Establishment of correlated pure states through decay in a squeezed reservoir

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(Received 28 November 1988)

Our normal understanding of the dissipative relaxation of a quantum system interacting with a reservoir or heat bath leads us to expect that the final state of the quantum system is a mixed state characteristic of one subsystem in equilibrium with the larger reservoir. We show that the broadband squeezed vacuum has such strong correlations in its quantum noise that, although it represents a true reservoir, the decay of subsystems, such as pairs of atoms or oscillators interacting with a squeezed reservoir, relaxes to a pure state that reflects these strong correlations.

An excited quantum subsystem, upon interaction with a much larger reservoir or heatbath not only acquires fluctuations from the reservoir interaction but also dissipates its energy. The reduced subsystem describing the initially excited state will, in general, decay to an equilibrium characterized, not by a pure-state wave function, but by a statistical mixture. A rather trivial exception is provided by the case of a decay in a vacuum with the subsystem relaxing to its ground state. In this paper we address ourselves to the nature of the equilibrium state generated by decay of quantum systems in a multimode squeezed vacuum.[1,2] The multimode squeezed vacuum is a most unusual reservoir which exhibits thermal fluctuations when quantities sensitive to single modes are studied, but exhibits extremely strong quantum correlation when quantities dependent on pairs of modes are studied.[3] We shall demonstrate that, for certain observables, decay in squeezed broadband light can behave precisely like the approach to conventional thermodynamic equilibrium with the squeezed light appearing to be Planckian with an effective temperature. Nevertheless we can also construct phase-sensitive observables which are sufficiently influenced by the reservoir correlations that they relax to correlated and nonthermal forms in which the correlations in the reservoir transfer to the quantum system. In particular, a pair of quantum systems may relax to a highly correlated pure state.

A broadband squeezed vacuum forms a reservoir characterized by a phases-sensitive white noise.[1,2] It is composed of many modes of the electromagnetic field that are strongly correlated in pairs around some central frequency. The interaction of quantized systems, such as oscillators or atoms with such a reservoir, has already been the subject of several papers but attention was concentrated mainly on the modification of the Lamb shift or the spontaneous decay constant in particular systems such as a two-level atom or a harmonic oscillator[4,5] and on developing new techniques for description of that interaction.[6-10] In the present paper we consider a transfer of correlations from the broadband squeezed vacuum to *two* subsystems which are initially uncorrelated. We will show that the two subsystems become highly correlated in a steady-state limit. Moreover, the correlated steady state of the subsystems is *not* a mixed state but a *pure* state which reflects these correlations.

We consider the following Hamiltonian for the total system:

$$H=\hbar\omega_1 s_1^\dagger s_1+\hbar\omega_2 s_2^\dagger s_2+\sum_\mu \hbar\omega_\mu b_\mu^\dagger b_\mu + \sum_\mu \hbar g_\mu[(s_1^\dagger+s_2^\dagger)b_\mu+\text{H.c.}] \; . \tag{1}$$

The summation here goes over continuous sets of modes. The operators s_1 ($s_1^\dagger$) and s_2 ($s_2^\dagger$) are annihilation (creation) operators for subsystems 1 and 2, respectively. They could be, for example, annihilation and creation operators for a harmonic oscillator or alternatively, the Pauli operators for a two-level atom. These are the two examples of quantum subsystems that we shall consider in interaction with the multimode squeezed vacuum. Let us introduce now new "normal mode" operators s and $\tilde{s}$ defined by

$$\tilde{s}=\frac{1}{\sqrt{2}}(s_2-s_1),\quad s=\frac{1}{\sqrt{2}}(s_1+s_2) \; , \tag{2}$$

and also the new frequencies

$$\Omega=\frac{(\omega_1+\omega_2)}{2}, \quad \Delta=\frac{(\omega_2-\omega_1)}{2} . \tag{3}$$

Using those new "modes" we can rewrite the Hamiltonian (1) in the following form:

$$\begin{aligned} H=&\hbar\Omega\tilde{s}^\dagger\tilde{s}+\hbar\Omega s^\dagger s+\hbar\Delta(s^\dagger\tilde{s}+\tilde{s}^\dagger s) \\ &+\sum_\mu \hbar\omega_\mu b_\mu^\dagger b_\mu+\sum_\mu \hbar\sqrt{2}g_\mu(s^\dagger b_\mu+\text{H.c.}) , \end{aligned} \tag{4}$$

or when we go to the frame rotating with frequency Ω

$$\begin{aligned} H_I=&\hbar\Delta(S^\dagger\tilde{S}+\tilde{S}^\dagger S) \\ &+\sum_\mu \hbar\sqrt{2}g_\mu(S^\dagger B_\mu e^{i(\Omega-\omega_\mu)t}+\text{H.c.}) , \end{aligned} \tag{5}$$

where S, $S^\dagger$, B_μ, and $B_\mu^\dagger$ are interaction picture operators.

So far we have not made any assumptions about our model of the reservoir except that it is composed of the set of harmonic oscillators. In a squeezed vacuum case these harmonic oscillators are correlated in pairs around some central frequency that we set equal to Ω. When the squeeze parameter r (see, for example, Ref. 11 and references therein) is independent of the frequency of the oscillators, the squeezed vacuum can be generated from the normal vacuum by application of the following unitary transformation:

$$|0_{\text{sq}}\rangle=K|0\rangle , \tag{6}$$

here K is the usual multimode squeezing transformation,[10] which correlates modes in pairs symmetrically displaced from the central frequency Ω

$$K=\prod_\kappa e^{-r(B_\kappa B_{-\kappa}-B_\kappa^\dagger B_{-\kappa}^\dagger)} , \tag{7}$$

where κ denotes the mode with frequency $\bar{\omega}_\kappa=\Omega-\omega_\kappa$ so that mode frequencies are redefined with respect to the carrier frequency. Note that for later convenience we set the squeeze parameter equal to $-r$ instead of r through a choice of overall phase of the squeezing.

The initial state vector of the total system (that is two subsystems plus reservoir) we will write as

$$|\psi_{\text{tot}}\rangle=|\psi_s\rangle\otimes|0_{\text{sq}}\rangle , \tag{8}$$

where $|\psi_s\rangle$ describes the state of the two subsystems and $|0_{\text{sq}}\rangle$ the broadband squeezed vaccum. After performing the unitary transformation $K^\dagger$ we obtain the state

$$K^\dagger|\psi_{\text{tot}}\rangle=|\psi_s\rangle\otimes|0\rangle , \tag{9}$$

where $|0\rangle$ is the usual vacuum state and the dynamics is now governed by a Hamiltonian where the states are "unsqueezed" but the system operators are transformed to their r-dependent dressed, or squeezed form,

$$\begin{aligned} K^\dagger H_I K&=\hbar\Delta(S^\dagger\tilde{S}+\tilde{S}^\dagger S)+\sum_\kappa \hbar\sqrt{2}g_\kappa\{S^\dagger[B_\kappa\cosh(r)+B_{-\kappa}\sinh(r)]e^{i\omega_\kappa t}+\text{H.c.}\} \\ &=\hbar\Delta(S^\dagger\tilde{S}+\tilde{S}^\dagger S)+\left[\left(\sum_\kappa \hbar\sqrt{2}g_\kappa B_\kappa e^{i\omega_\kappa t}\right)[S^\dagger\cosh(r)+S\sinh(r)]+\text{H.c.}\right] \\ &=\hbar\Delta(\underline{S}^\dagger\underline{\tilde{S}}+\underline{\tilde{S}}^\dagger\underline{S})+\sum_\kappa \hbar\sqrt{2}g_\kappa(\underline{S}^\dagger B_\kappa e^{i\omega_\kappa t}+\text{H.c.}) \end{aligned} \tag{10}$$

with

$$\underline{S}=S\cosh(r)+S^\dagger\sinh(r) , \tag{11a}$$

$$\underline{\tilde{S}}=\tilde{S}\cosh(r)-\tilde{S}^\dagger\sinh(r) . \tag{11b}$$

The transformation K acts on the reservoir variables but we have obtained a transformed Hamiltonian in which modified operators for the subsystems are coupled to a reservoir in its vacuum state. At this point we can examine the final state of the two subsystems interacting with the correlated multimode squeezed vacuum field.

If we assume that $\Delta\neq 0$ then the system goes to the steady state which, *if it exists as a pure state,* is defined by

$$\underline{S}|\psi\rangle=0, \quad \underline{\tilde{S}}|\psi\rangle=0 . \tag{12}$$

This state is a vacuum state with respect to the dressed operator $\underline{S}$ and $\underline{\tilde{S}}$. We stress that the final state given by Eq. (12) is generated for the correlated pair of subsystems. It is not satisfied in a simple case of one two-level atom interacting with a broadband squeezed vacuum as it is insensitive to the intermode correlations of the squeezed vacuum.[10] For this single-atom case the atom evolves towards a mixed state, characteristic of conventional thermodynamic equilibrium. We now consider two simple cases of correlated subsystems to illustrate this general result. Our examples are firstly two harmonic oscillators, and secondly, two two-level atoms.

Our first example considers two oscillators in interaction with the squeezed multimode vacuum. The system operators denote now annihilation (A) and creation ($A^\dagger$) operators for the two harmonic oscillators with frequencies ω_1 and ω_2. We set $S_1=A_1$ and $S_2=A_2$ and we write the transformation (11) as

$$\underline{A} = A\cosh(r) + A^{\dagger}\sinh(r) = UAU^{\dagger} , \tag{13a}$$

$$\underline{\tilde{A}} = \tilde{A}\cosh(r) - \tilde{A}^{\dagger}\sinh(r) = \tilde{U}\tilde{A}\tilde{U}^{\dagger} . \tag{13b}$$

Here the unitary transformation U is the usual *single-mode* SU(1,1) squeezing operator,[11]

$$U = e^{(r/2)(A^2 - A^{\dagger 2})}, \quad \tilde{U} = e^{(r/2)(\tilde{A}^2 - \tilde{A}^{\dagger 2})} . \tag{14}$$

Now it is easy to see the steady state will be given by

$$|\psi\rangle = U\tilde{U}|0\rangle = e^{r(A_1A_2 - A_1^{\dagger}A_2^{\dagger})}|0\rangle . \tag{15}$$

This state is the well known two-mode squeezed vacuum state[11] which exhibits the strong correlation between the two oscillators.

For our second example, we consider the case of two two-level atoms, interacting with a broadband squeezed vacuum. Both atomic ground states have energy equal to zero and excited states have energies $\hbar\omega_1$ and $\hbar\omega_2$, respectively. Now the system operators are $S_1 = \sigma_-^1$ and $S_2 = \sigma_-^2$ and the transformation (11) can be put in the following form:

$$\underline{\sigma}_- = \sigma_-\cosh(r) + \sigma_+\sinh(r)$$
$$= \alpha[\sigma_-\cos(\theta/2) + \sigma_+\sin(\theta/2)] = \alpha U^{\dagger}\sigma_- U^{\dagger} , \tag{16a}$$

$$\underline{\tilde{\sigma}}_- = \tilde{\sigma}_-\cosh(r) - \tilde{\sigma}_+\sinh(r)$$
$$= \alpha[\tilde{\sigma}_-\cos(\theta/2) - \tilde{\sigma}_+\sin(\theta/2)] = \alpha\tilde{U}\tilde{\sigma}_-\tilde{U} , \tag{16b}$$

with the unitary SU(2) squeezing operator U now given by

$$U = e^{(\theta/2)[(\sigma_-)^2 - (\sigma_+)^2]} , \tag{17}$$

$$\tilde{U} = e^{(\theta/2)[(\tilde{\sigma}_-)^2 - (\tilde{\sigma}_+)^2]} = U^{\dagger} , \tag{18}$$

$$\alpha = [\cosh^2(r) + \sinh^2(r)]^{1/2} . \tag{19}$$

We note that σ_- stands for $(1/\sqrt{2})(\sigma_-^1 + \sigma_-^2)$ and $\tilde{\sigma}_-$ for $(1/\sqrt{2})(\sigma_-^2 - \sigma_-^1)$; their squares are, of course, not equal to zero. The angle θ is defined by

$$\cos\left[\frac{\theta}{2}\right] = \frac{\cosh(r)}{[\cosh^2(r) + \sinh^2(r)]^{1/2}} , \tag{20}$$

$$\sin\left[\frac{\theta}{2}\right] = \frac{\sinh(r)}{[\cosh^2(r) + \sinh^2(r)]^{1/2}} . \tag{21}$$

As before it is easy to see that this time our steady state is given by

$$|\psi\rangle = U|0\rangle = e^{(\theta/2)(\sigma_-^2\sigma_-^2 - \sigma_+^1\sigma_+^2)}|0\rangle \tag{22}$$

which simply defines the two-atom squeezed state.[12] For a particular case $\Delta = 0$ the final state of the two subsystems depends on initial conditions this is because the subsystem described by operators $\tilde{S}^{\dagger}, \tilde{S}$ is not coupled to the reservoir. To conclude, we have shown that during the interaction with a broadband squeezed vacuum two initially uncorrelated systems may become strongly correlated in the steady state limit. The strong correlations are reflected in the equilibrium or final state of the two subsystems which despite their interaction with a dissipative reservoir, relax to a strongly correlated pure, rather than the expected mixed, state.

This work was supported in part by the U.K. Science and Engineering Research Council, Pirie-Reid Fund at Oxford University, the Polish Research Project CPBP 01.06, the U.K. Fellowship of Engineering and General Electric Company Limited and the Gruppo Nazionale Struttura della Materia, Consiglio Nazionale delle Ricerche, Italy.

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