Experimental studies on atomic form factors at 4.808-Å^{-1} photon momentum transfer

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(Received 27 September 1988)

Differential cross sections for coherent scattering of 84.3-keV gamma rays at 90° (scattering) by Cu, Mo, Ag, Cd, Sn, Pr, Sm, Gd, Dy, Ho, Yb, Ta, W, Pb, Th, and U were measured using a 133-cm^3 HpGe detector. From these accurately measured cross sections atomic form factors are extracted corresponding to 4.808-Å^{-1} photon momentum transfer. The results are compared with predictions of form-factor theories, and the appropriateness of these theories is discussed.

I. INTRODUCTION

Atomic form factors are important because of their relevance in basic atomic physics investigations. They also serve[1] as an input to the theoretical predictions of various physical quantities. Some of these quantities include coherent (Rayleigh) scattering, pair production, and bremsstrahlung cross sections required in such diverse applications as medical x-ray technology, power reactor shielding, industrial radiation processing, and analysis of nuclear physics experiments. Consequently, considerable efforts have been made to obtain accurate values of form factors. Theoretical predictions of form factors are based on nonrelativistic and relativistic individual electron and total atom wave functions. As a result, we have a nonrelativistic form-factor formalism[2,3] and a relativistic form-factor formalism.[1,4] There is also a relativistic Hartree-Fock-Slater modified form-factor formalism,[5,6] which accounts for the correction for binding of the atomic electrons. Hubbell *et al.*[1,2] and Schaupp *et al.*[6] have tabulated the relativistic, nonrelativistic, and modified form factors for a wide range of photon momentum transfer and for all elements in the Periodic Table. They have also compared these theoretical form factors with the experimental measurements which are available at low momentum transfer $x \leq 1\ \text{Å}^{-1}$ ($x = 1/\lambda[\sin(\theta/2)]$, where λ is the photon wavelength in Å units and θ is the scattering angle) and at high momentum transfer, $x \geq 10\ \text{Å}^{-1}$. There is a regular lack of experimental data for intermediate momentum transfer $1 \leq x \leq 10\ \text{Å}^{-1}$. In the present work we have conducted an experiment at $x = 4.808\ \text{Å}^{-1}$ and accurate values of form factors are reported for 16 elements in the region $29 \leq Z \leq 92$. We believe these results constitute the first report at this momentum transfer value. The experimental results are compared with theoretical predictions based on the three form-factor formalisms and the appropriateness of these three theories is discussed.

II. EXPERIMENTAL DETAILS AND METHODS

The scattering experiments were conducted in a reflection geometry[7–10] by allowing 84.3-keV gamma rays to undergo coherent scattering at 90°, corresponding to 4.808-Å^{-1} photon momentum transfer. Figure 1 shows the schematic arrangement of the experimental setup. A 50-mCi ^{170}Tm source obtained from Bhabha Atomic Research Centre, Bombay, India, was used to provide 84.3-keV gamma rays. High-purity thin elemental foils of Cu, Mo, Ag, Cd, Sn, Pr, Sm, Gd, Dy, Ho, Yb, Ta, W, Pb (all of purity better than 99.9%), and Th and

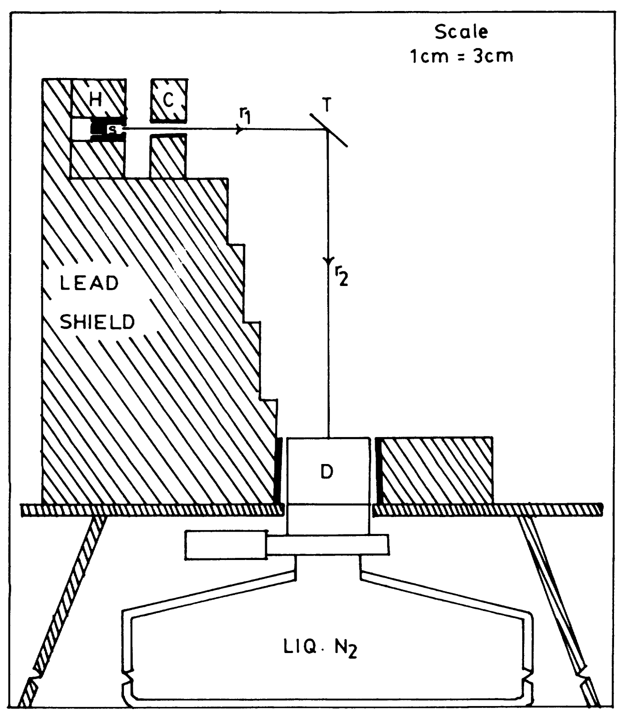

FIG. 1. Geometry and shielding arrangement of the experimental setup. S denotes source; T denotes scatterer; D denotes detector; r_1 denotes source-to-scatterer distance; r_2 denotes scatterer-to-detector distance; H denotes source collimator; and C denotes collimator.

U (purity better than 99.5%) covering the region $29 \leq Z \leq 92$ were used as scatterers. The thickness of these natural elemental foils ranged from 0.03 to 0.1 g/cm^2. By using Al and Cu as graded absorbers for collimation and Pb as shielding material, compact geometries were obtained by optimizing the source to scatterer and scatterer to detector distances. A 133 cm^3 HpGe detector was used to detect coherently scattered photons. The resolution of the detector [full width half maximum (FWMH)] was found to be 0.9 keV at 84.3 keV energy. The spectra were recorded in a 4-K analyzer. In Fig. 2 are shown representative plots of coherent peaks for Cu, Sn, Dy, and Pb.

The area under the coherent peak was accurately evaluated by fitting a Gaussian function and the differential cross section $(d\sigma/d\Omega)$ for coherent scattering of gamma rays was calculated using the relation

$$n_{\mathrm{coh}} = \frac{S}{4\pi}\left[\frac{d\sigma}{d\Omega}\right] n \epsilon \, d\Omega_1 d\Omega_2 \ , \tag{1}$$

where n_{coh} is the number of photons coherently scattered at 90° (area under the coherent peak), S is the source strength, n is the number of atoms in the scatterer, ϵ is the photopeak efficiency, $d\Omega_1$ is the source to scatterer solid angle, and $d\Omega_2$ is the scatterer to detector solid angle.

The source strength S was determined using an auxiliary weak source in an independent experiment. The

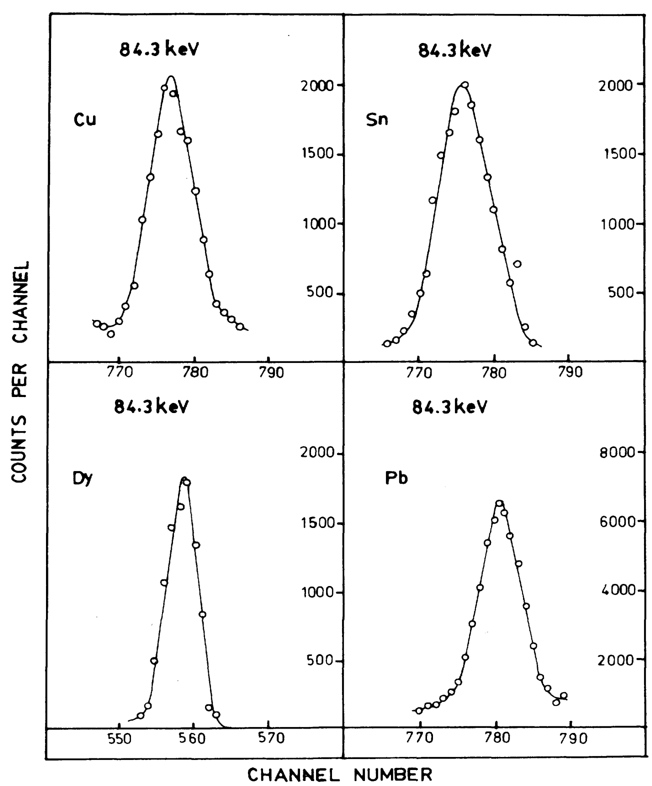

FIG. 2. Coherent peaks of 84.3-keV gamma rays scattered at 90° by Cu, Sn, Dy, and Pb.

strength of the weak source was determined using a 1.5×1 in^2 NaI(Tl) crystal whose efficiency is known accurately. If n and n_0 are the photopeak count rates obtained with the strong (experimental) source of strength S and weak source of strength S_0, then the strength S of the experimental source is obtained using the expression

$$\frac{n}{n_0} = \frac{S}{S_0}\left[\frac{r_0}{r}\right]^2 , \tag{2}$$

where r and r_0 are the distances of the strong and weak sources, respectively, from the detector in the independent experiment. (An estimate of attenuation in air has been made using data available in the literature[11] and it is found to affect the results only in the fourth decimal, which are certainly within the errors quoted in the paper.)

To avoid large uncertainties in the determination of ϵ and $d\Omega_2$, independently, the product $(\epsilon d\Omega_2)$, was determined[12] using a weak source of predetermined strength of the same isotope as the experimental source. The weak source was kept at the position of the scatterer so that the solid angle subtended by the detector remained the same and the spectrum was recorded. The photopeak count rate n_0 is then given by

$$n_0 = \frac{S}{4\pi}(\epsilon d\Omega_2) \ , \tag{3}$$

and, hence $(\epsilon d\Omega_2)$ was obtained.

The number of scattering atoms n in the scatterer was evaluated by weighing the scatterers using a microbalance. From the knowledge of the area of the scatterer and its distance from the source, the solid angle $d\Omega_1$ was computed. Finally differential cross sections for coherent scattering of gamma rays were calculated using the expression (1).

From the accurately measured differential cross sections coherent atomic form factors $F(x,Z)$ were extracted using the relation

$$\left[\frac{d\sigma}{d\Omega}\right] = \left[\frac{d\sigma}{d\Omega}\right]_T [F(x,Z)]^2 \ . \tag{4}$$

Here $(d\sigma/d\Omega)_T$ is the Thomson cross section (coherent contribution for scattering from a free electron) and is given by

$$\left[\frac{d\sigma}{d\Omega}\right]_T = \tfrac{1}{2} r_e^2 (1+\cos^2\theta) \ , \tag{5}$$

where r_e is the classical electron radius. Its value is known[1] very accurately ($2.817\,938\,0 \times 10^{-15}$ m). θ is the scattering angle.

III. CORRECTIONS AND ERRORS

In order to reduce the absorption in the scatterer, very thin foils of uniform thickness were used as scatterers. To take care of any small absorption in spite of these, corrections were applied taking the mass-attenuation coefficient values from the literature.[11] The selfabsorption in the source, if any, was neutralized in the source strength normalization experiments. For all the thin scatterers used in the experiments, the criterion $\mu t < 1$ (where μ is the mass-attenuation coefficient and t is the thickness of the scatterer) was satisfied. Hence the effects due to bremsstrahlung and multiple scattering were expected to be negligible. In order to reduce the statistical error in the count rate the spectra were recorded for a long time and about 10^4 counts (and more) were collected under the coherent peak. The photopeak area of the well-resolved coherent peaks was evaluated after fitting with a Gaussian function. The error associated in the evaluation of photopeak area was less than 1%.

The decay corrections were applied for the source. The foils were weighed accurately to 10 μg and, hence, the error in the determination of the number of atoms in the scatterer was negligible. In order to eliminate errors associated with the direct determination of photopeak efficiency and the detector solid angle individually, the product of the two quantities was determined as explained earlier. The associated error in the determination of source strength was estimated to be less than 4%. All the errors were compounded according to the well-known rules of propagation of errors and the resulting error is quoted on the measured cross-section values. The errors quoted on the extracted values of coherent atomic form factors were based on the uncertainties associated with the experimentally measured cross sections.

IV. RESULTS AND DISCUSSION

The results of the present investigations are summarized in Table I. To the best of our knowledge there are no experimental data on differential cross sections reported in literature for 84.3-keV gamma rays at 90° scattering for these elements except for Sn and Ta. It is, therefore, felt worthwhile to present the cross-section results also in addition to the form-factor results and are given in column 3. Our cross-section results 245±15 mb for Sn and 717±43 mb for Ta are in fair agreement with the experimental values reported by Raju *et al.*;[13] 270 (±1%) mb for Sn, and 810 (±3%) mb for Ta. Cross sections predicted by theory (Hubbell *et al.*[1,2]) are 260 mb for Sn and 760 mb for Ta and our results are in better agreement with these theoretical predictions.

It may also be noted that there does not exist in the literature the theoretical reports of cross sections for all these 16 elements computed on the basis of S-matrix theory. The S-matrix calculations are considered as "exact" calculations and, in a good majority of cases reported in the literature, good agreement has been observed between these S-matrix calculations and accurate experimental results obtained using solid-state detectors. It therefore will be interesting if the S-matrix calculations are made and compared with the present results of cross sections.

The atomic form factors derived from the measured cross sections are given in column 4. In column 5 are given the results of Tirsell *et al.*[14] which are available for Sn, Sm, and Ta measured using the Ge(Li) detector and at $x=5.0$ Å^{-1} for 74.15-keV gamma rays. Considering the factor that the energy and momentum transfer were slightly different than in the present work, the agreement of our results with those of Tirsell *et al.* is reasonably good.

The present experimental results of form factors are compared with the predictions of form factor theories in Fig. 3. It is clear from the figure that our experimental results differ with the predictions of relativistic form factor (RFF) theory. The RFF theory predicts much too large values for form factors particularly for high-Z ele-

TABLE I. Experimental differential coherent scattering cross sections ($d\sigma/d\Omega$) (mb/atom) and atomic form factors at $x=4.808$ Å^{-1}.

Sl. no.	Element with atomic no.	Experimental ($d\sigma/d\Omega$) (mb/atom)	Atomic form factors: Present work	Atomic form factors: Literature expt. value
1	^{29}Cu	45±4	1.065±0.047	
2	^{42}Mo	122±7	1.753±0.050	
3	^{47}Ag	171±9	2.075±0.054	
4	^{48}Cd	209±12	2.294±0.066	
5	^{50}Sn	245±15	2.484±0.076	2.67±0.05[a]
6	^{59}Pr	440±25	3.329±0.094	
7	^{62}Sm	530±30	3.654±0.103	4.05±0.08[a]
8	^{64}Gd	570±30	3.789±0.099	
9	^{66}Dy	573±36	3.799±0.119	
10	^{67}Ho	580±35	3.822±0.115	
11	^{70}Yb	685±40	4.154±0.121	
12	^{73}Ta	717±43	4.250±0.127	4.00±0.09[a]
13	^{74}W	734±44	4.300±0.129	
14	^{82}Pb	780±40	4.432±0.113	
15	^{90}Th	1184±60	5.461±0.138	
16	^{92}U	1330±66	5.788±0.143	

[a]Value at $x=5.0$ Å^{-1} for 74.15-keV photons. See Ref. 14.

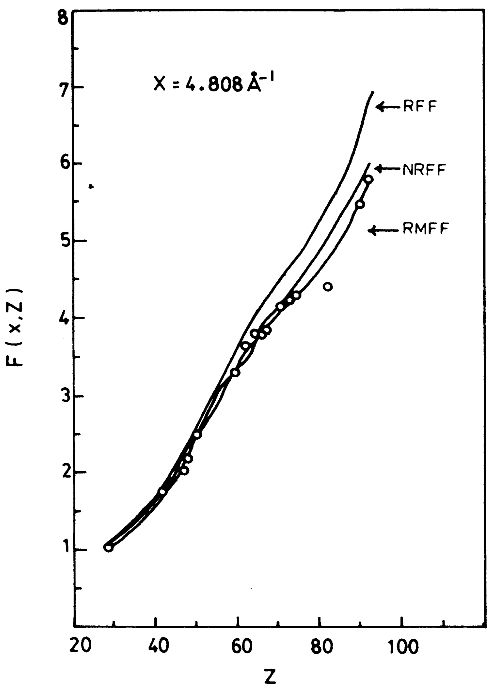

FIG. 3. Graphical comparison of present experimental results with the form-factor theories at $x = 4.808$ Å^{-1}.

ments. Although our experimental results agree with the nonrelativistic form-factor (NRFF) theory and relativistic modified form-factor (RMFF) theory, there is a better overall agreement with the RMFF theory. The agreement becomes conspicuously better for high-Z elements except for Pb. The discrepancy of our result for Pb with theory may be traced to the dispersion effects (the K edge of Pb is at 88 keV).

Similar findings suggesting the superiority of the RMFF theory have been reported in literature.[6,15–18] From the comparison of experimental results with form-factor theories at high momentum transfer values ($x > 10$ Å^{-1}), Kissel *et al.*[15] and Roy *et al.*[16] have shown that the RFF predictions are poorer than the predictions of the NRFF theory in the case of scattering from a heavy atom such as Pb. Kane *et al.*[17] reported excellent agreement of their differential cross-section data for Sn and Pb with RMFF theory at $x = 12$ Å^{-1} (from coherent scattering of 1.33-MeV gamma rays). Their experiments also indicated RFF theory predicting systematically too large values. Similarly Eichler *et al.*,[18] in comparison with form-factor theories of their experimental results for Cu, Cd, and Pb for 145-keV gamma rays at scattering angles of 25°–80°, have reported that the predictions of RFF theory are much larger. Their experimental results were reported to be in better agreement with RMFF theory. Schaupp *et al.*[6] have concluded from their systematic analysis that as a general rule the modified form factors may be considered better in predicting the scattering amplitude. Our present experimental results of form factors at the intermediate momentum transfer value 4.808 Å^{-1} confirm these earlier findings and clearly establish the appropriateness of the RMFF theory.

ACKNOWLEDGMENTS

The authors are very grateful to Dr. D. V. Gopinath, Head, Safety Research and Health Science Programme, Indira Gandhi Centre for Atomic Research, Kalpakkam, for his keen interest in the present work and for his constant encouragement. The authors also thank Dr. M. A. R. Iyengar, Head, ESL, Kalpakkam, and Professor S. Gopal, Department of Physics, University of Mysore, for their help and useful discussions. Two of the authors (N.G.N.) and (K.M.B.) are grateful to the Department of Atomic Energy, Government of India, for support.

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