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Comment on "Squeezing and frequency jump of a harmonic oscillator"

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The relevance of the conclusions of a paper by Fan and Zaidi [Phys. Rev. A **37**, 2985 (1988)] concerning the possibility of generating squeezed states of a harmonic oscillator by sudden change of its frequency is analyzed.

In a recent paper[1] (hereafter referred to as paper I) the results of our previous work[2] were criticized. There we showed that during a sudden frequency change of a quantum oscillator it mav become squeezed.

In Ref. 2 tne normally ordered characteristic function of a quantum oscillator was found after a change of the oscillator frequency for the case when before the frequency jump the oscillator was in a state corresponding to a superposition of a coherent signal with equilibrium thermal noise. The variances of the Hermitian field quadratures were evaluated and the effect of thermal fluctuations on squeezing was discussed. It was shown that at temperatures $T=0$ any sudden change of the oscillator frequency leads to squeezing. The considerations of Ref. 3 confirm this result. Graham[4] discussed the possibility of generating squeezed states from coherent states by external changes of the oscillator frequency and explored two limiting cases of an adiabatic and a sudden change. He found that only the latter results in squeezing, while in the former case symmetrical uncertainties of the field quadratures turn out to be adiabatic invariants.

On the other hand, the authors of paper I state that the suddenness of the frequency change is unessential to squeezing and also that the results of our paper[2] on this subject are based on an incorrect interpretation of the squeezing transformation. We cannot agree with such a statement and the purpose of the present Comment is to clarify the origin of this discrepancy.

The behavior of a quantum oscillator with time-dependent frequency and unit mass is described by the Hamiltonian ($\hbar=1$)

$$H(t)=\tfrac{1}{2}P^2+\tfrac{1}{2}\omega^2(t)Q^2 \; . \tag{1}$$

Here Q and P are the coordinate and momentum operators. The corresponding evolution operator in the Schrödinger picture satisfies the integral equation[5]

$$U(t,t_0)=1-i\int_{t_0}^{t}H(t')U(t',t_0)dt' \; . \tag{2}$$

In Ref. 2 we considered a harmonic oscillator having frequencies ω_1 at $t<0$ and ω_2 at $t>0$. The corresponding Hamiltonian is

$$H(t)=\tfrac{1}{2}P^2+\tfrac{1}{2}[\omega_1^2+\Theta(t)(\omega_2^2-\omega_1^2)]Q^2 \; , \tag{3}$$

where $\Theta(t)$ is 1 at $t>0$ and 0 at $t<0$. From (3) we have the following Heisenberg equations of motion:

$$\frac{dQ}{dt}=P \; , \quad \frac{dP}{dt}=-\omega_1^2 Q \; , \quad t<0 \tag{4}$$

$$\frac{dQ}{dt}=P \; , \quad \frac{dP}{dt}=-\omega_2^2 Q \; , \quad t>0 \; . \tag{5}$$

To solve these equations we need a condition for $t=0$. The Heisenberg operators $Q(t_2)$ and $Q(t_1)$ are connected ($t_2>t_1$):

$$Q(t_2)=U(t_2,t_1)Q(t_1)U^\dagger(t_2,t_1) \; . \tag{6}$$

From Eqs. (2) and (3) for the operator

$$U(+0,-0)\equiv\lim_{\tau\to 0}U(\tau,-\tau) \; , \quad \tau>0 \tag{7}$$

we have $U(+0,-0)=1$. This fact for a sudden excitation is well known in quantum mechanics.[5] This is the principal difference between the problem concerning the sudden change of the oscillator frequency and the situation considered in paper I where the squeezing operator is the evolution operator of the system.

Using (7), from (6) and an analogous expression for the momentum we can see that the right and left limits of the coordinate and momentum operators coincide at $t=0$:

$$Q(-0)=Q(+0)=Q(0) \; , \quad P(-0)=P(+0)=P(0) \; . \tag{8}$$

Introducing the creation and annihilation operators

$$a_j^\dagger=\frac{1}{\sqrt{2}}\left[(\omega_j)^{1/2}Q-\frac{i}{(\omega_j)^{1/2}}P\right],\quad j=1,2$$
$$a_j=\frac{1}{\sqrt{2}}\left[(\omega_j)^{1/2}Q+\frac{i}{(\omega_j)^{1/2}}P\right],\quad j=1,2 \tag{9}$$

from (4), (5), and (8) we find the solution for the operator $a_2(t)$ at $t>0$ in the Heisenberg picture,

$$a_2(t)=\frac{1}{\sqrt{2}}\left[(\omega_2)^{1/2}Q(0)+\frac{i}{(\omega_2)^{1/2}}P(0)\right]e^{-i\omega_2 t}$$
$$=[ua_1(0)+va_1^\dagger(0)]e^{-i\omega_2 t}, \tag{10}$$

$$u=\frac{\omega_2+\omega_1}{2(\omega_1\omega_2)^{1/2}},\quad u=\frac{\omega_2-\omega_1}{2(\omega_1\omega_2)^{1/2}},\quad u^2-v^2=1\ . \tag{11}$$

Let us suppose now that just before the frequency jump, i.e., at $t=-0$, the Heisenberg and Schrödinger states coincided. Considering the simplest case from Ref. 2 when the oscillator at $t=-0$ was in its ground state $|0\rangle_{\omega_1}$ defined by $a_1|0\rangle_{\omega_1}=0$ for the variances of the quadratures

$$X_j=a_je^{i\omega_j t}+a_j^\dagger e^{-i\omega_j t},\quad j=1,2$$
$$Y_j=-i(a_je^{i\omega_j t}-a_j^\dagger e^{-i\omega_j t}),\quad j=1,2 \tag{12}$$

and also for those of the coordinate and momentum we find at $t>0$

$$\Delta X_2=(\omega_2/\omega_1)^{1/2},\quad \Delta Y_2=(\omega_1/\omega_2)^{1/2}, \tag{13}$$

$$\Delta Q=\frac{1}{2(\omega_2)^{1/2}}\left[\frac{\omega_2}{\omega_1}[1+\cos(2\omega_2 t)]+\frac{\omega_1}{\omega_2}[1-\cos(2\omega_2 t)]\right]^{1/2}, \tag{14}$$

$$\Delta P=\frac{(\omega_2)^{1/2}}{2}\left[\frac{\omega_1}{\omega_2}[1+\cos(2\omega_2 t)]+\frac{\omega_2}{\omega_1}[1-\cos(2\omega_2 t)]\right]^{1/2}, \tag{15}$$

while for $t<0$ $\Delta Q=1/(2\omega_1)^{1/2}$ and $\Delta P=(\omega_1/2)^{1/2}$.

It is clear from Eqs. (13) in accordance with the results of Ref. 2 that at $t>0$ the oscillator is in a squeezed state[6,7] if $\omega_2\neq\omega_1$. It is also worth mentioning that at $t>0$ the state $|0\rangle_{\omega_1}$ is a state with unsymmetrical uncertainties for the operators X_2 and Y_2.[8]

Of course, the sudden change of the frequency is an idealization of the frequency change for small but finite time. One can consider, for example, a linear frequency change. For this purpose we substitute $\Theta(t)$ in (3) with a function $\vartheta(t)=0$ at $t<-\tau$, $\vartheta(t)=1$ at $t>\tau$, and $\vartheta(t)=\frac{1}{2}(1+t/\tau)$ at $|t|<\tau$. For small enough τ the evolution operator $U(\tau,-\tau)\simeq 1$. Using the method described in Ref. 5 for the value of τ for which the sudden change is a good approximation we have the estimate $\tau\ll\min(\omega_1,\omega_2)/|\omega_1^2-\omega_2^2|$.

If we compare our expression (3) with the Hamiltonian

$$H=\omega a^\dagger a+V(t)e^{-i2\omega t}(a^\dagger)^2+V^*(t)e^{i2\omega t}a^2\ , \tag{16}$$

used in paper I we can see that in paper I and in (2) completely different problems were considered. The authors of paper I first obtain an important and very elegant representation of the squeezing operator (see also Ref. 9). Secondly, they postulate that this operator is the evolution operator of their system, and thirdly, they find a Hamiltonian which satisfies this assumption. We think that in the case of a sudden change of the oscillator frequency their second step is not justified.

Unfortunately, the authors of paper I gave no example of a physical process which could be described by the Hamiltonian (16). We will try to do that: this Hamiltonian describes a degenerate parametric amplifier or oscillator, with time-dependent coupling constant $V(t)$. With this example we do not want to exclude the fact that the Hamiltonian (16) can describe a quantum oscillator with a time-dependent frequency if some canonical transformation[10] and rescaling of the time variable is carried out.

Regarding our Hamiltonian (3) we gave an example of a physical process in Ref. 2. This was the Franck-Condon transition in molecules or crystals during which the frequency of the vibration changes due to an electronic transition. Here we would like to give another example: a very fast change of the stiffness of a usual mechanical oscillator. Let us make a *Gedankenexperiment* during which the stiffness of a standard bar detector for gravitational radiation, which can be treated as a harmonic oscillator,[6,11] changes very quickly. Due to the sudden change of the frequency the bar will be in a squeezed state and one can measure the displacement of the oscillator resulting from the effect of the gravitational radiation in the quadrature with reduced fluctuations using strategies proposed by Hollenhorst.[6]

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