Study of the cusp for H⁰,He⁺,He²⁺(0.2 MeV/u)–Ne,Ar,Kr collision systems in the angular region from 0° to 4°

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The doubly differential energy and angular distribution of electrons ejected into the continuum from H^0 , He^+ , $He^{2+}(0.2 \text{ MeV/u})$ -Ne, Ar, Kr collision systems are studied. The measured electron distributions are analyzed in a three-dimensional presentation with a series expansion by using a three-dimensional fitting procedure. The coefficients are discussed. Some other parameters of the cusp are also determined and compared with the theory.

I. INTRODUCTION

Since the discovery of the so-called cusp in the electron spectrum taken at 0° with respect to the beam direction in atomic collision processes,^{1,2} a great number of experimental and theoretical works were devoted to clarify the details of the phenomenon (see, e.g., Refs. 3-5). Although the angular dependence of the cusplike formation in the neighborhood of 0° has also been studied in several papers, 6-10 the number of shape studies, at nonzero observation angles, however, are very few. Meckbach et al.¹¹ carried out a shape study for the H⁰(105 keV)-He by using the method of the three-dimensional presentation of the experimental data. A similar study was performed by Elston *et al.*¹² for $O^{5+}(2.4, 5.1, 6.5)$ MeV/u, $O^{8+}(5.9 MeV/u)$ – He, Ar systems. In both of these works the shape of the cusp was investigated by the study of the contour lines at different percentages of the peak height. Quantitative evaluation of the data and a comparison with the corresponding theory was given, however, only in the latter paper (Ref. 12) by determining the asymmetry parameters from the experimental distribution by means of the fitting procedure.

In such a situation it seems to be justified to determine similar additional data with a broader set of projectiles and targets, especially because of there are theoretical calculations available for the asymmetry parameters.^{13,14}

In the present work a study is carried out for three different light-ion projectiles (H^0, He^+, He^{2+}) and for three targets (Ne,Ar,Kr) at 0.2-MeV/u impact energy. The three-dimensional distribution of the forward ejected electrons was investigated by the method of series expansion¹⁵ (Legendre polynomilas) and in addition to the shape studies some other parameters (angular dependence of the half width and peak position) were also determined.

II. EXPERIMENTAL SETUP AND MEASURING PROCEDURE

The electron spectra were taken by a distorted-field double-pass cylindrical-mirror electron spectrometer

(ESA-13) made by ATOMKI. The experimental setup was similar to that used in our earlier work.¹⁶ The relative energy resolution and the half angular acceptance angle at 0° were set to be $\pm 0.5\%$ and 1.1°, respectively.

The projectiles were accelerated by the 2.4-MV Van de Graaff accelerator of the Institute for Nuclear Physics of the University of Frankfurt am Main. The purity of the beam was controlled by using a magnetic deflection system.

Measured spectra were corrected for the detection efficiency in the same way as was done earlier.¹⁶ The angular correction due to the different target lengths seen by the spectrometer at different angles is based on the measuring of the L_2 - $M_{2,3}^2$ isotropic Auger line of Ar.¹⁷ The cusps for studying the electron loss to the continuum (ELC) and the electron capture to the continuum (ECC) processes were measured at every 0.5° from 0° to 4°.

The accurate determination of the 0° ejection angle of electrons with respect to the projectile direction is very important in this kind of measurement. To satisfy this demand we evaluated the maximum yield of electrons measured at different angles that originated from the process of electron capture into the continuum states of a 0.5-MeV H⁺ projectile colliding with the Ar target with very fine angular steps around the supposed 0° direction. In order to avoid the systematic errors during the measurement, the electron distributions were measured in several separate runs at the same collision systems and they were summed if they satisfied the statistical test described in Ref. 18. The relative target density was determined by measuring the Rutherford scattering of the 0.5-MeV-energy protons on the above-mentioned different targets.

III. EVALUATION PROCEDURE AND THEORETICAL DESCRIPTION

A. Evaluation procedure

In this work, the method developed by Meckbach et al.¹⁵ was used for the evaluation of the measured elec-

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tron distribution. They proposed to expand the double differential cross section (DDCS) for ECC and ELC process into a finite series,

$$\frac{d\sigma}{d\mathbf{v}_e} = \frac{1}{v'_e} \sum_{n,j=0} (v'_e)^n B_{nj}(v_p) P_j(\cos\theta') , \qquad (1)$$

where \mathbf{v}_e is the electron velocity and \mathbf{v}_p the ion velocity in the laboratory frame and θ' the ejection angle, \mathbf{v}'_e is the velocity of the electron in the projectile frame, and P_j are the Legendre polynomials.

To compare this DDCS with the $Q(v_e, \theta)$ measured electron distribution (θ is the ejection angle of electron in the laboratory frame), we must consider the $S(v_e, \Omega_e)$ spectrometer transmission function and integrate the $d\sigma/dv_e$ over the experimental acceptances in velocity and angle. Thus

$$Q(v_e,\theta) = \sum_{n,j=0} B_{nj}(v_p) U_{nj}(v_e,\theta) , \qquad (2)$$

where

$$U_{nj}(v_e,\theta) = \int (v_e')^{n-1} P_j(\cos\theta') S(v_e,\Omega) d\mathbf{v}_e \quad . \tag{3}$$

In a number of previous works (e.g., Refs. 19–21) the electron distribution was measured only at a 0° ejection angle. In the present study we extend the measurement and treat the process as a three-dimensional phenomenon (measuring the electron intensity as a function of the electron energy and the detection angle around 0°) and thus we must regard $Q(v_e, \theta)$ as a two-dimensional distribution.

We supposed that the $S(v_e, \Omega)$ transmission function is separable according to the velocity and the angle at the present experimental condition and the integration in Eq. (3) was performed numerically. Thus

$$S(v_e, \Omega) = F(v_e)G(\theta)H(\phi) , \qquad (4)$$

where ϕ is the azimuth angle of the ejected electron.

The $F(v_e)$ is approximated by a Gaussian function, whose parameters are determined on the basis of the relative energy resolution $(\Delta E/E)$ of the spectrometer. The $G(\theta)$ and $H(\phi)$ functions could be calculated from the geometrical conditions of the spectrometer. $G(\theta)$ was well approximated by a trapezoidal function and $H(\phi)$ was a multiplication factor at the given measuring angle. We calculated the $Q(v_e, \theta)$ function for the abovementioned angles for n=0-2 and j=0-4 and fitted the measured and calculated electron distribution as a threedimensional distribution using a least-squares fitting routine.

It was found that the values of the B_{nj} parameters depend on the size of the velocity volume (in \mathbf{v}'_e space) used for the fitting. We used a spherical volume with a v'_e raidus around $\mathbf{v}'_e = \mathbf{0}$. In our case the parameters were nearly constant if $|\mathbf{v}'_e| \ge 0.07$ a.u. This is why the fitted B_{nj} values presented in this paper are referred to the fitting region, with $v'_e = 0.07$ a.u. radius.

B. Theoretical description

Recently Hartley and Walters^{22,23} have surveyed the former theoretical results for the ELC process and at the same time suggested a new model, which is more consistent than the former ones. They used the impulse approximation for the singly inelastic (SI) process. To carry out this procedure, first the knowledge of the cross sections for elastic electron scattering is needed. By using the published experimental values the best estimate is obtained for these cross sections. A new procedure to calculate the characteristic features of the doubly inelastic (DI) process involves the modification of the Born approximation. These results show a better agreement with the experimental data than the former ones. It was shown^{16,24,25} that the collisions mechanism at

It was shown^{16,24,25} that the collisions mechanism at 0.2-MeV/amu H⁰ and He⁺ impacts is not a pure ELC process; an ECC contribution also should be present. Thus it is not expected that even the Hartley-Walters theory should give good agreement here. That is why we used a first-order Born approximation [which is good enough for ELC (Ref. 26)] to describe the SI process for the sake of simplicity (see details in Ref. 27). The DI process, however, is treated in the present work according to the procedure of Hartley and Walters.²²

For describing the ECC cusps originating from bareion-atom collision (in our case: He²⁺-Ar,Kr), many theoretical attempts were published (see the details in Ref. 28). These models are mostly valid only for the three-body processes and they can be extended for the He target by applying H-like wave functions. Recently Jakubassa²⁹ has carried out calculations for the H⁺-Ne system by use of the impulse approximation. These theoretical approaches describe more or less the singly differential cross section as a function of the impact energy and the angular dependence of the location of the top of the ECC cusp in the electron-energy spectrum, but they do not give a satisfactory explanation for the asymmetry of the cusp. Thus it is not expected that the theories concerned with H-like wave functions can give satisfactory approximations for our collision systems of the type concerned, namely, for He^{2+} -Ar,Kr. That is why for these systems there were no calculations carried out in the present work.

IV. RESULTS AND DISCUSSION

Altogether eight three-dimensional cusps were studied in the present work. In the following the results on the shape of the cusps and some other parameters of them are given.

A. Contour lines of the cusp

In the case of a three-dimensional study of the cusp phenomenon, the most illustrative and direct way to investigate the shape of the cusp is the utilization of the contour lines obtained at some fractional levels of the height of the cusp. Figures 1 and 2 show the contour lines obtained from our deconvoluted three-dimensional experimental spectra [see Eq. (1)]. In Fig. 1 the corresponding theoretical contour lines are also given. The DDCS determined from Eq. (1) and from the theories are infinite at $\mathbf{v}'_e = 0$, so the contour lines presented in different figures refer to a calculated distribution which where determined at the same \mathbf{v}'_e coordinates in every

case (the smallest $|\mathbf{v}'_e|$ was 0.012 a.u.). The indicated numbers in these figures at the different contour lines mean the logarithm of the intensity ratio of the given and the top level (namely, -2.2, -1.8, -1.4, -1.0, and



FIG. 1. Deconvoluted experimental and theoretical contour lines for (a) He^+ and (b) H^0 projectiles at different targets. Horizontal and vertical scales indicate the parallel and transverse coordinates of the velocity of electrons (in atomic units) in the projectile frame with respect to the direction of the impact velocity. Contours represent the logarithmic ratio of the given and the top level (the difference between these lines is 0.4 in this scale).

-0.6).

All the results in this paper [full width at half maximum (FWHM), asymmetry, etc.] presented in Figs. 3-5are determined from the deconvoluted experimental and the theoretical distribution obtained in the abovementioned way. Therefore we cannot present data at 0° angle.

At first sight it can be stated on the basis of the figures



FIG. 1. (Continued).

that there is a significant difference in the patterns of the H^{0} - and He^{+} -impact cases. For the He^{+} projectile the "infolding," i.e., the special behavior around $\mathbf{v}'_{e} = 0$, is nearly completely missing for all the three targets and there is at least a rough approximate agreement with the corresponding theoretical pattern [see Fig. 1(a)]. For the H^{0} projectile, however, the experimental patterns are quite different from those for He^{+} impact and also from the corresponding theoretical ones. They show "infolding," and their patterns are more similar to those at He^{2+} impact (see Fig. 2) than at He^{+} impact. Furthermore, a superficial analysis of the contour lines shows that the patterns depend mainly on the projectile and not on the target. Possible explanations for these findings can be found in Ref. 25 and Sec. IV D.

The possibility for comparison of the present results on



FIG. 2. Experimental contour lines in the case of He^{2+} impact on Kr and Ar targets. Notations are similar to those in Fig. 1.

the contour lines with other published results is rather limited. Mechbach *et al.*¹¹ determined the contour lines of the cusp for the 105-keV H⁰-He collision system and Elston *et al.*¹² published the corresponding data for heavy-ion impact in the high-energy region. In the second one,¹² the incident bare O⁸⁺ on Ne, their contour line pattern is highly asymmetric but without the special shape at $\mathbf{v}'_e = 0$ (infolding) which we observed for H⁰ and He²⁺ impact. This special behavior was not found either by Meckbach *et al.*¹¹ in their measurement.

B. Asymmetry and FWHM of the cusp

We also studied the asymmetry and the FWHM of the DDCS at different angles. The asymmetry was determined as the ratio of the partial width at half maximum on the left (low-velocity) side of the peak to that on the right (see, e.g., Ref. 30). It can be seen in Fig. 3 that the experimental values of the asymmetry are much larger than the theoretical ones in general, and that the disagreement is larger the higher the angle value is. Only their slope, namely, the increase as a function of the observation angle, is comparable to the theory. Comparing the cases of He⁺ and of H⁰, the asymmetry values and the increase are higher for He⁺. In both cases, however, there is a definite dependence on the target species. As regards the absolute values of the asymmetry, they are nearly in agreement with those of Man *et al.*³⁰ for the Ne



FIG. 3. Angular dependence of the asymmetry (see the text) for He^+ and H^0 projectiles on different targets. The theory gives the same result for He^+ projectiles at different targets.

target (but not for Ar) after an extrapolation of their values.

A rough approximate agreement with the theory was found in the angular dependence of the FWHM (Fig. 4). This agreement, however, is better for He⁺ than for H⁰ impact. In the case of He⁺ a definite target dependence was also observed which is missing in the theory. The FWHM as a function of the observation angle was studied by Duncan and Menendez⁶ and Yu and Lapicki,¹⁹ but in the former case the projectiles (H⁺, H₂⁺), in the latter the targets (hydrocarbons), and in both cases the impact energies were different from the present ones. However, their data show a similar dependence as a function of the observation angle than ours.

C. Location of the maximum of the cusp

In the present study the location of the maximum as a function of the electron-ejection angle (in the laboratory frame) was also determined for H^0 and He^+ projectiles and the corresponding calculations were carried out also. The results are given in Fig. 5. It can be seen that the target dependence is much smaller for H^0 than for He^+ impact. The peak location, however, as a function of the observation angle is in an agreement with the theoretical prediction within the experimental errors for the H^0 projectile. For the He⁺ projectile, however, the theory does not predict any dependence on the target species, but



FIG. 4. Angular dependence of FWHM (in a.u.) of the cusp for He^+ and H^0 impact on different targets. The theory gives the same result in the case of the He^+ projectile at different targets.

here a definite dependence on target species is observed. The best agreement between the theoretical and experimental data as a function of the observation angle was found for the Ne target.

D. Shape study by means of series expansion

Day²⁶ and Burgodörfer *et al.*¹³ showed that the DDCS has the following form for the electron loss in the Born approximation if the velocity of ejected electron in the projectile frame is small:

$$\frac{d\sigma}{d\mathbf{v}_e} = \frac{B_{00}}{v'_e} \sum_{k=0}^{2n} \beta_k p_k(\cos\theta') \quad \text{for even } k \quad , \tag{5}$$

where $\beta_k = B_{0k} / B_{00}$ and *n* is the principal quantum number of the electron in the projectile.

The DDCS for ECC (Ref. 14) (also in the small-velocity limit of the electron in the projectile frame) can be expressed in terms of the isotropic cross section B_{00} and the anisotropy coefficients β_k as

$$\frac{d\sigma}{d\mathbf{v}_{e}} = \frac{B_{00}}{v'_{e}} \sum_{k=0}^{\infty} \beta_{k} P_{k}(\cos\theta'), \quad k = 0, 1, 2, \dots$$
 (6)

The spectra obtained with the He^{2+} projectile are expected to be a manifestation of the ECC process while the data obtained for the He^+ and H^0 projectiles are supposed to have a contribution from both ELC and ECC mechanisms. But the ratio of the different components depends very strongly on the impact velocity of the projectile.^{16,24,25}

In addition, a part of the measured electron yield (be-



FIG. 5. Angular dependence of the peak position (in a.u.) for He^+ and H^0 projectiles on different targets. The theory gives the same result on He^+ projectiles.

side the dominant process, i.e., ELC, ECC, or both) originates from other mechanisms (e.g., direct Coulomb ionization of the target, second-order effects, interferences between the different processes, etc.).^{14,26,31} In the following the processes different from ELC and ECC are called "background" processes. The estimation and calculation of the contribution of these "background" mechanisms in the measured electron spectra are rather difficult. Burgdörfer¹⁴ showed that the cross section for the ECC and the accompanying target ionization process have the following form in the v'_e expansion:

$$\frac{d\sigma}{d\mathbf{v}_{e}} \approx a_{n} (v_{e}')^{n/2}, \quad n = -1, 0, 1, 2, \dots$$
 (7)

Equation (7) contains terms $[(v'_e)^{-1/2}, (v'_e)^{1/2}, (v'_e)^{3/2}]$, etc.] which are not included in the expansion of Eqs. (1), (5), and (6). In this way, the background effects mentioned above (different from ELC or ECC process) could appear in the fitted values of B_{0j} . Therefore the uncertainty of these fitted values orginating from the experiments may be higher.

In our case the convoluted DDCS with β_k coefficients (k=1-4) was not enough to fit the measured electron yield at the three different projectile systems. We had to include the n=1 terms in order to take into consideration the effect of background processes. The best fit (the normalized χ_2) was between 1 and 2 if we took into consideration the B_{nj} , n=0, 1, j=0,1,2 terms. In Table I the numerical values of the fitted parameters relative to B_{00} is presented for the various projectile-target combinations. In analyzing the B_{nj} parameters, roughly speaking, we could not find any definite target dependence (as was already seen in the case of the contour lines), except for the B_{00} term, so in the following the averaged values of the individual coefficients over the targets are discussed.

The n = 0 terms in Eq. (2) are characteristic of the measured distribution at the neighborhood of the top of the peak, while the n = 1 terms probe it at the wings. So the above-mentioned "background" contributions appear mainly in the n = 1 terms. That is why the values of the β_k parameters are less sensitive to the effect of the "background" mechanism and they are determined mainly by the ELC and ECC mechanisms. The observed discrepancies for the values of the n = 1 parameters in the case of

different collision systems (see Ref. 19-21) might be caused by the different "background" contributions to the main processes. In the following we discuss the values of the fitted parameters obtained for characterizing the ELC and ECC process.

Coefficient B_{00} . The B_{00} coefficients are proportional to the single differential cross section (cf., e.g., Ref. 32). The increase of B_{00} relative to B_{00} for the Ne target as a function of the target nuclear-charge-number ratio relative to Ne is linear both for H⁰ and He⁺. However, the experimental slope values (1.1 and 0.13, respectively) are much lower than the corresponding theoretical ones (3.9 and 2.9, respectively) obtained on the basis of calculations according to Sec. III B.

Major asymmetric terms β_1 . For all three projectiles values for β_1 are negative, indicating an asymmetric velocity distribution toward the lower electron velocities (in \mathbf{v}'_e space) along the direction of the projectile velocity. It is, however, the largest in the case of He²⁺ projectile. A large amount of data can be found in the literature on He²⁺ projectiles mainly on a He target.^{20,33} The theoretical values from continuum distorted wave (CDW) theory¹⁴ -0.28 and -0.23 from a second Born calculation³⁴ for the He²⁺-He system (we obtained these values from figures presented in the referred articles) are close to our experimental values (mainly for Kr target) for the He²⁺ bombarding projectile.

The appearance of the nonzero values of this parameter for H⁰ and He⁺ projectiles can be due partly to the ECC contribution to the cusp, as it was mentioned before (cf. Ref. 16 and 24), and partly to a new observation according to which the shape of the cusp is decisively determined by the outgoing part of the projectile path (charge).²⁵ Recently Yu and Lapicki¹⁹ have obtained similar negative values for β_1 in the case of a 0.6-MeV/u He⁺ impact ion and hydrocarbon targets. Elston *et al.*¹² with an O⁵⁺ bombarding projectile at higher impact energies found smaller negative values for the β_1 parameter than ours; this might be explained partly on the basis of the theoretical model prediction according to which ECC decreases rapidly with the increasing impact energy (see Ref. 4).

Values of the β_2 parameter. The β_2 parameter values are negative for all the three projectiles. (The reason of the small positive value in the He⁺-Ar case is not clear.)

Proj.	Target	$B_{01}/B_{00} = \beta_1$	$\boldsymbol{B}_{02} / \boldsymbol{B}_{00} = \boldsymbol{\beta}_2$	B_{10} / B_{00}	B_{11}/B_{00}	B_{12} / B_{00}
He ⁺	Ne	-0.26	-0.30	-1.29	0.25	0.98
	Ar	-0.17	0.02	-0.81	-0.92	0.70
	Kr	-0.19	-0.16	-1.10	-1.10	-0.35
\mathbf{H}^{0}	Ne	-0.11	-0.72	-2.40	-0.27	3.32
	Ar	-0.22	-0.56	-1.79	-0.08	2.14
	Kr	-0.12	-0.82	-2.19	-0.35	3.66
He ²⁺	Ar	-0.45	-0.53	0.92	0.27	0.74
	Kr	-0.28	-0.80	2.54	-2.26	1.24
Error		20%	20%	40%	60%	20%

TABLE I. Fitted relative B_{nj} coefficients, for different projectiles and targets.

They are very similar for He²⁺ and H⁰ and lower for He⁺. The theoretical values of β_2 for ELC process²⁶ are negative in the high-energy limit and slowly approach the zero and the positive values with decreasing impact velocity. At lower impact velocities the change is stronger and the value of β_2 become positive around $v_p \approx Z_p$ (Z_p the nuclear charge of the projectiles) and increases very rapidly toward zero projectile velocity. The experimental β_2 values from Refs. 11, 12, and 19 (studied with different projectile-target combinations) and those from the present work are in an approximate agreement in tendency with the above-mentioned dependence with respect to the projectile velocity.

The differences in the values of this β_2 parameter at H⁰ and He⁺ projectiles can be explained in the following way. In the case of the H⁰ projectile the curve of β_2 crosses the $\beta_2 = 0$ axis at lower impact velocity than that for the He⁺ projectile, because it has a lower nuclear charge, resulting in larger negative values of the β_2 parameters in the present energy region (see Ref. 26). These differences are also seen in the theoretical curves presented in Figs. 1(a) and 1(b). However, as was seen for the case of the β_1 parameter, the ECC occurs for structured projectiles. The nearly equal β_1 values in both He⁺ and H⁰ impacts and the theoretical result from Ref. 14 for the He²⁺-He case indicate that the contribution of the ECC process to the β_2 values is rather small at these energies and projectiles. Consequently, the β_2 parameter must have higher negative values at H⁰ projectiles than for He⁺, due to the contribution of the ELC and ECC process. The high negative value of β_2 (close to -1) means a strong symmetric transverse-velocity distribution along the direction of the projectile velocity in the \mathbf{v}_{e}' space (prolate-contour-line pattern). However, in our cases, for H^0 and He^{2+} projectiles this symmetric distribution is "distorted" by the effect of the β_1 parameter, resulting in the "infolding" in the contour lines, as it can be seen in Figs. 1 and 2. The effect of the β_1 parameter is also seen on the contour lines corresponding to the He⁺ projectile (i.e., asymmetry towards smaller energies).

Regarding the parameter β_2 in the case of the He²⁺ projectile, we could not find theoretical values for ECC in the literature. For the H⁺-He case the CDW calculation has similar behavior¹⁴ to that described for ELC. The existing few experimental data for bare projectiles on

different targets (see Refs. 19 and 20) together with our results are not enough to find any tendency in them with respect to the projectile velocity.

V. CONCLUSIONS

We have studied the three-dimensional electron cusp (i.e., DDCS energy and angular distribution of the ejected electrons) yield for H^0 , He^+ , $He^{2+}(0.2 \text{ MeV/amu})$ – Ne,Ar,Kr collision systems and evaluated them with the series-expansion method.¹⁵

We found that the spectra induced by H^0 , He^+ , and He^{2+} projectiles do not fit well to the distribution obtained from the ELC and ECC theories [see Eqs. (5) and (6)]. The necessity of the inclusion of the n = 1 terms in order to get the best fit means the presence of other "background" effects appearing together with the dominant ELC or ECC mechanism.

The appearance of β_k terms (k odd) in the fitting of the electron spectra obtained from H⁰,He⁺-Ne,Ar,Kr collision processes, which are in contradiction with ELC theoretical prediction [see Eq. (5)] might be the consequence of the ECC mechanism at structured projectiles.

The observed difference in the special shape of the measured double differential electron distribution (see Fig. 1) at H^0 and He^+ projectiles can be understood qualitatively by the difference between the ELC process and the presence of the ECC mechanism at structured projectiles.

In this work the above-mentioned processes were considered in its completeness in the forward direction. Further investigations would be very important in understanding the mechanism which produces the cusp in more detail. It is also important to decrease the errors in the values of the B_{nj} parameters caused by different "background" effects. Coincidence measurements would be helpful. It should also be mentioned here that the approximation in the spectrometer transmission function could cause errors in the values of the parameters also.

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