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Stopping Power and Energy Straggling for Swift Protons

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Calculations and measurements of energy dissipation by protons at energies above ~ 100 keV are presented. The calculations, which make use of a statistical model of the atom, are based on a refinement of a procedure suggested by Lindhard and Scharff. The theoretical section of the present paper is concerned with energy straggling, as stopping powers were dealt with in an earlier publication. Measurements of stopping power and energy straggling for 100–500-keV protons have been made in various gases, viz. hydrogen, helium, air, neon, argon, and krypton. The stopping-power data are in good agreement with theory and earlier experimental work. For the heavy gases, the experimental straggling values are seen to be an increasing function of energy, as expected from theory. In a more quantitative comparison, however, some discrepancy between theory and experiment is observed.

I. INTRODUCTION

Consider a beam of heavy particles of low charge number traversing matter. The average energy loss is a quantity of great interest, and it has been studied extensively, both theoretically and experimentally. Sometimes, however, it becomes necessary to pay attention to the fact that during penetration, the energy distribution of the beam is broadened. This happens because slowing down is the result of individual collision events, the number of which is governed by statistical laws.

In the following we present a theoretical and experimental investigation of straggling, i.e., the mean-square deviation in energy loss. For a sufficiently fast particle, only collisions with electrons contribute appreciably to the slowing-down (electronic stopping, Sec. IV B). Only such cases will be treated below. On the other hand, we concentrate on such low energies that a well-known asymptotic expression for straggling does not apply.

II. THEORY

A general discussion of energy loss has been given by Bohr.[1] For the straggling Ω^2, one obtains in limiting cases

$$\Omega^2 = \Omega_B^2 = 4\pi Z_1^2 Z_2 e^4 N \Delta R \ , \qquad (1)$$

where $-e$ is the charge of the electron, Z_1 and Z_2, the atomic numbers of the projectile and the target atoms, respectively, N the number of atoms per unit volume, and ΔR the target thickness. The above expression is derived under the following conditions: (i) The target atoms are randomly distributed; (ii) the velocity of the projectile is high as compared to the orbital velocities of the target electrons; (iii) the energy of the projectile is changed only slightly during penetration. In the following, conditions (i) and (iii) will always be assumed to be fulfilled.

If, for some of the electrons, condition (ii) is not satisfied, a calculation of average energy loss and straggling becomes rather complicated. A fairly simple treatment of cases, where condition (ii) is not necessarily fulfilled, was suggested by Lindhard and Scharff and found to be useful.[2,3] The idea was to use, as far as possible, a comparison with an electron gas of constant density, for which exact results can be obtained. Such a treatment is of Thomas-Fermi type and thus contains the kind of similarity which is characteristic for a Thomas-Fermi description.

In the case where a heavy nonrelativistic particle of charge $Z_1 e$ and velocity v only exerts a perturbing influence on a free electron gas of density ρ, all quantities connected with the slowing down of the particle can be expressed in terms of the longitudinal dielectric constant $\epsilon^l(k, \omega)$ for the gas.

$\vec{k}$ and ω refer to Fourier components of functions of space and time, respectively.[4] Lindhard[4] found the following formula for the straggling Ω^2:

$$\Omega^2 = -\Delta R \frac{2Z_1^2 e^2 \hbar}{\pi v^2} \mathrm{Im}\left(\int_0^\infty \frac{dk}{k} \int_0^{kv} d\omega \frac{\omega^2}{\epsilon^I(k,\omega)} \right) . \tag{2}$$

By using the explicit expression for $\epsilon^I(k,\omega)$, asymptotic formulas were derived for the cases $v/v_F \gg 1$ and $v/v_F \ll 1$, v_F being the Fermi velocity belonging to the gas. For $v/v_F \gg 1$, Eq. (2) turned into (1) with the obvious replacement $\rho \to NZ_2$. In the other limit, Ω^2 was found to be proportional to v^2, implying a great reduction as compared to (1).

Lindhard and Scharff[2] treated the case of a real target material by dividing up the electron cloud of a target atom into "outer" and "inner" electrons, the outer ones being roughly those corresponding to a (local) Fermi velocity lower than v. Letting outer and inner electrons contribute according to asymptotic expressions for an electron gas and using a simple and rather crude model for the density $\rho(r)$ of atomic electrons at a distance r from the nucleus, Lindhard and Scharff arrived at the following formula:

$$\Omega^2 = \Omega_B^2 \begin{cases} \frac{L(x)}{2} & \text{for } x \lesssim 3 \\ 1 & \text{for } x \gtrsim 3. \end{cases} \tag{3}$$

Here, x is a reduced energy variable

$$x \equiv v^2/v_0^2 Z_2 \quad (v_0 \equiv e^2/\hbar) , \tag{4}$$

and $L(x)$ is given in terms of the stopping power dE/dR by

$$\frac{dE}{dR} \equiv \frac{4\pi Z_1^2 e^4}{mv^2} N Z_2 L(x) , \tag{5}$$

where m is the electron mass.

A similar calculation of the stopping power gave the following result for $L(x)$:

$$L(x) = 1.36 x^{1/2} - 0.016 x^{3/2} . \tag{6}$$

Actually, it is only to a first approximation that L, as defined in (5), can be written as a function of one variable.

Experimental measurements in metals by Madsen,[5] by Nielsen,[6] and by Chilton *et al.*[7] were consistent with (3). Particularly, there was an indication of a decrease in straggling at x values lower than ~3. However, the measured points scattered to an extent where a more detailed comparison with theory was not possible. The experiments to be reported in Sec. III are considered accurate enough to allow such a comparison, and it even seems worthwhile to try to improve upon the theoretical evaluations.

An obvious refinement of the Lindhard-Scharff calculation would be to use a more realistic formula for $\rho(r)$ together with a more accurate expression for the contribution to the straggling $\Omega^2(r,v)$ from the various parts of the electron cloud. Indeed, a similar refinement of the Lindhard-Scharff calculation for dE/dR turned out to be profitable.[8]

In order to obtain $\Omega^2(r,v)$, numerical calculations of Ω^2 in (2) were performed, using the exact expression for $\epsilon^I(k,\omega)$.[4,9] The results are shown in Fig. 1. The density is conveniently given in terms of the dimensionless parameter χ^2, where

$$\chi^2 \equiv e^2/\pi\hbar v_F, \quad v_F = (\hbar/m)(3\pi^2\rho)^{1/3} . \tag{7}$$

Typical electron densities in atoms correspond to $\chi^2 \sim 10^{-1}$–10^{-2}. In the same figure, the numerical results are compared with analytic, approximate formulas for Ω^2. For high velocities, the expression was derived from (2) by expanding $\epsilon^I(k,\omega)$ for large k. Explicitly, the following formula was obtained:

$$\frac{\Omega^2}{\Omega_B^2} \simeq 1 + \left[\frac{1}{5}\left(\frac{v_F}{v}\right)^2 + \frac{\hbar\omega_0}{2mv^2} \right] \ln\left(\frac{v}{v_F}\right)^2 \quad \text{for } v > v_F . \tag{8}$$

The term with the plasma frequency $\omega_0 \equiv (4\pi e^2\rho/m)^{1/2}$ is a contribution from resonance collisions due to collective excitations. However, this is only a small fraction of Ω^2/Ω_B^2, the main part of which comes from single-particle collisions. For dE/dR, the situation is different, the contributions from single-particle and resonance collisions being equally important at high velocities.[9]

For $v \ll v_F$, one has the asymptotic expression[4]

$$\Omega^2/\Omega_B^2 = A(\chi)(v/v_F)^2 , \tag{9}$$

with $A(\chi)$ of the order of 1 at ordinary electron densities. As shown in Fig. 1, a reasonably good fit to the numerical calculations was obtained by setting

$$A(\chi) = (1 + 13\chi^2)^{-1/2} . \tag{10}$$

It is seen that formula (9) together with formula (10) is a somewhat less accurate approximation than formula (8).

For the atomic case, we now suggest the following formula for Ω^2/Ω_B^2:

$$\frac{\Omega^2}{\Omega_B^2} = \frac{1}{Z_2} \int_0^\infty 4\pi r^2 \rho(r) \frac{\Omega^2(r,v)}{\Omega_B^2} dr , \tag{11}$$

where

$$\frac{\Omega^2(r,v)}{\Omega_B^2} = 1 + \left(\frac{1}{5} + \frac{\chi(r)}{\sqrt{3}}\right)\left(\frac{v_F(r)}{v}\right)^2 \ln\left(\frac{v}{v_F(r)}\right)^2 \tag{12a}$$

$$= \frac{1}{(1+13\chi^2)^{1/2}} \left(\frac{v}{v_F(r)}\right)^2 . \tag{12b}$$

For $v \leq v_F(r)$ we use (12b), and for $v \geq v_F(r)$, the expression with the lower value should be applied. The functions $v_F(r)$ and $\chi(r)$ are given in terms of

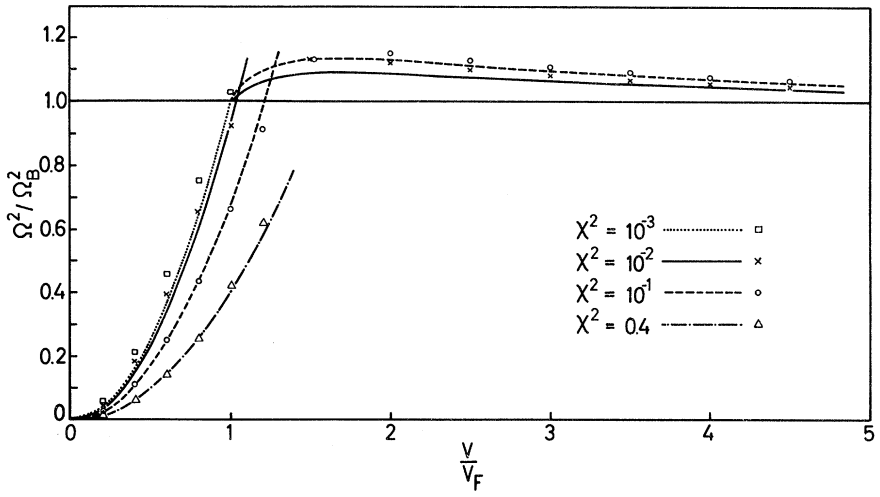

FIG. 1. Straggling as a function of projectile velocity at various densities of an electron gas. Points are taken from numerical calculations, whereas the curves represent analytic asymptotic expressions.

the electron density $\rho(r)$ by the quoted expressions for an electron gas.

In the actual calculations, an analytical form of Thomas-Fermi type for $\rho(r)$ was used, namely, the first-order Lenz-Jensen model.[10] As ρ is proportional to Z_2^2, and thus v_F to $Z_2^{2/3}$, the formulas suggest that Ω^2/Ω_B^2 should depend only weakly on Z_2, when expressed in terms of the reduced energy x_1, where

$$x_1 \equiv v^2/v_0^2 Z_2^{4/3} . \tag{13}$$

The Lindhard-Scharff formula could be written as a function of the reduced energy x, as the division between inner and outer electrons was made in terms of the variable $2mv^2/\hbar\omega_0$ rather than $2mv^2/2mv_F^2$. Below we shall display Ω^2/Ω_B^2 in both variables.

As to the validity of (11) and (12), the following remarks should be made: First, the approximation can only be expected to give good results when we obtain relatively small contributions from the inner electrons, i.e., from those that contribute according to (12b). As an example, for $Z_2 = 18$, $\Omega^2_{\text{inner}}/\Omega^2_{\text{total}}$ takes on the values 0.41, 0.22, and 0.09 at the x_1 values 0.23, 0.79, and 2.9, respectively. Thus, we should not try to use our procedure for x_1 values lower than, say, 0.1. Second, owing to the statistical nature of the treatment, our formula may not apply too well for low Z_2 values.

There is still one difficulty of a more fundamental character. According to well-known theory,[1] we account for straggling by dividing up collisions into a large number of basic statistically independent processes i, each process corresponding to an energy transfer lying in a small interval around T_i. This leads to the following formula for the straggling:

$$\Omega^2 = \sum_i T_i^2 \omega_i . \tag{14}$$

Here, the number of collisions of the ith kind only appear through their average value ω_i.

It was by means of formula (14) that Lindhard arrived at his expression for Ω^2 for the free electron gas.

The Lindhard-Scharff treatment of straggling for an atomic target material is justified if we are concerned with the same basic statistically independent excitations in the atom as in the electron gases by means of which we describe the electronic cloud. In these circumstances, one can attempt to express atomic cross sections in terms of excitation probabilities for the electron gas. Such a simple situation should occur if the average total number of single collisions, as calculated from the electron-gas picture, is small for a projectile penetrating the electron cloud of an atom. The number n of single collisions will, of course, depend on velocity v and impact parameter p. An upper bound to n can be found by means of the formulas on pp. 43–46 of Ref. 4. For $v = 2v_0$, such an upper bound, generous enough to apply for all values of p, could be set at $\sim \frac{1}{2} Z_2^{1/3} Z_1^2$. However, for most impact parameters, n will be well below this value. This makes it seem reasonable to apply the deduced formulas for proton energies down to ~100 keV. But notice that n increases as Z_1^2.

As discussed elsewhere,[8] a calculation based on results from a *perturbation* treatment of an electron gas should, in any case, be limited to proton energies above ~100 keV.

A calculation similar to the usual derivation of Bethe's stopping formula gives the following asymptotic formula[11]:

$$\frac{\Omega^2}{\Omega_B^2} = 1 + \frac{4}{3} \frac{\langle K \rangle}{mv^2} \ln \frac{2mv^2}{I_1} \quad , \quad v \to \infty . \tag{15}$$

Here, $\langle K \rangle$ is the average kinetic energy of a target electron and I_1 is given in terms of transition fre-

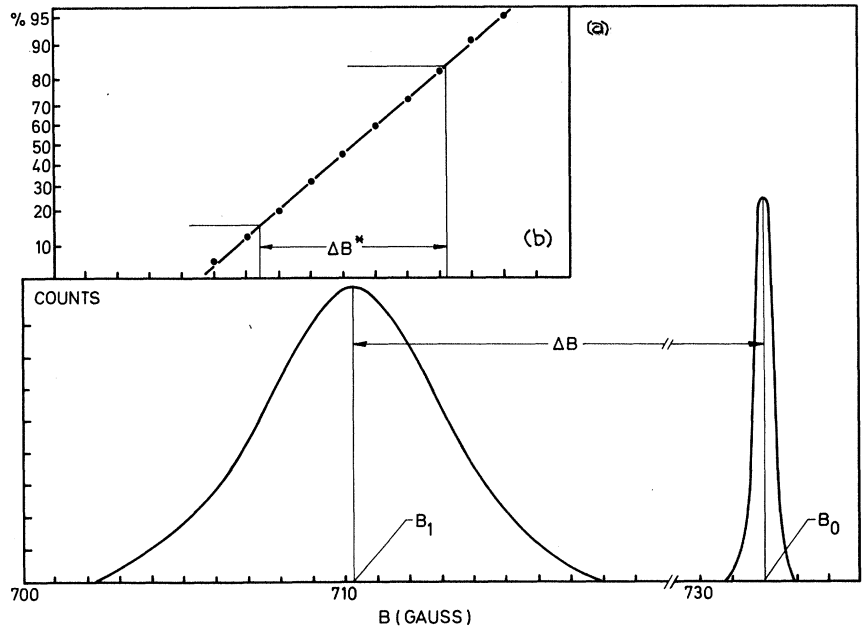

FIG. 2. (a) Momentum distribution for a proton beam with an initial energy of 200 keV before and after traversing 4.92×10^{17} atoms/cm^2 of neon. ΔB is the reduction in magnetic field corresponding to the two peak values. (b) The low-energy distribution shown on probability paper. ΔB^* is twice the standard deviation.

quencies ω_{nn_0} and corresponding dipole oscillator strengths f_{nn_0} by

$$\ln I_1 = \sum_n f_{nn_0}\hbar\omega_{nn_0}\ln\hbar\omega_{nn_0} / \sum_n f_{nn_0}\hbar\omega_{nn_0} . \quad (16)$$

Equation (15) can only be used if the logarithm is somewhat larger than 1, i.e., at somewhat higher energies than those corresponding to the maximum in Eq. (15).

Evidently, an evaluation of the quantities appearing in Eq. (15) can be made with most confidence for the lightest materials. For heavy substances, Livingstone and Bethe[12] have tried in a semiempirical way to rewrite Eq. (15) in terms of ionization potentials and effective nuclear charges for the various electronic shells. In the Lindhard-Scharff description, such difficulties are avoided.

III. EXPERIMENTAL

Reference has already been given to experimental investigations of energy straggling in solid targets. Such materials present difficulties, however, because foil inhomogeneities and crystalline structures tend to increase the straggling in an uncontrollable way.

Mason *et al.*[13] and Raminez *et al.*[14] have avoided this problem by using gas targets, but the mean energy loss in their experiments is considerable compared with the initial energy. This means that condition (iii) in Sec. II is not fulfilled and that a comparison with simple theory becomes difficult. (The measured straggling per path length will be larger than the one calculated for negligible energy loss, when the stopping power is a decreasing function of energy; see, e.g., Briggs[15] and Tschalär.[16])

In the present experiment, we have measured stopping power and energy straggling of protons at energies from ~100 to ~500 keV in hydrogen, helium, air, neon, argon, and krypton. In order to overcome the problems mentioned above, we have used differentially pumped gas targets and an analyzing magnet with an energy resolution of 0.1%.

Since details of the experimental procedure were presented in a previous publication,[17] only a brief description will be given here.

A magnetically analyzed proton beam from the Aarhus 600-keV accelerator was passed through a differentially pumped, 828-mm-long gas chamber via 0.3-mm-diam apertures. Having traversed the gas chamber, the ions which scattered to angles within a small solid angle around the forward direction were analyzed by means of an analyzing magnet and detected by an open electron multiplier.

Figure 2 shows a momentum spectrum taken with and without gas in the target chamber. In all cases, a pressure sufficient to give a Gaussian distribution was chosen (normally 0.3–1.5 Torr). In order to confirm this experimentally, the distributions were plotted on probability paper, where they appeared as straight lines.

From a momentum spectrum like the one in Fig. 2, the average energy loss can be found directly because the distributions are symmetric. Since the distributions are Gaussian, the straggling is found by means of the formula $\Omega^2 = \Omega_2^2 - \Omega_1^2$, where Ω_2 and Ω_1 are the standard deviations of the energy distribution corresponding to measurements with and without gas in the chamber.

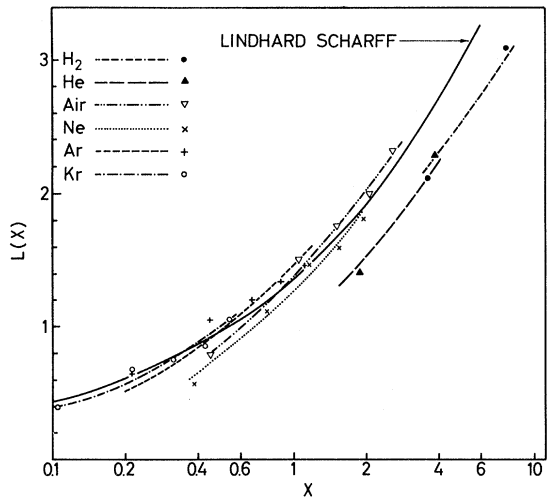

FIG. 3. Experimental values of $L(x)$ compared with theory (cf. text). In all cases, the curves represent the theoretical expressions.

IV. RESULTS AND DISCUSSION

A. Stopping Cross Section

The stopping cross section $S \equiv (1/N) \times dE/dR$ is found as $\overline{\Delta E}/N\,\Delta R$ at the energy $E = E_i - \frac{1}{2}\overline{\Delta E}$, where E_i is the initial energy. In all cases, the average energy loss $\overline{\Delta E}$ is less than 20% of E_i.

The experimental data are listed in Table I and represent points read off the smoothed-out curves through the measured values. The random errors in the individual measurements are estimated to be ~4%, the uncertainties in the determination of ΔB in Fig. 2 and of target pressure amounting to 1.5 and 3%, respectively. Systematic errors are due to uncertainties in target length (0.5%), in McLeod-gauge calibration (0.4%), in magnetic field (0.5%), and in energy (0.1%). The contribution of random errors to the uncertainty of the reported stopping cross sections is reduced by the process of drawing a smooth curve through the points from at least two experimental runs. The uncertainty of the stopping-cross-section values in Table I is therefore estimated to be ~4%.

TABLE I. Experimental stopping cross sections in units of 10^{-15} eV cm^2/atom.

Proton energy (keV)	H_2	He	Air	Ne	Ar	Kr
100	5.60	7.02	16.7	14.6	30.5	37.5
200	3.82	5.60	13.8	14.0	22.7	29.0
300			11.0	12.1	17.7	23.8
400			9.1	10.6	14.8	20.0
500			8.0	9.6	13.0	17.4

The stopping cross section may be compared with the results of Reynolds *et al.*[18] The agreement is within a few percent, except for argon and krypton, where the present data are 6–10% lower.

Figure 3 shows the stopping-power data, plotted as $L(x)$ vs x [cf. formula (5)]. As can be seen, the measurements are in good agreement with formula (6). In a more detailed treatment than that of Ref. 2, L becomes a function of the two variables x and Z_2. It is customary to split up the stopping power as follows:

$$\frac{dE}{dR} \equiv \frac{4\pi Z_1^2 e^4 N Z_2}{mv^2} L(x, Z_2),$$
$$L(x, Z_2) = \ln x + \ln \frac{2mv_0^2}{I(Z_2)/Z_2} - \frac{C(x, Z_2)}{Z_2} \quad . \tag{17}$$

The so-called shell corrections $C(x, Z_2)/Z_2$ are defined in a way so as to vanish in the mathematical limit $x \to \infty$.

A calculation within the Lindhard-Scharff model should be more reliable for $C(x, Z_2)/Z_2$ than for $\ln I(Z_2)$, the latter quantity being rather sensitive to the detailed distribution of the outermost electrons.[8] Using the shell corrections as derived in Ref. 8, a value of $I(Z_2)$ was chosen so as to give the best fit of formula (17) to the experimental dE/dR values. The fit was made visually and is shown in the form of $L(x, Z_2)$ vs x in Fig. 3. A change by 1% in $I(Z_2)$ will shift $L(x, Z_2)$ by 0.01.

Values of $I(Z_2)/Z_2$ thus determined are listed in Table II and are compared with estimates made by Fano[11] on the basis of high-energy experiments. For H_2 and He, theoretical values exist, and these are in agreement with experiments.[11] As Fano's list does not contain Ne, we quote the value $I(Z_2)/Z_2 = 13.1$ eV recommended by Turner.[19]

B. Energy Straggling

Although electronic collisions account for practically all of the average energy loss in the present velocity range, the nuclear contribution might still be important for the straggling because the more

TABLE II. Comparison of values of $I(Z_2)/Z_2$ from this work and from Ref. 11.

Gas	$I(Z_2)/Z_2$ (eV) (this work)	$I(Z_2)/Z_2$ (eV) (Ref. 11)
H_2	19.4	19 (theory)
He	20.0	21 (theory)
N_2		12.6
Air	12.4	
O_2		12.6
Ne	12.9	13.1[a]
Ar	10.6	10.6
Kr	9.7	10.0

[a]Taken from Turner, Ref. 19.

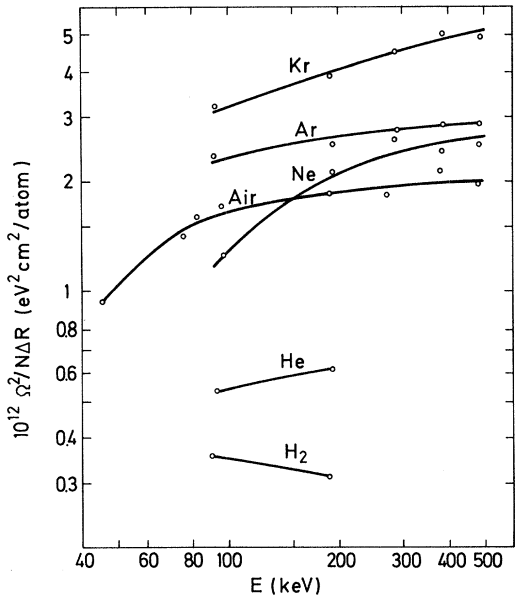

FIG. 4. Experimental energy straggling for various gases.

violent collisions have a greater influence on the straggling than on the stopping power.

If electronic and nuclear collisions are considered to be independent events, and if the Rutherford-scattering law is used, it is found that the nuclear straggling Ω_n^2 and the electronic straggling Ω_e^2 are related by the following formula:

$$\Omega_n^2/\Omega_e^2 = (M_1/M_1+M_2)^2 Z_2 , \qquad (18)$$

where M_1 and M_2 are the mass numbers of the projectile and the target atom, respectively. From formula (18) it is seen that for protons in hydrogen the electronic straggling is only four times the nuclear straggling. However, before we compare with measurements, it is important to notice the difference in energy-loss distributions resulting from electronic and from nuclear collisions, respectively. For our target thicknesses, the spectrum corresponding to electronic collisions alone would be a Gaussian distribution (many collisions with small energy transfers), whereas the nuclear encounters would give rise to an asymmetric distribution with a low-energy tail (few collisions with large energy transfers). According to Bohr, [1] the total spectrum can be described as a sum of a Gaussian distribution resulting from electronic and soft nuclear collisions and a tail of very low intensity resulting from violent nuclear collisions.

According to formula (2.4.7) of Ref. 1, the nuclear contribution Ω_n^* to the width of the Gaussian distribution amounts to less than one percent of Ω_e for the target thicknesses in question. Furthermore, all the recorded energy spectra were Gaussian. We therefore feel confident in attributing the measured straggling to pure electronic slowing down. A more detailed treatment of such problems is given in Ref. 17.

The experimental straggling values are listed in Table III, and Fig. 4 shows a plot of $\Omega^2/N\Delta R$ versus energy for different target gases. Each point represents the average of at least two measurements taken in two runs. The uncertainty of the quoted average values is estimated to be ~ 8%. In addition to the error sources listed in Sec. IV A, pressure fluctuations (less than 0.1%) contribute with 1% to the uncertainty of Ω^2.

Figure 5 shows the experimental values of Ω^2/Ω_B^2 as a function of x, and a comparison is made with the original Lindhard-Scharff expression as given by formulas (3) and (6), and with the present calculations. Figure 6 is a similar plot in the variable x_1.

Values of $\langle K \rangle$ and I_1 have been calculated by Bell and Dalgarno, [20] and for He they found $\langle K \rangle = 39$ eV and $I_1 = 81$ eV. Using these values in formula (15), one obtains $\Omega^2/\Omega_B^2 = 1.47$ and 1.41 at 94 and 192

TABLE III. Experimental straggling values.

Target gas	Proton energy (keV)	$10^{12}\Omega^2/N\Delta R$ $\frac{eV^2\ cm^2}{atom}$
Hydrogen	90	0.355
	190	0.315
Helium	94	0.535
	192	0.606
Air	45	0.93
	75	1.40
	82	1.60
	95	1.70
	187	1.85
	270	1.80
	374	2.15
	473	1.95
Neon	97	1.25
	193	2.15
	287	2.60
	386	2.40
	486	2.50
Argon	92	2.35
	194	2.50
	291	2.75
	386	2.85
	478	2.85
Krypton	93	3.20
	190	3.90
	283	4.50
	387	5.10
	485	5.00

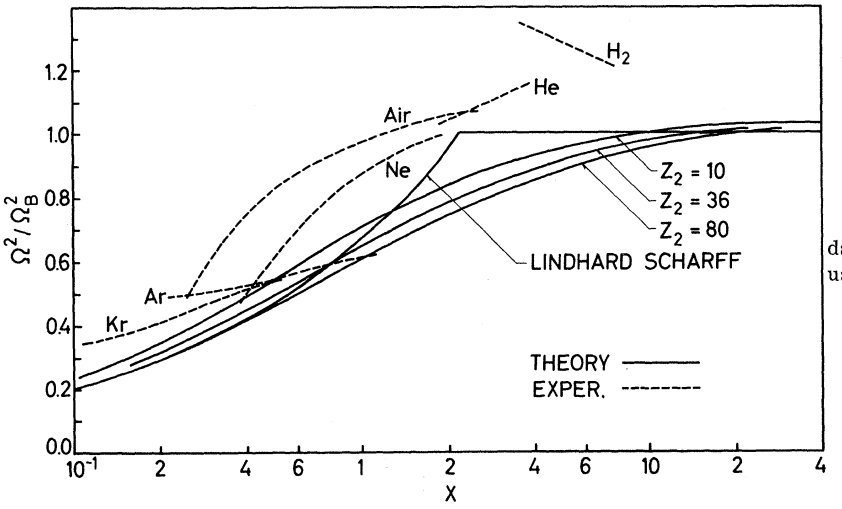

FIG. 5. Comparison of straggling data from Fig. 4 with theoretical evaluations.

keV, respectively, where the corresponding experimental values are 1.03 and 1.16. This discrepancy, not only in magnitude, but also in energy dependence, indicates that formula (15) fails at energies where the logarithm is close to one. Apparently, no theoretical values of $\langle K \rangle$ and I_1 exist for molecular hydrogen, where one would expect the logarithm to be somewhat larger than one at our energies. Actually, the experimental curve for hydrogen shows an energy dependence like the one expected from formula (15).

At the time the measurements were made, only the original Lindhard-Scharff curve was available. For the heavier elements, the experimental results agreed qualitatively with theoretical predictions. The question therefore arose whether one could obtain a better agreement by repeating the calculations with more accurate expressions for the various quantities entering the theoretical model. As can be seen from Fig. 5, the difference between curves based on new and old calculations is relatively small when compared to the difference between corresponding experimental and theoretical results. The measured curves do not agree too well with the theoretical prediction that Ω^2/Ω_B^2 is almost independent of Z_2 when expressed as a function of x or x_1. Figures 5 and 6 even show experimental curves that are somewhat different in shape.

In conclusion, the Lindhard-Scharff model gives a fair account of the over-all dependence of electronic straggling on energy. However, in a more quantitative comparison between theory and experiment, some discrepancy is revealed.

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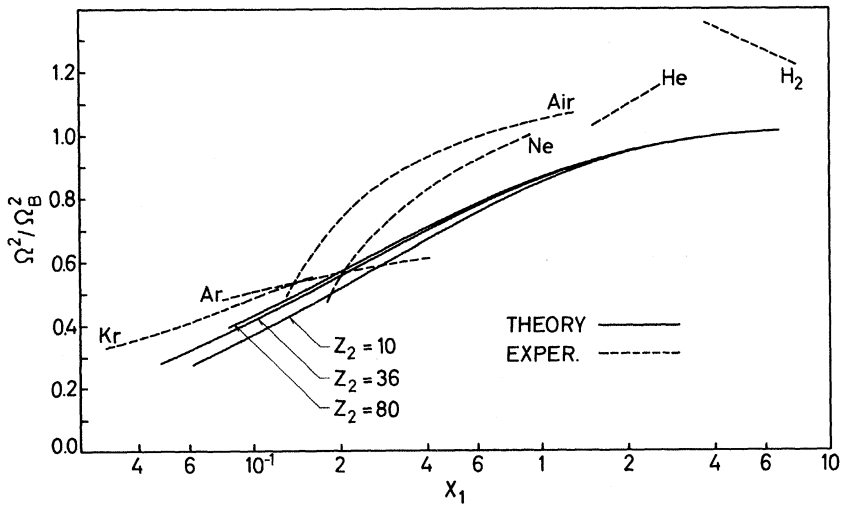

FIG. 6. Similar plot as that in Fig. 5, here in the variable x_1.

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Choosing Pseudostates in the Close-Coupling Formalism for the Electron–Atomic-Hydrogen System

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A modified close-coupling formalism has been applied to the electron–atomic-hydrogen system. We investigate various possible choices of pseudostates which are used to augment the eigenfunction expansion. As a test of the appropriateness of these choices we have evaluated various bound and scattering properties of the electron–atomic-hydrogen system and have computed photodetachment cross sections for H^- as well. We find that pseudostates which emphasize short-range interactions are superior to those which incorporate long-range polarization effects when one computes elastic scattering phase shifts in the nonresonant region below the $n=2$ threshold. Excellent photodetachment cross sections for H^- are also obtained using initial and final states obtained from a modified close-coupling formalism which includes these short-range pseudostates.

I. INTRODUCTION

The difficulty of the slow convergence of the close-coupling eigenfunction expansion for electron-hydrogen scattering was first discussed by Burke and Schey.[1] They observed that in all channels short-range correlations dominate the long-range polarization of hydrogen. Since the further addition of bound eigenfunctions into the expansion affects predominantly the long-range interactions, their inclusion leads to a slow convergence of the phase shifts. These observations were based on the following results. In the ^1S channel the $1s$-$2s$ expansion gave a larger correction to the static phase shift than did the $1s$-$2p$ expansion. Since the $2s$ state contributes nothing to the polarization of the ground state, the inference is clear. In the ^3S channel the static result itself is in good agreement with the "exact" result of Schwartz.[2] For multipoles higher than $L=0$ one would expect the difference between the exact phase shifts and the static exchange phase shifts in both the singlet and triplet channels to agree if polarization effects dominated. Burke and Schey[1] showed that this is not the case away from threshold ($k^2 \gtrsim 0.02$).

The eigenfunction expansion, therefore, is not appropriate for incorporating these short-range correlations, since the inclusion of continuum