Collective resonance fluorescence in a squeezed vacuum

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The resonance fluorescence of a small sample of N two-level atoms driven by a strong resonant coherent field and damped by a squeezed vacuum is examined. The steady-state spectrum is shown to be dramatically dependent on the relative phase of the pumping field and on the squeezed-vacuum field in both the strong- and weak-squeezing limits.

I. INTRODUCTION

The 1980's witnessed an intense search for squeezed states of the electromagnetic field (for a review see Refs. 1 and 2). Broadband squeezed light has been generated using four-wave mixing in atomic vapors[3] and fibers,[4] optical parametric oscillation,[5] and optical bistability.[6]

Having known the possibilities of obtaining squeezed fields it is quite natural to examine the interaction of such fields with matter. In general, the interaction can lead to quantitatively and qualitatively new phenomena. The phenomena of collapses and revivals for a single two- or three-level atom interacting with squeezed light have been discussed by Milburn[7] and Abdel-Hafez *et al.*,[8] respectively. The radiative decay of an atom interacting with a broadband squeezed-vacuum field has been considered by Gardiner.[9] Carmichael *et al.*[10] have examined resonance fluorescence from an atom damped by a squeezed vacuum. They have shown striking differences from the spectrum of ordinary resonance fluorescence in both the weak- and strong-driving-field limits. Quite recently, Ekert and Rzążewski[11] have shown that the squeezed-vacuum field is the most efficient in the production of the second-harmonic light.

In this paper we analyze the collective resonance fluorescence from N strongly driven two-level atoms which are damped by a squeezed vacuum. Interatomic interactions are neglected. The steady-state fluorescent spectrum is calculated.

II. MASTER EQUATION

We consider a system of N two-level atoms concentrated in a region small compared to the wavelength of all relevant radiation modes and interacting with a classical coherent driving field of frequency ω_L and with a quantized multimode radiation field.

In the electric dipole and rotating-wave approximations the interaction picture Hamiltonian of the system under consideration reads

$$H=\tfrac{1}{2}\delta(J_{22}-J_{11})+\Omega(e^{i\phi_L}J_{21}+e^{-i\phi_L}J_{12})+H_{\rm rad}+(J_{21}\Gamma+J_{12}\Gamma^\dagger)\ . \qquad (1)$$

$\delta=\omega_{21}-\omega_L$ is the detuning of the frequency ω_L from the atomic transition frequency ω_{21}. $\Omega\exp(i\phi_L)=\mu E$, where μ is the atomic dipole moment and E is the amplitude of the coherent driving field. $H_{\rm rad}$ denotes the free Hamiltonian of the quantized radiation field and Γ and $\Gamma^\dagger$ are operators corresponding to the positive- and negative-frequency components of this field, respectively. J_{ij} $(i,j=1,2)$ are the collective angular momenta of the atoms, obeying the commutation relation

$$[J_{ij},J_{kl}]=J_{il}\delta_{jk}-J_{kj}\delta_{il}\ . \qquad (2)$$

In the usual treatment of resonance fluorescence the quantized radiation field is assumed to be in the normal-vacuum state. Following the considerations presented in Refs. 9, 10, and 12 we assume here that the quantized radiation field is in a squeezed-vacuum state and the bandwidth of the squeezing is sufficiently broad. Hence the squeezed vacuum appears as δ-correlated squeezed white noise to the atoms and correlation functions for the free parts $\Gamma_{\rm free}$ and $\Gamma^\dagger_{\rm free}$ (the noise operators) of the operators Γ and $\Gamma^\dagger$ take the form[12]

$$\begin{aligned}\langle\Gamma^\dagger_{\rm free}(t)\Gamma_{\rm free}(t')\rangle&=\gamma P\delta(t-t')\ ,\\ \langle\Gamma_{\rm free}(t)\Gamma^\dagger_{\rm free}(t')\rangle&=\gamma(P+1)\delta(t-t')\ ,\\ \langle\Gamma_{\rm free}(t)\Gamma_{\rm free}(t')\rangle&=\gamma|Q|e^{i\phi_V}\delta(t-t')\ ,\\ \langle\Gamma^\dagger_{\rm free}(t)\Gamma^\dagger_{\rm free}(t')\rangle&=\gamma|Q|e^{-i\phi_V}\delta(t-t')\ .\end{aligned} \qquad (3)$$

γ is the single-atom decay rate for spontaneous emission into the unsqueezed vacuum. The parameters P and $Q=|Q|\exp(i\phi_V)$ characterize the squeezing and $|Q|^2\le P(P+1)$, where the equality takes place for a minimum-uncertainty squeezed state.

Using Eqs. (1) and (3) and making the unitary transformation

$$U=\exp[i\phi_L(J_{22}-J_{11})/2]$$

one finds a master equation for the reduced density operator ρ of the atomic system in the form[12]

$$\begin{aligned}\frac{\partial\rho}{\partial t}=&\,i[\rho,H_{\rm coh}]+\frac{\gamma}{2}(P+1)(J_{12}\rho J_{21}-J_{21}J_{12}\rho+\text{H.c.})\\ &+\frac{\gamma}{2}P(J_{21}\rho J_{12}-J_{12}J_{21}\rho+\text{H.c.})\\ &-\frac{\gamma}{2}|Q|e^{-i\phi}(J_{21}\rho J_{21}-J_{21}^2\rho+\text{H.c.})\\ &-\frac{\gamma}{2}|Q|e^{i\phi}(J_{12}\rho J_{12}-J_{12}^2\rho+\text{H.c.})=L_\rho\ .\end{aligned} \qquad (4)$$

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Here $\phi=2\phi_L-\phi_V$ is the relative phase of the driving field and the squeezed-vacuum field. In what follows we restrict ourselves to the case of exact resonance ($\delta=0$); then $H_{\rm coh}=\Omega(J_{12}+J_{21})$.

It is convenient to use the Schwinger representation[13] for the collective atomic operators:

$$J_{ij}=a_i^\dagger a_j \ , \tag{5}$$

where the operators a_i and $a_i^\dagger$ are treated as the annihilation and creation operators for the atoms being populated in the level $|i\rangle$ and obey the boson commutation relation $[a_i,a_j^\dagger]=\delta_{ij}$.

The canonical transformation

$$\begin{aligned} a_1&=(c_1+c_2)/\sqrt{2} \ , \\ a_2&=(c_2-c_1)/\sqrt{2} \ , \end{aligned} \tag{6}$$

splits the Liouville operator L in the master equation (4) into two parts, one containing the slowly varying terms and the other containing the terms oscillating at frequencies 2Ω and 4Ω. In the strong-driving-field limit the secular approximation[14] is justified. Then one can retain only the slowly varying part of the Liouville operator and the master equation (4) transforms to the following one:

$$\begin{aligned} \frac{\partial\tilde{\rho}}{\partial t}&=i\Omega[\tilde{\rho},R_3]+A(R_3\tilde{\rho}R_3-R_3^2\tilde{\rho}+\text{H.c.}) \\ &\quad +B(R_{12}\tilde{\rho}R_{21}+R_{21}\tilde{\rho}R_{12}-R_{21}R_{12}\tilde{\rho}-R_{12}R_{21}\tilde{\rho} \\ &\quad +\text{H.c.}) \ . \end{aligned} \tag{7}$$

When the squeezing is weak the secular approximation is justified at the same condition which holds in ordinary resonance fluorescence, i.e., for $\Omega\gg\gamma$.[14] In turn, for a highly squeezed vacuum ($P\gg1$) this approximation requires much stronger driving fields satisfying the inequality $\Omega\gg\gamma P$. In (7),

$$\begin{aligned} A&=\frac{\gamma}{4}(P+\tfrac{1}{2}-|Q|\cos\phi) \ , \\ B&=\frac{\gamma}{4}(P+\tfrac{1}{2}+|Q|\cos\phi) \ , \\ R_3&=R_{22}-R_{11} \ , \\ \tilde{\rho}&=T\rho T^\dagger \ , \end{aligned} \tag{8}$$

where T is the unitary operator representing the dressing transformation (6). $R_{ij}=c_i^\dagger c_j$ are the collective operators for the "dressed" atoms and satisfy the same commutation relation (2) as the operators J_{ij}.

The master equation (7) has the exact stationary solution

$$\tilde{\rho}_s=(N+1)^{-1}\sum_{n_1=0}^{N}|n_1\rangle\langle n_1| \ , \tag{9}$$

where $|n_1\rangle$ is an eigenstate of the operators R_{11} and $R_{11}+R_{22}$. The analogous solution for collective resonance fluorescence of normal radiatively damped atoms has been given earlier.[14,15] As a consequence, in the strong-driving-field limit all stationary one-time averages of the atomic and field observables preserve their normal-vacuum form, i.e., they are independent of the parameters P, $|Q|$, and ϕ_V of the squeezed vacuum.

III. STEADY-STATE SPECTRUM

With the help of the master equation (8) we derive the following equations of motion for the averages ($\langle\ \rangle$) of the atomic observables:

$$\begin{aligned} \frac{d}{dt}\langle R_3\rangle&=-\gamma(P+\tfrac{1}{2}+|Q|\cos\phi)\langle R_3\rangle \ , \\ \frac{d}{dt}\langle R_{12}\rangle&=-2i\Omega\langle R_{12}\rangle \\ &\quad -\frac{\gamma}{2}(3P+\tfrac{3}{2}-|Q|\cos\phi)\langle R_{12}\rangle \ , \\ \frac{d}{dt}\langle R_{21}\rangle&=2i\Omega\langle R_{21}\rangle-\frac{\gamma}{2}(3P+\tfrac{3}{2}-|Q|\cos\phi)\langle R_{21}\rangle \ . \end{aligned} \tag{10}$$

The above equations can easily be integrated.

In what follows, the steady-state averages will be denoted by $\langle\ \rangle_s$. Using the quantum regression theorem[16] and the solutions of Eqs. (10) we get the two-time atomic correlation function $\langle J_{21}(\tau)J_{12}\rangle_s$ in the form

$$\begin{aligned} \langle J_{21}(\tau)J_{12}\rangle_s&=\tfrac{1}{4}\langle R_3^2\rangle_s e^{-\beta_0\tau} \\ &\quad +\tfrac{1}{4}\langle R_{21}R_{12}\rangle_s e^{(2i\Omega-\beta_1)\tau} \\ &\quad +\tfrac{1}{4}\langle R_{12}R_{21}\rangle_s e^{-(2i\Omega+\beta_1)\tau} \ , \end{aligned} \tag{11}$$

where

$$\begin{aligned} \beta_0&=\gamma(P+\tfrac{1}{2}+|Q|\cos\phi) \ , \\ \beta_1&=\frac{\gamma}{2}(3P+\tfrac{3}{2}-|Q|\cos\phi) \ . \end{aligned} \tag{12}$$

The averages appearing in (11) may be expressed by the statistical moments $\langle R_{11}^q\rangle_s$ which, in turn, from (9) are

$$\langle R_{11}^q\rangle_s=(N+1)^{-1}\sum_{n_1=0}^{N}n_1^q \ , \tag{13}$$

where n_1 is the eigenvalue of R_{11} in the eigenstate $|n_1\rangle$. After simple algebra we find

$$\langle R_3^2\rangle_s=2\langle R_{21}R_{12}\rangle_s=2\langle R_{12}R_{21}\rangle_s=N(N+2)/3 \ . \tag{14}$$

The steady-state spectrum of the fluorescent light is proportional to the Fourier transform of the correlation function (11) and reads

$$\begin{aligned} S(\omega)&\simeq\tfrac{1}{2}\text{Re}\int_0^\infty e^{-i(\omega-\omega_L)\tau}\langle J_{21}(\tau)J_{12}\rangle_s d\tau \\ &=\frac{N(N+2)}{12}\left[\frac{\beta_0}{(\omega-\omega_L)^2+\beta_0^2}\right. \\ &\quad +\frac{1}{2}\frac{\beta_1}{(\omega-\omega_L-2\Omega)^2+\beta_1^2} \\ &\quad \left.+\frac{1}{2}\frac{\beta_1}{(\omega-\omega_L+2\Omega)^2+\beta_1^2}\right] \ . \end{aligned} \tag{15}$$

IV. DISCUSSION

Irrespective of whether the vacuum modes are squeezed or unsqueezed the incoherent steady-state fluorescent spectrum in the strong-driving-field limit contains three Lorentzians centered at $\omega=\omega_L$, $\omega_L \pm 2\Omega$. The central line has the spectral width β_0, while the sidebands have the linewidths β_1. From (11) or (15) it is evident that the integrated intensity under the central peak is always equal to the sum of the integrated intensities under the two sidebands and they are proportional to $N(N+2)$. The widths and heights of the spectral lines depend on the parameters characterizing the squeezed vacuum and are phase-sensitive quantities. The quantitative analysis of the spectrum is particularly clear for minimum-uncertainty squeezing. Then $P(P+1)=|Q|^2$ and for the relative phases $\phi=0$ and $\phi=\pi$ from (12) we have

$$\begin{aligned} \beta_0(P,0)&=\gamma(P+\sqrt{P(P+1)}+\tfrac{1}{2}) , \\ \beta_1(P,0)&=\frac{\gamma}{2}(3P-\sqrt{P(P+1)}+\tfrac{3}{2}) , \\ \beta_0(P,\pi)&=\gamma(P-\sqrt{P(P+1)}+\tfrac{1}{2}) , \\ \beta_1(P,\pi)&=\frac{\gamma}{2}(3P+\sqrt{P(P+1)}+\tfrac{3}{2}) . \end{aligned} \tag{16}$$

Obviously, for $P=0$ from (16) we find the well-known linewidths of the Mollow triplet in the case of ordinary fluorescence: $\beta_0=\gamma/2, \beta_1=3\gamma/4$.

For a highly squeezed ($P \gg 1$) minimum-uncertainty state from (16) we arrive at

$$\begin{aligned} &\beta_0(P,0)=2\gamma P, \quad \beta_1(P,0)=\gamma P , \\ &\beta_0(P,\pi)=\gamma/8P, \quad \beta_1(P,\pi)=2\gamma P , \end{aligned} \tag{17}$$

in full agreement with the results obtained by Carmichael *et al.*[10] for a single atom.

The linewidths of the sidebands will then be always broadened in comparison with those for the normal vacuum. In turn, the central line may have either a subnatural ($\beta_0<\gamma/2$) or supernatural ($\beta_0>\gamma/2$) linewidth depending on the phase ϕ.

In order for us to view the single-atom fluorescent spectrum for $P \gg 1$ there must exist a small "window" of unsqueezed modes.[10] In the case of collective resonance fluorescence from a large number of atoms ($N > P \gg 1$) the intensities of the spectral lines, proportional to N^2, will dominate over the squeezed noise. Hence in this case such a window is unnecessary.

In the weak-squeezing limit ($P \ll 1$) from (16) we get approximately

$$\begin{aligned} &\beta_0(P,0)=\gamma(\sqrt{P}+\tfrac{1}{2}), \quad \beta_1(P,0)=\frac{\gamma}{2}(-\sqrt{P}+\tfrac{3}{2}) , \\ &\beta_0(P,\pi)=\gamma(-\sqrt{P}+\tfrac{1}{2}), \quad \beta_1(P,\pi)=\frac{\gamma}{2}(\sqrt{P}+\tfrac{3}{2}) . \end{aligned} \tag{18}$$

It is readily seen that now not only the central line but also the sidebands may have either subnatural or supernatural linewidth depending on ϕ. The function $\beta_1(P,0)$ (16) reaches its minimal value $\sqrt{2}\gamma/2$ for $P=-\frac{1}{2}+3\sqrt{2}/8$. In general, $\beta_1(P,0)$ is less than $3\gamma/4$ within the range $P \in (0,\frac{1}{8})$. So, in the case of weak squeezing it is possible to get for the sidebands subnatural linewidths; however, the narrowing will be rather small.

The secular approximation does not include the small additional sidebands in collective resonance fluorescence.[17,18] Then on resonance ordinary cooperative fluorescence reminds us of the single-atom case.[19] Hence the results obtained here for the shape of the steady-state spectrum of collective fluorescence in a squeezed vacuum could be cautiously deduced from those derived by Carmichael *et al.*[10] for a single atom damped just by a squeezed vacuum. Here, in fact, we have proved that.

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