\sigma_D/d\Omega_1 d\omega_1 = R(d\sigma_0/d\Omega_1)$ as a function of ω_1/ω_s for some values of the primary energy ω_0 and the scattering angle θ_1 (lab system). Here ω_1 is the energy of one of the scattered photons in double Compton scattering, and ω_s is the energy of the scattered photon in single Compton scattering.

ω_1/ω_s	0.7	0.75	0.80	0.85	0.90	0.95	0.97	0.99	θ_1 (deg)	ω_0(MeV)
10^2R	0.134	0.146	0.169	0.241	0.412	0.935	1.63	5.13	30	0.662
10^2R	0.688	0.825	1.012	1.50	2.56	6.38	10.87	32.9	90	0.662
10^2R	1.04	1.26	1.76	2.53	4.10	10.35	19.25	52.8	150	0.662
10^2R	0.316	0.405	0.57	0.80	1.28	3.16	5.56	17.3	30	6.14
10^2R	1.38	2.41	3.19	5.13	8.48	17.7	34.3	95.5	90	6.14
10^2R	4.94	4.84	4.94	6.34	13.9	36.5	58.0	193	150	6.14
10^2R	0.515	0.71	0.98	1.42	2.22	5.33	8.60	24.9	30	100
10^2R	2.31	3.23	5.08	8.03	14.1	35.2	66.5	198	90	100
10^2R	2.5	3.78	7.2	14.4	26.0	51.3	123	341	150	100
10^2R	0.612	0.795	1.08	2.04	3.43	8.56	15.3	63.6	30	1000
10^2R	3.65	5.48	8.2	12.3	20.5	52.0	95.6	200	90	1000
10^2R	7.82	9.65	13.1	19.6	35.1	84.0	150	500	150	1000

double Compton effect by detailed examination of these cross sections. Thus in our approximation we can always put $\omega_0\theta_1 \gg 1$ in the lab system. In the c.m. system we find correspondingly that the cross section has a maximum at angles $\theta_1 \approx \pi$, and that angles in the region $\theta_1 \approx O(1/\omega_0)$ or $\pi - \theta_1 < O(1/\omega_0{}^2)$ need not be taken into account when we are interested in the total cross section. This means that $4\sinh^2 y = -(\kappa+\tau) \gg 1$, and according to Ref. 1, $h(y)$ is given approximately by $h(y) = \frac{1}{2}y + \pi^2/12y$. Therefore the Brown-Feynman expression for the radiative corrections can be written

$$d\sigma_{\mathrm{vir}} = d\sigma_0\,\delta_{\mathrm{vir}} = -2\alpha r_0{}^2 \frac{d\tau}{\kappa^2}\left\{2(1-2y)U\ln\lambda + \frac{\pi^2}{6}\left(4-5t-\frac{3}{t}-\frac{2}{\kappa^2 t^3}\right) + 4(2-U)y^2 - 4y + \tfrac{3}{2}U - \frac{2}{\kappa t^2}\right.$$
$$+\left(1-\frac{t}{2}-\frac{1}{t}\right)\ln^2|\kappa| + \left(2t+\frac{1}{t}-2+\frac{2}{\kappa t^3}\right)L_2(1+\kappa t) + \left[1-\tfrac{3}{2}t+4y\left(\frac{1}{t}+\frac{t}{2}-1\right)\right]\ln|\kappa|$$
$$\left.+\left[\left(1-\frac{2}{t}+\frac{2}{\kappa t^2}+\frac{\kappa}{1+\kappa t}\right)+4y\left(t-1+\frac{1}{2t}\right)\right]\ln\kappa t\right\}. \quad (4.1)$$

Here

$$y = \tfrac{1}{2}\ln[|\kappa|(1-t)], \quad t = -\tau/\kappa, \quad U = t + 1/t, \quad (4.2)$$

and in the lab system κ is $-2\omega_0$ and τ is $2\omega_0/[1+\omega_0(1-\cos\theta_1)]$, and in the c.m. system $\kappa = -2\epsilon_1\omega_0 \approx -2\omega_0{}^2$ and $\tau = 2\epsilon_1\omega_0(1+\beta_1\cos\theta_1) \approx 2\omega_0{}^2(1+\beta_1\cos\theta_1)$.

The approximate expression for the cross section for the soft-photon double Compton effect in the lab system becomes

$$d\sigma_{\mathrm{real,soft}} = (\alpha/\pi)d\sigma_0[2(1-2y)\ln(2\omega_{2\max}/\lambda) - 1 - 2y - 4y^2 + \tfrac{1}{6}\pi^2]. \quad (4.3)$$

The formula (2.17) can also be simplified considerably at high energies and not too small scattering angles. The quantities b and c become

$$b = \tan^2(\tfrac{1}{4}\theta_1) + \cos(\tfrac{1}{2}\theta_1)/[4\omega_0{}^2\sin^4(\tfrac{1}{4}\theta_1)] + O(1/\omega_0{}^4),$$

$$c = \tan^2(\tfrac{1}{4}\theta_1) + \cos(\tfrac{1}{2}\theta_1)/[8\omega_0{}^2\sin^4(\tfrac{1}{4}\theta_1)] + O(1/\omega_0{}^4).$$

Introducing these quantities in Eq. (A3.7) of Ref. 4 we find an expression for $F(\beta_1)$ which can be simplified further by means of relations between the dilogrithms.[8,9] The cross section in the c.m. system becomes finally

$$d\sigma_{\mathrm{real,soft}} = -(\alpha/\pi)d\sigma_0[2(1-2y)\ln(2\omega_{2\max}/\lambda) + 2\ln^2 E - 2\ln E + \tfrac{1}{3}\pi^2 - \tfrac{1}{2}\ln^2(1-t) - L_2(t)], \quad (4.4)$$

where $E = \epsilon_1 + \omega_0 \approx 2\omega_0$ is the total energy in the c.m. system.

V. CORRECTIONS IN c.m. SYSTEM WHEN BOTH ω_1 AND ω_2 MAY BE LARGE

The formula (4.4) for the double Compton cross section is only valid if $\omega_{2\max} \ll 1$. In order to find the cross section for the case in which both emitted photons may have high energies, but the energy of one of them is still much smaller than E, we return to Eq. (2.7).

Imposing the condition

$$\omega_{2\max} \le \omega_2 \le \Delta E, \tag{5.1}$$

where

$$1 \ll \Delta E \ll E, \tag{5.2}$$

we find from the conservation laws and Eq. (2.10) that $\omega_2 \ll \epsilon_1 \approx \omega_0 \approx \epsilon_2 \approx \omega_1 \approx \frac{1}{2}E$, and the expression for the cross section can be simplified considerably. The integrated cross section

$$d\sigma_{\text{real}}(\Delta E, \omega_{2\max}) = \frac{\alpha r_0^2}{E^2} \frac{d\Omega_1}{(4\pi)^2} \int_{\omega_{2\max}}^{\Delta E} \omega_2\, d\omega_2 X, \tag{5.3}$$

which contains part of both soft and hard photons, can be evaluated analytically in the high-energy approximation if we keep only terms which give a nonvanishing contribution to the total cross section. By detailed examination of each term in X [cf. Eq. (2.8)] we have shown that the integration of (5.3) gives just

$$d\sigma_{\text{real}}(\Delta E, \omega_{2\max}) = -(\alpha/\pi)\, d\sigma_0\, 2(1-2y)\ln(\Delta E/\omega_{2\max}). \tag{5.4}$$

This means that the effect of increasing the upper limit of integration of ω_2 from $\omega_{2\max}$ to ΔE is only to replace $\omega_{2\max}$ by ΔE in Eq. (4.4). In the laboratory system such a simple extension of $\omega_{2\max}$ from $\omega_{2\max} \ll 1$ to $1 \ll \omega_{2\max} \ll \omega_0$ evidently cannot be done, since the absolute upper limit of ω_2 is $\omega_0/[1+\omega_0 \times(1-\cos\theta_2)]$ which is smaller than one for angles θ_2 larger than $\frac{1}{2}\pi$. In the c.m. system we now have

$$d\sigma_{\text{real}}(\Delta E) = -\frac{\alpha}{\pi}\, d\sigma_0\left[2(1-2y)\ln\left(\frac{2\Delta E}{\lambda}\right) + 2\ln^2 E - 2\ln E + \frac{\pi^2}{3} - \frac{1}{2}\ln^2(1-t) - L_2(t)\right]. \tag{5.5}$$

This formula is not valid in the exact forward or backward directions. In the forward direction $p_1 = p_2$ and $d\sigma_{\text{real}}(\omega_{2\max}) = 0$ according to Eq. (2.13). We also find that $d\sigma_{\text{real}}(\Delta E, \omega_{2\max})$ vanishes, so that

$$d\sigma_{\text{real}}(\Delta E)_{\theta_1=0} = 0. \tag{5.6}$$

In order to get the cross section in the backward direction we put $\theta_1 = \pi$ in Eq. (5.3), and examine once more the integral for the case in which $1 \ll \Delta E \ll E$. We find that the cross section is again given by the formula (5.5) plus the term

$$-(\alpha/\pi) d\sigma_0 \ln^2(2\Delta E). \tag{5.7}$$

The formula (5.5) can also be used to get an analytic expression for the cross section in the lab system for the case in which both emitted photons are not soft. The cross section $d\sigma_{\text{real}}(\Delta E)$ is an invariant under a Lorentz transformation, and we therefore only need to express the quantities in Eq. (5.5) by the corresponding quantities in the lab system. The energy of the primary photon transforms like $\omega_0^{\text{c.m.}} = (\frac{1}{2}\omega_0^{\text{lab}})^{1/2}$, so $E = (2\omega_0^{\text{lab}})^{1/2}$. In the c.m. system ΔE is the isotropic upper limit of the energy of the photon k_2. In the lab system this corresponds to an anisotropic energy resolution in which the vector $\vec{k}_2$ is confined to an ellipsoid.

The cross section for the double Compton effect contains the nonphysical photon mass λ. In order to obtain the physical cross section to first order in α we have to add the double Compton cross section to the expression for the radiative corrections. Thus λ disappears as it should. We shall examine more closely the correction in the c.m. system at high energies. The expression for the correction

$$\delta^{\text{c.m.}} = \delta_{\text{vir}} + \frac{d\sigma_{\text{real}}(\Delta E)}{d\sigma_0} \tag{5.8}$$

is, according to Eqs. (4.1) and (5.5),

$$\delta^{\text{c.m.}} = -\frac{\alpha}{\pi U}\left\{2(1-2y)U\ln(2\Delta E) + \frac{\pi^2}{6}\left(4 - 3t - \frac{1}{t} - \frac{2}{E^4t^3}\right) + 4(2-U)y^2 - 4y + \frac{3}{2}U + \frac{2}{E^2t^3} + 4\left(1-\frac{1}{2t}\right)\ln^2 E \right.$$
$$+\left(2t + \frac{1}{t} - 2 - \frac{2}{E^4t^3}\right)L_2(1-E^2t) + \left[2 - 5t - \frac{2}{t} + 4y\left(\frac{2}{t} + t - 2\right)\right]\ln E - \frac{1}{2}U\ln^2(1-t) - U\,L_2(t)$$
$$\left. + \left[1 - \frac{2}{t} - \frac{2}{E^2t^3} - \frac{E^2}{1-E^2t} + 4y\left(t - 1 + \frac{1}{2t}\right)\right]\ln(E^2t)\right\}, \tag{5.9}$$

where $t = \frac{1}{2}(1+\beta_1\cos\theta_1)$, $y = \ln[E\sin(\frac{1}{2}\theta_1)]$, $U = t + 1/t$, and $\Delta E \ll E$.

The correction $\delta^{\text{c.m.}}$ above is valid for all angles which give a significant contribution to the total cross section. This correction contains terms proportional to $\ln^2 E$, and these terms will be very large at extremely high energies. To see what happens when $E \to \infty$ we consider the following four cases:

(a) $t=1$, $\theta_1 = 0$: In the forward direction $d\sigma_{\text{real}} \times(\Delta E) = 0$ according to Eq. (5.6) and only the radiative corrections contribute to $\delta^{\text{c.m.}}$. From the exact formula of Brown and Feynman, Eq. (2.6), we find at high energies

$$\delta_A^{\text{c.m.}} = \delta_{\text{vir}} = (\alpha/\pi)\,(2\ln^2 E + \ln E + \tfrac{1}{12}\pi^2 - \tfrac{3}{2}), \tag{5.10}$$

and the correction is seen to be proportional to

$(\alpha/\pi)\ln^2 E$ when $E\to\infty$.

(b) $t\sim O(1)$, θ_1 is not close to π and not very small: For this case Eq. (5.9) is valid and keeping only the $\ln^2 E$ terms we get

$$\delta_B = 4(\alpha/\pi)\ln(E/\Delta E)\ln E. \qquad (5.11)$$

The correction is of order $(\alpha/\pi)\ln E$ since $\ln(\Delta E/E)$ is taken to be of order one.

(c) $t\ll 1$, θ_1 is close to π: Keeping terms such as $\ln^2 E$ and $\ln^2 t$ in Eq. (5.9) we find

$$\delta_C = 4(\alpha/\pi)\left[\ln E\ln(\Delta E/E) + \tfrac{1}{8}\ln^2 t\right]. \qquad (5.12)$$

Here the first term is of the order $(\alpha/\pi)\ln E$, but the last term becomes of the order $(\alpha/\pi)\ln^2 E$ when θ_1 is of the order $\pi - 1/E$. As is shown in Sec. VI, Eq. (6.13), the $\ln^2 t$ term is large in a sufficiently large region of the angles to give a contribution of the order $(\alpha/\pi)\ln^2 E$ to the correction to the total cross section.

(d) $t\ll 1$, $\theta_1=\pi$: For this case formula (5.9) gives $\delta^{\text{c.m.}} = 4(\alpha/\pi)\left[\ln E\ln\Delta E - \tfrac{1}{2}\ln^2 E\right]$, but according to Eq. (5.7) we should add another term $-(\alpha/\pi)\times\ln^2(2\Delta E)$ from the double Compton effect.

Hence we have

$$\delta_D = (\alpha/\pi)\left[\ln^2 E + 2\ln E\ln(\Delta E/E) - \ln^2(\Delta E/E)\right]$$

$$\approx (\alpha/\pi)\ln^2 E, \qquad (5.13)$$

and we find that $\ln^2 E$ terms are also present in the backward direction.

VI. TOTAL CROSS SECTIONS

In the following we integrate the high-energy differential cross sections given above, and we obtain the corrections to the total Compton cross section to first order in α.

At high energies the integration of the Klein-Nishina formula Eq. (2.1) yields

$$\sigma_0 = (2\pi r_0^2/|\kappa|)(\ln|\kappa| + \tfrac{1}{2}), \qquad (6.1)$$

where $\kappa = -E^2$ in the c.m. system and $\kappa = -2\omega_0$ in the lab system.

The integration of the radiative corrections Eq. (4.1) can be performed analytically. The result is

$$\sigma_R = -(2\alpha r_0^2/|\kappa|)\left[\tfrac{1}{3}\ln^3|\kappa| - \tfrac{1}{2}\ln^2|\kappa| - \tfrac{5}{6}\ln|\kappa| + L_3(1) + \tfrac{1}{3}\pi^2 + \tfrac{19}{4} + (-2\ln^2|\kappa| + \ln|\kappa| + \tfrac{5}{2} + \tfrac{1}{3}\pi^2)\ln\lambda\right]. \qquad (6.2)$$

The trilogarithm L_3 is equal to the Riemann ζ function of argument 3[10]

$$L_3(1) = \zeta(3) = 1\,202. \qquad (6.3)$$

It is also possible to integrate the exact expression of Brown and Feynman, Eq. (2.6), analytically for all energies, expressing the result by dilogarithms and trilogarithms. To check our results we have performed this integration and made the high-energy approximation in the final result.

In the c.m. system ($\Delta E \ll E$) we get, by integration of the cross section in Eq. (5.5),

$$\sigma_{\text{real}}^{\text{c.m.}}(\Delta E) = -(\alpha r_0^2/E^2)\left[4\ln^3 E - 3\ln^2 E + (\tfrac{2}{3}\pi^2 - 1)\ln E + \tfrac{1}{12}\pi^2 - 2L_3(1) - \tfrac{1}{2} + (-8\ln^2 E + 2\ln E + \tfrac{5}{2} + \tfrac{1}{3}\pi^2)\ln(2\Delta E/\lambda)\right]. \qquad (6.4)$$

Integration of $d\sigma_{\text{real,soft}}^{\text{lab}}$ of Eq. (4.3) gives the total cross section for the soft-photon double Compton effect in the laboratory system,

$$\sigma_{\text{real,soft}}^{\text{lab}} = -(\alpha r_0^2/\omega_0)\left[\ln^3 2\omega_0 - \tfrac{1}{2}\ln^2 2\omega_0 - (3 + \tfrac{1}{6}\pi^2)\ln 2\omega_0 + 2L_3(1) + \tfrac{1}{4}\pi^2 + 2 + (-2\ln^2 2\omega_0 + 2\ln 2\omega_0 + \tfrac{5}{2} + \tfrac{1}{3}\pi^2)\ln(2\omega_{2\max}/\lambda)\right]. \qquad (6.5)$$

In order to obtain the total cross section for double Compton scattering in which both emitted photons may have high energies we integrate the exact expression of Eq. (2.7) numerically. Using the variables θ_1, θ_2, φ (of Fig. 4), and ω_2 we compute in the laboratory system

$$\sigma_{\text{real,hard}}^{\text{lab}} = \frac{1}{2}\alpha r_0^2\frac{1}{2\pi}\int_{-1}^{+1} d(\cos\theta_2)\int_{-1}^{+1} d(\cos\theta_1)\int_0^{2\pi} d\varphi \times\int_{\omega_{2\max}}^{\omega_{1\max}} d\omega_1\frac{\omega_1\omega_2 X}{\omega_0[1+\omega_0(1-\cos\theta_2)-\omega_1(1-\cos\theta_{12})]}. \qquad (6.6)$$

Here $\omega_{1\max}$ is given by Eq. (3.4). We have multiplied the cross section by a factor $\frac{1}{2}$ since by integration over all possible magnitudes and directions of $\vec{k}_1$ and $\vec{k}_2$ we include identical final states exactly twice. It is convenient to subtract the term containing the $\omega_{2\max}$ dependence in Eq. (6.6) before making the numerical integration. This procedure improves the accuracy of the method. We also computed the rms deviation to get a measure of the accuracy [see text following Eq. (6.19)].

The integrations in this section are quite similar to those made by Andreassi *et al.*[11] when calculating the corrections to the cross section for electron-positron pair annihilation. As a check of both their calculations and our method of integration we have also computed the three-photon annihilation total cross section by numerical integration. For this case, however, the domain of integration in phase space is more complicated, and we were not able to subtract the contribution from low-energy photons before making the numerical calculation. Consequently the accuracy of the method became much

TABLE III. Quantities $G_{VS}^{\text{c.m.}}$ and $G^{\text{c.m.}}$ as functions of E. The correction to the Compton scattering cross section in the c.m. system is given by $\Delta^{\text{c.m.}} = G_{VS}^{\text{c.m.}} + G^{\text{cm}} \times \ln 2\Delta E$.

E(MeV)	$10^2 G_{VS}^{\text{c.m.}}$	$10^2 G^{\text{c.m.}}$
10	−3.90	2.13
50	−12.0	3.70
100	−16.8	4.37
250	−24.4	5.25
500	−30.6	5.86
10^3	−37.9	6.53
10^4	−67.6	8.71

poorer, of the order 20–40% for energies below 100 MeV, and even larger for higher energies. Within the limits of error our results confirm those of Andreassi *et al.*

From the relations given above there now follow the corrections to the total cross section at high energies. The cross section including corrections to first order in α is

$$\sigma = \sigma_0(1+\Delta), \qquad (6.7)$$

where

$$\Delta = (1/\sigma_0)\,(\sigma_{\text{vir}} + \sigma_{\text{real}}). \qquad (6.8)$$

In the c.m. system we find

$$\Delta^{\text{c.m.}} = G_{VS}^{\text{c.m.}} + G^{\text{c.m.}} \ln(2\Delta E), \qquad (6.9)$$

where $\Delta E \ll E$ and

$$G_{VS}^{\text{c.m.}} = -(\alpha/\pi V^{\text{c.m.}})\,[\tfrac{20}{3}\ln^3 E - 5\ln^2 E - (1+\pi^2)\ln E - L_3(1) + \tfrac{5}{12}\pi^2 + \tfrac{17}{4}], \qquad (6.10)$$

$$G^{\text{c.m.}} = (\alpha/\pi V^{\text{c.m.}})\,[-8\ln^2 E + 2\ln E + \tfrac{5}{2} + \tfrac{1}{3}\pi^2], \qquad (6.11)$$

$$V^{\text{c.m.}} = 2\ln E + \tfrac{1}{2}. \qquad (6.12)$$

The functions $G_{VS}^{\text{c.m.}}$ and $G^{\text{c.m.}}$ are tabulated in Table III, and $\Delta^{\text{c.m.}}$ is plotted in Fig. 6 as a function of E for several values of $\Delta E/E$. At extremely high energies the $\ln^2 E$ and $\ln E \ln \Delta E$ terms in $\Delta^{\text{c.m.}}$ are dominant. Hence we have

$$\Delta_{E\to\infty}^{\text{c.m.}} = (\alpha/\pi)\,[\tfrac{2}{3}\ln^2 E + 4\ln E \ln(\Delta E/E)]$$

$$\approx (2\alpha/3\pi)\ln^2 E, \qquad (6.13)$$

since $\ln(\Delta E/E)$ is of order one.

In the laboratory system we obtain the correction

$$\Delta^{1\text{ab}}(\omega_{2\text{max}}) = G_{VS}^{1\text{ab}} + G^{1\text{ab}} \ln(2\omega_{2\text{max}}), \qquad (6.14)$$

where

$$G_{VS}^{1\text{ab}} = -(\alpha/\pi V^{1\text{ab}})\,[\tfrac{4}{3}\ln^3 2\omega_0 - \ln^2 2\omega_0 - (3+\pi^2)\ln 2\omega_0 + 3L_3(1) + \tfrac{7}{12}\pi^2 + \tfrac{27}{4}], \qquad (6.15)$$

$$G^{1\text{ab}} = -(\alpha/\pi V^{1\text{ab}})[-2\ln^2 2\omega_0 + 2\ln 2\omega_0 + \tfrac{5}{2} + \tfrac{1}{3}\pi^2], \qquad (6.16)$$

$$V^{1\text{ab}} = \ln 2\omega_0 + \tfrac{1}{2}, \qquad (6.17)$$

and $\omega_{2\text{max}}$ is the isotropic upper limit of the energy of one of the emitted photons. The functions $G_{VS}^{1\text{ab}}$ and $G^{1\text{ab}}$ are tabulated in Table IV and $\Delta^{1\text{ab}}(\omega_{2\text{max}})$ is plotted as a function of ω_0 for several values of $\omega_{2\text{max}}$ in Fig. 7.

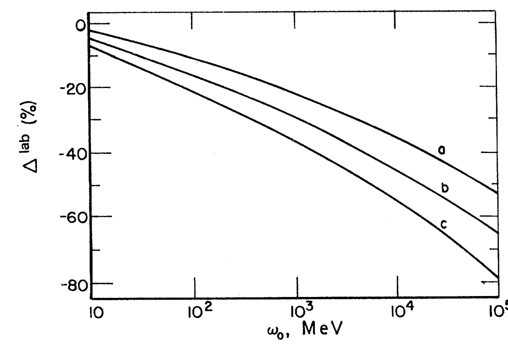

FIG. 7. Correction Δ^{lab} to the total cross section for Compton scattering as a function of ω_0. The curves a, b, and c correspond to $\omega_{2\text{max}} = 10^{-1}$, 10^{-2}, and 10^{-3}, respectively.

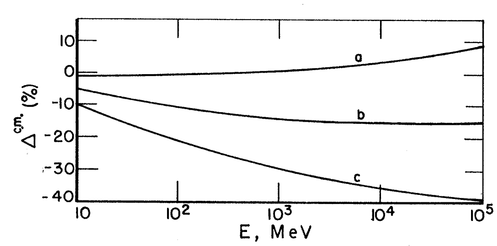

FIG. 6. Correction $\Delta^{\text{c.m.}}$ to the total cross section for Compton scattering as a function of E. The curves a, b, and c correspond to $(\Delta E/E) = 10^{-1}$, 10^{-2}, and 10^{-3}, respectively.

TABLE IV. Quantities G_{VS}^{lab}, G^{lab}, and G_H^{lab} as functions of primary photon energy ω_0. The quantity $G_{VS}^{\text{lab}} + G^{\text{lab}} \times \ln 2\omega_{2\,\text{max}}$ gives the correction to the total Compton cross section from radiative corrections and soft photons, while $G_H^{\text{lab}} - G^{\text{lab}} \ln \omega_{2\,\text{max}}$ gives the correction from hard photons.

ω_0(MeV)	$10^2 G_{VS}^{\text{lab}}$	$10^2 G^{\text{lab}}$	$10^2 G_H^{\text{lab}}$
10	−1.21	0.78	1.00
50	−4.68	1.58	4.33
100	−6.73	1.92	6.40
250	−9.90	2.38	10.05
500	−12.80	2.70	14.30
10^3	−15.80	3.03	18.15
10^4	−28.20	4.37	...

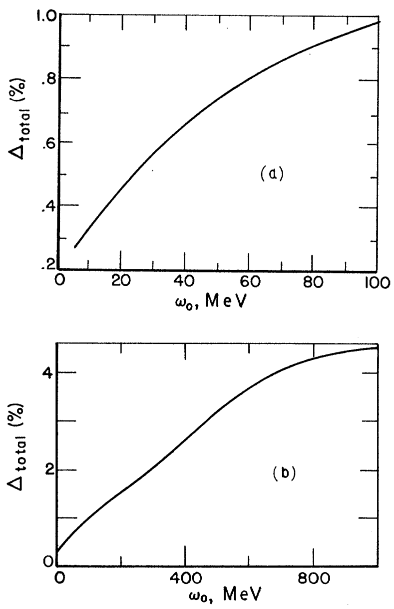

FIG. 8. (a) Correction Δ_{total} to the total cross section for Compton scattering as a function of the photon energy ω_0 for ω_0 in the range 10–100 MeV. The correction includes contributions from lowest-order radiative corrections and the total cross section for the double Compton effect. (b) Correction Δ_{total} [cf. Fig. 8(a)] for ω_0 in the range 10–1000 MeV.

The correction $\Delta^{1ab}(\omega_{2max})$ of Eq. (6.14) contains only the contribution from radiative corrections and soft photons. Part of the contribution from hard photons can be obtained from the cross section in the c.m. system by a Lorentz transformation as explained in Sec. V. If we want to include the complete contribution from double Compton scattering we have to add to $\Delta^{1ab}(\omega_{2max})$ the quantity

$$\Delta_H^{1ab} = (1/\sigma_0)\sigma^{1ab}_{real,hard}, \tag{6.18}$$

where $\sigma^{1ab}_{real,hard}$ is given by Eq. (6.6). We write

$$\Delta_H^{1ab} = G_H^{1ab} - G^{1ab}\ln(\omega_{2max}), \tag{6.19}$$

where G_H^{1ab} is the quantity computed numerically and it is given in Table IV. The rms error in this quantity increases from 3% at 500 MeV and about 20% at 1000 MeV.

The correction including radiative corrections to first order in α and the complete double Compton effect is then

$$\Delta_{total} = G_{VS}^{1ab} + G^{1ab}\ln 2 + G_H^{1ab}. \tag{6.20}$$

This quantity is plotted as a function of ω_0 in Fig. 8.

VII. DISCUSSION

In the previous sections we have established expressions for the corrections to the Klein-Nishina formula arising from lowest-order radiative corrections and double Compton scattering.

The corrections as a function of the relative energy resolution $\Delta E/\omega_s$ (ω_s is the maximum energy of the emitted photon) for a rectangular-type energy resolution is shown in Fig. 5 for several energies and scattering angles. An exponential-type energy resolution would not change these results much, but would tend to reduce the contribution from the double Compton scattering. At energies of 1 MeV or lower the correction is below 1%, and it varies only some tenths of a percent if $\Delta E/\omega_s$ varies by 10 to 20%. It is probably hard to detect this effect since it will be masked by other effects such as background, multiple scattering, electron motion, atomic bindings, etc. The correction is large only for extremely high energy resolutions. The corrections increase with increasing energy and also the dependence on $\Delta E/\omega_s$ becomes stronger. This general behavior was also found by Anders[12] who calculated the corrections to Compton scattering for the case in which the energy of the recoiling electron is recorded. Present experiments on the differential Compton cross section are not very accurate, see, e.g., the experiment of Peyman *et al.*[13] In this experiment photons of 17 MeV are scattered at small angles and recorded in coincidence with the outgoing electrons. The uncertainty in the experimental results is 15%, while the authors calculate the corrections to be about 5%.

The corrections to the cross section in the c.m. system show a behavior which seems to be quite regular for high-energy processes in quantum electrodynamics. At scattering angles which are not too close to 0 or π the corrections are of order $(\alpha/\pi)\ln E$ where E is the total energy, while at angles close to 0 or π the corrections are of order $(\alpha/\pi)\ln^2 E$. The same type of behavior is also found for the corrections to the electron-positron pair-annihilation cross section.[11] Eriksson[14] has shown that for electron scattering in a potential the effective expansion parameter at high-momentum transfers is $(\alpha/\pi)\ln q^2$, where q is the invariant momentum transfer of the process. According to our results for Compton scattering a high-momentum transfer is not sufficient to make the lowest-order corrections of order $(\alpha/\pi)\ln q^2$, but if not only $q^2 = (p_1 - p_2)^2 = (k_0 - k_1)^2$ is large, but also $(p_1 - k_1)^2 = (k_0 - p_2)^2$ is large, then the corrections reduce to the order $(\alpha/\pi)\ln E$. This means that all angles between incoming and outgoing particles must be large. For the case of pair annihilation the concept of momentum transfer is somewhat obscure, but also here the corrections are of order

$(\alpha/\pi)\ln E$ if all angles between incoming and outgoing particles are large. If one of the angles is small, then the corrections become of order $(\alpha/\pi)\ln^2 E$. Since both for Compton scattering and pair annihilation, small angles give an important contribution to the total cross section, the corrections to the total cross sections for both processes are of order $(\alpha/\pi)\ln^2 E$. The similarity between these processes is underlined by the fact that the coefficient multiplying the largest terms turns out to be the same, namely, $(2\alpha/3\pi)\ln^2 E$ in the c.m. system.

Recently Cheng and Wu[15] have made a systematic study of all two-body elastic scattering amplitudes in quantum electrodynamics at high energies. According to these authors the differential cross section $d\sigma/dt$, where $-t$ is the square of the momentum transfer, is finite for all processes in the limit of infinite energy. For the Compton effect the second- and fourth-order diagrams considered in the present article give a vanishing cross section $d\sigma/dt$ at infinite energies. According to Cheng and Wu the sixth-order diagrams lead to a constant cross section, and these diagrams therefore become dominant at a certain very high energy. This energy is not known since numerical results have not yet been given by Cheng and Wu.

The correction to the total cross section for Compton scattering is shown in Fig. 8. This correction which is the sum of the radiative corrections and the total cross section for double Compton scattering, should be included in the absorption coefficient for γ rays. Compton scattering does not contribute much to the absorption coefficient for energies over about 500 MeV, but for lower energies the process is important, and therefore also its radiative corrections. Photon absorption measurements may be performed very accurately, accuracy better than 1%, and they give a valuable check on quantum electrodynamics. The various electrodynamic processes which contribute to the absorption, photoelectric effect, Compton effect, pair production, triplet production, etc., are now known with an accuracy of about 1% or better for a large range of energies and materials. For a review of theoretical and experimental data on the photon absorption process see the paper of Hubbel[16] in which the effects of the present corrections to Compton scattering are included.

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