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Dielectronic recombination rate coefficient for some selected ions in the helium isoelectronic sequence

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Dielectronic recombination (DR) rate coefficients are calculated for selected heliumlike ions with atomic number $Z=10$, 14, 18, 22, 24, 26, and 28 at plasma temperatures 0.1–10 keV. The total DR rate coefficient has been obtained by summing over partial DR rates of recombination processes that proceed via intermediate lithiumlike resonance states $|1s2lnl'LSJ\rangle$. The effects of configuration interaction and spin-orbit coupling have been included for all states with $n\leq 8$; for higher-lying autoionizing states, the $1/n^3$ scaling was employed. The results from the present calculation are compared with available theoretical and experimental values of DR rate coefficients.

I. INTRODUCTION

In high-temperature laboratory plasmas several elements, viz., silicon,[1] calcium,[1,2] titanium,[1-6] chromium,[7-9] iron,[3,4,10,11] and nickel[12,13] have been found as impurities. Neon[14-16] and argon[17-20] have been injected into certain fusion plasmas for diagnostics purposes. Silicon,[21] calcium,[22-27] and iron[22-30] are the common naturally occurring elements in solar corona. At an electron temperature of several keV,typical of solar active regions and today's laser fusion devices, these elements remain mostly as hydrogenlike and heliumlike ions and radiate predominantly in the x-ray region. These x-ray spectra have various diagnostic applications: The ion temperature can be determined from Doppler broadening; the density of impurity ions can be inferred from a measurement of absolute radiation intensity; the electron temperature and density and relative charge state of the plasma can be obtained by comparing intensities of certain x-ray lines.[31-33]

In low-density high-temperature plasmas dielectronic recombination (DR) is the dominant recombination channel. In DR an electron is captured by the ion in one of its outer subshell with simultaneous excitation of one of its inner-shell electrons, followed by radiative decay to a lower-energy state of the recombined ion which is stable against autoionization. For heliumlike ions the DR process can schematically be represented as

$$1s^2+\epsilon l \rightarrow (1s2lnl')^* \rightarrow 1s^2nl'+\gamma \ . \tag{1}$$

In this process the kinetic energy ε of the free electron ϵl is lost in the form of x-ray photons. In tokamaks with high-Z impurity atoms this may constitute the dominant energy-loss mechanism. The various intermediate resonance states of the configuration $(1s2lnl)^*$ in Eq. (1) can also be produced by electron collisional excitation of lithiumlike ions, and radiative recombination of heliumlike ions. The x-ray photons emitted from these lithiumlike resonance states constitute satellite lines of the $1s2p\ ^1P\rightarrow 1s^2$ resonance line. The ratio of intensities of the resonance line to satellite lines has been used[34,35] to determine the electron temperature in the plasma.

Theoretical values of DR rate coefficients are needed for modeling of laboratory plasmas[36,37] and for diagnostics of both laboratory and astrophysical plasmas. The difficulty in calculating DR rate coefficients lies in the enormous number of intermediate resonance states $|1s2lnl'LSJ\rangle$ over which summation has to be carried out. Recently, Nilsen,[38] Younger,[39] Nasser and Hahn[40] and Bely-Dubau[41] have reported on DR rate coefficients of selected heliumlike ions at various plasma temperatures. In all these calculations approximate methods were adopted to compute DR rate coefficients for all higher-lying intermediate resonance states with $n>4$. We have recently reported[42-44] DR coefficients of hydrogenlike ions of elements with $Z=10$, 14, 18, 20, 24, 26, and 28 at plasma temperature of 0.6–10 keV.

In this paper we present DR rate coefficients of heliumlike ions for all the above elements at electron temperature of 0.1–10 keV. We have computed the relevant atomic parameters explicitly for all intermediate resonance states $|1s2lnl'LSJ\rangle$ with $n\leq 8$ and $l'\leq 6$. The results are compared with available experimental values of DR rate coefficients of heliumlike iron[11] and the theoretical values reported by other authors.[38-41] We have employed the self-consistent Hartree-Fock-Slater atomic model in calculating single-particle wave functions. The effects of relativistic mass corrections, and the spin-orbit coupling contributions are included.

II. THEORY

A. DR rate coefficient

Consider a DR process in which a free electron with kinetic energy ε is captured by a heliumlike ion in its ground state $|g\rangle=|1s^2\,^1S_0\rangle$, and let $|s\rangle=|1s2lnl'SLJ\rangle$ be an intermediate resonance state which decays radiatively to $|1s^2nl'S'L'J'\rangle$ state producing a dielectronic satellite line. The intensity of this satellite line is given by

$$I_d(s-f)=N_eN_g\alpha_d(s-f) \ , \tag{2}$$

where N_e and N_g are, respectively, the density of electrons and heliumlike ions in the ground state $|g\rangle$ in the plasma, and $\alpha_d(s-f)$ is the DR rate coefficient. Assuming a Maxwellian electron-energy distribution one can obtain[45,46]

$$\begin{aligned}\alpha_d(s-f)&=(\tfrac{1}{2})(2\pi\hbar^2/mkT_e)^{3/2}F_2^*(S-f)\\&\quad\times\exp(-E_a/kT_e)\\&\equiv 1.656\times10^{-22}(kT_3)^{-3/2}F_2^*(s-f)\\&\quad\times\exp(-E_a/kT_e)\ ,\end{aligned}\tag{3}$$

where $E_a=E_s-E_g$ is the Auger electron energy and kT_e, the product of electron temperature T_e and Boltzman's constant k, is in eV. The satellite intensity factor $F_2^*(s-f)$ is defined as

$$F_2^*(s-f)=(g_s/g_g)\Gamma_a(s)\Gamma_r(s-f)/\Gamma(s)\ .\tag{4}$$

Here g_s and g_g are, respectively, the statistical weights of the autoionizing state $|s\rangle$ and the ground state $|g\rangle$; $\Gamma_a(s)$ is the autoionization rate of $|s\rangle$, $\Gamma_r(s-f)$ is the rate for radiative transition for $|s\rangle\rightarrow|f\rangle$, and

$$\Gamma(s)=\Gamma_a(s)+\sum_f\Gamma_r(s-f)\ .\tag{5}$$

To calculate total DR rate coefficient Eq. (3) has to be summed over all possible intermediate resonance states $|s\rangle$ and all possible radiative channels $|f\rangle$. Computations of DR rate coefficients thus basically reduce to the calculations of transition rates for each radiative and nonradiative deexcitation channel of a large number of intermediate resonance states $|s\rangle$.

B. Autoionization rates

The autoionization transition rates are given by

$$\Gamma_a=(2\pi/\hbar)\left|\left\langle\psi_f\left|\sum_{\substack{i,j\\i>j}}V_{ij}\right|\psi_i\right\rangle\right|^2\rho(\varepsilon)\ .\tag{6}$$

Here, ψ_i and ψ_f are, respectively, the antisymmetrized many-electron wave functions of the initial and final states, $\rho(\varepsilon)$ is the density of final state, and $V_{ij}=\sum_{i,j}1/r_{ij}$ is the two-electron electrostatic operator. The matrix element of V_{ij} can be expressed as

$$\begin{aligned}\langle\psi_f|V_{ij}|\psi_i\rangle=\sum_k[&a_kR^k(n_1l_1,n_2l_2;n_3l_3,n_4l_4)\\&+b_kR^k(n_1l_1,n_2l_2;n_4l_4,n_3l_3)]\ ,\end{aligned}\tag{7}$$

where n_il_i represents the appropriate one-electron orbital. The generalized Slater integral $R^k(n_1l_1,n_2l_2;n_3l_3,n_4l_4)$ is defined as

$$\begin{aligned}&R^k(n_1l_1,n_2l_2;n_3l_3,n_4l_4)\\&\quad=\int_{s=0}^{\infty}\int_{t=0}^{\infty}P(n_1l_1;s)P(n_2l_2;s)\frac{r_<^k}{r_>^{k+1}}\\&\qquad\times P(n_3l_3;t)P(n_4l_4;t)ds\,dt\ ,\end{aligned}\tag{8}$$

where the notation $r_<$ and $r_>$ indicates, respectively, the smaller and larger of s and t.

We have used Racah algebra[47] and Fano's ansatz[48] to derive the matrix element of Eq. (7),

$$\begin{aligned}&\langle 1s^2\epsilon l_cSLJ|V_{ij}|1s2l(S_1L_1)nl'SLJ\rangle\\&\quad=(2)^{1/2}\sum_k[a_kR^k(1s\epsilon l_c;2lnl')\\&\qquad+b_kR^k(1s\epsilon l_c;nl'2l)]\ ,\end{aligned}\tag{9}$$

where

$$\begin{aligned}a_k&=(-1)^{l'}(2l'+1)^{1/2}\delta_{kl}\delta_{S_10}\\&\quad\times\begin{bmatrix}0&k&l\\0&0&0\end{bmatrix}\begin{bmatrix}l_c&k&l'\\0&0&0\end{bmatrix}\end{aligned}\tag{10}$$

and

$$\begin{aligned}b_k&=-(\tfrac{1}{2})(-1)^{l_c+l'}[(2l'+1)(2S_1+1)]^{1/2}\\&\quad\times\delta_{kl'}\delta_{l_cL}\begin{bmatrix}0&k&l'\\0&0&0\end{bmatrix}\begin{bmatrix}l_c&k&l\\0&0&0\end{bmatrix}\ .\end{aligned}\tag{11}$$

C. Radiative rates

The transition probability for spontaneous emission of a photon of angular frequency ω is

$$\Gamma_r(\gamma'J'-\gamma J)=\frac{4\omega^3}{3\hbar c^3}\frac{1}{2J+1}|\langle\gamma'J'\|D\|\gamma J\rangle|^2\ ,\tag{12}$$

where γJ and $\gamma'J'$ represent, respectively, the initial and final states of the system, D is the electric-dipole operator, and $\langle\gamma'J'\|D\|\gamma J\rangle$ is the reduced matrix element. The reduced matrix element can be written as[49]

$$\begin{aligned}\langle S'L'J'\|D\|SLJ\rangle&=(-1)^{t_>-l}(l_>)^{1/2}R_{\text{line}}(S'L'J'-SLJ)\\&\quad\times R_{\text{mult}}(\alpha'L'-\alpha L)I(n'l'-nl)\ ,\end{aligned}\tag{13}$$

where

$$\begin{aligned}R_{\text{line}}(S'L'J'-SLJ)&=[(2J+1)(2J'+1)]^{1/2}\\&\quad\times(-1)^{S+L+J'+1}\begin{Bmatrix}J'&1&J\\L&S&L'\end{Bmatrix}\ ,\end{aligned}\tag{14}$$

$$I(n'l'-nl)=\int_0^{\infty}P(nl,r)rP(n'l',r)dr\ ,\tag{15}$$

and $l_>$ is the larger of l and l'.

The lithiumlike configuration $1s2lnl'$ with $n=2$, 3, and 4 can decay by electric dipole transition through the following channels:

(i) $|1s2l^2(S_1L_1)SLJ\rangle\rightarrow|1s^22lSL'J'\rangle$,

(ii) $|1s2l(S_1L_1)nl'SLJ\rangle\rightarrow|1s^2nl'S'L'J'\rangle$,

(iii) $|1s2l(S_1L_1)nl'SLJ\rangle\rightarrow|1s^22l'SL'J'\rangle$,

(iv) $|1s2l(S_1L_1)nl'SLJ\rangle\rightarrow|1s2l^2(S_2L_2)SL'J'\rangle$, and

$$\text{(v)}\ |1s2l(S_1L_1)nl'SLJ\rangle \rightarrow |1s2l(S_1L_1)ml''SL'J'\rangle,\quad m<n\ .$$

The reduced matrix elements for the above transitions can be obtained from a general expression of reduced matrix elements reported by Karim *et al.*[50] for lithiumlike ions of configuration $l_1^n l_2^m l_3^t$.

III. NUMERICAL CALCULATIONS

A self-consistent Hartree-Fock-Slater[51] method was employed to generate bound-state one-electron atomic orbitals $P(nl;r)$ of Sec. II. The wave function of the continuum electron was obtained by solving numerically the appropriate Hartree-Fock-Slater equation in the field of the final-state configuration. It was assumed that the wave functions of the bound core electrons are not perturbed by the presence of the continuum electron. Angular momentum coupling in $|LSJ\rangle$ scheme of the configurations $1s2lnl'$, $n=2$–8 produced configuration-state functions (CSF) ϕ_i which served as basis states for expansion of the atomic-state functions (ASF) ψ_i as

$$\psi_i=\sum_j C_{ij}\phi_j\ .$$

The mixing coefficients were obtained by usual diagonalization of the atomic Hamiltonian. The relativistic mass correction and spin-orbit interaction were included in the energy matrix elements. All the radial energy and transition matrix elements were calculated using numerical Hartree-Fock-Slater wave functions. The transition energies and rates for each radiative and autoionization channel of all the intermediate resonance states $|s\rangle$ were calculated through the formulation of Sec. II.

IV. RESULTS AND DISCUSSION

The total DR rate coefficients for heliumlike ions with atomic number $Z=10$, 14, 18, 20, 22, 24, 26, and 28 are presented in Table I for electron temperature of 0.1–10 keV. The total DR rate coefficient is obtained by summing over partial DR rates of large number of intermediate lithiumlike states from configurations of the type $1s2lnl'$. Since the complexity of the calculation increases rapidly with n, we have restricted our calculation to $n\le 8$. In Table II we list the partial rate coefficients for heliumlike neon and heliumlike nickel of DR processes which proceed via the excited lithiumlike configurations $1s2lnl'$ with $n=2$–8. In Fig. 1 we plot the DR rate coefficients of heliumlike silicon, argon, titanium, chromium, and nickel as a function of electron temperature. The variation of DR rate coefficients of heliumlike ions with respect to variation in atomic number Z and plasma temperature T_e exhibit the following behavior: (i) for a particular Z the DR rate coefficient increases sharply with temperature, reaches a maximum at about $kT_e=(\frac{2}{3})E_a$ (E_a is the energy of the lithiumlike autoionizing state $|s\rangle$ with respect to the ground-state energy of the $1s^2$ heliumlike ion) and falls off gradually; (ii) the electron temperature corresponding to the maximum values of DR rate coefficients increases with Z; (iii) the maximum value of DR rate coefficient decreases with Z; and (iv) at low electron temperatures DR rate coefficients of lighter elements are larger than the rate coefficients of heavier ions, at very high temperatures this trend is reversed.

These features are very similar to the DR rate coefficients of hydrogenlike ions[38] and can be understood as follows: The temperature dependence of DR rate coefficient primarily comes from the terms

$$(kT_e)^{-3/2}\exp(-E_a/kT_e)\ . \tag{16}$$

The effects of the first term of Eq. (16) is a decrease in DR rate coefficient with temperature which is the same for all elements and for all DR channels. The exponential term lies between 0 and 1 corresponding to $kT_e=0$, and ∞; this term increases very rapidly until about $kT_e\approx E_a$ after which the growth becomes extremely slow. The overall variation of DR rate coefficients with T_e is a compromise between these two opposing trends; at low energies the exponential term is responsible for the steep rise in the curves of Fig. 1, at higher energies the $(1/kT_e)^{3/2}$ term falls off at a much faster rate than the increase in the exponential term, the net effect being a gradual decrease in DR rate coefficients as kT_e increases. The rate of growth of the exponential term at low energies is faster if E_a is smaller as is evident from Fig. 1 which shows that the curves rises less steeply as the atomic number Z increases. The value of Auger electron energies E_a for a typical $1s2l^2\rightarrow 1s^2\epsilon l$ transition are approximately 0.6, 1.4, 2.2, 2.8, 3.3, 4.0, 4.7, and 5.4 keV, respectively, for $Z=10$, 14, 18, 20, 22, 24, 26, and 28.

Differentiating Eq. (3) with respect to kT_e it can easily be seen that for a particular Z DR rate coefficient is maximum at $kT_e=(2/3)E_a$. As E_a increases with Z, the position of the maxima shift towards higher energy for heavier elements. The reasons for the decrease in DR rate coefficients with Z at temperatures corresponding to the maximum DR rate coefficients are rather involved.

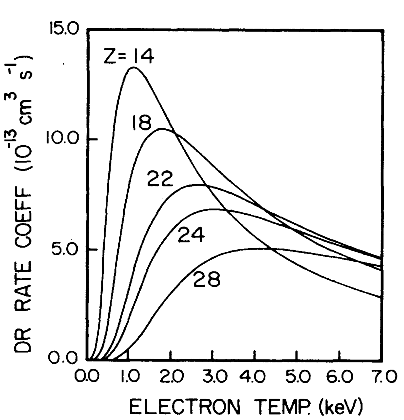

FIG. 1. Total dielectronic recombination (DR) rate coefficients for heliumlike ions with $Z=14$, 18, 22, 24, and 28 as a function of electron temperature.

TABLE I. Total dielectronic recombination (DR) rate coefficients for heliumlike ions with $Z=10$, 14, 18, 20, 22, 24, 26, and 28 as a function of electron temperature T_e. The plasma temperature are in keV and rate coefficients are in units of 10^{-13} cm^3 s^{-1}.

$kT_e \backslash Z$	10	14	18	20	22	24	26	28
0.1	0.282	0.001	0.000	0.000	0.000	0.000	0.000	0.000
0.2	5.408	0.292	0.005	0.001	0.000	0.000	0.000	0.000
0.3	11.590	1.992	0.146	0.031	0.005	0.001	0.000	0.000
0.4	15.032	4.751	0.709	0.218	0.058	0.013	0.003	0.000
0.5	16.318	7.502	1.744	0.688	0.239	0.073	0.022	0.005
0.6	16.403	9.718	3.067	1.433	0.597	0.223	0.082	0.022
0.7	15.888	11.304	4.459	2.357	1.121	0.482	0.209	0.068
0.8	15.104	12.339	5.770	3.352	1.765	0.846	0.413	0.152
0.9	14.224	12.945	6.923	4.333	2.471	1.291	0.691	0.280
1.0	13.332	13.234	7.887	5.246	3.192	1.787	1.029	0.454
1.1	12.473	13.296	8.665	6.061	3.891	2.308	1.410	0.666
1.2	11.667	13.199	9.271	6.765	4.545	2.829	1.816	0.910
1.3	10.921	12.993	9.728	7.360	5.139	3.335	2.231	1.177
1.4	10.234	12.716	10.058	7.850	5.668	3.813	2.642	1.457
1.5	9.605	12.392	10.282	8.246	6.130	4.255	3.039	1.743
1.6	9.030	12.040	10.418	8.558	6.528	4.658	3.417	2.030
1.7	8.504	11.674	10.484	8.796	6.865	5.020	3.769	2.311
1.8	8.023	11.303	10.492	8.971	7.146	5.342	4.094	2.582
1.9	7.582	10.934	10.454	9.093	7.376	5.624	4.390	2.841
2.0	7.179	10.569	10.381	9.168	7.561	5.869	4.658	3.085
2.2	6.465	9.869	10.153	9.209	7.817	6.259	5.110	3.526
2.4	5.859	9.216	9.856	9.141	7.949	6.533	5.461	3.901
2.6	5.338	8.615	9.521	8.999	7.988	6.711	5.723	4.211
2.8	4.888	8.064	9.167	8.807	7.956	6.814	5.911	4.464
3.0	4.496	7.560	8.808	8.584	7.875	6.857	6.036	4.663
3.2	4.154	7.102	8.454	8.341	7.756	6.854	6.111	4.816
3.4	3.852	6.683	8.108	8.088	7.611	6.814	6.144	4.929
3.6	3.585	6.300	7.775	7.832	7.449	6.745	6.144	5.008
3.8	3.347	5.951	7.455	7.577	7.275	6.657	6.117	5.059
4.0	3.134	5.631	7.151	7.325	7.094	6.552	6.070	5.085
4.5	2.688	4.940	6.456	6.725	6.634	6.249	5.888	5.071
5.0	2.338	4.374	5.851	6.177	6.183	5.919	5.653	4.977
5.5	2.059	3.906	5.326	5.685	5.759	5.585	5.399	4.838
6.0	1.829	3.516	4.869	5.245	5.366	5.262	5.128	4.672
6.5	1.640	3.184	4.469	4.852	5.007	4.954	4.866	4.493
7.0	1.482	2.901	4.119	4.501	4.680	4.668	4.616	4.310
7.5	1.347	2.657	3.811	4.189	4.382	4.401	4.377	4.128
8.0	1.231	2.445	3.538	3.908	4.111	4.155	4.153	3.950
8.5	1.131	2.260	3.925	3.656	3.865	3.927	3.942	3.779
9.0	1.044	2.097	3.077	3.429	3.641	3.717	3.746	3.615
9.5	0.967	1.953	2.883	3.223	3.437	3.523	3.564	3.461
10.0	0.899	1.823	2.708	3.038	3.249	3.344	3.393	3.313

At a particular temperature T_e the DR rate coefficients are proportional to the terms in Eq. (16) and the satellite intensity factors $F_2^*(s-f)$. It has been pointed out by Karim and Bhalla[38] that $F_2^*(s-f)$ functions are expected to be large when the Auger and fluorescence yields are comparable. Autoionization rates are nearly Z independent while the radiative rate increase with Z approximately as Z^4. For lighter elements autoionization rates are much higher than the radiative rates, but because of the Z^4 dependence the radiative channel becomes comparable to the autoionization channel near $Z=30$. For the elements considered here the dominant $F_2^*(s-f)$ functions are seen to increase sharply with Z from $Z=10$ to $Z=20$; for heavier elements the rate of increase gradually slows down and reaches a plateau at about $Z=28$. For heliumlike ions the most prominent satellite line is the J line which originates from the transition $1s2p^2\,{}^2D_{5/2} \rightarrow 1s^2 2p\; 2P_{3/2}$. The $F_2^*(s-f)$ functions for this line are, respectively, 2.11, 8.81, 21.90, 29.91, 37.88, 44.80, 50.31, and 53.49 (in units of 10^{13} s^{-1}) for $Z=10$, 14, 18, 20, 22, 24, 26, and 28. If one were to consider only the Z dependence of the $F_2^*(s-f)$ functions alone an increase in DR rate coefficients with increasing Z would be expected, contrary to observations in Fig. 1. At

TABLE II. Partial and total dielectronic recombination (DR) rate coefficients α_d (in units of 10^{-13} $\mathrm{cm}^3\,\mathrm{s}^{-1}$) for heliumlike neon and heliumlike nickel as a function of electron temperature kT_e. Column 1 gives the electron temperature in keV, and columns 2, 3, 4, 5, 6, 7, and 8 list, respectively, the partial rate coefficients for DR processes which proceed via the excited intermediate lithiumlike configurations $1s2lnl'$ with $n=2$, 3, 4, 5, 6, 7, and 8. In column 9 we list the DR rate coefficients for all states with $n>8$. The total DR rate coefficient obtained by summing over all partial DR rate coefficients is listed in column 10.

					α_d				
kT_e	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$	$n>8$	Total
$Z=10$									
0.1	0.118	0.079	0.032	0.019	0.011	0.006	0.004	0.013	0.282
0.2	1.241	1.685	0.839	0.538	0.321	0.200	0.131	0.453	5.408
0.4	2.392	4.614	2.570	1.709	1.049	0.675	0.453	1.569	15.032
0.5	2.402	4.972	2.832	1.897	1.172	0.758	0.512	1.772	16.317
0.7	2.137	4.793	2.801	1.891	1.176	0.766	0.521	1.802	15.888
1.0	1.674	3.988	2.375	1.614	1.009	0.661	0.451	1.561	13.332
1.5	1.142	2.852	1.724	1.177	0.739	0.486	0.333	1.152	9.604
2.0	0.831	2.123	1.294	0.885	0.556	0.367	0.252	0.871	7.179
2.5	0.636	1.649	1.009	0.691	0.435	0.287	0.197	0.683	5.588
3.0	0.506	1.325	0.813	0.558	0.351	0.232	0.159	0.552	4.496
4.0	0.348	0.921	0.568	0.390	0.246	0.163	0.112	0.387	3.134
5.0	0.258	0.687	0.424	0.291	0.184	0.122	0.084	0.290	2.339
6.0	0.200	0.537	0.332	0.228	0.144	0.095	0.066	0.227	1.829
7.0	0.162	0.434	0.269	0.185	0.117	0.077	0.053	0.184	1.482
8.0	0.134	0.361	0.224	0.154	0.097	0.064	0.044	0.153	1.232
9.0	0.113	0.306	0.190	0.130	0.082	0.055	0.038	0.130	1.044
10.0	0.097	0.264	0.163	0.112	0.071	0.047	0.032	0.112	0.899
$Z=28$									
0.8	0.126	0.016	0.004	0.002	0.001	0.001	0.000	0.001	0.151
1.0	0.349	0.063	0.019	0.009	0.004	0.003	0.002	0.005	0.453
1.2	0.654	0.148	0.048	0.022	0.012	0.007	0.004	0.014	0.909
1.5	1.154	0.326	0.112	0.054	0.029	0.017	0.011	0.037	1.741
2.0	1.848	0.653	0.240	0.121	0.066	0.040	0.026	0.085	3.080
2.5	2.273	0.917	0.351	0.181	0.101	0.062	0.040	0.131	4.056
3.0	2.481	1.095	0.431	0.226	0.127	0.078	0.051	0.166	4.654
3.5	2.548	1.198	0.480	0.255	0.144	0.089	0.058	0.189	4.960
4.0	2.530	1.248	0.508	0.272	0.154	0.095	0.062	0.203	5.072
4.5	2.464	1.262	0.519	0.280	0.159	0.098	0.065	0.211	5.057
5.0	2.373	1.252	0.520	0.281	0.160	0.100	0.066	0.213	4.965
5.5	2.270	1.227	0.513	0.279	0.159	0.099	0.065	0.212	4.825
6.0	2.162	1.193	0.502	0.274	0.157	0.098	0.064	0.209	4.659
6.5	2.056	1.153	0.488	0.267	0.153	0.095	0.063	0.205	4.480
7.0	1.952	1.111	0.472	0.259	0.149	0.093	0.061	0.199	4.296
7.5	1.853	1.069	0.456	0.251	0.144	0.090	0.059	0.193	4.115
8.0	1.760	1.026	0.439	0.242	0.139	0.087	0.057	0.187	3.938
8.5	1.672	0.985	0.423	0.233	0.134	0.084	0.055	0.181	3.768
9.0	1.590	0.945	0.406	0.225	0.130	0.081	0.054	0.174	3.604
9.5	1.514	0.906	0.319	0.216	0.125	0.078	0.052	0.168	3.450
10.0	1.442	0.869	0.376	0.208	0.120	0.075	0.050	0.162	3.027

the temperature $kT_e=(\frac{2}{3})E_a$, corresponding to maximum DR rate coefficient, the exponential function of Eq. (16) would be the same for all elements; the $(1/kT_e)^{3/2}=(3/2E_a)^{3/2}$ factor, however, will decrease rapidly with Z. The net effect is a decrease in DR rate coefficient with Z at the maxima as is shown in Fig. 1. At very high temperatures the population of resonance states $|s\rangle$ decreases more rapidly for low-Z elements so that the total DR rate coefficient of a heavier element becomes larger than the rate coefficient of a lighter element.

Bely-Dubau *et al.*[41,52] have calculated DR rate coefficients of heliumlike iron using Thomas-Fermi model and compared their results with experimental values of Bitter *et al.* and Bely-Dubau *et al.*[11] The experimental DR rate coefficients were obtained by using theoretical $1s^2-1s2p$ electron excitation rate coefficients for the resonance line. We find an excellent agreement between the DR rate coefficient from the present calculation of heliumlike iron with the measured values. In Table III, we list the total DR rate coefficients for $Z=10$, 14, 18, 20,

TABLE III. Comparison of the total dielectronic recombination rate coefficients from the present calculation for heliumlike ions (in units of 10^{-13} cm^3 s^{-1}) with other theoretical results.

Z	kT_e	This work	Nilsen[a]	Bely-Dubau *et al.*[b]	Nasser and Hahn[c]	Younger[d]
10	2.0	7.18	6.74			
14	2.0	10.57	10.39			
18	4.0	7.15	6.96		6.30	5.50
22	5.0	6.18	5.98			
26	4.0	6.07	5.78	5.0	5.88	5.60

[a] Reference 38.
[b] Reference 41.
[c] Reference 40.
[d] Reference 39.

22, 24, 26, and 28 at selected electron temperatures T_e from the present calculation, and the calculations of Nilsen,[38] Younger,[39] Nasser and Hahn[40] and Bely-Dubau *et al.*[41] Nilsen[38] used relativistic multiconfiguration wave functions to calculate explicitly the contributions from $2lnl'$ manifolds for $n \leq 4$. For $n > 4$ the DR rate coefficients were estimated by extrapolating from the results for the $2l4l'$ manifold assuming $1/n^3$ scaling for Auger transition rates. Younger[39] employed a single-configuration frozen-core Hartree-Fock approximation in *LS* coupling to all intermediate states with $n = 2$–7 and $l = 0$–3. The calculation of Nasser and Hahn[40] is based on single-configuration Hartree-Fock wave functions. Here the contributions from all possible intermediate states are first estimated in the angular-momentum-averaged scheme, a set of dominant states are then chosen, and the contributions from these states are then calculated in the *LS* coupling scheme. Bely-Dubau *et al.*[41] have employed the multiconfiguration Thomas-Fermi model in intermediate coupling to calculate DR rate coefficients for $n \leq 4$ and the approximate $1/n^3$ scaling law for $n > 4$. For electrons for $Z \geq 26$ the total DR rate coefficient from the present calculation are in good agreement with all these theoretical results. Our results are within 7% of the total rates coefficients reported by Nilsen[38] for all the ions considered here.

We see from Table II that for heliumlike nickel the partial DR rate coefficients rapidly decreases with n so that the neglect of higher-n configurations does not affect the total DR rate by more than 4% or so. This observation is valid in general for $Z \geq 20$. For heliumlike neon the partial DR rate coefficients from $n = 3$ terms are in general larger than the $n = 2$ terms and the decrease in rate coefficients after $n = 3$ is rather slow. This accounts for somewhat larger discrepancies in DR rate coefficients for lighter elements between the present calculation and the calculation of Nilsen[38] in which approximate methods were employed to calculate DR rates beyond $n > 4$. Inclusion of resonance states $|1s2lnl'LSJ\rangle$ beyond $n = 4$ is therefore warranted in the computation of total DR rate coefficients for heliumlike ions with $Z \leq 18$.

In conclusion we have presented total DR rate coefficient for heliumlike neon, silicon, argon, calcium, titanium, chromium, iron, and nickel which can be used in modeling and diagnostics of high-temperature fusion plasmas.

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