Second side-mode instability in optical bistability with a two-level, homogeneously broadened medium

B. Ségard, W. Sergent, B. Macke, and N. B. Abraham*
Laboratoire de Spectroscopie Hertzienne, Université de Lille I, F-59655 Villeneuve d'Ascq, CEDEX, France
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We report the observation of an instability in a bistable optical system with a homogeneously broadened two-level medium involving the onset of the cavity side mode that is two modes away from the source frequency and three modes away from the molecular resonance frequency. These measurements extend the range of a recent report [Phys. Rev. A **39**, 703 (1989)] of observations of the multimode instability in optical bistability predicted by Lugiato and Bonifacio [Lett. Nuovo Cimento **21**, 505 (1978); **21**, 510 (1978)], with the key difference being that the experimental observations in the previous report were limited to instabilities of the side mode adjacent to the source frequency.

Part of the revitalization of the study of optical instabilities in recent years owes much to the application to the study of optical bistability of the methods of stability analysis developed for multimode lasers.[1,2] Using these methods in 1978, Bonifacio and Lugiato[3] predicted a multimode instability in the Maxwell Bloch equation of optical bistability. At nearly the same time, Ikeda[4] simplified the equations to a delay-differential equations and predicted not only instabilities but also chaos. [It is worth noting that in recent years it has become clear that delay-differential equations rather than simple delay equations (one-dimensional maps) are required to accurately simulate the experimental systems.[5(a)] In this form the delay-differential equations correspond to the coupling of an infinite number of modes.[5]] While the delay-induced instabilities were soon observed[6] and have been well studied,[5] it was not until recently[7] that the particular form of multimode instability predicted by Bonifacio and Lugiato for an optically bistable system using a homogeneously broadened, two-level medium (where the instability arises from coherent effects) was observed, albeit in the dispersive (detuned) case. A systematic study of the experimental characteristics of this instability and a comparison with numerical simulations using plane-wave models has been recently reported.[8]

A similar instability has been observed by Khitrova, Valley, and Gibbs[9] using sodium vapor pumped off-resonance to generate Raman gain for side modes 9–12 free spectral ranges away from the pumping laser frequency. Their experiments differ from those of Ref. 8 and those reported here in that the Doppler broadening of the sodium transition was large and comparable to the Rabi frequency and that the pumping laser field excited the medium without circulating the laser cavity. In such a case the authors describe the system as a "Raman laser."

In the previous experimental studies (Ref. 8) similar to those reported here, no instabilities were observed for resonant tuning of the source frequency and the molecular resonance frequency although instabilities had been predicted for those experimental operating conditions using a plane-wave model. The absence of instabilities in the purely absorptive case was attributed[8] to the suppression of instabilities by the transverse profile of the externally injected field as had been predicted previously.[10] Experimentally this is difficult to study because the strong absorption on resonance and finite source power make it difficult to reach large values of the excitation or of the Rabi frequency. However, when the frequency of the driving field was detuned from the molecular resonance frequency, instabilities were observed which involved the onset of the cavity mode adjacent to the external driving field. This led to the development of two rules of thumb for the instabilities: first, that the pulsing frequency would be given approximately by the Rabi frequency (as the band of frequencies receiving gain by the Raman process has its center frequency separated from the frequency of the driving field by the Rabi frequency); and second, that the pulsing frequency would be given approximately by the separation of the nearest cavity mode from the driving frequency. It is, in fact, the generalized Rabi frequency which determines the separation between the source frequency and the peak of the band of frequencies where there is Raman gain for a homogeneously broadened medium, but for the small detunings in the experiment of Ref. 8 the two were relatively equal in predictive power. Actually, it is the coincidence of the gain (rule 1) and resonance with some mode of the cavity (rule 2) that leads to the onset of the instability. (It is worth noting that similar rules of thumb exist for the experiment of Khitrova *et al.*[9] but in their case the Doppler broadening of the transition and the velocity of the optically excited atoms cause the peak of the Raman gain to be separated from the frequency of the driving field by the Rabi frequency rather than the generalized Rabi frequency.)

Since in the dispersive case the instability predominantly involves the weak excitation of a single unstable mode (as demonstrated in Ref. 8 by heterodyne detection), the resulting intensity pulsations are nearly periodic and correspond to a beat between the frequency of the driving field and that of the excited mode.

It is easy to see how the rules of thumb should be generalized. Rule 1 remains unchanged because it locates the frequency range where the Raman process provides gain, though as the strength of the driving field increases, the width of this range grows. Hence the Rabi frequency does not precisely determine the pulsing frequency. Rule 2 is also not exact because the strongly driven medium causes mode pulling of the empty cavity resonance frequencies, though these effects are usually small. Modifications can easily occur in principle to rule 2, as the criterion for an instability is simply that some cavity mode fall within the region of frequencies where there is gain. Which mode satisfies this condition depends on the detunings of the source frequency and the molecular resonance frequency from the cavity frequencies and from each other, and on the free spectral range.

We have extended the measurements in Ref. 8 to operating conditions which lead to an instability of the cavity mode that is the second one away from the source frequency and the third one away from the molecular resonance frequency. As the detuning of the source frequency is relatively far from the molecular frequency and relatively close to one cavity resonance frequency, the intracavity field is larger than in the previous experiments leading to a higher Rabi frequency that excites an instability with a pulsing frequency of the order of 1500 kHz in contrast to the nearest sideband instabilities which appeared in the range of 400–700 kHz.

The experimental setup is as given in Ref. 8. Briefly, the optical cavity is a waveguide Fabry Perot, 182 m in length, filled with 0.9 to 1.5 mTorr of HCN. The $J=0$ and $M=0$ to $J=1$ and $M=1$ rotational transition used is at a frequency of approximately 86 GHz. Although the transition has a narrow hyperfine structure spanning 15 kHz, a collisional linewidth of 22 kHz, and a Doppler linewidth of 100 kHz (all values HWHM) for the strengths of the fields used in this experiment (which lead to a Rabi frequency larger than 400 kHz), the system behaves as a homogeneously broadened, two-level medium. The source is a phase-locked klystron with an output power of 500 mW of which approximately 160 mW is coupled to the lowest-order mode of the waveguide. The signal is detected by a Schottky-barrier diode mixer.

Figure 1 gives a plot of the output power versus the source frequency which corresponds to the experimental Fig. 15 of Ref. 8. Figure 2(a) schematically locates the molecular resonance frequency in the comb of resonance frequencies of the longitudinal cavity modes and indicates the range of the source frequency scanned in Fig. 1. The two steps in output power in Fig. 1 are parts of the multiple hysteresis loops that are found as the source frequency is increased and decreased.

In the lower step in Fig. 1 at the location marked *B*, an instability occurs with pulsing characteristics shown in inset *B* in the figure. The pulsing frequency of 469 kHz corresponds to the excitation of the mode nearest the source frequency. This mode is marked ν_p, and the source frequency ν_0 is located between the molecular frequency ν_m and the mode ν_p. The Rabi frequency is rela-

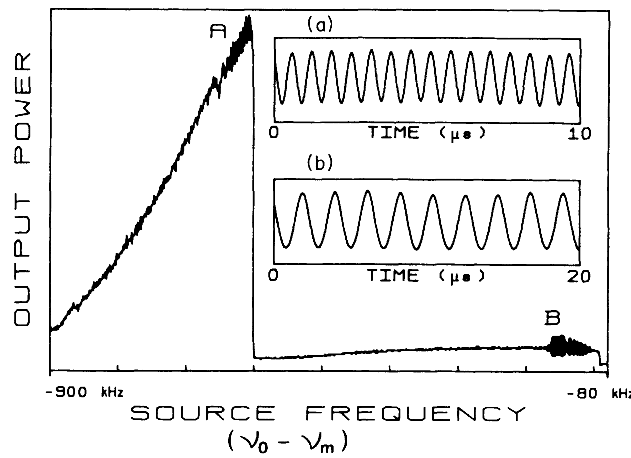

FIG. 1. Plot of output power vs source frequency with two regions of instability indicated by *A* and *B* with insets showing the intensity pulsations in these two regions.

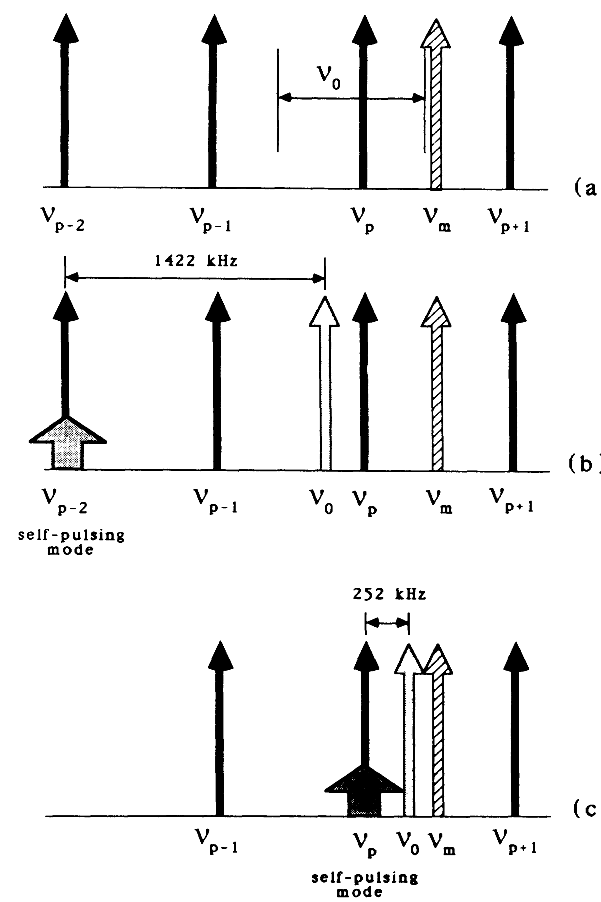

FIG. 2. Schematic drawing locating the cavity frequencies ν_i, the source frequency ν_0, and the molecular resonance frequency ν_m for the experiments shown in Fig. 1. (a) Cavity and molecular frequencies with range of source frequency detuning spanned by Fig. 1, (b) with source frequency at the specific value corresponding to *A* in Fig. 1, (c) with source frequency at the specific value corresponding to *B* in Fig. 1.

TABLE I. Observations of the second side-mode instability.

$\nu_p-\nu_0$ (kHz)	$\nu_m-\nu_0$ (kHz)	ν_{osc} (kHz)	$\nu_{th}=\lvert\nu_0-\nu_{p\pm2}\rvert$ (kHz)	$\nu_{gen\text{-}Rabi}$ (kHz)	ν_{Rabi} (kHz)	Unstable mode
315	780	1450	1335	1093	766	$p-2$
285	750	1480	1365	1122	836	$p-2$
−270	−864	1508	1380	1266	925	$p+2$
−216	−804	1535	1434	1318	1044	$p+2$
228	660	1533	1422			$p-2$
276	645	1461	1474			$p-2$
−198	−672	1520	1450	1200	980	$p+2$

tively low in this case because the near coincidence of the source and molecular frequencies causes large absorption and a relatively low intracavity field strength. The difference between the pulsing frequency at 469 kHz and $\nu_0-\nu_p=252$ kHz is caused by mode pulling.

On the upper step of Fig. 1 at the location marked A, another instability occurs with pulsing characteristics as shown in inset A. The pulsing frequency of 1533 kHz corresponds to the excitation of the mode that is second from the source frequency, namely, the mode labeled ν_{p-2} in Fig. 2(b) where the location of the source frequency is indicated as being between ν_{p-1} and ν_p. The larger separation between the source and molecular frequencies leads to a larger intracavity field due to a lower absorption. The generalized Rabi frequency is therefore larger due to a combination of a stronger field and larger detuning which give the Raman gain at a correspondingly larger distance from the source frequency.

A variety of similar observations of the second side-mode instability are presented in Table I. We see that rule 2 remains a useful predictor of the pulsing frequency and that the mode-pulling effects are relatively smaller than for the nearest side-mode instability observed here and in Ref. 8. Rule 1 is not as robust, and unlike the case of the nearest side-mode instability in Ref. 8 (where both the Rabi frequency and the generalized Rabi frequency were effective predictors of the pulsing frequency) the generalized Rabi frequency is a better predictor. The departure by as much as 30% arises from the specific location of the mode ν_{p-2} within the band of frequencies having Raman gain.

Similar instabilities involving the second side mode have been observed in the theoretical studies of this problem. For example, in Ref. 8, Fig. 24(b), one sees two regions of instability for a resonantly tuned (purely absorptive) case. These pulsations appear at the cavity mode spacing for low intracavity Rabi frequencies and at twice the cavity mode spacing for higher intracavity Rabi frequencies. Similar effects are to be expected in the theoretical studies of the dispersive case.

As in the comparison reported in Ref. 8, however, we found no evidence of a second side-mode instability in the purely absorptive case within the range of source powers accessible with our source.

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*Present address: Department of Physics, Bryn Mawr College, Bryn Mawr, PA 19010-2899.

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