tained numerically by Takeo[11] appear to belong to a very general type of effect, namely, the interference resulting from the existence of several paths, in this case two, connecting initial and final states. This "two path interference," the archetype of which is met in the diffraction of particles (or waves) through two slits, is also responsible for the nodal structure of the wave function, as seen in Sec. II, and for interference fringes in the differential scattering cross section.[1] What is perhaps particular to the case of the spectrum is that the different paths are distinguished by a temporal parameter rather than by a spatial one as is more common, and one might speak of a "two slits in time" interference effect, as is suggested by the pictorial representation of Fig. 2.

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[*] Present address: Centre de Recherches Mathématiques, Université de Montréal, Case Postale 6128, Montréal 101, Canada.

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PHYSICAL REVIEW A VOLUME 4, NUMBER 2 AUGUST 1971

K-Shell-Fluorescence Yield for Carbon and Other Light Elements

W. Hink and H. Paschke
Physikalisches Institut der Universität Würzburg, 87 Würzburg, Germany

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By recording the absolute number of *K* x-ray quanta emitted from a thin graphite target bombarded with electrons of energy *E* between 2 and 30 keV, the quantity $\omega_K \cdot Q_K(E)$ for carbon has been measured, where ω_K is the *K*-shell-fluorescence yield and $Q_K(E)$ the ionization cross section for the *K* shell. $Q_K(E)$ was calculated with the formula of Gryzinski, and the fluorescence yield of carbon was obtained as $\omega_K = 0.0035$. This value is much higher than the two other experimental values, but in agreement with a semiempirical formula of Byrne and Howarth and in near agreement with a recent calculation of McGuire. It is shown that this formula and this calculation seem to be valid for elements with atomic numbers down to $Z \approx 6$.

I. INTRODUCTION

The *K*-shell-fluorescence yield ω_K is defined as the number of radiation transitions per vacancy in the *K* shell of an atom. The method of production of this vacancy is of no significance. Measurements of ω_K for elements of atomic numbers $Z \gtrsim 15$ give a uniform picture,[1] but for low-*Z* elements the measured values in part differ greatly from one another (by a factor of 5 for Ne and Al). The recent tabulation of ω_K data for elements with $Z \leq 13$ is given by Dick and Lucas.[2]

Formulas for the evaluation of ω_K are always fitted to experimental values,[1,3,4] and therefore they cannot give additional information. We note that these formulas contain parameters which low-

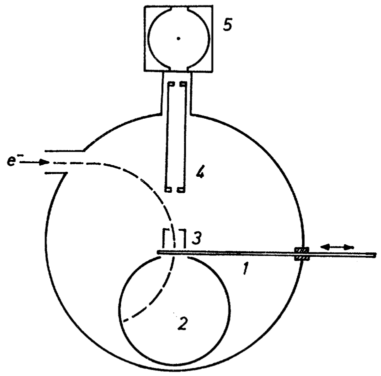

FIG. 1. Schematic drawing of the experimental arrangement for measuring carbon K radiation. Electron beam is deflected by a homogeneous magnetic field. 1 represents the target holder; 2, the Faraday cup; 3, the backscattering cup; 4, the pair of diaphragms; and 5, the gas-flow proportional counter.

er the $\omega_K(Z)$ curves for low Z in order to be in accordance with the measured values in the average. This lowering is explained with screening effects[1,3] or the shell structure of atoms.[4]

Recently, McGuire[5,6] has quantum-mechanically calculated K-shell Auger transition rates, and from these obtained the fluorescence yields for the elements from boron to xenon. His results for the medium-Z elements are in good agreement with experiments (especially the more recent ones); for $Z \lesssim 15$ they are clearly above the experimental values.

In our experiment we ionize the K shell of carbon by electron impact and measure the resulting K radiation, obtaining absolute values of $\omega_K Q_K(E)$. $Q_K(E)$ is the K-ionization cross section of carbon for electron energies E. With the known cross section $Q_K(E)$ we then obtain the K-shell-fluorescence yield for carbon. The discussion of the result is extended to fluorescence yields of other elements of low atomic number.

II. EXPERIMENTAL METHOD

The experimental arrangement employed for the measurements is shown schematically in Fig. 1. A beam of monoenergetic electrons (energy between 2 and 30 keV) enters the evacuated target chamber and is deflected on a horizontal circle by a homogeneous magnetic field. The beam is focused with diameter less than 1.5 mm on a self-supporting thin-film carbon target at the center of the chamber. The targets are produced by evacuation of graphite from an arc and have mass thicknesses between 4 and 30 μg/cm^2. The electrons enter the target normally, and the generated x rays are detected in the backward direction with a flow proportional counter.[7]

The solid angle of observation $[\Omega = (9.8 \pm 0.1) \times 10^{-5}$ sr$]$ is defined by a pair of diaphragms, which also prevents stray electrons and stray radiation from reaching the counter. The counting gas is methane at 700 Torr, and for the counter window we use foils of 2-μm Makrofol with an evaporated layer of 300 Å of aluminum.

The counting efficiency ϵ of the counter (ϵ is the product of window transmission and counting-gas absorption) was calculated using values of the mass-absorption coefficients tabulated by Henke *et al.*,[8] and as a control the window transmission was measured separately. The result for carbon K radiation is $\epsilon = 0.36 \pm 0.01$.

For evaluating the measurements we must know the number of electrons impinging on the target during the time of measurement (~ 100 sec per target and energy value E). This number is given by the charge q collected on the target holder, Faraday cup, and backscattering cup, and can be measured with an accuracy of $\pm 1\%$. To reduce the target contamination the backscattering cup is cooled with liquid nitrogen. The charge serves as a control for the registration of pulse-height spectra with a Laben 512-channel analyzer, thereby excluding errors from current fluctuations. Figure 2 shows a pulse-height spectrum from a target of mass thickness 17.5 μg/cm^2 at $E = 10$ keV. The solid line gives the bremsstrahlung background, calculated with an approximate formula given by Kirkpatrick and Wiedmann,[9] and then fitted to the measured spectrum at the upper pulse heights. Because the background correction is only about 1%, this calculation procedure is not necessary. Nevertheless, background calculation was carried out, because

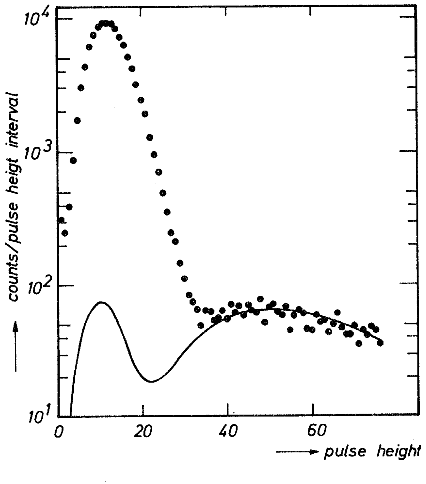

FIG. 2. Proportional counter pulse-height spectrum from a carbon target of 17.5 μg/cm^2 mass thickness at 10 keV primary electron energy. The line represents the calculated bremsstrahlung background. Pulse height is given in arbitrary units.

TABLE I. Measured values $\omega_K Q_K(E)$ for the electron energies E used in experiment and the corresponding errors δ. Q_{Gryz} is the K-shell-ionization cross section calculated with formula (2).

E (keV)	$\omega_K Q_K(E)$ (10^{-21} cm^2)	δ (%)	Q_{Gryz} (10^{-19} cm^2)
30	0.141	±5	0.405
25	0.161	±5	0.474
20	0.194	±5	0.573
15	0.244	±5	0.735
10	0.347	±5	1.025
7	0.475	±5	1.35
5	0.609	±5	1.75
4	0.748	±7	2.04
3	0.91	±10	2.52
2	1.22	±15	3.14

one may find invisible target impurities by comparing measured and calculated background spectra outside the line.

The difference of the measured pulse-height spectrum and the calculated background spectrum gives the number of recorded quanta N_{rec} (with an error of ±2–±5% depending upon the target thickness). With N_{rec} we calculate the quantity $\omega_K Q_K(E)$ with the formula

$$\omega_K Q_K(E) = \frac{A}{N_A \rho d}\,\frac{4\pi}{\Omega}\,\frac{N_{\mathrm{rec}}}{(q/e)\epsilon}\ , \tag{1}$$

where A is the atomic mass, N_A is Avogadro's number, $(N_A\rho)/A$ is the number of target atoms per unit volume, ρd is the mass thickness of the target, Ω is the solid angle of observation, q/e is the number of electrons impinging on the target, and ϵ is the counting efficiency of the proportional counter.

The mass thicknesses of the targets were determined by the weight of known areas of the target films[10] with errors between ±1 and ±3%, depending upon the film thickness. The measured ρd values must be corrected for two reasons: (a) Thickness grows by contamination, and (b) the electron path within the target is greater than the measured thickness because of scattering.

The determined values of $\omega_K Q_K(E)$ are shown in Table I together with their errors δ.

To evaluate the desired value of ω_K from the measured values $\omega_K Q_K(E)$, we must know the K-shell-ionization cross section for carbon as a function of electron energy E.

III. IONIZATION CROSS SECTION FOR THE *K* SHELL

There are some calculations[11-16] and one empirical formula[17] for determining cross sections. But these cross sections partly differ by factors up to 2. Because of two facts we can decide for one of these calculations: (a) There exists a measurement of the K-shell cross section of carbon from Glupe and Mehlhorn[18,19] for electron energies between 1 and 5 keV. They did not detect the characteristic x rays, but rather the Auger electrons. This has the great advantage that their evaluation of the measurement does not depend on an uncertain value for the fluorescence yield, but they can use the value for the Auger yield, which is known with great accuracy for light elements. The error of the values from Glupe and Mehlhorn[18] is ±15%. Because of the small E region used for measurement, the relative shape of the $Q_K(E)$ curve seems to be uncertain. (b) Because of the independence of the fluorescence yield from electron energy E, the experimental values $\omega_K Q_K(E)$ give the relative shape of $Q_K(E)$ (within the error limit, which is small for $E > 4$ keV).

Only one of the above-mentioned calculations[11-17] of $Qu(E)$ meets both the absolute values from Glupe and Mehlhorn[16] and the relative dependence from electron energy E, found by our measurement—i.e., the Gryzinski[14] formula

$$Q_K = \frac{13.12\times 10^{-20}}{E_K{}^2 U}\left(\frac{U-1}{U+1}\right)^{3/2} \times\left[1+\frac{2}{3}\left(1-\frac{1}{2U}\right)\ln[2.7+(U-1)^{1/2}]\right]\ , \tag{2}$$

where E_K is the ionization energy of the K shell in keV, $U = E/E_K$, and E is the energy of incident electrons in keV.

The K-shell cross sections calculated with formula (2) for the electron energies used in the experiment are listed in the last column of Table I. The probability for K ionization of more than one atom caused by one primary electron is small, and may be neglected even for the most unfavorable target-energy combinations used in our experiment.

IV. RESULTS AND DISCUSSION

Division of the experimental $\omega_K Q_K(E)$ values by the appropriate Gryzinski cross sections (Table I) yields values of the K-shell-fluorescence yield of carbon for the electron energies we used. These values are shown as crosses in Fig. 3. The mean

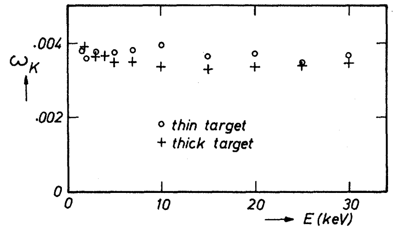

FIG. 3. K-shell-fluorescence yield ω_K for carbon obtained from this experiment for various energies E of incident electrons. There is good agreement between the results of thin- and thick-target measurement.

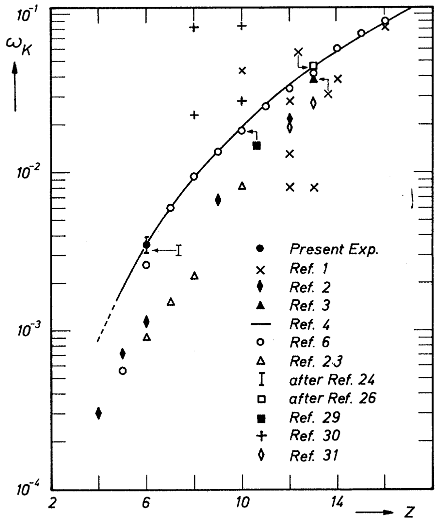

FIG. 4. Values of the K-shell-fluorescence yield ω_K for elements from beryllium to sulfur.

value is $\omega_K = 0.0035$, the estimated error being about ±10%.

In proof of this result we performed a measurement of the characteristic x-ray yield from thick carbon targets.[20] We found agreement with the thick-target yield of other authors.[21,22] It is possible to calculate this yield with the corresponding formula containing the product $\omega_K Q_K(E)$. If we now conceive this formula as an equation to determine ω_K, and substitute for $Q_K(E)$ the Gryzinski formula (2) and for the yield our measured values, then we obtain values for ω_K of about 0.0037 (circles in Fig. 3). This confirms the result of the thin-target measurement within the error limit.

The value 0.0035 for the K-shell-fluorescence yield of carbon strongly differs from the values 0.0009 and 0.0013 hitherto given[2,23] and therefore (after what has been mentioned in the Introduction) also from the older calculations.[1,3] We will proceed to show that one can perceive interesting relationships by assuming the high-ω_K value.

Byrne and Howarth[4] derived a semiempirical formula for ω_K by fitting it to experimental data for elements with atomic numbers between $Z \approx 15$ and $Z \approx 60$, which is supposed to be valid only for this Z region:

$$\omega_K = \{1 + [(1.16 \times 10^5)/Z^{3.36}]\}^{-1}. \tag{3}$$

If we apply this formula, however, for low-Z elements, we obtain the curve shown in Fig. 4. Formula (3) yields for carbon $\omega_K = 0.0035$.

The results from the calculation of McGuire[6] are given in Fig. 4 as open circles. They are in good agreement with the extrapolated curve according to formula (3), except that the values for carbon and boron are clearly lower. For carbon McGuire[6] derives $\omega_K = 0.0026$.

Khan *et al.*[24] have measured the K-shell-ionization cross section of carbon for proton impact. Their results, derived with $\omega_K = 0.007$, are different from theoretical calculations (Born approximation, calculation of Garcia[25]). If we assume that these calculations are correct for high energies ($E > 0.5$ MeV) and the differences in this region are due to an incorrect value for the fluorescence yield in the evaluation of Khan *et al.*,[24] then we obtain for ω_K a value between 0.0030 and 0.0035.

Hink and Ziegler[26] measured with thin targets by the x-ray method (described above for carbon) the K-shell-ionization cross section of aluminum by electron impact. Their measured $\omega_K Q_K(E)$ curve has the same shape as the Gryzinski curve (2). However, assuming that $\omega_K = 0.038$ for aluminum, the absolute values of $Q_K(E)$ are 23% higher than those given by (2). Agreement is observed for $\omega_K = 0.046$. Formula (3) yields $\omega_K = 0.045$ for aluminum.

The high value for the aluminum fluorescence yield is also derived by considering the thick-target yield for aluminum characteristic x rays. Agreement between the measurements of Hink, Paschke, and Ziegler[20] and the above-mentioned theory is again obtained for $\omega_K = 0.046$ by using Gryzinski cross sections. Bishop[27] calculated the yield with a Monte Carlo method, using $\omega_K = 0.028$ and cross sections from Bethe,[16] and compared it with measurements from Green.[28] His results show the same dependence on E as those of Green, but the absolute values are different. This difference vanishes with $\omega_K = 0.045$ for the fluorescence yield of aluminum.

The K-shell-fluorescence yields of carbon and aluminum found in this work, together with the calculations of McGuire,[6] give strong argument for the enlargement of the region of validity of formula (3) down to $Z \approx 6$. This makes doubtful the assumption of Byrne and Howarth[4] concerning the influence of the shell structure of atoms on the fluorescence yield.

Figure 4 shows, besides the curve (3) and the values from McGuire,[6] the experimental data of fluorescence yields for $Z \leq 16$. The values for the lowest atomic numbers are from Crone[23] and Dick and Lucas.[2] Crone in 1936 measured only relative values and fitted the one for neon to the formula

$$\omega_K = (Z-1)^4/[a + (Z-1)^4], \quad a = 0.8 \times 10^6. \tag{4}$$

Also by fitting the neon value to formula (3), the values for carbon, nitrogen, and oxygen still remain below curve (3). The statistical error for neon is ± 3% and for carbon ± 50%.

The recent absolute values were measured in 1970 by Dick and Lucas[2] by the fluorescence method.

This method has the great advantage of insensitivity to errors in two quantities directly entering the evaluation: mass-absorption coefficients and absorption jumps. All their values are clearly below the curve (3). The deviation of their carbon value from the value $\omega_K = 0.0035$ measured by us is greater than is to be expected from the given errors. We are not able to state a reason for this great discrepancy. We think, however, that our result is correct, and that we have given good reasons. It must be reserved for further work to bring greater clarity into this field.

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Determination of Rotational *g* Factors from Gas Transport Properties

H. Moraal and F. R. McCourt
Chemistry Department, University of Waterloo, Waterloo, Ontario, Canada

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A procedure is given whereby for simple diatomic molecules the rotational g factor can be determined from transport measurements alone. The values so obtained agree to within 15% with those obtained from molecular-beam techniques. For N_2 and CO, the values of g_{rot} obtained are -0.285 and -0.301, respectively, to be compared with, respectively, -0.2593 and -0.2691 as obtained from molecular-beam magnetic resonance.

I. INTRODUCTION

That all polyatomic molecules possess a magnetic moment due to their rotational motion has been known since the classic work of Wick[1] and Ramsey.[2] As for electronic and nuclear magnetic moments, the rotational magnetic moment is characterized by a quantity called the rotational (Landé) g factor. The determination of this rotational g factor g_{rot} has lain in the realm of molecular-beam magnetic resonance since a molecular beam of essentially pure rotational states can be prepared. Until the last decade, it appeared that the determination of such a molecular parameter was restricted to molecular-beam measurements. With the advent of studies of the Senftleben-Beenakker (SB) effect,[3] a second possible experimental source of g values became available. The use of these effects was not, however, trivial insofar as the g factor occurred together with certain collision cross sections and, in order to extract the g value, independent determinations or estimates of the