electron-molecule interaction potential, agrees with the "measured" elastic cross section to within 13% for the energies used in this study.

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Eikonal Theory of Intermediate-Energy Electron-Atom Scattering*

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An eikonal theory of elastic electron-atom scattering in the region of intermediate energies is proposed. In addition to the effects of the static and polarization potentials, we also take into account the leading absorption corrections, using the equivalent-potential method. Detailed calculations are performed for elastic electron-helium scattering in the energy range 100–500 eV. Our results are in good agreement with the recent experimental data.

I. INTRODUCTION

Present calculational techniques yield reliable results for electron-atom scattering processes when the relative incident energy is either low enough or sufficiently large with respect to typical target binding energies. In the former case only a few channels play an important role, so that close-cou-

pling, polarized-orbitals, variational, or related methods are applicable.[1] At high-incident energies the Born series or its modifications have been used extensively.

In the region of intermediate energies we do not expect these techniques to give accurate results. An alternative approach, long popular in nuclear collisions, is then to use the optical potential method in which a reasonable form is chosen for the real and imaginary parts of the interaction between the projectile and the target, and parameters are adjusted to fit the experimental results. Such an approach has been used lately in the case of elastic electron-helium scattering.[2]

A less phenomenological point of view, in which one attempts to *derive* the optical potential from first principles, was presented[3] some time ago and recently applied[4] to the case of intermediate electron-atom scattering. The theory describes elastic scattering by the equivalent potential method which explicitly accounts for the imaginary part of the potential. The basic equations and the approximations involved in the theory are briefly recalled in Sec. II. In Sec. III, we show that the eikonal approximation considerably simplifies the solution of the equations and allows the removal of several approximations. Incidentally, we construct an eikonal solution to the special form of nonlocal potential appearing in the problem. In Sec. IV, we apply our eikonalized equivalent potential method to the case of elastic electron-helium scattering at intermediate energies, a process which has recently attracted considerable interest, both experimental[5-7] and theoretical.[8-11] We show that not only *polarization effects* but also *absorption corrections*—induced by unitarity from the open channels—are important in the energy range considered (100–500 eV). Our results are in good agreement with the recent experimental data[6,7] and yield differential cross sections much larger at small momentum transfers than those obtained from the first Born approximation.

II. BASIC EQUATIONS

We consider an elastic electron-atom scattering process for which the equivalent one-body Schrödinger equation reads

$$(p^2 - T - \mathcal{U}_{\rm opt})\psi_{\vec{p}_i} = 0 \,, \tag{2.1}$$

where p^2 is the energy of the incident electron (in rydbergs), T is its kinetic-energy operator, $\mathcal{U}_{\rm opt}$ is the optical potential and $\psi_{\vec{p}_i}$ is the scattering wave function corresponding to an incident electron with momentum $\vec{p}_i$. Under the approximations of (i) neglecting the Pauli principle between the incident and target electrons and (ii) evaluating the equivalent potential to second order in the multiple-scattering expansion, one can write the optical potential as[3]

$$\mathcal{U}_{\rm opt} = V^{(1)} + V^{(2)} \,. \tag{2.2}$$

The first-order part of the optical potential, expressed in configuration space, is simply the static potential of the atom,

$$V^{(1)}(\vec{r}) = \langle 0 | V | 0 \rangle = \int |\Phi_0(x)|^2 V(\vec{r}, x)\, dx \,, \tag{2.3}$$

where $\Phi_0(x)$ is the initial wave function of the target, x denotes all the target coordinates, and $V(\vec{r}, x)$ is the electron-atom interaction potential.

The second-order part of the optical potential operator is given by[12]

$$V^{(2)} = \sum_{n\neq 0} \frac{\langle 0|V|n\rangle\langle n|V|0\rangle}{p^2 - T - (w_n - w_0) + i\epsilon} \,, \tag{2.4}$$

where the summation runs over all the intermediate states of the target. Here w_0 and w_n represent the internal target energies, respectively, in the initial and in the intermediate state $|n\rangle$, while $\epsilon \to 0^+$, as usual. The summation appearing in Eq. (2.4) can be simplified if a further approximation is made, namely (iii) replacing the excitation energies of the target by an "average excitation energy" Δ. This yields for $V^{(2)}$ the nonlocal potential

$$\langle \vec{r}\,| V^{(2)} | \vec{r}\,' \rangle = G_{p'}(\vec{r}, \vec{r}\,') A(\vec{r}, \vec{r}\,') \,, \tag{2.5}$$

where $G_{p'}$ is the Green's function describing the free[13] propagation of the projectile in some average intermediate state with energy

$$p'^2 = p^2 - \Delta \tag{2.6}$$

and the function $A(\vec{r}, \vec{r}\,')$ is defined by[4]

$$A(\vec{r}, \vec{r}\,') = \int \Phi_0^*(x) V(\vec{r}, x) [\delta(x, x') - \Phi_0^*(x)\Phi_0(x')] \times V(\vec{r}\,', x')\Phi_0(x')\, dx\, dx' \,. \tag{2.7}$$

The resulting equation [Eq. (I. 6) from Ref. 4], obtained by substituting in Eq. (2.1), is then

$$[p^2 - T - V^{(1)}(\vec{r})]\psi_{\vec{p}_i}(\vec{r}) - \int G_{p'}(\vec{r}, \vec{r}\,') A(\vec{r}, \vec{r}\,')\psi_{\vec{p}_i}(\vec{r}\,')\, d^3r' = 0 \,. \tag{2.8}$$

The approximations (i), (ii), and (iii), leading to the basic equation (2.8), are reliable at sufficiently high energies but the details of each and the three in combination have yet to be investigated In Ref. 4, hereafter referred to as paper I, three further approximations were made in order to solve Eq. (2.8): (iv) a separable approximation to the function $A(\vec{r}, \vec{r}\,')$, namely,

$$A(\vec{r}, \vec{r}\,') \simeq v(\vec{r})v(\vec{r}\,') \,, \tag{2.9}$$

which thereby replaces all inelastic channels by a single average one.[14] (v) The quantity Δ was set equal to zero and the potential in $G_{p'}$ chosen to be $V^{(1)}$. The choice of potential is not important as we shall show below, but neglecting Δ results in a logarithmic divergence of the cross section in the

forward direction which turned out to be numerically significant in the electron-helium example worked out in I. (vi) The resulting decoupled equations were treated in the eikonal approximation.

The first purpose of this paper is the removal of approximations (iv) and (v). This is accomplished in Sec. III, where the eikonal approximation[15] is applied directly to the basic equation (2.8). The difficulty in the forward direction will thereby be eliminated.

III. EIKONAL APPROXIMATION

Let us first analyze the free propagator $G_{p'}$ appearing in Eq. (2.8). We write it in momentum space as

$$G_{p'}(\vec{r},\vec{r}') = -(2\pi)^{-3}\int d^3k\,\frac{e^{i\vec{k}\cdot(\vec{r}-\vec{r}')}}{k^2-p'^2-i\epsilon}\ , \qquad (3.1)$$

or

$$G_{p'}(\vec{r},\vec{r}') = -(2\pi)^{-3}\,e^{i\vec{p}'\cdot(\vec{r}-\vec{r}')}\int d^3q\,\frac{e^{i\vec{q}\cdot(\vec{r}-\vec{r}')}}{2\vec{p}'\cdot\vec{q}+q^2-i\epsilon}\ , \qquad (3.2)$$

where we have changed the variable of integration to $\vec{q}=\vec{k}-\vec{p}'$. We take $\vec{p}'$ in the direction of the incident momentum so that for small-angle scattering—consistent with the eikon approximation—the vector $\vec{q}$ is essentially the momentum transfer during a scattering. Since $q \ll p'$ in the region contributing to the scattering we may neglect the q^2 term in the denominator.[16-18] With this approximation[19] the linearized propagator $G_{p'}$ may be written as

$$G_{p'}(\vec{r},\vec{r}') = -(i/2p')\,e^{i\vec{p}'\cdot(\vec{r}-\vec{r}')}\,\delta(\vec{b}-\vec{b}')\,\Theta(z-z')\ , \qquad (3.3)$$

where z is the component of $\vec{r}$ along the incident direction and the impact parameter vector $\vec{b}$ is perpendicular to it. The step function $\Theta(z-z')$ is unity for positive argument and zero otherwise. If we now substitute Eq. (3.3) into Eq. (2.8) and use the eikonal form of the scattering wave function

$$\psi_{\vec{p}_i} = e^{i\vec{p}_i\cdot\vec{r}+i\Lambda(\vec{b},z)}\ , \qquad (3.4)$$

where $\vec{p}_i$ is the incident momentum (of length p), we obtain

$$-2\vec{p}_i\cdot\vec{\nabla}\Lambda - V^{(1)}(\vec{r}) - (\vec{\nabla}\Lambda)^2 + i\nabla^2\Lambda + (i/2p')\times\int_{-\infty}^{z} e^{i(p'-p)(z-z'')}A(\vec{b},z;\vec{b},z')\,e^{i[\Lambda(\vec{b},z'')-\Lambda(\vec{b},z)]} = 0\ . \qquad (3.5)$$

Now, consistent with our previous approximations, we assume that Λ is slowly varying on the scale of the de Broglie wavelength of the incident particle. Then the quantity $\Lambda(\vec{b},z')-\Lambda(\vec{b},z)$ may be dropped inside the integral in Eq. (3.5) and the higher-order terms $(\vec{\nabla}\Lambda)^2$ and $\nabla^2\Lambda$ may be neglected. The remaining first-order equation for Λ is then easily integrated with the result

$$\Lambda(\vec{b},z) = -(1/2p)\int_{-\infty}^{z} dz'\,V^{(1)}(\vec{b},z') + (i/4pp')\times\int_{-\infty}^{z}dz'\int_{-\infty}^{z'}dz''\,e^{-i\xi(z'-z'')}A(\vec{b},z';\vec{b},z'')\ , \qquad (3.6)$$

where $\xi = p-p' \simeq \Delta/2p$. We now proceed in the standard manner by writing the scattering amplitude as

$$f = -(1/4\pi)\int e^{-i\vec{p}_f\cdot\vec{r}}[V^{(1)}(\vec{r})\delta(\vec{r}-\vec{r}') + G_{p'}(\vec{r},\vec{r}')A(\vec{r},\vec{r}')]\times e^{i\vec{p}_i\cdot\vec{r}'+i\Lambda(\vec{b}',z')}\,d\vec{r}\,d\vec{r}'\ , \qquad (3.7)$$

where $\vec{p}_f$ is the final momentum. If we again use Eq. (3.3) and make the Glauber approximation[15] of neglecting the longitudinal component of the momentum transfer in the exponent, then the z integration may be performed with the result

$$f = -(ip/2\pi)\int d^2\vec{b}\,e^{i\vec{K}\cdot\vec{b}}[e^{i\chi(\vec{b})}-1]\ , \qquad (3.8)$$

where $\vec{K}=\vec{p}_i-\vec{p}_f$ is the momentum transfer and the phase $\chi(\vec{b})$ is given by

$$\chi(\vec{b}) = -(1/2p)\int_{-\infty}^{+\infty} V^{(1)}(\vec{b},z)\,dz + (i/4pp')\times\int_{-\infty}^{+\infty}dz\int_{-\infty}^{z}dz'\,e^{-i\xi(z-z')}A(\vec{b},z;\vec{b},z')\ . \qquad (3.9)$$

Direct integration of the last term in Eq. (3.9) for even the simplest target is laborious and not completely consistent. In making this lowest-order approximation for the eikonal, additional real higher-order contributions from the static potential $V^{(1)}$ are neglected. The real part of the second term of Eq. (3.8) would combine with these terms contributing to the same order (in p^{-1}). We therefore neglect the real part of this second term, since we have already dropped higher-order terms in $V^{(1)}$, so that the eikonal becomes

$$\chi(\vec{b}) = -(1/2p)\int_{-\infty}^{+\infty} V^{(1)}(\vec{b},z)\,dz + (i/8pp')\times\int_{-\infty}^{+\infty}dz\int_{-\infty}^{+\infty}dz'\,e^{-i\xi(z-z')}A(\vec{b},z;\vec{b},z')\ , \qquad (3.10)$$

where we have used the fact that A is real and symmetric.

Before we proceed with Eq. (3.10) let us digress shortly on the separable approximation (2.9). With this approximation for A the scattering amplitude (3.8) becomes

$$f = -(ip/2\pi)\int d^2\vec{b}\,e^{i\vec{K}\cdot\vec{b}}\times[e^{-(i/p)u_1(\vec{b})}\,e^{-(1/2p^2)|u_2(\vec{b})|^2}-1]\ , \qquad (3.11)$$

where

$$u_1(\vec{b}) = \tfrac{1}{2}\int_{-\infty}^{+\infty} V^{(1)}(\vec{b},z)\,dz \qquad (3.12)$$

and

$$\bar{u}_2(\vec{b}) = \tfrac{1}{2}\int_{-\infty}^{+\infty} e^{-i\xi z} v(\vec{b}, z)\, dz \ . \tag{3.13}$$

In I the separable approximation for A and the remaining approximations resulted in the form (3.11) with the factor $\exp[-|\bar{u}_2(\vec{b})|^2/2p^2]$ replaced by $\cos[|u_2(\vec{b})|^2/p^2]$, where $u_2(\vec{b})$ is given by Eq. (2.13) with $\xi = 0$. These two expressions are identical up to terms of order p^{-4} and a numerical evaluation of the two forms (with $\xi = 0$) yields very similar results. The only difference in the assumptions used in deriving the two forms is that in I the propagator $G_{p'}$ was chosen to correspond to the potential $V^{(1)}$ instead of being the free propagator. This choice is therefore seen to be unimportant, as we had anticipated. We note, however, that we are now able to retain nonzero values of the quantity Δ, so that the difficulty in the forward direction encountered in I is avoided here.

We now return to Eq. (3.10) and proceed without the separable approximation to the function $A(\vec{r}, \vec{r}')$. We choose a Hartree-Fock wave function Φ_0 so that only the ground-state orbitals are necessary to compute A. As an illustration, we discuss the case of elastic electron-helium scattering where only one orbital $g(x)$ is required. For simplicity we shall analyze in detail the one-parameter approximation for this orbital

$$g(x) = (\alpha^3/\pi)^{1/2} e^{-\alpha x}, \quad \alpha = \tfrac{27}{16} \ . \tag{3.14}$$

We mention, however, that all the results of the next section have been obtained by using an analytical fit[20] to the Hartree-Fock orbital of Roothaan *et al.*,[21] as shown in Sec. IV.

We now write the phase shift $\chi(\vec{b})$ of Eq. (3.10) as

$$\chi(\vec{b}) = -(1/p)\, u_1(\vec{b}) + (i/2pp')\, W(\vec{b}) \ , \tag{3.15}$$

where we have used Eq. (3.12) and defined

$$W(\vec{b}) = \tfrac{1}{4}\int_{-\infty}^{+\infty} dz \int_{-\infty}^{+\infty} dz'\, e^{-i\xi(z-z')}\, A(\vec{b}, z; \vec{b}, z') \ . \tag{3.16}$$

Returning to the definition of A, Eq. (2.7), we write

$$W(b) = W_1(b) - W_2(b) \ , \tag{3.17}$$

where

$$W_1(b) = 2\int_{-\infty}^{+\infty} dz \int_{-\infty}^{+\infty} dz'\, e^{-i\xi(z-z')} \times \int d^3x\, \frac{g^2(x)}{|\vec{x}-\vec{r}|\,|\vec{x}-\vec{r}'|} = 16\int_0^{\infty} du\, \frac{u J_0(2\xi b \sinh u)}{[1+(\xi/\alpha)^2 \sinh u]^2} \tag{3.18}$$

and

$$W_2(b) = 2\left|\int_{-\infty}^{+\infty} dz\, e^{i\xi z}\int d^3x\, \frac{g^2(x)}{|\vec{x}-\vec{r}|}\right|^2 = 8\left|K_0(\xi b) - K_0(ab) - (2\alpha^2 b/a)\, K_1(ab)\right|^2 \ . \tag{3.19}$$

Here $a = (\xi^2 + 4\alpha^2)^{1/2}$, J_0 is an ordinary Bessel function, and K_0 and K_1 are modified Bessel functions.

The average excitation energy Δ, which acts as a cutoff parameter for the long-range behavior of W [for large b, one has $W \sim \exp(-2\xi b)$] has yet to be defined. An estimate of it can be obtained by requiring that it be chosen in such a way as to make the next-order correction term vanish. If the method of I is followed, the correction to the potential in Eq. (2.8) is obtained as

$$\delta V = -(2\pi)^{-3}\int d^3k\, \frac{e^{i\vec{k}\cdot(\vec{r}-\vec{r}')}}{k^2 - p^2 - i\epsilon} \times \sum_{n\neq 0}\int dx\, \Phi_0^*(x) V(\vec{r}, x)\Phi_n(x)[w_n - w_0 - \Delta] \times \int dx'\ \Phi_n^*(x') V(\vec{r}', x')\Phi_0(x) \ , \tag{3.20}$$

where w_n is the energy of the nth atomic state. To obtain a crude estimate of Δ, we note that this

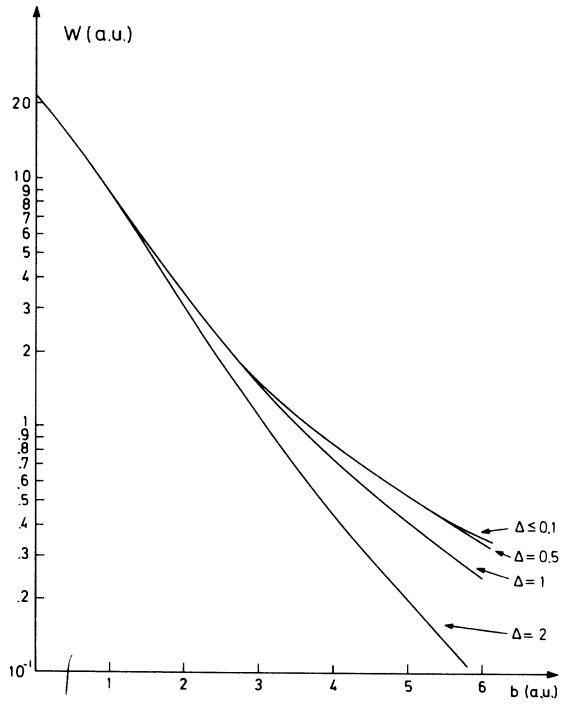

FIG. 1. The function W of the text vs the impact parameter b for elastic electron-helium scattering at $E = 200$ eV and for various values of the average target excitation energy Δ.

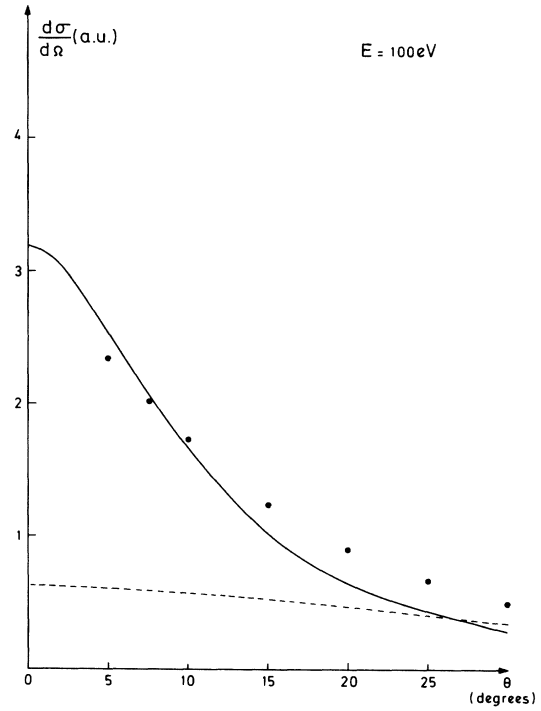

FIG. 2. Differential cross section for elastic electron scattering by helium at an incident electron energy E = 100 eV. The dots are the experimental points of Chamberlain *et al.* (Ref. 7). The curve is obtained from the present theory with the choice $d^2 = 2.40$ of the cutoff distance in the polarization potential and $\Delta = 2$ of the average target excitation energy. The dashed curve corresponds to the first Born approximation. The wave function used to describe the helium ground state is that of Byron and Joachain (Ref. 20).

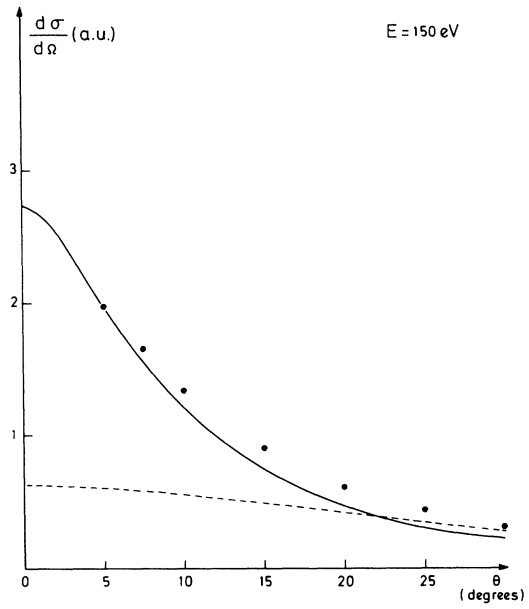

FIG. 3. Same as Fig. 2, but for $E = 150$ eV. Here $d^2 = 2.40$ and $\Delta = 2$.

quantity only plays a significant role at long range so that we specialize Eq. (3.20) to the case r, $r' \to \infty$. Upon taking this limit and requiring that this correction vanish, we obtain

$$\Delta = \frac{\int dx\, \Phi_0^*(x)\, \vec{x} \cdot (H_t - w_0)\vec{x}\Phi_0(x)}{\int dx\, \Phi_0^*(x)\, x^2 \Phi_0(x)} , \qquad (3.21)$$

where H_t is the target Hamiltonian and $\vec{x}$ is the dipole moment operator of the atom. This can be reduced to

$$\Delta = 3N/\langle x^2 \rangle_0 = 3[\langle x_1^2 \rangle_0 + (N-1)\langle \vec{x}_1 \cdot \vec{x}_2 \rangle_0]^{-1} , \qquad (3.22)$$

where the x_i are the coordinates of the ith electron in the atom, N is the number of electrons, and the subscript 0 denotes a ground-state expectation value. A crude evaluation of this expression for helium yields $\Delta \simeq 2.5$ indicating that the average excitation energy is somewhat larger than the ionization energy, a not unreasonable result.

Before we turn to our numerical results for helium we consider a problem discussed briefly in I. The expression containing the quantity A in Eq. (2.8) comes from the second-order term in the equivalent potential. This term also gives the long-range (local) polarization potential $V_p(r) \sim r^{-4}$. We have lost the latter in the approximations leading to Eq. (2.8). At sufficiently large r these approximations are bad and the corrections would lead to the polarization potential. We thus amend Eq. (3.10) by writing

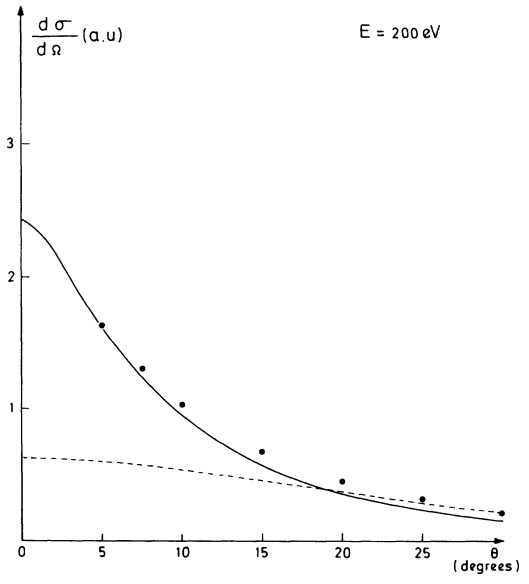

FIG. 4. Same as Fig. 2, but for $E = 200$ eV. Here $d^2 = 2.65$ and $\Delta = 2$.

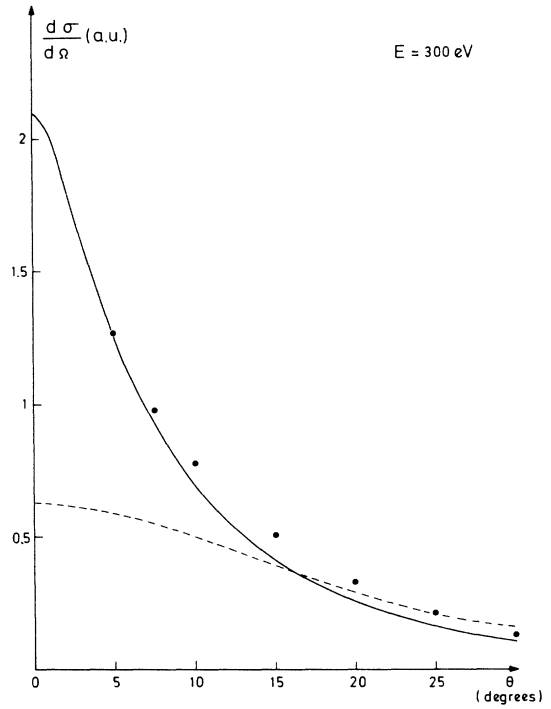

FIG. 5. Same as Fig. 2, but for $E = 300$ eV. Here $d^2 = 3.10$ and $\Delta = 2$.

$$\chi(\vec{b}) = -(1/2p)\left[\int_{-\infty}^{+\infty} V^{(1)}(\vec{b}, z)\,dz + \int_{-\infty}^{+\infty} V_p(\vec{b}, z)\,dz\right] + (i/8pp')\int_{-\infty}^{+\infty} dz \int_{-\infty}^{+\infty} dz'\, e^{-i\xi(z-z')} A(\vec{b}, z; \vec{b}, z') \,. \tag{3.23}$$

Choosing a polarization potential of the Buckingham[22] form (in rydbergs)

$$V_p(r) = -\bar{\alpha}/(r^2 + d^2)^2 \,, \tag{3.24}$$

where $\bar{\alpha}$ is the polarizability of the atom and the length d is a phenomelogical parameter, we may rewrite Eq. (3.15) as

$$\chi(\vec{b}) = -(1/p)\left[u_1(\vec{b}) + u_p(\vec{b})\right] + (i/2pp')W(\vec{b}) \,, \tag{3.25}$$

where

$$u_p(\vec{b}) = -\pi\bar{\alpha}/4(b^2 + d^2)^{3/2} \,. \tag{3.26}$$

We note that the parameters Δ and d have significance only near the forward direction. An encouraging fact is that the function $W(b)$ given by Eq. (3.16) is fairly insensitive to the value of Δ, except for large impact parameters b. This is shown in Fig. 1, where the quantity $W(b)$ is displayed as a function of b for electron-helium scattering at an incident electron energy $E = 200$ eV and for various values of Δ.

An alternative way of gaining information on the values of the parameters Δ and d, which takes advantage of the fact that the eikonal approximation preserves unitarity at small scattering angles, is to use the total (complete) cross section which can be deduced from the absorption coefficient and is related to $\mathrm{Im} f(\theta = 0)$ by the optical theorem.

IV. ELASTIC ELECTRON-HELIUM SCATTERING

We now apply the theory presented above to analyze the elastic scattering of electrons by helium in the intermediate energy range 100–500 eV. We use the following analytical fit[20] to the Hartree-Fock orbital:

$$g(x) = (4\pi)^{-1/2}\left[Ae^{-\alpha x} + Be^{-\beta x}\right] \,, \tag{4.1}$$

where $A = 2.605\,05$, $B = 2.081\,44$, $\alpha = 1.41$, and $\beta = 2.61$. This function gives a Hartree-Fock energy which agrees with that calculated from the 12-parameter function of Roothaan *et al.*,[21] to a few parts in 10^4. The equations of Sec. III are only slightly complicated when the form (4.1) is used instead of (3.14). We have computed the differential cross sections arising from the eikonal amplitude (3.8) in which we have taken into account the effects of the static potential, the polarization potential and the leading absorption corrections as discussed in Sec. III. In contrast to previous approaches,[8,9] we first concentrate on the contribution of the static potential and the *absorption effects*. Since the angular distribution is rather insensitive to the values of the parameter Δ (except at very small

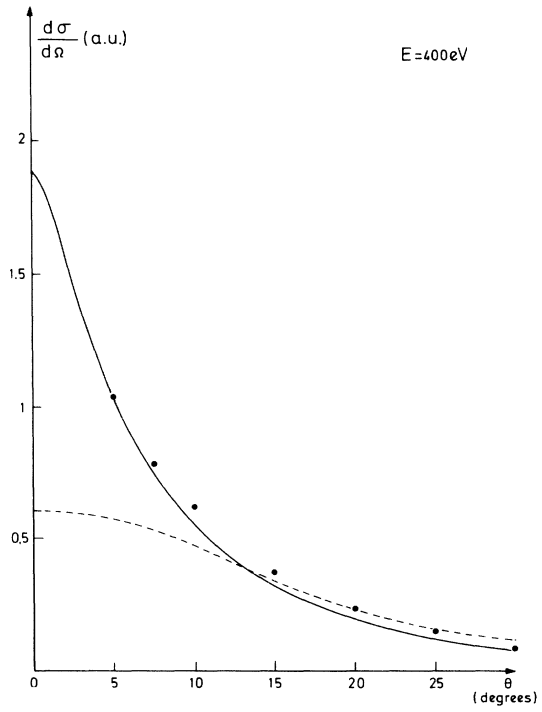

FIG. 6. Same as Fig. 2, but for $E = 400$ eV. Here $d^2 = 3.60$ and $\Delta = 2$.

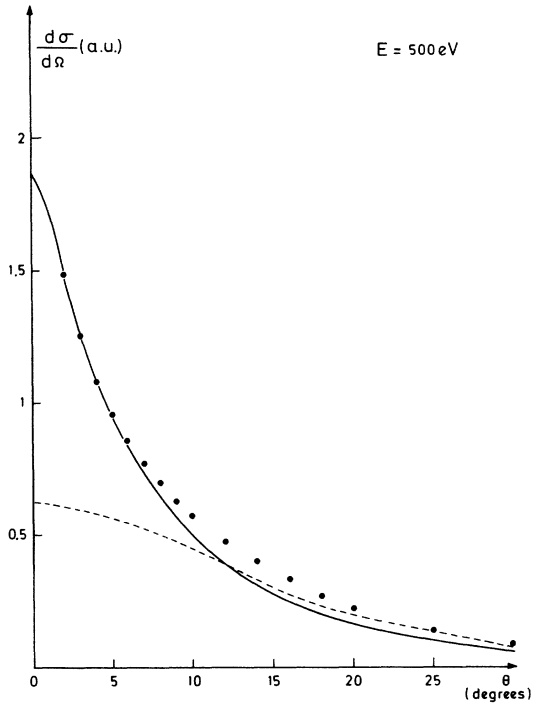

FIG. 7. Same as Fig. 2, but for $E = 500$ eV. The dots are the experimental data of Bromberg (Ref. 6). Here $d^2 = 3.40$ and $\Delta = 2$.

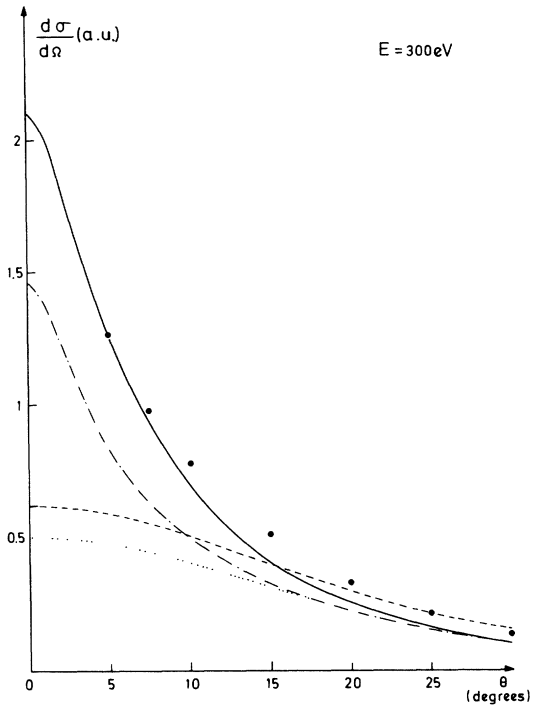

FIG. 8. Comparison of various theoretical predictions for the differential cross section of elastic electron-helium scattering at $E = 300$ eV. Solid line: present theory, including the polarization potential and absorption effects; dash-dot line: polarization potential neglected, absorption effects included; dotted line: both polarization and absorption effects neglected; dashed line: first Born approximation. The experimental points are those of Chamberlain *et al.* (Ref. 7).

scattering angles), the absorption corrections are determined without ambiguity. The cutoff parameter in the polarization potential is then adjusted in order to fit the absolute magnitude of the experimental data at a scattering angle $\theta = 5°$.

Our results are displayed in Figs. 2–7 where they are compared with the experimental data[6,7] and with the predictions of the Born approximation. Our values represent a considerable improvement over the Born approximation, which even at 500 eV

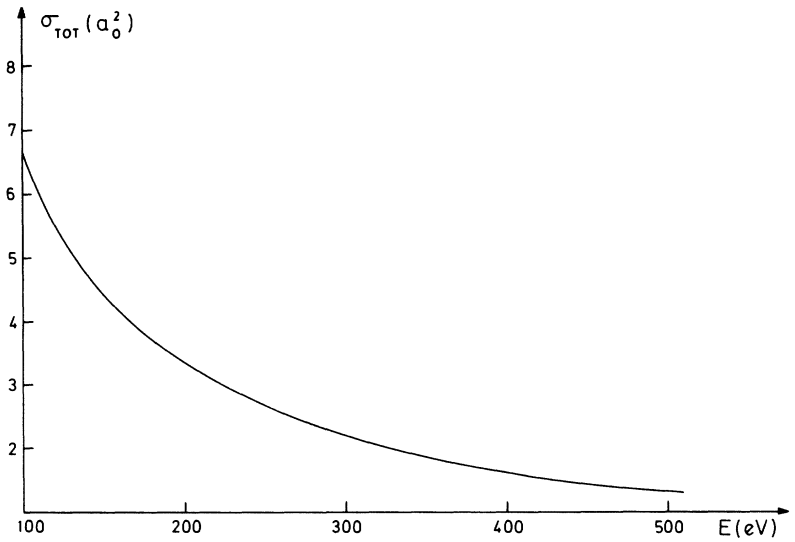

FIG. 9. Total cross section for electron-helium scattering (in units a_0^2) as a function of the incident electron energy, for $\Delta = 2$ and the values of d^2 reported in Figs. 2–7.

yields much too small intensities at small momentum transfers. Owing to the presence of Δ, our angular distributions are free of the forward direction divergence which appeared in I and also when the many-body eikonal approximation of Glauber[15] is applied to the elastic scattering of a charged particle by an atom.[11] The fact that our angular distribution falls off too rapidly at larger angles is probably due to the breakdown of the eikonal approximation which is certainly less accurate in this angular region.

In order to display separately the various contributions to the differential cross section, we show in Fig. 8 the results of several calculations at $E = 300$ eV which omit the polarization potential, or the absorption corrections, or both. The dash-dot line, which accounts for the effects of the static potential together with the absorption corrections, exhibits the correct trend near the forward direction. It emphasizes the importance of *absorption effects* which are ignored in such treatments as the "extended polarization approximation."[9] A recent analysis[10] of elastic electron-helium scattering, based on forward dispersion relations, lends strong support to the results obtained here in showing that absorption corrections play a key role in understanding elastic electron-atom scattering at intermediate energies.

Finally, we display in Fig. 9 the total cross section σ_{tot} (electron + helium → anything) as a function of the incident energy, using our best values of Δ and d. Measurements of σ_{tot}, together with additional experimental data on the angular distributions at small angles, would obviously be of great interest in order to determine accurately the quantities Δ and d.

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