Scattering with Charge Exchange to the State H(2p) for the Systems H^+ + He and H^+ + Ar†

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Studies have been made of differential scattering with charge exchange to the $2p$ state of hydrogen for the systems H^+ + Ar and H^+ + He. Fast hydrogen atoms in the state H(2p) resulting from the collision process were detected by making coincidence measurements between the Ly-α photon emitted in the decay of the $2p$ state and the resultant fast neutral particle. The probability of transfer to the state H(2p), P_{2p}, is presented for H^+ + Ar for the condition ΘT = 20 keV deg, where Θ is the scattering angle and T the incident-ion kinetic energy, which represents an impact parameter of 0.53 a.u. For the H^+ + He system a comparison between experimental results for P_{2p} and the many-state calculation of Sin Fai Lam shows the theory to overestimate P_{2p} by an order of magnitude for ΘT = 20 keV deg as well as for measurements where the impact parameter varied from 0.05 to 0.25 a.u.

I. INTRODUCTION

The present paper describes measurement of the probability of charge exchange into the state H(2p) for specific impact parameters during collisions involving the H^+ + Ar and H^+ + He systems. The incident proton energy T varied from 4 to 20 keV while the scattering angle Θ ranged from 1° to 5°. Measurements were made for the condition ΘT = 20 keV deg for H^+ + Ar and H^+ + He and for T = 6.25 keV for H^+ + He. When the product ΘT is held fixed, the impact parameter associated with the collision remains approximately constant for small-angle collisions such as those studied here, and only the relative velocity of the collision changes as the energy is varied.

There are two classes of related experiments. The first deals with measurement of charge exchange into the $n = 2$ states of hydrogen. These measurements may be typified by the work of Geballe and co-workers at the University of Washington[1] and Andreev, Ankudinov, and Bobashev of the A. F. Ioffe Institute in Leningard.[2] Total cross sections for charge exchange into the states H(2p) and H(2s) were determined for protons incident on the rare gases. There is generally good agreement between the results obtained at the two laboratories. More recently, Bayfield has measured the total cross sections for charge exchange into the metastable state H(2s) in an energy range which overlaps both the University of Washington and Leningrad results.[3] These recent results agree well with the previous measurements. Differences appear to be in magnitude only which would suggest uncertainty in absolute calibration of the detector used to observe the Ly-α photon resulting from the decay of the $n = 2$ states. Other uncertainties may arise because the radiation from the decay of the $2p$ and quenched $2s$ states has a nonisotropic angular distribution around the beam.

A second series of measurements of charge-exchange processes were made by Everhart and his colleagues at the University of Connecticut. In these experiments, the probability of charge exchange was determined for particles which were scattered through some angle with respect to the incident beam. A number of collision systems were investigated, and in some systems, notably H^+ + H and H^+ + He, an oscillatory structure was observed in the charge-exchange probability P_0 when plotted as a function of the reciprocal of the incident-particle velocity.[4] Such oscillatory behavior in H^+ + H is predicted by the results of a two-state calculation based on the perturbed stationary-state approximation for resonant charge exchange, but the agreement between this theory and experiments is not good.[5] More recent calculations in which basis functions composed of many states were used show excellent agreement with experiment.[6,7] In a series of papers, Lichten developed semiempirical expressions which agreed remarkably with experimental results.[8,9] His approach was based on application of molecular wave functions and use of appropriate potential energy curves for the collision system.

The present experiment combines features of both classes of experiments in that charge-exchange probabilities are measured for collision processes in which the incident particle undergoes a violent collision, is scattered through a finite angle, and picks up an electron in the excited state H(2p).

II. EXPERIMENTAL METHOD

The experimental method employed all the basic elements of differential scattering experiments. A beam of particles was directed into a cell containing the target gas, and a particle detector which could be positioned at various angles with respect to

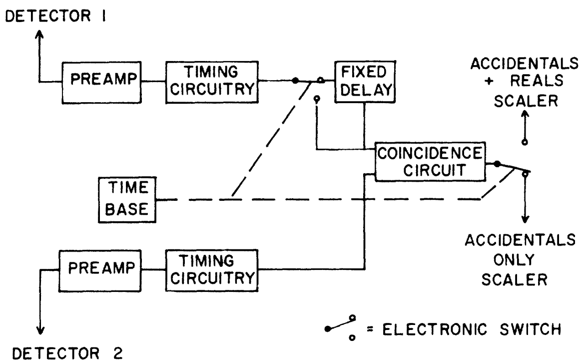

FIG. 1. Electronic circuitry used in coincidence measurements.

the incident beam detected particles which were scattered. To identify a collision event in which charge exchange to an excited state occured, a photomultiplier which was sensitive to the photon emitted in the decay of the excited state was positioned above the collision region and at 90° to the scattering plane. The collision events of interest are those in which a proton simultaneously picks up an electron in the state $H(2p)$ and is scattered through some angle with respect to the incident beam. To identify these events the signals from the photon detector and particle detector are processed by suitable coincidence circuitry by means of which correlations between photon and particle signals may be established.

The proton involved in the collision emerges from the collision region as a neutral hydrogen atom in the $2p$ state. This state has a lifetime of approximately 1.6 nsec and decays emitting a Ly-α photon, which is detected by the photomultiplier. The resulting fast hydrogen atom is detected by the particle detector after some finite flight time. If a fixed delay equal to the time of flight of the neutral particle is introduced in the photon channel, coincidences will be observed which correspond to the scattering event of interest.

Most of the charge-exchange events correspond to collisions in which there is little deflection of the incident particle. Consequently there is a large background of Ly-α photons produced by events which are not of interest. Similarly, most of the particles which are scattered into the particle detector do not result from the process which is being investigated.

These uncorrelated events produce signals in the two channels which may result in accidental coincidences. The real signal may be obtained by subtracting the results from two consecutive measurements, the first of which has correct time alignment and the second a large time delay so that only accidental coincidences are observed. Analysis of the statistical uncertainty introduced in the real signal has been detailed previously.[10]

Since the signal rates in each channel depend on beam current and target gas pressure in a collision experiment, the accidental coincidence rate depends quadratically on both current and pressure. If a subtraction of consecutive runs were to be made to determine the number of real coincidences, beam current and target gas pressure would have to be very stable over an extended period of time, an operating condition which is extremely difficult to obtain.

Simultaneous measurement of real-plus-accidental and accidental-only signals may be made by using two coincidence modules driven in parallel with a large fixed delay preceding one module. To obtain reliable results, the modules must have nearly identical electronic characteristics, particularly as regards temperature effects and over-all stability. Although other methods have been reported[11,12] a scheme using electronic switching was incorporated in the present experiment. This technique was developed at the University of Alaska by Sheridan and his co-workers, who also presented a discussion of the proper analysis of coincidence experiments in atomic physics in an earlier paper.[13] The electronics used in the experiment are shown schematically in Fig. 1. The switching module, consisting of a series of electronic switches, sends the signal in the photon channel directly to the coincidence module during one-half cycle of the switching wave form and through a fixed delay during the other one-half of the cycle. Simultaneously the output of the coincidence module is switched back and forth between scalers which record accidental-plus-real coincidences and accidental-only coincidences. This switching, which is done at a frequency of 12.5 Hz, averages out beam fluctuations and pressure drifts. The difference in counts in the two scalers at the termination of a data run corresponds to the number of real coincidences.

In any differential scattering measurement, the acceptance angles for scattered particles will be determined by the collimator or slit assembly for the detector. A finite segment of the incident beam will be viewed by the detector, and this segment defines the interaction region. The scattering events detected are proportional to the product of differential scattering region dx and the solid angle subtended by the detector assembly at that point. The summation over the entire interaction region observed is called the effective solid-angle–scattering region product and is usually written as $\int l d\Omega$.

The situation is somewhat different for the coincidence experiment. Only scattering events occurring in the common viewing region of the photon and particle detectors can give rise to real coincidences. Signals originating from events outside the coinci-

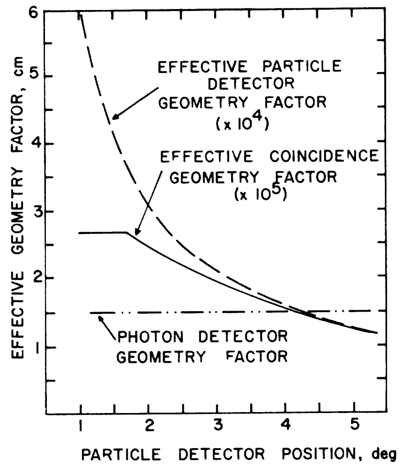

FIG. 2. Effective geometry factors for the coincidence experiment.

dence region contribute only accidental coincidences. For an infinitesimally small interaction region, the number of coincidences detected is proportional to the produce $dx d\Omega_1 d\Omega_2$, where dx is the differential interaction region and $d\Omega_1$ and $d\Omega_2$ are the solid angles associated with the two detectors. In any practical apparatus, the scattering region is finite, and to calculate the appropriate geometry factors, a sum is made over the common viewing region $\sum dx_i (d\Omega_1)_i (d\Omega_2)_i$, which is written symbolically as $\int_C 1 d\Omega_1 d\Omega_2$. A computer program was developed to do this summation taking into account the umbra and penumbra sections of each detector collimator as well as the angular position of the particle detector. The geometry factors for the experiment are displayed in Fig. 2. At small angles, the coincidence region is determined by the extremities of the photomultiplier collimator while, above 2°, the particle detector assembly determines the common viewing region. The effective angular resolution of the experiment is much better at small angles than would be expected on the basis of the particle detector assembly alone.

The interpretation of photon-particle coincidence experiments must take into consideration a proper treatment of the angular distribution of the collision-induced radiation. A detailed discussion of the theory of photon-particle coincidence techniques which is based on the quantum theory of radiation is presented elsewhere.[14] A coordinate system suitable for decreasing the experiment may be generated with the polar axis defined by the incident beam and the polar coordinates of the particle and photon detectors being θ, ϕ and θ', ϕ', respectively. An expression for the coincidence rate for Ly-α transitions in atomic hydrogen may be written as[14]

$$dN_C = (\text{const})\tfrac{1}{9}\,[7\sigma_0 + 11\sigma_1 - 3(\sigma_0 - \sigma_1)\cos^2\theta' - 3\sigma_1 \sin^2\theta' \cos 2(\phi - \phi') + 3\sqrt{2}\,\mathrm{Re}\langle a_0 a_1^*\rangle \times \sin 2\theta' \cos(\phi - \phi')]\, d\Omega\, d\Omega' .$$

Here σ_0 and σ_1 are cross sections for populating the magnetic substates of the level H(2p) corresponding to $M_L = 0$ and ± 1, respectively. The quantity $a_0 a_1^*$ is a density-matrix element and not a cross section. By definition, the total differential cross section is

$$\left(\frac{d\sigma}{d\Omega}\right)_{2p} = \sigma_0 + 2\sigma_1 .$$

For the present experiment, the neutral detector and incident beam are coplanar, and the photon detector is located at 90° to this plane. Assuming an infinitesimal scattering region, we have

$$dN_c = (\text{const})\ \tfrac{7}{9}\ (\sigma_0 + 2\sigma_1)\ d\Omega\, d\Omega' .$$

The factor $\frac{7}{9}$ represents the reduction in intensity measured perpendicular to the beam that is due to precession of the angular momentum vectors. This results from the fine structure associated with the H(2p) level. An integration over the finite detector geometries of the present experiment for a scattering angle of 1° yields a result which is

$$dN_c = (\text{const})\ \tfrac{7}{9}\ (0.99\sigma_0 + 1.87\sigma_1)\, d\Omega\, d\Omega' .$$

For this, the most extreme case, the number of coincidences is proportional to a real cross section, but not directly proportional to the total cross section. The lack of proportionality becomes less as the scattering angle increases. The functional relationship between σ_0 and σ_1 is unknown so that no corrections have been made in the results presented later. Extending the relation $\sigma_0 \approx 2\sigma_1$ shown in Ref. 1 to the case of differential scattering, a correction of approximately 10% would result for 1° scattering. The important conclusion is that the number of coincidences is proportional to a real cross section for the reaction $H^+ + He \to H(2p) + He^+$. For many experiments, this result is not valid. A further discussion of this subtle point may be found in Ref. 14.

III. APPARATUS

A. General Description

A top view of the scattering chamber used in the experiment is shown schematically in Fig. 3. A proton beam, obtained by analyzing the positive ion beam extracted from a duoplasmatron, was introduced into the 18-in.-diam scattering chamber through 1-mm-diam beam collimating apertures. The calculated divergence of the incident proton beam was ±0.23° but beam profile measurements indicated the actual beam diameter was essentially 1 mm through the scattering region. This is discussed more fully in the preceding paper.[15] Shown in Fig.

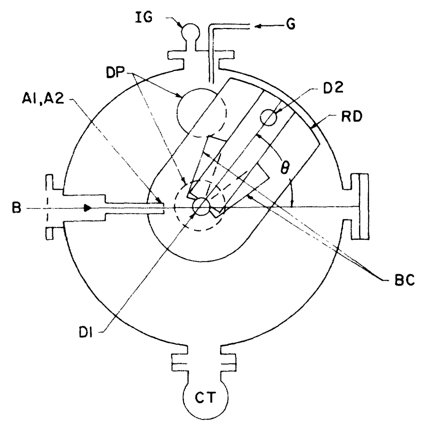

FIG. 3. Scattering chamber diagram. IG, ionization gauge; G, gas inlet; DP, diffusion pump; A1, A2, beam defining apertures; B, incident beam; D1, photon detector; D2, particle detector; RD, rotating detector assembly; BC, beam collectors; CT, cold trap.

3 is the particle detector assembly which may be rotated from outside the scattering chamber. The entrance slit of the collimator for the particle detector is defined by the sharp inside edges of the beam collection cups. This detector assembly encloses the particle detector, a set of deflection plates, and a second defining slit which is part of the collimator for the particle detector. The resultant angular half-width of the scattered particle collimator was 0.33°. During initial assembly of the apparatus, special care was taken to assure that the beam axis and the axis of the particle detector assembly intersected at the center of rotation of the assembly.

Also mounted on the detector assembly were a quench capacitor and photomultiplier viewing the quench region, which were used in a companion experiment, the purpose of which was to study differential scattering with charge exchange to the metastable state $H(2s)$.[15] This photomultiplier was also utilized in determining the efficiency of the particle detector as will be discussed later.

During a data acquisition run the entire chamber was filled with target gas. Oil diffusion pumps, baffled by liquid-N_2 traps located on the beam tube and below the center of the scattering chamber, differentially pumped the scattering region. The third diffusion pump located on the scattering chamber maintained a base pressure of $(2-5)\times 10^{-8}$ Torr when no target gas was present. The detector assembly pressure was at least a factor of 100 below the target gas pressure.

B. Electronics

The coincidence circuitry illustrated in Fig. 1 consisted of commercial NIM units. Detector outputs were amplified and shaped by charge-sensitive preamplifiers. Further amplification and shaping was accomplished by double-delay-line amplifiers. The resulting bipolar pulses were fed into timing single-channel analyzers which operated in a cross over timing mode. A coincidence module and the switching module described earlier completed the signal processing unit. Such a system is capable of resolving times of approximately 20 nsec but system performance was degraded by electrical noise and resolving times of 80 – 100 nsec were used during the measurements.

A commercial integrator which was used to integrate the incident beam current during a data run also served as a control to terminate the data run after a predetermined amount of charge had been collected.

Target gas pressure was measured with an ion gauge which had been calibrated against a McLeod gauge. The McLeod gauge was cooled during the calibration procedure to eliminate mercury streaming effects. Target gas pressures were in the $10^{-3}-10^{-4}$-Torr range. Photon and scattered-particle signals were linear with target pressure and ion beam current.

The Ly-α radiation was observed by an EMR 542 G photomultiplier. This "solar blind" tube has very low dark current and a photocathode with response which peaks near the Ly-α line. To determine the absolute efficiency for detecting radiation a comparison was made between the total cross section σ_{2p} for the reaction $H^+ + Ar$ determined during the present experiment and the previous results obtained by the University of Washington group and Leningrad workers. Figure 4 shows the results of this comparison for a tube efficiency of 0.053. Because of the secondary nature of this measurement, the largest absolute uncertainty of the experiment is associated with this efficiency value. A conservative estimate of this uncertainty is ±40% including the uncertainties quoted in Refs. 1 and 2.

Particles scattered into the detector assembly were detected by a 13-stage secondary electron

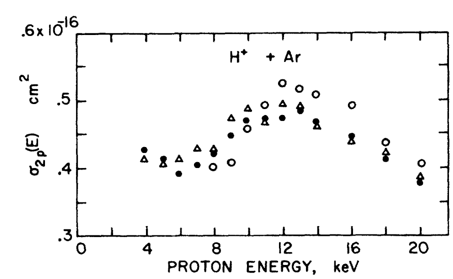

FIG. 4. Total cross sections for charge exchange to the state $H(2p)$ for $H^+ + Ar \rightarrow H(2p) + Ar^+$. Δ, present data for tube efficiency of 0.053; ●, data of Gaily, Jaecks, and Geballe, Ref. 1; O, data of Andreev, Ankudinov, and Bobashev, Ref. 2.

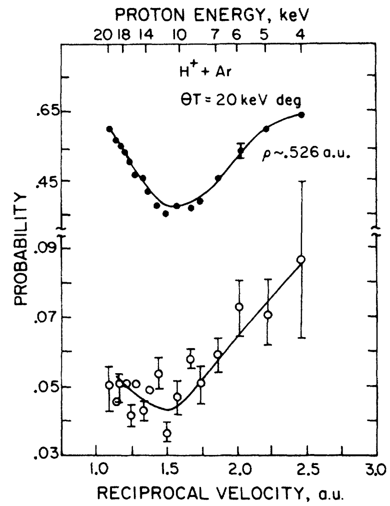

FIG. 5. Probability of charge exchange for the system H^+ + Ar. ●, total probability P_0; ○, probability of charge exchange into the state H(2p), P_{2p}.

multiplier with Cu-Be dynodes. The method by which the absolute efficiency of the multiplier for the detection of fast neutral atoms was measured has been described in a previous publication.[16] Those atoms entering the detector assembly which are in the metastable state H(2s) may be detected by applying an electric field along the scattered beam. This field causes quenching of the metastable state by Stark mixing of the H(2s) and H(2p) states. The photons emitted in the decay are detected by a photomultiplier mounted on the detector assembly.

If the output of the photomultiplier and the particle detector are connected to the coincidence circuitry described earlier, real coincidences should be observed for each photon detected if the particle detector has an efficiency of one. If not, the ratio of real coincidence counts to photon counts is a direct measure of the detector efficiency. This technique is particularly attractive because the efficiency measurement does not depend on the absolute determination of other quantities such as pressure or charge.

IV. RESULTS

The differential cross section for charge exchange into the state H(2p) and the differential scattering cross section for all particles were determined from the formulas

$$\left(\frac{d\sigma}{d\Omega}\right)_{2p} = \frac{eN_c}{\rho Q\epsilon_1\epsilon_2}\,\frac{4\pi}{\int_c l\,d\Omega_1 d\Omega_2}$$

and

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm tot} = \frac{eN_{\rm tot}}{\rho Q\epsilon_1 \int l\,d\Omega_1},$$

where N_c is the number of real coincidence counts, e is the electronic charge, the target gas density is ρ, Q is the total charge collected, and ϵ_1 and ϵ_2 are the detector efficiencies. The number of scattered particles of all charge states is $N_{\rm tot}$. The quantities

$$(1/4\pi)\int_c l\,d\Omega_1 d\Omega_2 \text{ and } \int l\,d\Omega_1$$

are the geometrical factors defining the effective coincidence scattering region and the total scattering region, respectively.

The probability P_{2p} is defined as

$$P_{2p} = \frac{(d\sigma/d\Omega)_{2p}}{(d\sigma/d\Omega)_{\rm tot}},$$

and the total probability of charge exchange P_0 is given by $P_0 = N_{\rm neut}/N_{\rm tot}$, where $N_{\rm neut}$ is the number of neutral particles scattered through some fixed angle and $N_{\rm tot}$ is the number of all the particles scattered through the same angle. Experimental values were determined for each of the quantities listed above.[10] The most significant information is seen in P_0 and P_{2p} which are shown in detail for both collision systems studies. The differential cross section for transfer to the state H(2_p) for the system H^+ + He will also be presented while the total scattering measurements will be presented elsewhere.[17] In the following results, the error bars on P_{2p} and the differential cross section represent 70% confidence limits as determined by "student's" t test while those of P_0 are 1 standard deviation. Curves drawn through experimental values are freehand fits to the data unless otherwise noted.

A distinct oscillatory structure is evident for P_0 for the H^+ on Ar system as is seen in Fig. 5. Data were not available for a sufficiently wide range of collision velocities to allow satisfactory analysis based on Lichten's theory of quasiresonant charge exchange. No definite structure is present in P_{2p} although there is a broad minimum at about the same reciprocal velocity as that seen in P_0. The impact parameter for these results was approximately 0.53 a.u.[15] The probability of charge exchange to the 2p state accounts for about 10% of the total charge-exchange probability for this impact parameter.

The only theoretical calculations for the H^+ + He system which treat charge transfer into excited states are those of Sin Fai Lam.[18] These calculations were made using the impact-parameter formalism and a basis consisting of 1s, 2s, and 2p hydrogenic wave functions. A simple variational wave function was used for the helium atom. Calculations of total cross sections for charge transfer into the 2s state agree well with experiment but similar calculations for the 2p state give a cross section which is a factor of 2 larger than the experimental values.[18] A more striking disagree-

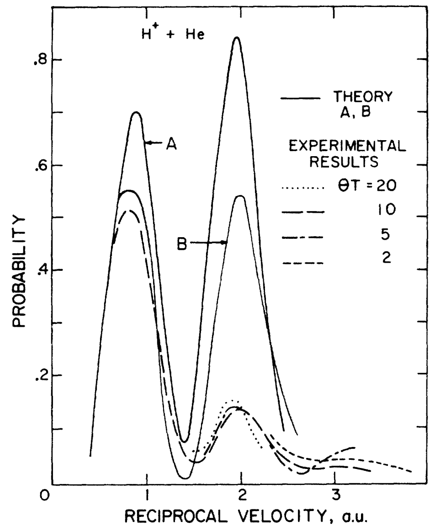

FIG. 6. Comparison of theory and experiment for total probability of charge exchange for H⁺ + He. The experimental results are those of Helbig and Everhart (Ref. 4) for various impact parameters. The theoretical calculation A is due to Sin Fai Lam and B is that of Green.

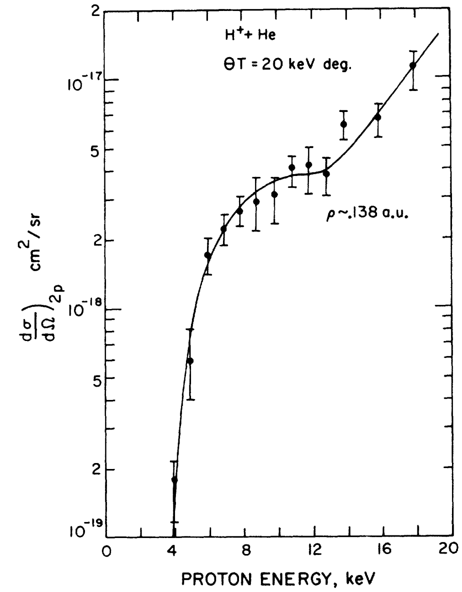

FIG. 7. Differential cross section for charge exchange to the state H(2p).

ment between theory and experiment is evident in Fig. 6 which presents the total capture probability for H⁺+ He. The theoretical treatments shown overestimate P_0 significantly for slower collisions ($1/v$ greater than 1.5 a.u.), while the agreement is much more favorable for faster incident protons. The results of Sin Fai Lam include transfer into the 1s, 2s, and 2p states while those of Green are for transfer into the 1s state only.[19] In his treatment, Green used the impact-parameter method and a two-state atomic wave-function expansion. An application of the perturbed stationary-state method with neglect of momentum transfer was made by Colegrave and Stephens.[20] Their results, which are not included here, agree reasonably well with experiment for energies less than 5 keV. Experimental values shown are those from Ref. 4, but essentially identical results were obtained during the present investigation for a more restricted range of impact parameters and incident proton energies.[15]

The differential cross section for charge transfer into the state H(2p) for the H⁺+ He system is shown in Fig. 7. Here the impact parameter was 0.138 a.u., and the shoulder evident at an incident proton energy of 12 – 14 keV is evidenced more clearly in Fig. 8, in which is seen the structure in P_{2p} when plotted as a function of $1/v$, where v is the incident proton energy in a.u. Presented for comparison with P_{2p} is the probability calculated by Sin Fai Lam as well as the experimentally determined P_0. The maximum and minimum as predicted by theory are coincident with experiment to a remarkable degree.

As was the case for P_0, as shown in Fig. 6, the divergence between theory and experiment is most severe for slower collisions, and for incident proton energies above 15 keV the disagreement is much less. The prominent peak at $1/v \approx 1.9$ a.u. and the minimum in P_0 correspond almost identically

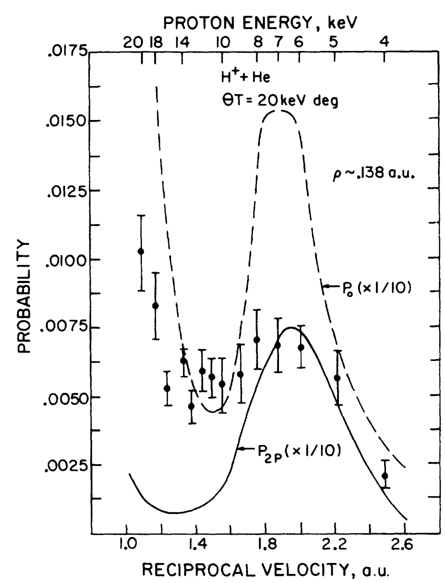

FIG. 8. Probability of charge exchange for the system H⁺ + He, $E = 6.25$ keV deg. ●, present results for P_{2p}; ---, present results for P_0 (multiplied by $\frac{1}{10}$); ——, theoretical calculation of Sin Fai Lam (Ref. 18) for P_{2p} (multiplied by $\frac{1}{10}$).

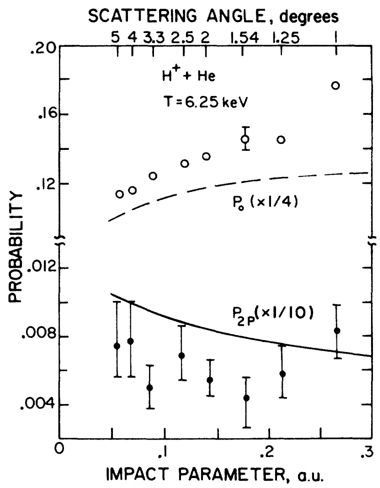

FIG. 9. Probability of charge exchange for the system $H^+ + He$, $E = 6.25$ keV. O, present results for P_0; ---, theoretical calculation of Sin Fai Lam for transfer to the state H(1s) (multiplied by $\frac{1}{4}$); ●, present results for P_{2p}; ——, theoretical calculation of Sin Fai Lam (multiplied by 1/10).

to the oscillation observed in P_{2p}. This strongly suggests a correlation between the processes which produce charge exchange to the 1s state and those which populate the excited states in collisions of the type observed. If P_{2p} and P_0 are simply related, Lichten's theory would predict the next peak in P_{2p} to occur at $1/v \approx 0.8$ a.u., which corresponds to a proton energy of 45 keV.

Figure 9 presents data for which the incident proton energy was 6.25 keV but for which the scattering angle was varied from 1° to 5°. By making this measurement, an indication of the variation of P_{2p} and P_0 with impact parameters may be obtained. For the range of impact parameters studied, there is no significant variation. As in earlier low-energy results the theory considerably overestimates P_0 and P_{2p}.

V. SUMMARY AND DISCUSSION

The results of the present study indicate that for the range of impact parameters investigated, the variation of probability of charge transfer into the 2p state with incident proton energy is essentially like that for total charge transfer. This result supports the suggestion that the charge transfer to excited states proceeds in an intermediate step through the ground state. For the system $H^+ + Ar$, charge exchange into the $n = 2$ states represents a significant fraction of the total probability for the violent collisions studied.

Measurements of differential cross section in general afford a more sensitive test of theoretical calculations than do total cross section measurements. In the case of charge-transfer processes, the contribution to the total cross section from those violent collisions which result in scattering through detectable angles is quite small. Because of this, a calculation may give acceptable results for total-cross-section values but be inadequate in describing differential scattering processes.

Agreement between present experimental results and theory for the system $H^+ + He$ is not good. Green has shown that reasonable results are obtained for P_0 above 10 keV if distortion effects and momentum transfer are included. For energies below 10 keV, the magnitude of both P_0 and P_{2p} (for the calculation of Sin Fai Lam) are grossly overestimated. Recent theoretical studies of the scattering system $H^+ + H$ would indicate that for lower energies, proper choice of the eigenfunction expansion is most important to obtain good agreement with experiment.[21,22] The calculation of Ref. 18 which used an atomic eigenfunction expansion would be expected to yield less satisfactory results for lower energies.

The atomic eigenfunction expansion does agree fairly well with measurements of total charge probability for higher energies (above 10 keV), while the calculation of Colegrave and Stephens, utilizing molecular eigenfunctions, correlates quite closely with experiment below this energy. Clearly, the theoretical situation is quite complex and a single calculation may not be expected to give satisfactory results over a large energy range.

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Theory of Atomic Photon-Particle Coincidence Measurements*

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The theory of measurements in which photons are detected in delayed coincidence with a scattered particle is developed in a form specifically applicable to atomic collisions. Equations are obtained which relate the measured coincidence rate to excitation amplitudes. These equations incorporate the polarization of the radiation, the fine and hyperfine structure of the atomic levels, the coherence of the radiation, and the time dependence of the radiation intensity. A semiclassical model for certain transitions is introduced to illustrate new features of coincidence measurements. The Ly-α transitions in hydrogen and $^1P \to {}^1S$ transitions in He are treated in detail to illustrate the general theory.

I. INTRODUCTION

Recently, coincidence techniques have been used to measure the amplitudes that describe atomic scattering processes. Three different types of measurements, distinguished by the types of particles detected, have been employed. Erhardt[1] has detected scattered electrons in coincidence with secondary electrons ejected in an ionizing collision of electrons with atomic helium. This represents a particle-particle coincidence measurement, and the data from such experiments can be directly interpreted in terms of differential ionization cross sections. Sheridan[2] has proposed measurements of photons emitted in the $D \to P$ transition of He in coincidence with a photon emitted in the subsequent $P \to S$ decay. This constitutes a photon-photon coincidence measurement, and the data can be interpreted in terms of the probabilities for these radiative transitions.[3] Jaecks *et al.*[4] have measured Ly-α photons emitted in the decay of the $2p$ states of hydrogen atoms formed by the charge exchange reaction

$$H^+ + A \to H^* + A^+$$

where A represents any target atom. This represents a photon-particle coincidence measurement, and, although similar experiments have been employed in nuclear physics, a theory relating experimental data to scattering amplitudes has not been given in a form directly applicable to atomic collisions. The theory[5] developed for nuclear studies is not directly applicable to atomic studies since nuclear studies have concentrated on determinations of the multipolarity of the electromagnetic transitions. In experiments of interest in atomic collision studies, the multipolarity of the radiative transitions is known, and experiments are conducted to determine the parameters of the population of states produced by the collision.

Recently, the theory of photon-particle coincidences has been developed in a form directly applicable to elementary particle reactions.[6] This theory emphasizes the determination of scattering amplitudes (or density matrix elements) from experimental data, and is more directly applicable