tulate of phase incoherence of components of the Schrödinger wave function made in quantum statistical mechanics. Since the total radiation density obeys Planck's law, there is no conflict with experimental data on blackbody radiation in which total radiant energy is measured.

As has been shown by Jaynes and co-workers,[1] the semiclassical theory describes dynamical radiative level shifts as well as radiative lifetimes due to spontaneous emission. These two phenomena both follow from Eq. (48), when applied to the total effect of all levels E_β on a given level E_α. This equation becomes

$$i\hbar\dot{c}_\alpha(t) = (\Delta E_\alpha - \tfrac{1}{2} i\hbar\Gamma_\alpha)\, c_\alpha(t) \tag{75}$$

$$= \sum\nolimits_\beta (\Delta E_{\alpha\beta} - \tfrac{1}{2} i\hbar\Gamma_{\alpha\beta}) |c_\beta(t)|^2 c_\alpha(t), \tag{76}$$

where

$$\Delta E_{\alpha\beta} - \tfrac{1}{2} i\hbar\Gamma_{\alpha\beta} = -\frac{1}{c^2}\iint \vec{j}^{\,\mathrm{tr}}_{\alpha\beta}(P)\cdot R^{-1} e^{i\omega_{\alpha\beta}R/c}\, \vec{j}^{\,\mathrm{tr}}_{\beta\alpha}(Q)\, d\tau_P\, d\tau_Q \,. \tag{77}$$

This gives a general formula for the dynamical level shift $\sum_\beta \Delta E_{\alpha\beta} |c_\beta|^2$ and the dynamical width $\sum_\beta \Gamma_{\alpha\beta} |c_\beta|^2$. For two levels, the width formula is equivalent to Eq. (50) for the time rate of change of $|c_1|^2$.

Crisp and Jaynes[1] found that for electric dipole radiation the $1s$-$2p$ level shift obtained from Eq. (77) for nonrelativistic wave functions agrees within experimental error with the experimental value, assuming that $|c_{1s}|^2$ is unity. The $2s$-$2p$ level shift (the Lamb shift) is roughly two-thirds of the experimental value when computed with nonrelativistic wave functions, assuming that $|c_{2s}|^2$ is unity under the conditions of the Lamb-shift experiments. These calculations need to be refined before they could be considered to provide a definitive test of the semiclassical theory.

It is important to note that irreversibility is built into the theory of spontaneous emission given here by assuming retarded solutions of the inhomogeneous wave equation, in Eq. (15). In consequence, the rate of energy production given by Eq. (21) is positive definite, at least for electric dipole radiation. If advanced potentials had been used, the energy production rate would change sign, and it vanishes if half-advanced, half-retarded potentials are used. Thus the choice of retarded potentials establishes an "arrow of time," in that energy is dissipated from an excited material system as time progresses, and an eventually established equilibrium would not be disrupted by reversing the time sense.

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He II-He I Transition in a Heat Current: Model Calculations*

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The Ginzburg-Pitaevskii phenomenological theory has been extended to include the effects of vortex lines. The model calculations are compared with some of our recent experimental results on the He II–He I transition in the presence of a heat current.

I. INTRODUCTION

Several years ago Ginzburg and Pitaevskii[1] proposed a phenomenological theory of superfluidity near T_λ. Some of the results of this theory have been discussed in recent papers by Mamaladze,[2] Kramer,[3] and Mikeska[4] and in an unpublished report by Burkhardt and Stauffer.[5] A salient feature of the theories is the prediction that the superfluid density ρ_s should be reduced for finite values of the counter-flow velocity $w = (v_s - v_n)$. Concomitantly, this implies an instability in a driven superfluid current $\rho_s w$. In other words, for any given temperature there should exist a maximum current consistent with superfluidity. In addition, the theory predicts that, barring the instability, the superfluid density should vanish at high enough values of w, i.e., the He II–He I transition temperature should be lowered for finite w. To our knowledge, until recently there has been no unequivocal experimental evidence to support this view. In a recent paper[6] (hereafter referred to as I) we have reported an experiment in which the He II–He I transition was investigated in the presence of a heat current. Figure 1 [cf. Fig. 3(b) of I] shows the thermograms observed at two points along a thermally isolated

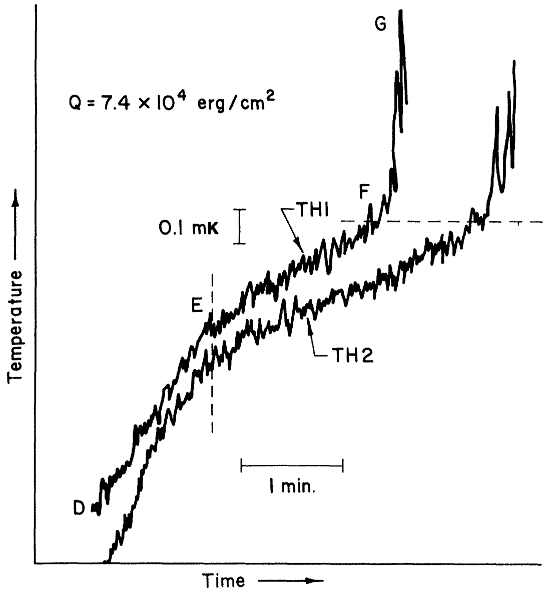

FIG. 1. Thermogram for two thermometers, TH 1 and TH 2, placed 5 and 10 cm away from the heater. As noted in I the points E are simultaneous and the points F occur at the same temperature.

column of He II when the main helium bath was allowed to warm up slowly. As explained in I, the points E were identified as the instant at which the He II–He I transition occurred at the heater and the points F measured the temperature of this transition[7] for an applied heat current Q. The results were summarized by the relation

$$T_F \equiv T_\lambda(Q) = T_\lambda(0) - (5.9\times 10^{-9})Q \quad \text{for } 10^4 \lesssim Q \lesssim 10^6\ \text{erg/cm}^2 . \quad (1)$$

This equation represents a least-squares fit to several hundred data points taken with cylindrical flow channels of 0.4- and 0.6-cm diameter, the thermometers being placed 3, 5, 10, and 12 cm away from the heater. Equation (1) differs considerably from the results predicted in Refs. 2 and 4. It was proposed in I that in order to account for such discrepancies, one has to take explicit account of the fact that in the experimental situation there is likely to be a considerable density of vortex lines in the superfluid. In this paper we present further details of a model calculation based on this idea and show that by introducing a reasonable number of vortex lines one can account for the observations. Admittedly, the model is very idealized and, although no approximations are made, the results are no better than the model itself. Further, we have used the model to calculate the temperature distribution in the fluid incorporating some of Ahlers's[8] suggestions regarding the behavior of the Gorter-Mellink mutual friction force[9] near T_λ. Unfortunately, such a calculation involves many adjustable parameters. However, for a reasonable choice one again gets a satisfactory description for the measurements. There is one experimental situation for which the present model should be quite accurate, i.e., when the He II–He I transition is studied in the presence of a uniform rotation. It is very gratifying that our model accounts for the negative results of the experiments of Andelin *et al.*[10]

II. MODEL CALCULATIONS

A. Critical Heat Current

As described in the Introduction, the present model differs from that used in Refs. 2–5 in that we wish to include the effects of vortex lines in the superfluid. The effects of having vortex lines in a rotating mass were discussed by Ginzburg and Pitaevskii.[1] In the present problem we wish to consider the superfluid flowing with a uniform velocity v_z through a cylindrical tube of radius a. In addition, we will superpose on this motion a velocity field due to N vortex lines each of strength $\pm\kappa\hat{z}$. The vortices are assumed to be uniformly distributed in the fluid. In addition, since there is no net flow in the plane perpendicular to the z axis, one is also assuming that neighboring vortices have opposite circulations. In essence this implies that on the large scale the presence of the vortices does not spoil the simple hydrodynamic situation.

With these assumptions, the only effect of introducing the vortices is to deplete the superfluid density. Presumably, this effect is independent of the orientation of the line and is controlled only by the magnitude of the circulation κ, the number of lines N, the correlation distance ξ, and the radius of the Bernoulli hole a_0. Thus, if the distance between two vortex lines, $a/\sqrt{N}$, is much larger than ξ, the hydrodynamic interaction between them will not contribute to our results.[11]

It is clear that the actual distribution of vortex lines in the superfluid is likely to be very complex and will perhaps bear no simple relationship to our model. Simplicity is, of course, the main justification for taking such a model although one could also argue that a picture in which the vortex lines are anchored at one or two points on the tube walls and are sensibly parallel to the walls over considerable lengths is not entirely invalid.

The assumption of a uniform v_z may also seem surprising since there is likely to be coupling between normal and superfluid flow, the former velocity having the familiar parabolic variation along the radius. However, for T close to T_λ and flow induced by a heat current, $v_n \ll v_z$, and the variation of v_n across the cross section will become unimportant. Presumably, at the tube wall itself v_z will go to zero in some manner. In calculating the free energy this could, in principle, be taken into account by reducing the effective radius of the channel. This effect may also be neglected because the excluded region will have a thickness of order ξ and in our regime $\xi/a \ll 1$. As shown in I [Eq. (7)], the free energy per unit volume may be written as

$$\Delta F_N = (-\alpha + \tfrac{1}{2} m v_z^2)\chi_\infty^2(1-N\epsilon) + \tfrac{1}{2}\beta\chi_\infty^4(1-N\delta) + \frac{N\kappa^2}{4\pi^2 a^2} m\left(\ln\frac{a}{\sqrt{N}a_0}+\theta\right)\chi_\infty^2 , \quad (2)$$

where

$$\epsilon = \frac{a_0^2}{a^2} + \frac{3a_0\xi}{a^2} + \frac{7}{2}\frac{\xi^2}{a^2} ,$$

$$\delta = \frac{a_0^2}{a^2} + \frac{25}{6}\frac{a_0\xi}{a^2} + \frac{415}{72}\frac{\xi^2}{a^2} ,$$

$$\theta = \left[\frac{a_0}{2\xi} + \frac{1}{4} - 2P\left(\frac{a_0}{\xi}\right) + P\left(\frac{2a_0}{\xi}\right)\right] ,$$

where $\kappa = h/m$ and the other symbols are as defined in I. The term involving κ has essentially the same form as that derived earlier by Ginzburg and Pitaevskii.[1] Anticipating the results of the sequel, $N\epsilon \ll 1$ and $N\delta \ll 1$, and are therefore neglected. Minimizing the free energy gives[2]

$$\chi_\infty^2 = \frac{1}{\beta}\left[\alpha - \tfrac{1}{2}mv_z^2 - \frac{N\kappa^2}{a^2 4\pi^2} m\left(\ln\frac{a}{\sqrt{N}a_0}+\theta\right)\right] , \quad (3)$$

which gives for the free energy at equilibrium

$$(\Delta F_N)_{\text{eq}} = -\frac{1}{2\beta}\left[\alpha - \tfrac{1}{2}mv_z^2 - \frac{N\kappa^2}{a^2 4\pi^2} m\left(\ln\frac{a}{\sqrt{N}a_0}+\theta\right)\right]^2 .$$

Again, we have the superfluid density

$$\rho_s(v_z) = \frac{1}{v_z}\frac{\partial(\Delta F_N)_{\text{eq}}}{\partial v_z} = \rho_s(0)\left[1 - \frac{mv_z^2}{2\alpha} - \frac{N\kappa^2}{4\pi^2 a^2\alpha} m\left(\ln\frac{a}{\sqrt{N}a_0}+\theta\right)\right] , \quad (4)$$

where $\rho_s(0) = C[T_\lambda(0) - T]^{2/3}$ is presumably the density measured in the experiments of Clow and Reppy[12] and of Tyson and Douglass.[13] As before, the maximum heat current $[Q_c]_{\text{Th}}$ (see Fig. 1 in I and Fig. 2 below) that the superfluid can carry corresponds to the point $\partial^2(\Delta F_N)_{\text{eq}}/\partial v_z^2 = 0$, and is given by

$$[Q_c]_{\text{Th}} = \frac{\beta^{1/2}ST}{m}\left\{\rho_s(0)\left[1 - \frac{m(w_c)_{\text{Th}}^2}{2\alpha} - \frac{N_c\kappa^2}{4\pi^2 a^2\alpha} m\left(\ln\frac{a}{\sqrt{N_c}a_0}+\epsilon\right)\right]\right\}^{3/2} . \quad (5)$$

Here, we have utilized the fact that $w = v_z - v_n \simeq v_z$. N_c represents the number of lines at the critical point. It is interesting to compare this result with that of Mamaladze,[2] namely,

$$[Q_c]_{\text{Th}} = \frac{\beta^{1/2}ST}{m}\left[\rho_s(0)\left(1 - \frac{m(w_c)_{\text{Th}}^2}{2\alpha}\right)\right]^{3/2} . \quad (6)$$

For nonzero N, the value of $[Q_c]_{\text{Th}}$ predicted by Eq. (5) is likely to be smaller than that obtained from Eq. (6). In addition, the temperature dependence of $[Q_c]_{\text{Th}}$ may also be altered by the presence of vortex lines.

In order to compare Eq. (5) with the experimental results, as summarized in Eq. (1), one proceeds as follows. In a driven situation, if the applied heat current Q exceeds $[Q_c]_{\text{Th}}$ at any point in the fluid, the excess heat $Q - [Q_c]_{\text{Th}}$ must be transported by other means. This is likely to raise the local temperature, thereby lowering $[Q_c]_{\text{Th}}$ further, which will cause a further increase in temperature. One gets a cascade of processes such that $\rho_s(Q, T)$ vanishes locally. Thus one should expect that once $[Q_c]_{\text{Th}}$ is exceeded, the He II-He I transition will ensue. Obviously, this will occur first of all in the neighborhood of the heat source, as observed in I. If this interpretation is valid, Eqs. (5) and (1) describe essentially the same physical discontinuity. Since there is no *a priori* way of calculating N_c, we have used the data to determine the value of N_c which brings Eqs. (5) and (1) into agreement, and obtain

$$N_c\left(\ln\frac{a}{a_0\sqrt{N_c}}+\theta\right) = \frac{4\pi^2\alpha a^2}{m\kappa^2}\left[1 - \frac{3}{2m^{1/3}\alpha}\left(\frac{\beta[T_\lambda(0)-T]}{(5.9\times10^{-9})ST}\right)^{2/3}\right] . \quad (7)$$

In other words, N_c is the number of vortex lines required to give the observed shift in the transition. Following Mamaladze $\alpha = (1.11\times10^{-16})\,[T_\lambda(0)-T]^{4/3}$ erg and $\beta = (3.52\times10^{-39})\,[T_\lambda(0)-T]^{2/3}$ erg/cm^3. With $\rho_s(0)/\rho = 1.44[T_\lambda(0)-T]^{2/3}$ one gets

$$N_c\left(\ln\frac{a}{a_0\sqrt{N_c}}+\theta\right) = (6\times10^{13})\,[T_\lambda(0)-T]^{4/3} \times\{1 - 0.045[T_\lambda(0)-T]^{-2/9}\} . \quad (8)$$

For the range of $[T_\lambda(0) - T]$ covered by our experiment this equation can be approximated by

$$n_c = N_c/\pi a^2 \simeq (4\times10^{13})\,[T_\lambda(0)-T]^{4/3} . \quad (9)$$

There is no calculation or measurement in the literature to which this result can be compared directly. To our knowledge the only attempt at trying to obtain the equilibrium density of vortex lines in liquid He II is due to Vinen.[14] He showed, from simple hydrodynamical considerations, that for $T \lesssim 2\,°\text{K}$ and $v_s \sim 10$ cm/sec the density of vorticity is $\sim 10^7$ lines/cm^2. At higher temperatures one should expect a somewhat larger density although no direct extrapolation is possible since all his assumptions break down for T close to T_λ. Further, it is important to note that near T_λ the generation of vorticity also becomes favorable due to thermal fluctuations.[15] The values implied by Eq. (9) therefore do not appear to be unreasonable. As expected, Eq. (9) shows that the farther one is from the $T_\lambda(0)$ the larger the density of vorticity required to cause the transition.

Next, it is interesting to note that $N_c\xi^2 \simeq 8\times10^{-4}$ cm^2, i.e., the transition appears to take place when the average distance between vortex lines is about 20ξ. Also, since $N_c\xi^2/a^2 \sim N_c a_0^2/a \ll 1$, neglect of $N\epsilon$ and $N\delta$ is indeed reasonable.

It is instructive to note from Eqs. (5) and (6) that the superfluid density for $Q=[Q_c]_{\rm Th}$ has the form

$$\rho_{s,c}=\tfrac{2}{3}\rho_s(0)\,(0.045)\,[T_\lambda(0)-T]^{-2/9} \qquad (10)$$

while the critical counterflow velocity $[w_c]_{\rm Th}$ has the value

$$[w_c]_{\rm Th}=(2\alpha/3m)^{1/2}\{0.045\,[T_\lambda(0)-T]^{-2/9}\}^{1/2}\,. \qquad (11)$$

In the Mamaladze theory these quantities were given by $\rho_{s,c}=\frac{2}{3}\rho_s(0)$ and $(w_c)_{\rm Th}=(2\alpha/3m)^{1/2}$. Thus, the presence of vortex lines reduced $\rho_{s,c}$ by a larger factor than it does $(w_c)_{\rm Th}$. The usual Q-vs-w curve, shown as a dotted curve in Fig. 2, gets replaced by the full curve. It has to be admitted that only the end points in the full curve have been computed by us. However, the trend is quite clear.

B. Temperature Distribution in Fluid

As mentioned earlier, E in Fig. 1 marks the instant (same for all thermometers) at which the He II–He I transition occurs in the liquid close to the heater. The temperature T_E can thus be used to describe the temperature variation in liquid He II as a function of l, the thermometer-heater distance. Since the temperature at the heater is $T_\lambda(Q)$ one may be inclined to study the relationship between $[T_\lambda(Q)-T_E]$ and the applied heat current Q. This was tried first. It was found that in principle one could write $Q=f[T_\lambda(Q)-T_E]^g$. However, in such an expression both f and g turned out to be functions of l. Following the results of earlier measurements by Leider and Pobell[16] and Ahlers,[17]

TABLE I. Observed values of $f(l)$ and g, along with values computed from Eq. (14) for two sets of the adjustable parameters.

	$p=5$, $\gamma=1.14$, $A_0=0.023$		$p=4$, $\gamma=0.67$, $A_0=1.08$			
l (cm)	$f_{\rm calc}$[a]	$g_{\rm calc}$	$f_{\rm calc}$[a]	$g_{\rm calc}$	$f_{\rm expt}$[a]	$g_{\rm expt}$
3	11.3	1.07	11.4	1.06	12.4	1.09
5	10.3	1.08	10.2	1.07	11.5	1.08
10	9.1	1.09	8.8	1.08	9.8	1.08
12	8.7	1.09	8.4	1.08	8.4	1.08

[a] f is in units of mW/cm^2/(m °K)^g.

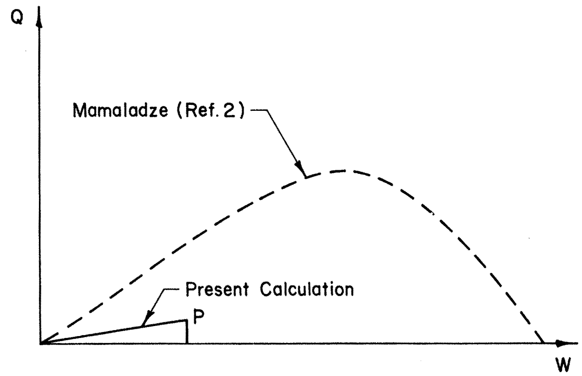

FIG. 2. Q (heat current) vs w (counterflow velocity) for a temperature $T=T_\lambda-0.001$ °K. The dotted curve was calculated using the theory of Mamaladze (Ref. 2) and the full curve from Eqs. (5), (10), and (11). The sharp drop at P follows from the discussion in the text.

we next tried fitting the data to the form

$$Q=f(l)[T_\lambda(0)-T_E]^g \qquad (12)$$

and were very gratified to note that the present results gave for g the value 1.08 ± 0.01, which is in extremely good agreement with the results of Refs. 8 and 16. Also it was found, in agreement with the earlier measurements, that $f(l)$ (Table I) is a slowly varying function of l. Ahlers[8] has suggested that the relationship given by Eq. (12) can be understood in terms of a mutual friction term of the Gorter-Mellink form, i.e., $\nabla T=-A\rho_n Q^p/S(\rho_s ST)^p$, provided that $A\propto[T_\lambda(0)-T]^{-2/3}$ and $p=4$. However, in his analysis Ahlers took no account of the possible Q (or w) dependence of ρ_s. In addition, one must include the fact that the heater temperature does not exceed $T_\lambda(Q)$. This is accomplished by writing

$$\rho_s(w)=\rho_s(0)\left[1-\frac{mw^2}{2\alpha}-\frac{N\kappa^2 m}{4\pi^2\alpha a^2}\left(\ln\frac{a}{a_0\sqrt{N}}+\theta\right)\right] \quad \text{for } T<T_\lambda(Q)\,,$$
$$\rho_s(w)=0 \quad \text{for } T\geq T_\lambda(Q)\,. \qquad (13)$$

The temperature at a distance l from the heater will thus be given by the equation

$$\int_0^l \frac{\rho_n Q^p}{S^{p+1}T^p}\,dl=-\int_{T_\lambda(0)}^{T(l)} A^{-1}(T)\,\rho_s^p(w)\,dT\,. \qquad (14)$$

We retain Ahlers's assumption that $A=A_0[T_\lambda(0)-T]^{-\gamma}$ although, of course, this relation is meaningful only for $T<T_\lambda(Q)$. Unfortunately, Eq. (14) contains a large number of undetermined parameters. In order to compare with experiment the following steps were taken: (i) Since the temperature distribution is measured when the heater end is at $T_\lambda(Q)$, one can take for N in Eq. (13) the value $N=N_c(Q)$. (ii) On the right-hand side of Eq. (13)

the second term in parenthesis is small and was therefore neglected. (iii) The resulting $[T_\lambda(0) - T(l)]$-vs-Q relationship was expressed in the power-law form [Eq. (12)]. The results for two choices of the parameters, along with the experimental values, are exhibited in Table I. In hindsight, it appears as if the extra terms have not altered Ahlers's conclusions too drastically, since he obtained good fits for $p=4$, $\gamma=+0.67$.

C. He II-He I Transition in Rotating Liquid Helium

The model proposed here should also find application when discussing the He II–He I transition in the presence of a uniform rotation with angular velocity ω. In this case, indeed, one expects a uniform distribution of vortex lines in the fluid. All the vortex lines will have their circulation in the same direction, which is also the direction of macroscopic rotation. It is well known that for $N\kappa/\pi a^2 = 2\omega$ such a vortex pattern will simulate solid-body rotation. Since $v_z = 0$ it follows from Eq. (13) that for a bucket of radius a, one should expect a shift in T_λ given by

$$\Delta T_\lambda = T_\lambda(0) - T_\lambda(\omega) = (5.39\times 10^{-9})\omega^{3/4} \times \left[\ln\left(\frac{(2.8\times 10^{+6})(\Delta T_\lambda)^{2/3}}{\omega^{1/2}}\right) + 0.36\right]^{3/4}, \quad (15)$$

which, apart from logarithmic factors, is the same as Mamaladze's result $T_\lambda(0) - T_\lambda(\omega) = (5.4\times 10^{-9}) \times \omega^{3/4}$. This equation should also be compared with the results of an experiment due to Andelin *et al.*[10] They found that for $\omega \sim 35$ sec^{-1} no observable shift in T_λ took place. For this value of ω and their geometry, Eq. (15) would predict a shift of 0.2 μdeg. (Mamaladze's result would be 0.1 μdeg.) Since Andelin *et al.* had a precision of only 10^{-5} °K, it is not surprising that they obtained a negative result.

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