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Quantum optical test of observation and complementarity in quantum mechanics

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We propose and analyze experiments designed to probe the way in which the measurement process (the presence of a detector) influences the investigated system. These experiments are based on the fact that number states of the radiation field can be generated by the use of the micromaser and cavity quantum electrodynamics. It is shown that "which-path" (particle) information rules out interference (wave) effects due to the system-detector correlations and not due to a randomization of phase. Specific experiments based on neutron interferometry and quantum-beat techniques taken together with the micromaser are suggested and analyzed.

I. INTRODUCTION

Complementarity, e.g., the wave-particle duality of nature, lies at the heart of quantum mechanics. As we learn[1] from textbooks, matter sometimes displays wavelike properties (e.g., interference phenomena) and at other times displays particlelike behavior (e.g., "which-path" information). At the very beginning of such discussions it was made clear that the dual wave-particle aspects of quantum mechanics are complementary, but not contemporary.

The classic example of this merger of wave and particle behavior is provided by Young's double-slit experiment. There we find that it is impossible to tell which slit light went through and still observe an interference pattern. In other words, any attempt to gain which-path information will disturb the light so as to wash out the interference fringes. This point is made especially clear in the Einstein-Bohr dialogues,[2] whose arguments we recall in the following paragraphs.

Einstein invites us to consider a Young's double-slit experiment in which the slits can recoil, as indicated in Fig. 1. The interference pattern is constructed by, for example, measuring the output of a photodetector array due to light passing through the slits. Now if the mass of the optical baffle (Double-slit assembly) is small enough, it will recoil when the light is "emitted" by a given slit, then by conservation of momentum, we could tell which wave vector $\mathbf{k}_1$, or $\mathbf{k}_2$, the "photon" has, see Fig. 1. That is, we would then have which-path information.

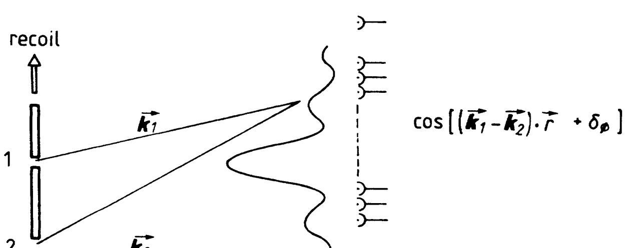

FIG. 1. Figure depicting the Einstein-Bohr recoiling-slit *Gedanken* experiment. Light emerging from slits 1 and 2 is collected by an array of photodetectors displaying the usual Young's double-slit interference pattern.

However, Bohr points out that we must also treat the recoiling plate by the rules of quantum mechanics. Specifically, Bohr argues that the physical position of the recoiling plate is only known to within Δx due to the uncertainty principle. This error will contribute a random phase shift $\delta\phi$ to our light beams which will destroy the interference pattern.

Such random-phase arguments, showing how which-path information destroys the coherent-wave-like interference aspects of a given experimental setup, are appealing. This is in the spirit of Heisenberg's[3] "γ-ray microscope." In all such arguments, one notes that the act of measuring invariably disturbs the system being measured and the loss of coherence is the inevitable result of such disturbance.

However, such arguments are incomplete. As has been shown elsewhere,[4] and further discussed in this paper, it is possible in principle and in practice to design experiments which provide which-path information via detectors which *do not disturb the system* in any noticeable way. Such *Welcher Weg* (German for "which-path") detectors have been recently considered within the context of studies involving spin coherence.[4]

In the present paper we present a simple experiment, which is being constructed in our laboratory, which shows that the loss of coherence occasioned by which-path information, i.e., by the presence of a *Welcher Weg* detector, is due simply to the establishing of quantum correlations and is in no way associated with large random-phase factors as in Einstein's recoiling slits or Heisenberg's microscope. The essence of the present *Welcher Weg* detector is the micromaser.[5] Such masers are unique in that they can operate with very few atoms (e.g., one) in the cavity at any given time. This permits many interesting experiments, e.g., using such a device we can, in principle, prepare number states of the maser field in a high-Q cavity.[6]

In Sec. II we review a sequence of neutron experiments which have recently been carried out by Rauch and co-workers,[7] and which are very much in the spirit of the present studies. We will see that while a micromaser *Welcher Weg* detector could be, in principle, used in these neutron experiments, in practice it is probably not feasible. In Sec. III we turn to the presently envisioned experiments, in which we use a quantum-beat configuration, together with the above-mentioned micromaser *Welcher Weg* detector, to provide a new test of complementarity in quantum mechanics.

II. YOUNG'S DOUBLE-SLIT-TYPE EXPERIMENTS WITH NEUTRONS

In this section we consider the problem of neutron interferometry. The fact that the neutrons are massive spin-$\frac{1}{2}$ particles provides, as we shall see, new possibilities and insights above and beyond that of optical interference and diffraction. We begin by describing the neutron experiments of Rauch and co-workers which set the stage for the present paper.

To properly appreciate these neutron interferometric experiments we first recall their optical ancestor,[8] i.e., the Mach-Zehnder or Twyman-Green interferometer as per Fig. 2(a). There we see that a light beam, characterized by an electric field $E_{\rm in}$, incident on a beam splitter. The output of the beam splitter, in region I, is two beams denoted by E_1 and E_2. After reflection by the mirrors m_1 and m_2 there are now two beams in region II, in direct analogy to Young's double-slit experiment. The two beams are recombined by the second beam splitter and in region III we have E_1+E_2.

The corresponding experiment has been carried out with neutrons as in Fig. 2(b). There we see that the neutron wave function $\psi(\mathbf{r},t)$ replaces the electric field $E(\mathbf{r},t)$ and the neutron "mirrors" are actually silicon crystals, otherwise the analogy is complete. If we regard the upper and lower crystal in Fig. 2(b) as sources S_1 and S_2, we may think of the right-hand side (regions II and III) as being essentially a neutron version of Young's experiment.

In the neutron experiments a phase "shifter" is placed in one arm, as in Fig. 2(b). Then, after recombining the beams, the state of the system is given by

$$\psi(\phi)=\psi_1+e^{i\phi}\psi_2 , \qquad (1)$$

and the probability of detecting a neutron goes as

$$I(\phi)=|\psi_1+e^{i\phi}\psi_2|^2=|\psi_1|^2+|\psi_2|^2+\psi_1^*\psi_2e^{i\phi}+\text{c.c.} , \qquad (2)$$

where $\psi_1^*\psi_2e^{i\phi}+$c.c. results from interference.

In later experiments they take advantage of the fact that the neutron has a spin magnetic moment to perform a neutron "NMR spin-flip" experiment, as depicted in Fig. 3(a). Concerning this work they say the following.[7]

"In continuation of previous work explicitly demonstrating the basic quantum-mechanical principles of the static and the time-dependent superposition of spinors, a new double-resonance coil experiment is described, where a spin-flip process associated with an exchange of energy quanta happens in both beams of a perfect-crystal neutron interferometer. It is shown that under the given circumstances of neutron self-interference coherence is preserved in spite of the energy transfer to every neutron, ..."

The main point is that the "rf spin flipping" does not destroy the coherence, i.e., the interference terms remain even after spin flip.

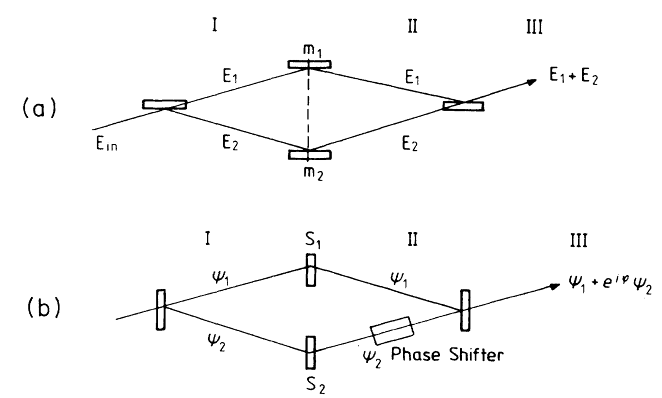

FIG. 2. (a) Incident light beam is split into beams E_1 and E_2, reflected from mirrors m_1 and m_2, and recombined in a region III by means of a second beam splitter. (b) Incident neutron wave is split by beam splitter and reflected by crystals designated S_1 and S_2, which may be thought of as sources of neutron waves. These neutron beams are then recombined by another crystal which produces a coherent superposition of the two beams in region III. The second beam is shifted by a phase factor accumulated in passing through the phase shifter located in lower arm of neutron interferometer.

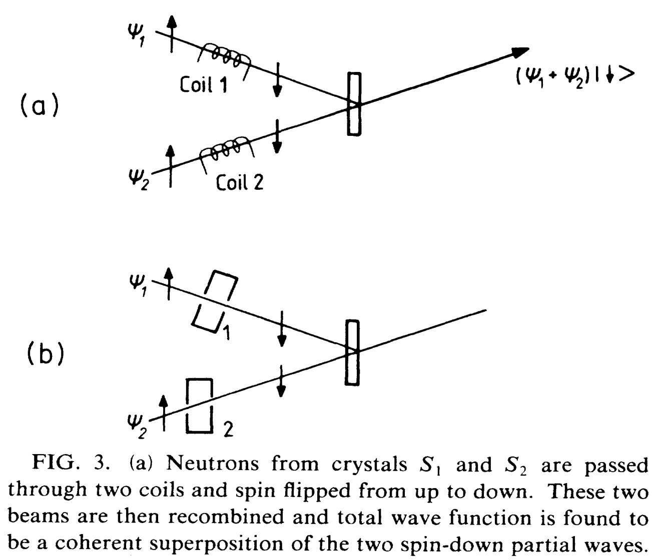

FIG. 3. (a) Neutrons from crystals S_1 and S_2 are passed through two coils and spin flipped from up to down. These two beams are then recombined and total wave function is found to be a coherent superposition of the two spin-down partial waves. (b) In this figure we replace the coils of (a) by micromaser cavities.

Let us next replace the spin-flip coils by two micromasers as in Fig. 3(b). The state of the quantized field in the ith micromaser is initially given by $|\Phi_i^0\rangle$, $i=1,2$. Thus the neutron-micromaser-state vector is then

$$|\Psi(0)\rangle=[\psi_1(r,0)+\psi_2(r,0)]|\uparrow\rangle\otimes|\Phi_1^0\Phi_2^0\rangle \ , \quad (3)$$

and, as is shown in Appendix A, after the spins pass through the micromaser cavities, Eq. (3) becomes

$$|\Psi(t)\rangle=\psi_1(r,t)|\downarrow\rangle\otimes|\Phi_1^f\Phi_2^0\rangle+\psi_2(r,t)|\downarrow\rangle\otimes|\Phi_1^0\Phi_2^f\rangle \ , \quad (4)$$

where $|\phi_k^f\rangle$, $i=1,2$, is the state of the maser field after interaction with the atom. The detection probability now goes as

$$I(\phi)=|\psi_1|^2+|\psi_2|^2+\psi_1^*\psi_2\langle\Phi_1^f\Phi_2^0|\Phi_1^0\Phi_2^f\rangle+\text{c.c.} \ , \quad (5)$$

where $\psi_1^*\psi_2\langle\Phi_1^f\Phi_2^0|\Phi_1^0\Phi_2^f\rangle+\text{c.c.}$ results from interference.

To make contact with the "rf-coil spin-flip" experiment we should prepare our maser cavities in coherent states,[9] since the coils in Fig. 3(a) will generate such coherent states. In such a case the initial state of the maser is taken as

$$|\Phi_i^0\rangle=|\alpha_i\rangle, \quad i=1,2 \quad (6)$$

and since the classical coherent field is not changed much by the addition of a single photon associated with spin flip, we may write

$$|\Phi_i^f\rangle\cong|\alpha_i\rangle, \quad i=1,2 \quad (7)$$

so that to a very good approximation we have

$$\langle\Phi_1^f\Phi_2^0|\Phi_1^0\Phi_2^f\rangle\cong\langle\alpha_1\alpha_2|\alpha_1\alpha_2\rangle = 1 \ , \quad (8)$$

and the interference cross term in Eq. (5) is identical to that of (2), i.e., the "spin flipping" has changed nothing.

These measurements are interesting in several different ways. First they show that the process of dynamically flipping the spins need not (in fact, *does* not) destroy the interference fringes. Furthermore, we should not be surprised that the introduction of a π (spin-flip) interaction leaves the quantum coherence intact. There is no which-path information left in the micromaser after passage of the spins since the coherent-photon distribution is essentially unchanged.

There is, however, a way to qualitatively (and quantitatively) change all this. If instead of preparing the masers in coherent states, $|\alpha_i\rangle$, $i=1,2$, we were to prepare them in numbers states,[6] $|n_i\rangle$, $i=1,2$, things are very different. In such a case, the initial state of the neutron-beam or maser system is given by

$$|\Psi(0)\rangle=[\psi_1(\mathbf{r},0)+\psi_2(\mathbf{r},0)]|\uparrow\rangle\otimes|n_1,n_2\rangle \ , \quad (9)$$

then, see Appendix A, after the spin-flip interaction we have

$$|\Psi(t)\rangle=\psi_1(\mathbf{r},t)|\downarrow\rangle\otimes|n_1+1,n_2\rangle + \psi_2(\mathbf{r},t)|\downarrow\rangle\otimes|n_1,n_2+1\rangle \ . \quad (10)$$

But now the inner product (8) is replaced by

$$\langle\Phi_1^f\Phi_2^0|\Phi_1^0\Phi_2^f\rangle=\langle n_1+1,n_2|n_1,n_2+1\rangle=0 \ , \quad (11)$$

and the coherence cross terms vanish.

This might seem a bit surprising. In the previous case in which $|\Phi_i\rangle=|\alpha_i\rangle$, flipping and spins did not destroy the coherent cross terms, i.e., did not affect our interference fringes. Similarly in the case of number-state preparation $|\Phi_i\rangle=|n_j\rangle$, the final state is $|\downarrow\rangle$ just as before. That is, the neutrons emerge from their respective cavities in the down configuration just as when we used coherent states, $|\alpha_i\rangle$, to effect the spin flip. However, in the case of the "number-state spin flipper" we have a *Welcher Weg* detector. By simply "looking" (actually it does not matter if we look or not, having the information there is enough) at the micromaser, we can tell which path our neutron took. If we find, for example, the first maser cavity to now be in a state having n_1+1 photons, we known the upper path was followed.

Thus we have a new and potentially practical (i.e., potentially experimental) example of wave-particle duality and observation in quantum mechanics. There is a basic difference, however, between the present situation and that of the Einstein recoiling-slit experiment. In that case the coherence (interference) was lost due to a phase disturbance of the light beams. In our case, the loss of coherence is due to the correlation established between the system and micromaser detector. Random-phase arguments never entered the discussion. (The fact that the number state has no phase is not relevant here; the important dynamics is due to the spin-flip transition.) In other words, the fact that which-path information is made available is enough to wash out the interference cross terms.

We note, however, that such experiments would be very difficult. In particular, since the neutron magnetic moment is so small, a number state having a very large number of photons would be required in order to produce a π "pulse."[10]

It is possible to generate number states via a micromaser,[6] but such number states can persist only for a time on the order of the microwave-cavity-ring down time. Such cavity-decay times for the micromaser are presently in the region of hundreds of milliseconds to a second, a very long time as such times go.

Thus, the generation of such an $|n\rangle$ state, and its application in the corresponding neutron experiments would not (to say the least) be easy. It is better to investigate situations in which the basic physics is preserved but the experiments are made simpler by utilizing, for example, the large transition-matrix elements associated with the Rydberg atom. These experiments are described in Sec. III where it will be shown that even the *vacuum* can serve as an *effective π-pulse mechanism* in such experiments.

III. PROPOSED QUANTUM-BEAT MICROMASER EXPERIMENT

In the usual quantum beat[11] experiments atoms are excited to a coherent superposition of states and allowed to decay to a lower level as in Fig. 4. The spontaneously emitted radiation will then show "beats" (temporal fringes) between the two possible transition paths $a \to c$ and $b \to c$ as indicated in Fig. 4. The initial state

$$|\Psi(0)\rangle = [\alpha(0)|a\rangle + \beta(0)|b\rangle] \otimes |0_1 0_2\rangle \ , \tag{12}$$

decays into a state in which the atoms (which are taken for simplicity to be in a small volume at the origin) are in the ground state c and photons[12]

$$|\gamma_i\rangle = \sum_k \frac{g_k^2}{(\omega_i - \nu_k) - i\Gamma_i} |1_k\rangle \ , \tag{13}$$

are emitted. In the above the index i refers to transitions 1 and 2 as in Fig. 4, g_k is the coupling constant between the atom and the kth mode of the radiation field, ω_i is the atomic frequency associated with the transitions 1 and 2, each of which decays at a rate Γ_1, ν_k is the frequency of the kth mode, and $|1_k\rangle$ denotes the single photon eigenstate.

After several decay times, the state of the coherently excited atom evolves into

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}} |c\rangle \otimes [|\gamma_1\rangle + |\gamma_2\rangle] \ . \tag{14}$$

Now the photocurrent will be determined by the correlation function[13]

$$G(\mathbf{r}_d,t) = \langle \Psi | \hat{E}^{(-)}(\mathbf{r}_d,t) \hat{E}^{(+)}(r_d,t) | \Psi \rangle \ , \tag{15}$$

where the positive frequency (annihilation) electric field operator, in the interaction picture, is given by

$$\hat{E}^{(+)}(\mathbf{r}_d,t) = \sum_k \mathcal{E}_k \hat{a}_{\mathbf{k}} e^{-i(\nu_k t - \mathbf{k}\cdot\mathbf{r}_d)} \ , \tag{16}$$

in which $\mathcal{E}_k = (\hbar\nu_k/\epsilon_0 V)^{1/2}$ is the electric field per photon, $\hat{a}_{\mathbf{k}}$ is the usual field-annihilation operator, and $\mathbf{r}_d$ determines the detector position.

From Eqs. (13)–(16) we calculate the photocurrent correlation function to be

$$G(\mathbf{r}_d,t) = \frac{I}{2r_d^2} \left| \theta\left[t - \frac{r_d}{c}\right] e^{-\Gamma_1(t - r_d/c)} e^{i\nu_1(t - r_d/c)} + \theta\left[t - \frac{r_d}{c}\right] e^{-\Gamma_2(t - r_d/c)} e^{i\nu_2(t - r_d/c)} \right|^2 \ , \tag{17}$$

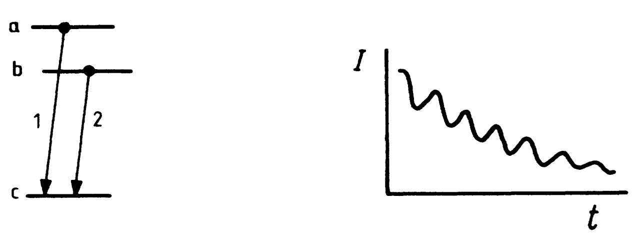

FIG. 4. Scheme for quantum-beat experiments. Coherent superposition of upper levels a and b decay to ground state c. The detector current shows a modulation in addition to the usual exponential decay.

where $\theta(x)$ denotes the usual step function. If, for simplicity, we take $\Gamma_1 = \Gamma/2$, this becomes

$$G(t) = \frac{I}{2r_d^2} \theta\left[t - \frac{r_d}{c}\right] e^{-\Gamma(t - r_d/c)} \times [1 + e^{i(\nu_1 - \nu_2)(t - r_d/c)}] + \text{c.c.} \ , \tag{18}$$

where the last exponential term results from temporal interference. Such quantum-beat phenomena have been seen in a variety of experiments in atomic and laser physics.

The connection between the quantum-beat and neutron-interferometry experiments is clear. In the neutron experiments we have two paths corresponding to ψ_1 and ψ_2 in Fig. 2(b). In the quantum-beat experiments the two "paths" are associated with the γ_1 and γ_2. In both cases we have interference cross terms.

We now wish to extend these considerations in order to include a *Welcher Weg* as in Sec. II. To this end, consider the experimental arrangement of Fig. 5. There we depict a more complicated atomic configuration such that, upon passage through cavity 1, the atoms will be "flipped" from $a \to a'$, and upon passing through cavity 2, will be flipped from $b \to b'$. Afterward, both a' and b' the atom will decay to state c'. See Appendix B for mathematical details and further discussion.

Thus we may write the final-state vector of the atom, the visible photon, and the micromaser system as

$$|\Psi\rangle = \frac{1}{\sqrt{2}} |c'\rangle \otimes [|\gamma_1'\rangle \otimes |\Phi_1^f \Phi_2^0\rangle + |\gamma_2'\rangle \otimes |\Phi_1^0 \Phi_2^f\rangle] \ , \tag{19}$$

as is shown in Appendix B. Now if we calculate the correlation function $G(\mathbf{r}_d,t)$ by using (19) and (15) we find

$$G(\mathbf{r}_d,t) = \frac{I}{2r_d^2} \theta\left[r - \frac{r_d}{c}\right] e^{-\Gamma(t - r_d/c)} \times [1 + e^{i(\nu_1 - \nu_2)(t - r_d/c)} \langle \Phi_1^f, \Phi_2^0 | \Phi_1^0, \Phi_2^f \rangle] + \text{c.c.} \tag{20}$$

Hence we see, as in the neutron problem of Sec. II, that

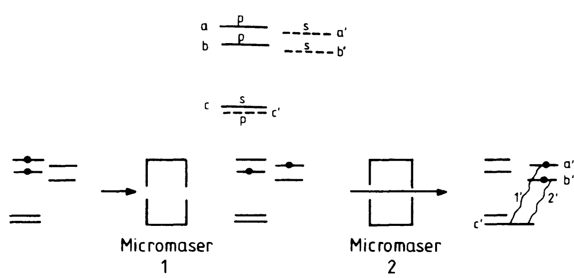

FIG. 5. Figure depicting the quantum-beat experiment with micromaser *Welcher Weg* detector. In passing through the first micromaser, atom makes transition from a to a' and in the second micromaser makes transition from b to b'. These transitions are analogous to the spin-flip transitions in the neutron experiment Fig. 3(b).

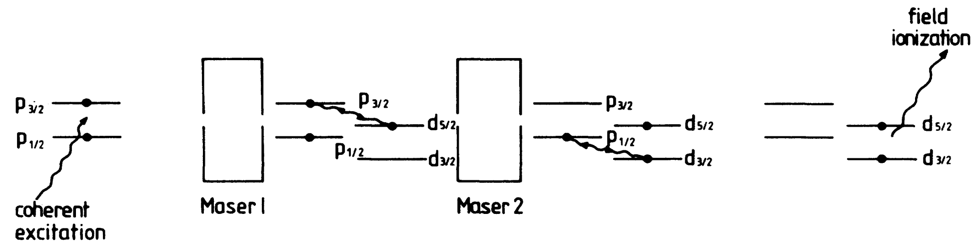

FIG. 6. Quantum-beat experiment using micromaser *Welcher Weg* detectors in which quantum beats are detected via field ionization rather than spontaneous emission.

the quantum beats will be present in the case of coherent-state maser fields $|\Phi_i\rangle = |\alpha_i\rangle$. However, if we prepare the micromasers in number states, the interference (quantum-beat) cross term will be multiplied by $\langle n_1+1, n_2 | n_1, n_2+1\rangle$, and will vanish just as in the neutron experiments.

In the present case, however, we do not need such large photon numbers to produce the π-spin flip from $a \rightarrow a'$ and from $b \rightarrow b'$ since the atomic matrix elements are much larger than those encountered in the neutron experiments. In fact, when using Rydberg atoms as we do, the vacuum interaction (i.e., spontaneous emission) is sufficient to insure π-spin flip. Thus we are relieved of the necessity of preparing number states $|n_i\rangle$ having large photon number, and instead prepare our masers in the easiest number state of all, the vacuum; see Appendix B.

In the envisioned experimental situation, the Rydberg atoms, as used in the micromaser, provide a very nice means for observing the quantum beats. The lifetime of the highly excited levels increases with the third power of the main quantum number of the states. This leads to lifetimes for the micromaser levels in the millisecond range. As a consequence, the spontaneous decay of the highly excited atoms can be neglected. Moreover, an efficient and very convenient detection is possible using field ionization.[14] This has the advantage that the probing of the atoms can be performed at any suitable time.

For this purpose the time delay between the coherent excitation of the levels a and b and the field ionization of the levels $a'b'$ (Fig. 5) has to be varied In the actual experimental setup this timing can be performed very accurately within a fraction of a nanosecond, since the ionization pulse can be triggered very accurately by the laser pulse producing the coherent population of the levels a and b.

An actual level scheme of this type of experiment is shown in Fig. 6. The $63p_{1/2}$ and $63p_{3/2}$ levels of Rb are very suitable; and they are used in the present Rydberg maser experiments as well. Their splitting is 396 MHz so that coherent excitation can easily be performed via a mode-locked dye laser. The envisioned *Welcher Weg* maser cavities could be identical with those used in the ongoing micromaser experiments. The low temperatures achieved with a He^3 cryostat would allow us to perform the experiment with the vacuum state, as mentioned above.

The velocity of the atoms must be adjusted so that the "spin flip" of $p_{3/2} \rightarrow d_{5/2}$ is guaranteed in the first cavity; cavity 2 must induce "spin-flip" transitions for $p_{1/2} \rightarrow d_{3/2}$. The frequency difference between $d_{5/2}$ and $d_{3/2}$ is 50 MHz and thus, in principle, easily detectable in the field-ionization quantum-beat experiment.

There is a small time spread in the velocity distribution of the Rb atoms (about 1%), however, this does not affect the timing of the field ionization, since this can be controlled by the dye-laser pulse used for the excitation. The field plates of the ionizing field can be arranged to be large enough so that the geometric spread of the atoms due to the slightly different time of flight is of no significance. One important condition which has to be fulfilled in this type of experiment is that the beam density be very low: there must be time enough so that the cavities return to their vacuum state before the next atom arrives, otherwise a field different from the vacuum would build up in the cavities and the which-path information would be lost. Experiments of this type could provide new tests of and insights into the foundations of quantum mechanics.

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APPENDIX A: THE NEUTRON-MICROMASER *WELCHER WEG* DETECTOR

Let us consider the experiment in which we use two microwave cavities as indicated in Fig. 3(b). In these cavities the microwave fields will cause the spin-$\frac{1}{2}$ system to flip from up to down to up as indicated. We note that this combination of "flips" can be arranged by choosing the field strengths and/or interaction times to be such that the spin flips occurs with 100% probability (i.e., $\mu B t/\hbar = \pi$). This is demonstrated in the following paragraphs.[15]

The interaction Hamiltonian for the spin plus microwave field is given (in the rotating-wave approximation) by

$$H=\mu\sigma_z B_0+\hbar\Omega_1 a_1^\dagger a_1+\hbar\Omega_2 a_2^\dagger a_2 + \tfrac{1}{2}g\sigma[a_1^\dagger u_1(x)+a_2^\dagger u_2(x)]+\text{H.a.} \ , \tag{A1}$$

where Ω_i, a_i ($a_i^\dagger$), and $u_i(x)$ are the microwave cavity-frequency annihilation (creation) operators and mode functions for the ith ($i=1,2$) cavity, H.a. denotes the Hermitian adjoint. The coupling constant g is given by the magnetic moment μ times the magnetic field "per photon" and σ is the lowering operator for the spin, which exists in the (large) magnetic field B_0.

To proceed we assume that the spin-$\frac{1}{2}$ particles have constant velocity v as they pass through cavities 1 and 2. Then the x coordinate of the spin is given by vt and the mode functions $u_i(x)\to u_i(t)$. Thus we may regard the mode functions as time-dependent "switching" functions, and the Hamiltonian takes the form

$$H=H_0+\tfrac{1}{2}g_1(t)\sigma a_1^\dagger+\tfrac{1}{2}g_2(t)\sigma a_2^\dagger+\text{H.a.} \ , \tag{A2}$$

where H_0 is the interaction free Hamiltonian and $g_i(t)$ denotes the (time-dependent) coupling constant.

Let us first consider the situation as the spin exits cavity 1. The initial state before entering the cavity is

$$|\Psi_{\rm I}\rangle=\frac{1}{\sqrt{2}}[\psi_1(\mathbf{r},0)+\psi_2(\mathbf{r},0)]|\uparrow\rangle\otimes|\Phi_1^0,\Phi_2^0\rangle \ , \tag{A3}$$

where Φ_i^0 is the state of the microwave field in the ith cavity. Upon entering cavity 1 we have

$$|\Psi_{\rm I}\rangle_1=\frac{1}{\sqrt{2}}\sum_{n_1}\psi_{1,n_1}(r,0)|\downarrow\rangle\otimes|n_1\rangle\Phi_2^0\rangle \ , \tag{A4}$$

where the probability amplitudes $\psi_{1,n_1}(r,0)$ are initially (i.e., at a time $t=0$ just before entering the cavity) given by

$$\psi_{1,n_1}(r,0)=\psi_1(r,0)\alpha_{n_1}(0) \ , \tag{A5}$$

in which $\alpha_{n_1}(0)$ is the probability amplitude for having n_1 photons in cavity 1.

Now we may reasonably assume (and it can be rigorously shown to a good approximation) that the center-of-mass part of the spin wave function is unchanged by the microwave field interaction, then the state of the upper beam plus maser system in region II (i.e., upon leaving cavity 1) is given by

$$|\Psi_{\rm II}(t)\rangle_1=\frac{1}{\sqrt{2}}\psi_1(r,t)\sum_{n_1}[\alpha_{n_1}(t)|\uparrow\rangle|n_1\rangle + \beta_{n_1}(t)|\downarrow\rangle|n_1\rangle]|\Phi_2^0\rangle \ , \tag{A6}$$

We note that only the state of maser 1 is changed, the second cavity is, of course, left unchanged. The time evolution of the center-of-mass wave function $\psi_1(r,t)$ is now determined solely by the free-particle Hamiltonian. The probability amplitudes α_{n_1} and β_{n_1} are found from the Schrödinger equation formed from (A2) and (A6), we find

$$\dot{\tilde{\alpha}}_{n_1}=\frac{-i}{2\hbar}g_1(t)\sqrt{n_1+1}\,\tilde{\beta}_{n_1+1}(t) \ , \tag{A7}$$

$$\dot{\tilde{\beta}}_{n_1+1}=\frac{-i}{2\hbar}g_1(t)\sqrt{n_1+1}\,\tilde{\alpha}_{n_1}(t) \ , \tag{A8}$$

where the high-frequency time dependence has been removed in the usual way, i.e., the probability amplitudes $\tilde{\alpha}_n$ and $\tilde{\beta}_{n+1}$ are in the interaction picture. In Eqs. (A7) and (A8), we have taken

$$\omega-\Omega=0 \ , \tag{A9}$$

where

$$\omega=2\mu B_0/\hbar \ , \tag{A10}$$

in order to arrive at the simple equations of motion (A7) and (A8). For the initial (spin-up) conditions

$$\tilde{\alpha}_{n_1}(0)=A_{n_1}^0, \quad \tilde{\beta}_{n_1}(0)=0 \ ,$$

Eqs. (A7) and (A8) yield

$$\tilde{\alpha}_{n_1}(t)=A_{n_1}^0\cos\theta_{n_1}(t) \ , \tag{A11}$$

$$\tilde{\beta}_{n_1+1}(t)=-iA_{n_1}^0\sin\theta_{n_1}(t) \ , \tag{A12}$$

where

$$\theta_{n_1}(t)=\frac{1}{2\hbar}\int_0^t g_1(t')dt'\sqrt{n_1+1} \ . \tag{A13}$$

Now if the microwave field is in a state whose photon distribution is sharply peaked about some value of n_1 (call it $\bar{n}_1$) then to a good approximation

$$\theta_{n_1}(t)\cong\frac{1}{2\hbar}\int_0^t g_1(t')dt'\sqrt{\bar{n}_1+1} \ . \tag{A14}$$

We want the spins to be flipped as they pass through the cavity, therefore we choose our parameters to yield the π-pulse condition

$$2\theta_{n_1}(\tau)\cong\frac{1}{\hbar}\int_0^\tau g_1(t')\sqrt{\bar{n}_1+1}dt'=\pi \ . \tag{A15}$$

Then, after leaving cavity 1, we have from (12) and (15)

$$\tilde{\alpha}_{n_1}(t>\tau)=0 \ , \tag{A16}$$

$$\tilde{\beta}_{n_1}(t>\tau)=-iA_{n_1-1} \ , \tag{A17}$$

and inserting (A16) and (A17) into (A6) we have

$$|\Psi_{\rm II}(t>\tau)\rangle_1=\psi_1(r,t)|\downarrow\rangle\otimes\sum_{n_1}A_{n_1-1}^0|n_1\rangle|\Phi_2^0\rangle = \psi_1(r,t)|\downarrow\rangle\otimes|\Phi_1^f\rangle|\Phi_2^0\rangle \ , \tag{A18}$$

where we have absorbed uninteresting factors, such as $1/\sqrt{2}$, etc., into the $\psi_i(r,t)$ probability amplitudes, and the final state of the maser is denoted by

$$|\Phi_1^f\rangle=\sum_{n_1}A_{n_1-1}^0|n_1\rangle \ . \tag{A19}$$

A similar analysis for the lower path yields

$$|\Psi_{\rm II}(t>\tau)\rangle_2=\psi_2(r,t)|\downarrow\rangle\otimes|\Phi_1^0\rangle|\Phi_2^f\rangle \ , \tag{A20}$$

and final the state of the total (both neutron beams and masers) system is

$$|\Psi\rangle=\psi_1(r,t)|\downarrow\rangle\otimes|\Phi_1^f\rangle|\Phi_2^0\rangle + \psi_2(r,t)|\downarrow\rangle\otimes|\Phi_1^0\rangle|\Phi_2^f\rangle \ . \quad \text{(A21)}$$

Note that if the maser state is initially $|n\rangle$ then after spin flip the maser is in state $|n+1\rangle$. Hence in the case of number-state preparation (A21) becomes

$$|\Psi\rangle=\psi_1(\mathbf{r},t)|\downarrow\rangle\otimes|n_1+1\rangle|n_2\rangle + \psi_2(\mathbf{r},t)|\downarrow\rangle\otimes|n_1\rangle|n_2+1\rangle \ . \quad \text{(A22)}$$

APPENDIX B: A MICROMASER *WELCHER WEG* DETECTOR SETUP FOR A QUANTUM-BEAT EXPERIMENT

As is well known, all two-level systems are equivalent. Thus the preceding spin-$\frac{1}{2}$ treatment applies directly to the quantum-beat problem in which microwave transitions between pairs of levels (a,a') and (b,b') now correspond to the neutron spin flip in the upper (1) and lower (2) paths of Fig. 3. That is, a and b are analogous to spin-up neutrons in the upper and lower paths, respectively. The a' and b' states are analogous to a *spin-down* neutron moving along upper and lower paths.

Thus the state of the atom-micromaser system before entering masers 1 and 2 is given by

$$|\Psi\rangle_0=\frac{1}{\sqrt{2}}[|a\rangle+|b\rangle]\otimes|\Phi_1^0\rangle|\Phi_2^0\rangle \ , \quad \text{(B1)}$$

after passing through maser 1 and "flipping" from $a\rightarrow a'$ the state of the total system becomes

$$|\Psi\rangle_{\rm I}=\frac{1}{\sqrt{2}}[|a'\rangle\otimes|\Phi_1^f\rangle|\Phi_2^0\rangle+|b\rangle\otimes|\Phi_1^0\rangle|\Phi_2^0\rangle] \ , \quad \text{(B2)}$$

and upon passing through the second maser wherein the atom experiences a π interaction such that $b\rightarrow b'$ we have

$$|\Psi\rangle_{\rm II}=\frac{1}{\sqrt{2}}[|a'\rangle\otimes|\Phi_1^f\rangle|\Phi_2^0\rangle+|b'\rangle\otimes|\Phi_1^0\rangle|\Phi_2^f\rangle] \ . \quad \text{(B3)}$$

Finally, we allow enough time to pass so that the decay from $|a'\rangle$ and $|b'\rangle$ to $|c'\rangle$ will take place with the emission of a photon $|\gamma_1\rangle$ and $|\gamma_2\rangle$, that is

$$|a'\rangle\rightarrow|\gamma_1'\rangle|c'\rangle \ , \quad \text{(B4)}$$

$$|b'\rangle\rightarrow|\gamma_2'\rangle|c'\rangle \ . \quad \text{(B5)}$$

Inserting (B4) and (B5) into (B3) we have the final state of our system

$$|\Psi\rangle=\frac{1}{\sqrt{2}}|c'\rangle\otimes[|\gamma_1'\rangle\otimes|\Phi_1^f\Phi_2^0\rangle+|\gamma_2'\rangle\otimes|\Phi_1^0\Phi_2^f\rangle] \ . \quad \text{(B6)}$$

In particular, we note that if $|\Phi_1^0\rangle$ and $\Phi_2^0\rangle$ are the vacuum, then

$$|\Psi\rangle=\frac{1}{\sqrt{2}}|c'\rangle\otimes[|\gamma_1'\rangle\otimes|1_1 0_2\rangle+|\gamma_2'\rangle\otimes|0_1 1_2\rangle] \ . \quad \text{(B7)}$$

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[1]A good discussion of this and related issues is given by C. Cohen-Tannoudji, B. Diu, and F. Laloë, *Quantum Mechanics* (Wiley, New York, 1977). Throughout this paper we will adhere to the Lamb operational point of view in measurement theory; see W. E. Lamb, Jr., Phys. Today **22**, 23 (1969).

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[3]W. Heisenberg, *Die Physikalischen Prinzipien der Quantentheorie* (Hirzel, Leipzig, 1930).

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[8]M. Born and E. Wolf, *Principles of Optics,* 6th ed. (Pergamon, New York, 1980).

[9]For an excellent overview of the subject see R. Glauber, in *Quantum Optics and Electronics,* 1964 Les Houches Summer Lectures, edited by C. Dewitt, A. Blandin and C. Cohen-Tannoudji (North-Holland, Amsterdam, 1964).

[10]In the present work several simplifying assumptions were possible due to the nature of the problem, e.g., we postulate a square-wave "pulse" and identical atomic-transition frequencies. However, these assumptions are not necessary. As has been shown in the pioneering work of Hahn and co-workers, a spread in atomic frequencies and generalized pulse shapes (i.e., other than square) can, under some condition, still yield π-pulse spin flip and for *all* atoms; see, e.g., E. Hahn, Phys. Rev. **80**, 580 (1950); S. McCall and E. Hahn, Phys. Rev. **183**, 457 (1969).

[11]The pioneering work in quantum-beat phenomenon was carried out by Series and co-workers. Later atomic-foil experiments of Bashkin provide a nice example of this effect. Specific reference to papers relevant to the present discussion can be found in W. Chow, M. Scully, and J. Stoner, Phys. Rev. A **11**, 1380 (1975).

[12]See for example, M. Hillery and M. Scully, *Quantum Optics Experimental Gravity and Measurement Theory,* edited by P. Meystre and M. Scully (Plenum, New York, 1983).

[13]See R. Glauber, Ref. 9.

[14]G. Leuchs, S. Smith, and H. Walther, in *Laser Spectroscopy IV,* Vol. 21 of *Springer Series in Optical Sciences,* edited by H. Walther and K. W. Rothe (Springer-Verlag, Berlin, 1979), p. 255; G. Leuchs and H. Walther, Z. Phys. A **293**, 93 (1979).

[15]For a treatment of the two-level-system problem along the lines of the present work see M. Sargent, M. Scully, and W. Lamb, *Laser Physics* (Addison-Wesley, Reading, MA, 1973); or L. Allen and J. Eberly, *Optical Resonance and Two-Level Atoms* (Wiley, New York, 1975).