Motion of muons in heavy hydrogen in an applied electrostatic field

K. F. Ness and R. E. Robson
Department of Physics, James Cook University, Townsville, Queensland 4811, Australia
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Transport and reactive properties of a muon swarm in deuterium gas in an electrostatic field are analyzed by solving Boltzmann's equation over a wide range of E/n_0 (ratio of the applied electric field to the gas number density). We find that both the muon-capture rate and the recoil energy of the deuterium molecule after muon capture show significant variation with E/n_0. The implications for muon-catalyzed-fusion research are outlined. Muon transport properties are given as functions of E/n_0.

Recent interest in muon-catalyzed fusion as an energy source[1,2] has followed the prediction[3–5] and later experimental verification[6–8] that a single muon can catalyze 100 or more fusions. As pointed out previously,[9] the kinetic theoretical methods of swarm physics can be readily adapted to muon swarms in hydrogen, and we thus analyze the first step in the cycle,[1] muonic-atom formation, using an established and highly accurate procedure for solving Boltzmann's equation. The next, and perhaps most crucial step in the cycle is the resonant formation of the mesomolecule due to interactions between hydrogen molecules and the muonic atoms.[1] Cohen and Leon[10] proposed as epithermal model to explain the temperature dependence of the mesomolecule-formation rate, in which the more energetic muonic atoms lead to faster formation rates. Rather than enhancing the available energy in the center of mass by increasing the gas temperature (and therefore also the pressure, thereby adding to containment problems) we consider here the possibility of achieving the same result using an electric field E instead to increase the steady-state temperature of the muons with a view to raising the average recoil energy of the muonic atoms formed following capture. For density $\lesssim 0.1$ liquid-hydrogen density, a significant fraction of "hot," recoiling muonic atoms, produced initially in a highly-excited state ($n \gtrsim 14$) can be expected[10–12] to reach the ground state and subsequently participate in mesomolecular formation. Furthermore, as the capture cross section is energy dependent, the capture rate itself λ_a will vary with E and hence, the rate of formation of muonic atoms, in addition to their recoil energy, can be controlled, at least in principle, by the electric field. However, the variation of λ_a is probably of little practical significance in muon-catalyzed-fusion experiments, where the "bottleneck" in the cycle is the mesomolecular formation rate. We also believe that, apart from this potential application, the theoretical calculation of muon transport properties, as presented here, is of interest in its own right, since the possibility may soon arise, as is indeed already the case with positrons,[13] of performing muon swarm experiments in order to determine low-energy muon-matter cross sections.

Boltzmann's equation for the distribution function, $f(\mathbf{r},\mathbf{c},t)$ of a tenuous swarm of muons of charge e, mass m, number density n, and velocities $\mathbf{c}$, moving through a gas of neutral (deuterium) molecules of number density n_0, is

$$\left[\frac{\partial}{\partial t}+\mathbf{c}\cdot\frac{\partial}{\partial \mathbf{r}}+\frac{e\mathbf{E}}{m}\cdot\frac{\partial}{\partial \mathbf{c}}\right]f(\mathbf{r},\mathbf{c},t)=-[{}^{C}J(f)+{}^{R}J(f)] \ . \tag{1}$$

The collision integral on the right-hand side of (1) is assumed linear on the basis that $n \ll n_0$. ${}^{C}J(f)$ denotes the collision integral of Wang-Chang *et al.*,[14] for conservative collisions and ${}^{R}J$ denotes the reactive part of the collision integral leading to changes in the charge particle number, given by

$$J^R(f)=f(\mathbf{c})\int f_0(\mathbf{c}_0)\sigma_a(g)g\,d\mathbf{c}_0 \ , \tag{2}$$

where $\sigma_a(g)$ denotes the capture cross section averaged over internal neutral states, $f_0(\mathbf{c})$ is a Maxwellian distribution of neutral velocities $\mathbf{c}_0$, and g is the relative speed before capture. We ignore the finite lifetime of the muons for purposes of the present calculation. Furthermore, we assume the neutral molecules to be in thermal equilibrium at temperature T_0 and that the swarm has achieved steady-state conditions in which energy gained from the field is balanced by energy lost in collisions with the neutral molecules, leading to time-independent transport coefficients which are functions of E/n_0. Note, however, that even for moderate values of E/n_0 the steady-state temperature T of the charged particles can be much greater than T_0. (Even for zero field, reactive effects[9] lead to $T \neq T_0$.)

Numerical solution of (1) is achieved using the two-temperature moment method detailed in Refs. 15 and 16. For muons, however, we find that it is necessary to retain terms to second order in the general mass-ratio expansion[17] of J, unlike the case of electron swarms,[16] where only terms to first order in m/m_0 are needed. The muon-capture rate coefficient is given by

$$k_a=(nn_0)^{-1}\int\int f(\mathbf{c})f_0(\mathbf{c}_0)g\sigma_a(g)d\mathbf{c}d\mathbf{c}_0=\langle g\sigma_a\rangle \ , \tag{3}$$

and the capture rate by $\lambda_a=n_0k_a$.

Assuming that all the initial kinetic energy of the muon and deuterium molecule goes into the kinetic energy of the composite particle, μD_2^+ and the ejected electron, then the recoil energy of μD_2^+ in terms of the center of mass and relative velocities, **G** and **g**, respectively, is given by

$$\epsilon_R = \frac{m_c}{2}G^2 + \frac{m m_0 m_e}{2M^2}g^2 - \frac{m_e m_c}{M}\mathbf{G}\cdot\mathbf{g}' , \qquad (4)$$

where m_e is the electron mass, m_c is the mass of μD_2^+, $M = m + m_0 = m_c + m_e$, $\mathbf{G} = (m\mathbf{c} + m_0\mathbf{c}_0)/M$, and $\mathbf{g}'$ is the relative velocity after capture. The average recoil energy following capture is given by

$$u_{\rm rec}^{\dagger} = \langle \epsilon_R g \sigma_a(g) \rangle / \langle g \sigma_a(g) \rangle , \qquad (5)$$

where $\langle\ \rangle$ denotes an average over $f(\mathbf{c})$ and $f_0(\mathbf{c}_0)$ as in (3). If one assumes that the electron is ejected isotropically than the last term in (4) makes no contribution to the average.

Also of interest are the transport coefficients which determine the steady-state behavior of the swarm. These are the drift velocity W, and the diffusion coefficients perpendicular and parallel to the field $D_\perp$ and $D_\parallel$, respectively. Expressions for these in terms of averages of the distribution function are given by Eqs. (53) of Ref. 15.

In the absence of complete information, we have used a rather simple model of muon-deuterium interaction, based in part on electron-deuterium cross sections.[18,19] Our code can easily be adapted to incorporate more realistic cross sections, if known. For the elastic-scattering cross section a constant value of 10 Å^2 is used, and all inelastic-scattering cross sections are assumed to be constant above threshold (Table I). The muon-capture cross section was modeled from Fig. 2 of Ref. 20 as $\sigma_a = 3.5$ Å^2 up to 25 eV and zero beyond that. We have thus calculated transport coefficients for muons in deuterium for E/n_0 in the range $1-10^4$ Td (1 Td = 1 townsend = 10^{-21} V m^2) at $T_0 = 293$ K. In Fig. 1 we see that the capture rate of muons for the model deuterium peaks at approximately 850 Td and then drops off rapidly as the mean c.m. energy exceeds the upper threshold of σ_a of 25 eV. At $E/n_0 = 850$ Td the capture rate and therefore the production of muonic atoms is about a factor of 20 larger than that in the absence of a field. As E/n_0 increases from 1 to 10^4 Td, the mean recoil energy (Fig. 2) of the composite particle μD_2^+ increases by an order of magnitude. The saturation for $E/n_0 > 10^3$ Td is due to the upper threshold of σ_a. If we assume that μD_2^+ splits up into μd and d almost instantaneously following capture then the muonic atom μd will take away approximately half the recoil energy shown in Fig. 2. Thus for $E/n_0 > 10^3$ Td the mean recoil energy of μd is ~0.2 eV.

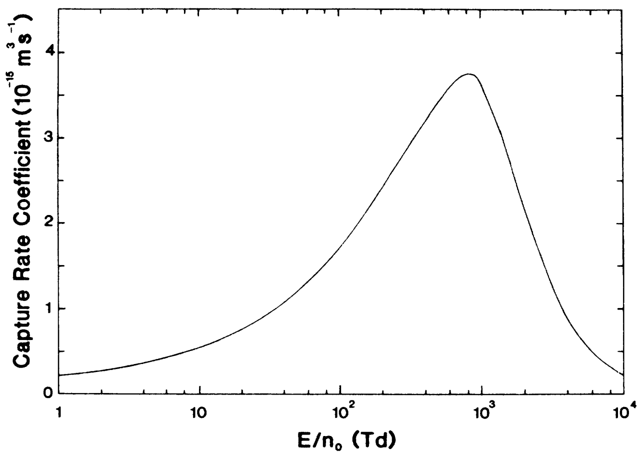

FIG. 1. Muon-capture rate coefficient as a function of E/n_0, $T_0 = 293$ K.

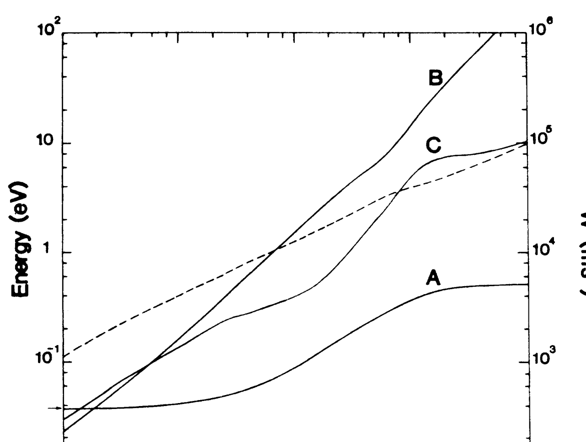

FIG. 2. Average recoil energy of μD_2^+ (A), average muon energy u_μ (B), and muon drift velocity W (C) as a function of E/n_0, $T_0 = 293$ K. The dashed curve gives the quantity ω. Thermal energy is indicated by the arrow on the energy axis.

TABLE I. Inelastic muon-deuterium processes used in the present study. The rotational cross sections are denoted by σ_J with the levels involved indicated in the parentheses, e.g., $\sigma_J(0\rightarrow2)$ is the cross section for the transition from the ground state to the second rotation level. The vibrational processes are modeled by the single cross section σ_v, while σ_d and σ_p model the electronic excitation processes (Ref. 19). The ionization cross section σ_i has been modeled from Fig. 2 of Ref. 20.

Process	Threshold energy (eV)	Cross section (Å^2)
$\sigma_J(0\rightarrow2)$	0.023	0.3
$\sigma_J(1\rightarrow3)$	0.035	0.25
$\sigma_J(2\rightarrow4)$	0.053	0.2
$\sigma_J(3\rightarrow5)$	0.068	0.15
σ_v	0.365	0.2
σ_d	9.0	0.2
σ_p	12.0	1.0
σ_i	15.4	1.5

Note that for $E/n_0 < 1.8$ Td the mean muon energy u_μ falls below the thermal-equilibrium energy $3kT_0/2 = 0.038$ eV. This is due to a strong "capture-cooling" effect caused by the very large capture cross section. For zero applied field, $u_\mu = 0.0125$ eV, corresponding to a steady-state muon temperature of $T = 97$ K, well below the equilibrium temperature of 293 K.

Figure 2 shows that $u^{\dagger}_{\text{rec}}$ increases by an order of magnitude as E/n_0 increases up to 10^3 Td. Such an increase in recoil energy, if translated to the closed fusion cycle in D-T mixtures,[9] would produce an increase in cycling rate of almost 30% for a 60% tritium mixture at room temperature.[21] However, at near-liquid-hydrogen densities, very strong fields $E \gtrsim 10^9$ V/m would be needed to produce this effect. Such fields are presently achievable in small regions of intense laser focus.

In Figs. 2 and 3, the drift velocity, and the longitudinal and transverse diffusion coefficients are shown as functions of E/n_0. Such transport coefficients are of interest because they indicate the rate at which the muons drift from anode to cathode and diffuse to the walls of a containing vessel. Hence it is important to know their behavior with E/n_0 to aid in the design of a "field-tuned" muon-catalyzed fusion vessel. Also shown in Figs. 2 and 3 are the quantities w, $\mathcal{D}_\perp$, and $\mathcal{D}_\parallel$. These denote, respectively, the drift velocity, and the traverse and longitudinal diffusion coefficients neglecting the explicit effect of muon loss due to capture.[17] These quantities are defined by Eqs. (56) of Ref. 17, where their relevance to transport processes is discussed in some detail.[22] In Fig. 2 we note that for $E/n_0 \lesssim 800$ Td, for which $u_\mu \lesssim 10$ eV (see Fig. 2), $W < w$. If we envisage an isolated swarm of muons, this indicates that the preferential capture of the more energetic muons at the front of the swarm effectively retards center-of-mass motion. For $E/n_0 > 800$ Td, corresponding to $u_\mu > 10$ eV, the slower muons at the back of the swarm are preferentially captured and hence the center-of-mass drift velocity is enhanced, i.e., $W > w$. As E/n_0 increases above a few thousand Td the capture rate rapidly drops off (see Fig. 1) and hence $W \to w$, $D_\perp \to \mathcal{D}_\perp$, and $D_\parallel \to \mathcal{D}_\parallel$, as the loss of muons through capture has a diminishing effect upon the transport characteristics.

In conclusion, we have demonstrated that an electric field can be used in principle to alter both the capture rate of muons and the subsequent recoil energy of the muonic atoms. This was done using model muon-deuterium cross sections. Of course, isolation of any segment of the catalysis cycle, in this or any other way (e.g., Ref. 23), will not necessarily supply precise answers to questions concerning actual operating conditions for a reactor. In that case, the *closed* cycle must be considered and the kinetic equation to be solved is (2.11) of Ref. 9, which is effectively (1) above plus a source term accounting for regeneration of muons following fusion. The source term itself is dependent upon an unknown mesomolecule-velocity distribution function which in turn is furnished by another member of the hierarchy [Ref. 9, Eqs. (2.11)–(2.14)] of kinetic equations, and so on. The system will indeed relax to a steady energy state[9] after the initial transients have died away, but how closely this corresponds to the situation discussed here remains to be seen. This question is part of our ongoing program, which aims at solution of the complete kinetic-equation hierarchy describing the velocity distribution functions of all the participant species in the catalysis chain. It is emphasized that such a program is essential for a proper understanding of the cycle, whether or not an electric field is used to tune muon energy, and is indispensable for answering questions concerning optimal operating conditions.

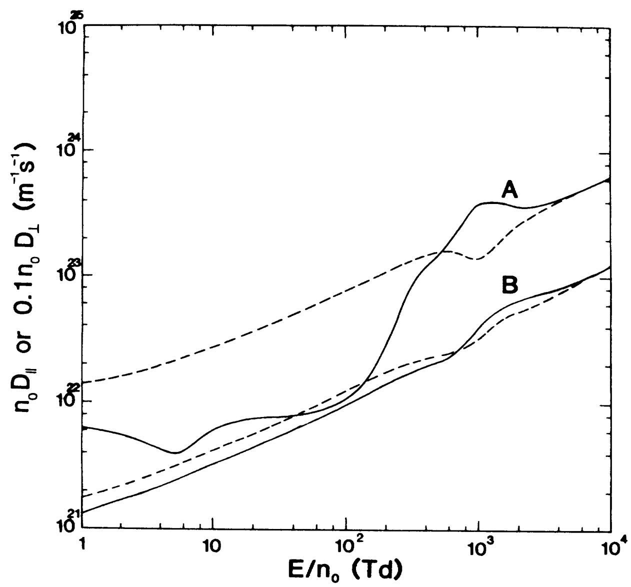

FIG. 3. $n_0 D_\parallel$ (A) and $0.1 n_0 D_\perp$ (B) as a function of E/n_0, $T_0 = 293$ K. The dashed curves give the quantities $n_0 \mathcal{D}_\parallel$ and $0.1\, n_0 \mathcal{D}_\perp$.

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