Enhanced squeezing with mixed two-photon and four-photon processes in two external pump fields

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By making use of two external pumps here it is shown that a given medium, with high-order optical susceptibility tensor, is a good candidate to get enhanced squeezing, by considerably reducing the interaction length.

In light of the recent investigation of nonclassical light sources[1] there is much interest in finding systems where it is possible to generate squeezed states of light.[2] The most exciting experiment to date, which may generate such states with quantum fluctuations three times less than the quantum limit, was performed on a parametric oscillator configuration in two-photon media.[3] The strength of the squeezing depends on the interaction time of the electromagnetic field with the parametric medium then, the use of resonant cavities has been a must. In order to get a great amount of squeezing, very high-finesse cavities have to be used. This is an experimental difficulty that makes squeezed light generation a noneasy task. The optical cavity, moreover, increase the total linear loss reducing the obtainable squeezing.[4,5]

In order to reduce the interaction times and consequently the importance of the linear loss, Yuen and the present author,[6] a few years ago, considered the temporal development of the squeezing resulting from the interaction of single-mode coherent light with an optically bystable two-photon medium. Recently such result was confirmed by Gerry and Rodrigues,[7] in the initial transient regime, by using a numerical technique on the same system. Furthermore, Mecozzi and the author showed[8] that enhanced squeezing is obtained with a four-photon process for a nonempty initial signal. Although particular, the last model is exactly solved.

In the present paper I wish to show that by using two external coherent pump fields, the usual two-photon medium, with added four-photon interaction, is a good candidate for the production of a great amount of squeezing; thus reducing linear losses since shorter interaction lengths are required.

By passing over its microscopic nature, the macroscopic description of the interaction of the electromagnetic field with matter can be written down by means of the polarizability[9]

$$P_i=\chi^{(1)}\delta_{ij}E_j+\chi^{(2)}_{ijl}E_jE_l+\chi^{(3)}_{ijlm}E_jE_lE_m+\cdots\ , \qquad (1)$$

where $\chi^{(k)}_{ij,l\ldots}$ represents the kth order optical susceptibility tensor and its subscripts correspond to spatial and polarization components. For simplicity, the attention is restricted to a single mode of the electromagnetic field only, ignoring all but one spatial component and optical polarization. Thus, the model Hamiltonian can be written as (setting $\hbar=1$)

$$H=\omega a^\dagger a+\gamma^{(2)}(E^*(t)a^2+E(t)a^{\dagger 2}) +\gamma^{(3)}(a^{\dagger 2}a^2+a^2a^{\dagger 2})+\gamma^{(4)}(B^*(t)a^4+B(t)a^{\dagger 4})\ . \qquad (2)$$

$\gamma^{(i)}$ is proportional to the ith order susceptibility tensor. $E(t)$ and $B(t)$ are two external pump fields assumed ideal, for which there is no depletion, and are so intense to be considered classical; ω is the frequency of the signal mode taken into account, which is described by means of the boson operators $a,a^\dagger$. The dissipation and any coupling with a thermal bath are also neglected. It will be later shown that the interaction length, to get a very good squeezing, could become very small and, consequently, losses are less important on such a short time scale. The second term represents the usual two-photon interaction, the third is a self-phase-modulation term, the last term describes a four-photon interaction. The model could describe the frequency splitting of two very intense coherent light beams in a nonlinear medium via two-photon and four-photon processes. For the sake of simplicity, only the degenerate case will be considered here whereas the extension to the nondegenerate case can be easily performed. The derivation of the results, however, turns out to be a little more complicated.

We assume perfect matching conditions and we choose the two external pump fields such as

$$B(t)=B_0e^{-2i\omega_p t+i\phi} \qquad (3)$$

and

$$E(t)=\frac{E_0}{2}e^{-i\omega_p t+i\psi} \qquad (4)$$

with E_0 and B_0 real and $\omega_p=2\omega$. The Hamiltonian (2) describes a mixing of two-photon and four-photon processes. We set the phases of the external fields such as

$$\phi=\pi,\quad \psi=\pi/2\ . \qquad (5)$$

Furthermore, by choosing the amplitude of the higher-frequency pump field such that

$$B_0=\gamma^{(3)}/\gamma^{(4)}\ , \qquad (6)$$

in a frame rotating at ω, the Hamiltonian becomes

$$H=-\mu H_0+\chi H_0^2 , \tag{7}$$

where

$$\mu=\gamma^{(2)}E_0, \quad \chi=4\gamma^{(3)} , \tag{8}$$

and

$$H_0=\frac{i}{2}(a^2-a^{\dagger 2}) \tag{9}$$

represents the two-photon Hamiltonian, extensively discussed by Yuen,[4] which is suitable to get two-photon coherent states and the squeezing of quantum noise fluctuations.

Defining the dimensionless time variable $\tau=\chi t$ and the dimensionless coupling $\eta=\mu/\chi$, the equation of motion for the mode operator a reads

$$\frac{d}{d\tau}a=-ia-a^{\dagger}(\eta+2H_0) . \tag{10}$$

By making use of Eq. (10) and its Hermitian conjugate, for the quadrature operators $a_1=(a+a^{\dagger})/2$, $a_2=i(a^{\dagger}-a)/2$, the following evolution equations hold:

$$\frac{da_1}{d\tau}=-a_1(i+\eta+2H_0) , \tag{11a}$$

$$\frac{da_2}{d\tau}=-a_2(i-\eta-2H_0) . \tag{11b}$$

Since H_0 is a constant of motion, the solutions of Eqs. (11a) and (11b) read

$$a_1(\tau)=a_1(0)e^{-(i+\eta+2H_0)\tau} , \tag{12a}$$

$$a_2(\tau)=a_2(0)e^{-(i-\eta-2H_0)\tau} , \tag{12b}$$

where $a_i(0)$ represents the initial value of the ith quadrature.

The expectation values with respect to any initial coherent state $|\alpha\rangle$ are then easily performed, and for the in-phase quadrature we get

$$\langle\alpha|a_1(t)|\alpha\rangle=e^{-(i+\eta)\tau}\langle\alpha|a_1(0)e^{-2H_0\tau}|\alpha\rangle =\frac{e^{-(i+\eta)\tau}}{2}\left[\alpha^*+\alpha+\frac{\partial}{\partial\alpha^*}\right]\langle\alpha|e^{-2H_0\tau}|\alpha\rangle , \tag{13}$$

where the results of Ref. 6 have been used. Completely in the same fashion one gets

$$\langle\alpha|[a_1(t)]^2|\alpha\rangle=\frac{e^{-2(2i+\eta)\tau}}{4}\left[\alpha^*+\alpha+\frac{\partial}{\partial\alpha^*}\right]^2 \times\langle\alpha|e^{-4H_0\tau}|\alpha\rangle . \tag{14}$$

The variance

$$\langle\alpha|[\Delta a_1(t)]^2|\alpha\rangle=\langle\alpha|[a_1(t)]^2|\alpha\rangle-\langle\alpha|a_1(t)|\alpha\rangle^2 \tag{15}$$

thus becomes

$$V_1(\tau)\equiv\langle\alpha|[\Delta a_1(t)]^2|\alpha\rangle=\frac{e^{-i2\eta\tau}}{4}f(\tau) \tag{16}$$

with

$$f(\tau)=e^{-4i\tau}\left[\alpha^*+\alpha+\frac{\partial}{\partial\alpha^*}\right]^2\langle\alpha|e^{-4H_0\tau}|\alpha\rangle -e^{2i\tau}\left[\left[\alpha^*+\alpha+\frac{\partial}{\partial\alpha^*}\right]^2\langle\alpha|e^{-2H_0\tau}|\alpha\rangle\right]^2 . \tag{17}$$

The function $f(\tau)$ represents the contribution of the four-photon process to the usual two-photon coherent states variance. As may be inferred from Ref. 6, for an initial coherent state with $\alpha=|\alpha|e^{-i\pi/4}$,

$$f(\tau)=\left[\frac{1}{\cos(4\tau)}+2|\alpha|^2\frac{1-\sin(4\tau)}{\cos^2(4\tau)}\right]\times\frac{\exp[-|\alpha|^2 s(4\tau)]}{\sqrt{\cos(4\tau)}} -2|\alpha|^2\frac{1-\sin(2\tau)}{\cos^3(2\tau)}\exp[-2|\alpha|^2 s(2\tau)] , \tag{18}$$

where

$$s(n\tau)=\frac{\cos(n\tau)+\sin(n\tau)-1}{\cos(n\tau)} \tag{19}$$

with $0\le\tau<\pi/8$.[8]

In Fig. 1 the variance $V_1(\tau)$ is plotted for various values of $|\alpha|^2$ and for a small squeezing parameter ($\mu=0.1$) for conventional two-photon media. It is immediately seen that as soon as the average number of photons in the initial state of the signal is different from zero, the interaction time, needed to get very high squeezing, is considerably reduced with respect to the usual two-photon medium, represented in the figure by the slow descending line. The divergence of the variance is only a consequence of considering the two pumps undepleted. Indeed, as the number of photons in the signal

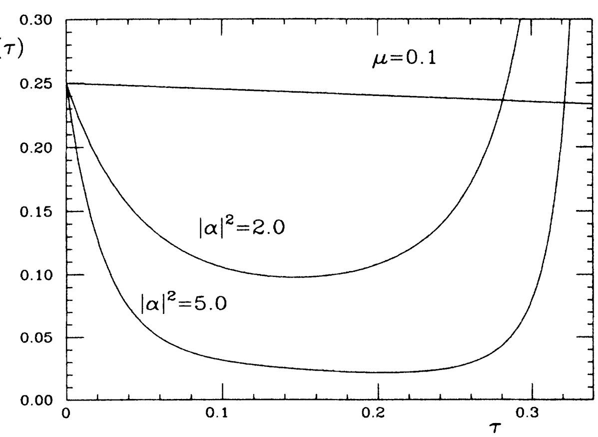

FIG. 1. Variance $V_1(\tau)$ is plotted vs the dimensionless time $\tau=\chi t$ for various values of the average number of photons in the initial state of the signal. The slowly descending line represents the usual two-photon result.

mode becomes very large, one should consider the whole quantum problem.

Due to the shortening of the interaction time, linear losses are less important and the feasibility of the experiment becomes much easier. Of course, the practical realizability of such an experiment relies on the fulfilling of Eq. (6). Several years ago $\chi^{(4)}$ was measured[10] in crystals without inversion center and the modern technology of materials science should not have any problem in obtaining a medium with these characteristics. When the B pump is switched on, the interaction time (length) to get almost perfect squeezing becomes very short. One could then be more confident that losses are negligibly small. The pump's phase fluctuations are also less important at short times. By populating the initial signal mode, even pulsed light could well be used in suitable materials.

As stated before, the nondegenerate case could also be described and the effect of the detuning considered. However, since what I wish to outline here is just the idea of using two pump fields in this medium, the detailed presentation of the nondegenerate case will be considered elsewhere.

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